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Trend inflation meets macro-finance: the puzzling behavior of price dispersion

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Trend inflation meets macro-finance: the puzzling behavior of price dispersion*

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Abstract

Motivated by recent empirical findings that emphasize low-frequency movements in inflation as a key determinant of term structure, we introduce trend inflation into the workhorse macro-finance model. We show that this compromises the earlier model success and delivers implausible business cycle and bond price dynamics. We document that this result applies more generally to non-linearly solved models with Calvo pricing and trend inflation and is driven by the behavior of price dispersion, which is *i*) counterfactually high and *ii*) highly inaccurately approximated. We highlight the channels behind the undesired performance under trend inflation and propose several remedies.

Keywords: trend inflation; Calvo pricing; price dispersion; macro-finance; non-linear solution methods;
JEL-Codes: E13, E31, E43, E44

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1 Introduction

This paper lays out the economic channels by which trend inflation impacts the real economy and bond pricing in a general equilibrium model with endogenous risk premia. Trend inflation increases the dispersion of prices and firms cost conditions in the economy, which magnifies the impact of productivity shocks and raises the required compensation for risk. In addition, standard model solution methods poorly capture the increased non-linearities of policy functions implied by trend inflation which leads to episodes of unrealistic realizations of price dispersion. Understanding the economic channels and numerical inaccuracies related to trend inflation in non-linear macro models helps us to suggest modeling tools that realign the model with observed data.

Macroeconomic models have long struggled to explain why the yield curve is upward-sloping, a puzzle that has been labeled the "Bond Premium Puzzle" (cf. Backus, Gregory, and Zin (1989), and Den-Haan (1995)). Workhorse structural macro models, such as the celebrated New Keynesian models of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) do not serve as suitable frameworks to price financial assets such as bond yields, implying term premia close to zero or negative, as opposed to the empirically observed premia that lie above 100 basis points. Extensions of macro models with habits (e.g. Hordahl, Tristani, and Vestin (2008)) or recursive preferences (Rudebusch and Swanson (2012) model (henceforth, RS)) can better address macro and finance stylized facts simultaneously, however, at the cost of having to assume high degrees of risk aversion.

The common feature for all these models is that they are solved around zero trend inflation, as in much of the macroeconomic literature in general. This simplifying assumption is, however, not in line with the data, as average trend inflation rates across developed countries in the post war period have been above two percent. Several theoretical contributions (cf. Ascari and Sbordone (2014); Ascari and Ropele (2009)) show that assuming zero trend inflation is not innocuous for the conduct of monetary policy. As is well known, monetary policy is a leading determinant of the shape of the term structure of interest rates (i.e. Rudebusch and Swanson (2012), Gurkaynak, Sack, and Swanson (2005) among others). This is because monetary policy pins down the short tail of the term structure of interest rates by setting the policy rate, and long-term interest rates are nothing else but risk-adjusted expectations of future short-term rates. Relaxing the assumption of zero trend inflation is therefore consequential also for the macro-finance literature: a model version where monetary policy reflects the non-zero inflation target strongly impacts bond prices.

Also, a growing body of the empirical asset pricing literature highlights the importance of trend inflation (i.e. Kozicki and Tinsley (2001), Cieslak and Povala (2015), Bauer and Rudebusch (2017)) in explaining the behavior of the U.S. treasury yield curve.[1] This literature shows that accounting for time-varying trend inflation stands as the key element in understanding the empirical dynamics of U.S. Treasury yields.[2]

Motivated by these crucial empirical findings and with the intent to improve the modeling frameworks that jointly address a macro and a finance side, we incorporate trend inflation into the workhorse macro-finance model of Rudebusch and Swanson (2012). Consequently, we study in detail the economic channels through which positive trend inflation impacts macro moments and bond prices. The RS model builds on a textbook New Keynesian model, enriched with Epstein-Zin preferences and long-run inflation risks. The mechanism that generates the sizable and time-varying term premia –without compromising the model's ability to fit key macroeconomic variables– relies mostly on technology shocks, which give rise to large inflation risks for bond holders at business cycle frequencies. A positive steady-state inflation rate plays no role as, in fact, the model is approximated around a zero-inflation steady state. The model, however, captures long-term natured monetary policy shifts by allowing for a time-varying inflation target, represented by a highly persistent stochastic process in the spirit of Ireland (2007) or Cogley, Primiceri, and Sargent (2010). In our model extension we add a drift to this process by introducing non-zero steady state inflation.

We find, similarly as the empirical literature, that accounting for positive trend inflation in the calculation of nominal term premia substantially impacts bond yields and implies significantly different means and volatilities of term premia. However, contrary to what the empirical literature suggests, we find that the incorporation of trend inflation actually compromises the model performance in matching both macro and bond price stylized facts. For instance, the volatilities of the two main variables which enter the bond pricing equation, consumption and inflation, are magnified to extreme levels. When we increase trend inflation from zero to, e.g., 1.6%, the volatility of inflation rises from an original three percent to almost forty

[1] Note that the term 'trend inflation' is somewhat differently used in different streams of the literature. In particular, the empirical macro literature understands under the 'term trend inflation' the change in the level (mean) of inflation rather than just a positive level (mean) of inflation, as typically referred to in the theoretical macro literature.

[2] Prior to the empirical literature on asset pricing the importance of trend inflation was also emphasized for understanding inflation dynamics. Stock and Watson (2007) provide strong evidence that the dynamics of inflation have been largely dominated by the trend component. Further, Cogley, Primiceri, and Sargent (2010) and Ascari and Sbordone (2014) demonstrate that inflation innovations account for a small fraction of the unconditional variance of inflation, implying that most of the volatility stems from the trend component of inflation.

percent, and the volatility of consumption is about sixteen times higher. Moments from other model simulated data become similarly implausible, price dispersion and the implied output losses of price dispersion rise to unrealistic values, and price dispersion itself is inaccurately approximated. The reason for these findings are drastically amplified inefficiencies from price rigidity as well as numerical inaccuracies that arise when the Calvo pricing mechanism meets trend inflation – in particular when using higher order approximations. Our paper offers an understanding of the channels behind these results and provides possible remedies to restore the original model performance. It is important to emphasize that the encountered problems are not specific to the asset pricing related features of the RS model, but, more generally, apply to the class of macro models with a Calvo pricing mechanism and positive trend inflation, when solved non-linearly (under both second or third order approximations). In fact, as we show throughout the paper, similar results can be obtained from a higher-order solution of the otherwise standard New Keynesian (NK) model of Clarida, Galí and Gertler (1999, hereafter CGG) under certain specifications and parameterizations, albeit generally to a lesser degree. The findings of this paper are, thus, relevant to more than just the macro-finance asset pricing literature and have broader applicability.[3,4]

The key contribution of our paper lies in the detailed formal explanation of the transmission and amplification mechanism between trend inflation, the distribution of prices, the real economy and bond prices. At the heart of the amplification is the impact of trend inflation on the distribution of prices in the economy. Trend inflation widens the left tail of price distribution in the economy as firms facing Calvo contracts cannot change their price to reflect the growing aggregate price level. The right tail of the distribution also shifts further to the right because firms internalize trend inflation into their optimization problem and set their prices higher in a precautionary manner. We emphasize three channels through which the dispersion of prices has a key influence on model dynamics: i) a marginal-cost channel, ii) a trend-inflation-markup channel, and iii) an price-inflation spiral channel of Andreasen and Kronborg (2017). Under the marginal-cost channel we understand the fact that firms which are stuck with a (too) low price will face higher demand for their products and in turn employ more inputs than firms which can set their prices

[3]E.g., the literature on globally solved ZLB-models under Rotemberg or Calvo, such as in Boneva, Braun, and Waki (2016) and Miao and Ngo (2019).

[4]There are arguably more realistic setups than the Calvo mechanism to capture nominal rigidities, such as discussed by the on state-dependent pricing (see, e.g., among others Golosov and Lucas (2007), Midrigan (2011), or Costain and Nakov (2011). Nonetheless, the Calvo mechanism, which belongs to the class of time-dependent pricing mechanisms, continues to remain the most widely used device to introduce nominal rigidities.

optimally; as these firms move along the concave production function to the right, their marginal cost will increase compared to marginal costs of price resetting firms. With trend inflation there will be more firms with too high marginal costs and the overall marginal cost will be more dispersed on the economy-wide level. As more firms have to face the less favorable cost conditions under trend inflation they will have less space to accommodate exogenous shocks and real quantities will need to adjust by more for markets to clear, contributing the elevated economic dynamics. Under the trend-inflation-markup channel we understand that firms incorporate trend inflation into their forward-looking pricing decision. They know that prices will go up and that they will not be able to change current prices for some period, so they set their optimal prices higher (at a markup in addition to the one from monopolistic competition) in the case of positive trend inflation compared to the case when trend inflation is zero. The trend inflation mark up leads to, on average, higher changes in the price level than in case of zero trend inflation which in turn leads to higher adjustment of the real economy. The price-inflation spiral channel is relevant when inflation reaches a maximum feasible upper bound on inflation implied by the Calvo pricing assumption. This upper bound on feasible inflation values introduces a kink into the underlying model's policy functions, a discontinuity which is poorly approximated by perturbation methods. For inflation values beyond the upper bound, firms would maximize profits by not producing at all and the optimal relative price approaches infinity. We demonstrate that the above channels lead to levels of price dispersion and implied output-losses from dispersion that lie significantly above values typically obtained in the case of zero trend inflation and become counterfactually high. In addition, we show that the Calvo price dispersion equation also becomes poorly approximated by local perturbation methods.[5] This is why we later on use the extended perturbation of Andreasen and Kronborg (2017) which is better suited to deal with the non-linearity of the model.

The typical remedy in the literature for the outlined unrealistic model behavior is to introduce full price indexation (Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2018), Kliem and Meyer-Gohde (2017)). The empirical literature has however shown that full price indexation is not supported by the data (Nakamura, Steinsson, Sun, and Villar (2018), Uribe (2020)). Alternatively, an otherwise equivalent setup with Rotemberg price adjustment costs instead of Calvo pricing or a linear-in-labor production function can also discipline the behavior of price dispersion. All

[5] We should note, that the problem of counterfactual price dispersion and its poor approximation, is present already in the original RS specification, without trend inflation. Positive steady state inflation, however, aggravates the problem substantially, up to the point that simulated model moments stop making sense.

these modeling devices can to a large degree restore the performance of the RS model (or more generally, a trend-inflation-augmented Calvo pricing model) in matching the data. The key contributions of this article are thus, to offer both a warning about potential pitfalls of the Calvo setting and some guidance to the macroeconomic modeler for avoiding these pitfalls.

The rest of the paper proceeds as follows. Section 2 develops the main body of the paper. Section 2.2 documents in detail how simulated model moments are affected by the incorporation of trend inflation and discusses model devices that help remedy this situation. Section 2.3 is dedicated to a discussion of the channels that lead to high levels of price dispersion under the presence of trend inflation, and to the large inefficiencies they create. Section 2.4 uses model simulations to further develop an understanding of the behavior of price dispersion and studies its numerical properties. Section 3 concludes.

2 The baseline Rudebusch and Swanson model with trend inflation

Our example model, the Rudebusch and Swanson model with trend inflation, is in many aspects a standard model in the New Keynesian tradition. A continuum of firms operate under monopolistic competition and are subject to nominal rigidities à la Calvo. Households have preferences over consumption and labor –albeit in the form of Epstein-Zin preferences instead of the more conventional CRRA preferences. The central bank follows a Taylor rule, with a time-varying inflation target that is centered around a positive steady-state inflation level, instead of a zero trend inflation as in the original article.[6] Throughout the paper, Π_t denotes the gross inflation rate, defined as $\Pi_t = P_t/P_{t-1}$; lower case variable π_t instead denotes the (annualized) net inflation rate in percent, $\pi_t = 100\log(\Pi_t^4)$.

2.1 Model sketch, RS model

2.1.1 Households

The description of the households and firms' problems below closely follows RS.

The household maximizes the continuation value of its utility (V), which is of the

[6] For more detailed exposition on the model we refer the reader to Appendix A or to the original article of Rudebusch and Swanson (2012).

Epstein-Zin form and follows the specification of RS:

$$V_t = \left\{ \begin{array}{c} U(C_t, N_t) + \beta \left[E_t V_{t+1}^{1-\alpha}\right]^{\frac{1}{1-\alpha}} \text{ if } U(C_t, N_t) \geq 0, \\ U(C_t, N_t) - \beta \left[E_t(-V_{t+1})^{1-\alpha}\right]^{\frac{1}{1-\alpha}} \text{ if } U(C_t, N_t) < 0. \end{array} \right\}. \tag{1}$$

The households' problem is subject to the flow budget constraint:

$$B_t + P_t C_t = W_t N_t + D_t + R_{t-1} B_{t-1}. \tag{2}$$

In equation (1), β is the discount factor. Utility (U) at period t is derived from consumption (C_t) and leisure ($1 - N_t$). E_t denotes expectations conditional on information available at time t. As the time endowment is normalized to one, leisure time ($1 - N_t$) is what remains after spending some time working (N_t). $W_t N_t$ is labor income, R_t is the return on the one-period nominal bond, B_t, D_t is dividend income.

To be consistent with balanced growth, RS impose the following functional form on U:

$$U(C_t, N_t) = \frac{C_t^{1-\varphi}}{1-\varphi} + \chi_0 Z_t^{1-\varphi} \frac{(1-N_t)^{1-\chi}}{1-\chi}, \quad \varphi, \chi > 0, \tag{3}$$

where Z_t is an aggregate productivity trend, and φ, χ, $\chi_0 > 0$. The intertemporal elasticity of substitution (IES) is $1/\varphi$, and the Frisch labor supply elasticity is given by $(1 - \overline{N})/\chi\overline{N}$, where $\overline{N}$ is the steady state level of hours worked.

2.1.2 Firms

Final good firms operate under perfect competition with the objective to minimize expenditure subject to the aggregate price level $P_t = \left[\int_0^1 P_t^{1-\epsilon}(i)(di)\right]^{\frac{1}{1-\epsilon}}$, where $P_t(i)$ is the price of intermediate good produced by firm i, using the technology $Y_t = \left[\int_0^1 Y_t^{\frac{\epsilon-1}{\epsilon}}(i)di\right]^{\frac{\epsilon}{\epsilon-1}}$. Final good firms aggregate the continuum of intermediate goods i on the interval $i \in [0, 1]$ into a single final good. Parameter ϵ determines the elasticity of substitution between goods variety. The cost-minimisation problem of final good firms delivers demand schedules for intermediary goods of the form:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t. \tag{4}$$

A continuum of intermediate firms operates in the economy. Intermediate firm i produces according to a Cobb-Douglas production function, where θ denotes the capital share. Aggregation across firms, yields:

$$S_t Y_t = A_t \bar{K}^{\theta} (Z_t N_t)^{1-\theta}. \tag{5}$$

$\bar{K}$ refers to the fact that firms have fixed capital[7] and S_t is the cross-sectional price dispersion. Technology follows the autoregressive process:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_A \epsilon_t^A, \tag{6}$$

where ϵ_t^A is an i.i.d. shock with zero mean and constant variance.

Intermediate firms maximize the present value of future profits facing Calvo contracts by choosing price, $P_t(i)$,

$$E_t \left\{ \sum_{k=0}^{\infty} \zeta^k Q_{t,t+k} \frac{P_t}{P_{t+k}} \left[P_t(i) Y_{t+k}(i) - W_{t+k} N_{t+k}(i) \right] \right\}, \tag{7}$$

where $Q_{t,t+j}$ is the real stochastic discount factor from period t to $t+k$. The term $W_{t+j}N_{t+j}(i)$ represents the cost of labor. The optimal relative price, $p_t^* = \frac{P_t^*}{P_t}$ is a weighted average of current and future expected real marginal costs,

$$p_t^* = \frac{\epsilon}{\epsilon - 1} \sum_{k=0}^{\infty} \Upsilon_{t+k} MC_{t+k}, \tag{8}$$

Where $\Upsilon_{t+k} = \frac{\sum_{k=0}^{\infty} \theta^k E_t Q_{t,t+k} \pi_{t+k}^{\epsilon+1} Y_{t+k}}{\sum_{k=0}^{\infty} \theta^k E_t Q_{t,t+k} \pi_{t+k}^{\epsilon} Y_{t+k}}$ is the time varying mark-up implied by price rigidity and $\frac{\epsilon}{\epsilon-1}$ is the mark-up implied by monopolistic competition.

Average real marginal cost is defined as

$$MC_t = \frac{1}{1-\theta} \left(\frac{W_t}{A_t} \right) \left(\frac{Y_t}{\bar{K} A_t} \right)^{\frac{\theta}{1-\theta}}. \tag{9}$$

2.1.3 Fiscal policy and monetary policy

Government spending follows the process:

$$\log(g_t/\bar{g}) = \rho_G \log(g_{t-1}/\bar{g}) + \varepsilon_t^G, \quad 0 < \rho_G < 1, \tag{10}$$

where $\bar{g}$ is the steady-state level of $g_t \equiv G_t/Z_t$, and ε_t^G is an i.i.d. shock with mean zero and variance σ_G^2.

The model is closed by a monetary policy rule:

$$4i_t = 4\rho_i i_{t-1} + (1-\rho_i) \left[4(\bar{i} - \bar{\pi}) + \pi_t^{avg} + \phi_\pi (4\pi_t^{avg} - \pi_t^*) + \phi_Y \left(\frac{\mu_t Y_t}{\bar{\mu} \bar{Y}} - 1 \right) \right], \tag{11}$$

[7] Fixed capital can be interpreted as a model with endogenous investment that features high adjustment costs in investment.

where i_t is the (net) policy rate, $i_t = log(1+i_t)$, π_t^{avg} is a four-quarter moving average of (net) inflation (defined below), and Y_t^* is the trend level of output $\bar{Y}Z_t$ (where $\bar{Y}$ denotes the steady-state level of Y_t/Z_t). π_t^* is the target rate of inflation, and ε_t^i is an i.i.d. shock with mean zero and variance σ_i^2. ρ_i allows for interest rate smoothing. The four-quarter moving average of inflation (π_t^{avg}) is approximated by a geometric moving average of inflation:

$$\pi_t^{avg} = \theta_{\pi^{avg}} \pi_{t-1}^{avg} + (1-\theta_{\pi^{avg}})\,\pi_t, \tag{12}$$

where $\theta_{\pi^{avg}} = 0.7$ ensures that the geometric average of inflation has an effective duration of approximately four quarters. The inflation target π_t^* is time-varying and driven by the following process,

$$\pi_t^* = (1-\rho_{\pi^*})\,4\pi_t^{avg} + \rho_{\pi^*}\pi_{t-1}^* + \zeta_{\pi^*}\left(4\pi_t^{avg} - \pi_t^*\right) + \sigma_{\pi^*}\varepsilon_{\pi^*,t}. \tag{13}$$

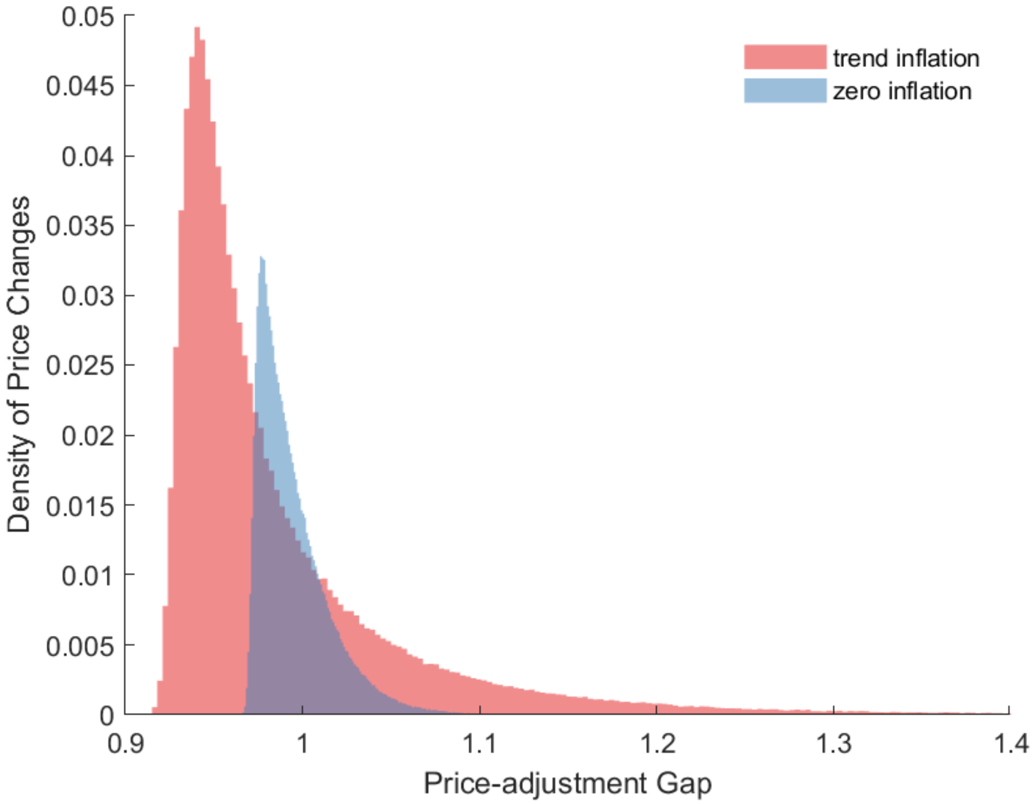
2.2 Model moments

Table 1 illustrates that relaxing the assumption of zero trend inflation ($\bar{\pi} = 0$), and, instead, allowing for positive trend inflation ($\bar{\pi} > 0$) in the RS model, produces unreasonable, largely inflated macro and finance unconditional second moments. The mechanism that accelerates the model dynamics is closely linked to the distribution of prices in the model economy. Figure 1 displays the simulated distribution of optimal relative prices, $p_t^* = \frac{P_t^*}{P_t}$, which captures the changes in the prices of the optimizing firms relative to the aggregate price in the economy at every period. This ratio is also often referred to as the price-adjustment gap in the literature (c.f. Ascari and Sbordone (2014)) and we follow this terminology further on in the paper. The main effect of trend inflation is to make large price changes more likely because the subset of firms that can change the price needs to react to positive trend in inflation when changing the price. Some firms with relatively low prices fixed far in the past will have to make large price changes to compensate for the rise in the price level that took place over time due to trend inflation. The distribution of prices can be described succinctly by the measure of the price dispersion, S_t, defined as

$$\begin{aligned} S_t^{\frac{1}{1-\theta}} &\equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\varepsilon}{1-\theta}} di \\ &= (1-\zeta)\,(p_t^*)^{\frac{-\varepsilon}{1-\theta}} + \zeta\,(\Pi_t)^{\frac{\varepsilon}{1-\theta}}\,S_{t-1}^{\frac{1}{1-\theta}}; \end{aligned} \tag{14}$$

where ζ is the Calvo parameter (the per-period probability that the price cannot be changed), $1-\theta$ is the labor income share, and ε is the elasticity of substitution

Figure 1: Simulated Distribution of Price-adjustment Gap



Note: The shaded areas plot the simulated distribution of optimal relative prices, $p_t^* = \frac{P_t^*}{P_t}$*, which captures the changes in the prices of the optimizing firms relative to aggregate price in the economy at every period with (red) and without (blue) trend inflation.*

between varieties. Given the definition in equation (14), price dispersion, S_t, is bounded by 1 from below, or, equivalently, inverse price dispersion S_t^{-1} is bounded by 1 from above which means that when $S_t = 1$ all firms have the same prices in the economy.

The first column of table 1 reports targeted empirical moments. The subsequent columns are model-based unconditional moments, calculated from third-order approximated and pruned model simulations of several model versions of the RS model. Column RS1 reports simulated moments from the original baseline RS model with zero trend inflation, using the RS best fit calibration from Table 3 of their paper[8].

[8]The model calibration is summarized in Table A.2.

Table 1: Empirical and Model-Based Unconditional Moments

Unconditional Moment	US data 1961-2007	RS1 $\bar{\pi}=0\%$	RS2 $\bar{\pi}=1.6\%$	RS3 $A_tN_t^{1-\theta}$	RS4 $A_tK_t^{\theta}N_t^{1-\theta}$	RS5 $\pi_t^*=\bar{\pi}^*\%$
SD(dC)	2.69	0.72	8.29	7.83	5.89	7.42
SD(C)	0.83	0.88	12.88	11.13	447.44	9.60
SD(N)	1.71	2.51	38.16	30.99	482.68	27.27
Mean(i)	5.72	3.06	0.46	4.12	-787.40	1.98
SD(i)	2.71	3.41	49.39	41.39	2212.94	34.86
Mean(π)	3.50	-0.54	-2.12	1.42	-791.14	-0.66
SD(π)	2.52	3.01	40.84	36.47	2211.99	29.83
SD($i^{(40)}$)	2.41	2.33	31.25	29.62	2215.71	23.67
Mean($NTP^{(40)}$)	1.06	0.91	2.50	3.41	0.55	3.23
SD($NTP^{(40)}$)	0.54	0.42	7.21	6.63	5.94	6.27
Mean($R^{(40)}-R$)	1.43	0.88	2.72	3.24	0.98	2.90
SD($R^{(40)}-R$)	1.33	1.59	26.57	21.95	36.31	19.98
Mean(S^{-1})	< 1.00	0.99 [0.83,1.07]	1.05 [0,56]	1.01 [0.83,1.07]	1014.74 [0,7e^5]	1.01 [0,47]

Note: All variables are quarterly values expressed in percent. Inflation, interest rates and the term premium are expressed at an annual rate. The red colored numbers represent values of the inverse price dispersion that violate the economically feasible range, as S^{-1} is bounded from above by one. The interval indicated below row 'Mean(S^{-1})' reports the range (minimum and maximum values) of S^{-1} observed over the simulation. ***RS1*** *is the original RS model which has following features: fixed capital $Y_t = A_t\bar{K}^{\theta}N_t^{1-\theta}$, time-varying inflation target, π_t^*, zero trend inflation, $\bar{\pi}=0\%$.* ***RS2*** *is RS1 with positive trend inflation $\bar{\pi}=1.6$.* ***RS3*** *is RS1 with trend inflation $\bar{\pi}=1.6$ and a labor-only-DRS production function, $Y_t = A_tN_t^{1-\theta}$.* ***RS4*** *is RS1 with trend inflation $\bar{\pi}=1.6$ and variable capital $Y_t = A_tK_t^{\theta}N_t^{1-\theta}$.* ***RS5*** *is RS1 with trend inflation $\bar{\pi}=1.6$ and a constant inflation target in Taylor rule, $\pi_t^*=\bar{\pi}^*$.*

Column RS2 reports results for the RS model with an annualized steady-state inflation of 1.6%.[9]

Even this very moderate level of trend inflation inflates the model moments, both macro and finance, to unrealistic values. Whereas, under the assumption of zero trend inflation, the mean and standard deviation of S_t^{-1} stays in the economically justifiable range (a value of 0.99 can be interpreted as an output loss of 1 percent due

[9]Note that this is an only very modest assumed level of annualized trend inflation. Empirically, the observed value of annualized trend inflation lies well above 2% for most of the sample periods since the second world war. However, we confirm Ascari and Ropele's (2009) result that positive trend inflation significantly shrinks the determinacy region (see figure C.2) in a NK DSGE model. In the RS model with trend inflation at a rate higher than 1.6% the model solution becomes indeterminate for the empirically relevant calibration of the Taylor rule.

to price dispersion), with trend inflation (column RS2) the mean of the inverse price dispersion becomes economically unfeasible. Also a large standard deviation and the wide range over which values of S_t^{-1} are observed in a simulation (reported in the squared brackets below the values of column 'Mean(S^{-1})') documents that periods where almost all output is lost due to price dispersion are frequent. Columns RS3-RS5 document that the problems of counterfactually high levels of price dispersion persist for a number of model modifications. Column RS3 reports moments for a model version where the feature of fixed capital is removed and replaced by a labor-only DRS production function; column RS4 is a version when capital is allowed to be variable, as in a standard Cobb-Douglas production function. Column RS5 relaxes RS's assumption of a time-varying inflation target and replaces it with a fixed target (as is more common in standard New Keynesian (NK) models). In Appendix B we develop a set of results for a nonlinear version of the standard CGG NK model (Clarida, Gali, and Gertler (1999)) that mirrors the findings just described, documenting that poor model performance is not specific to the asset pricing features of our example model.[10] Another important issue typical for non-linearly solved NK models which violates the empirical observations is the negative mean of inflation. In case of non-linearly solved models the variances of the shocks are reflected in the expectation of agents, which increases the equilibrium level of savings, lowers average yields and inflation through Fisher equation. The precautionary saving effect thus pushes the stochastic steady state of the model bellow the zero inflation deterministic steady state.[11]

After we identified an unreasonable behavior of price dispersion as the main culprit in the poor performance of models with a Calvo pricing mechanism with trend inflation, we now turn to a number of candidates of model specifications that restore the model's moments fit, which are reported in Table 2. Table 2 presents simulated moments from versions of the trend-inflation-augmented-RS model, where, in column RS6, the assumption about how prices are set in the economy is modified to a setting

[10]To be precise, while the channels (to be described in detail in section 2.3 below) that drive up price dispersion are present at all times, whether or not they lead to the aforementioned problems also in the NK model is a quantitative matter. For example, we describe a model version of the NK model with difference-stationary technology shocks, i.e. shocks to the economy's growth rate. With persistent difference-stationary technology shocks, we can demonstrate that the same set of problems as in RS also arises in the NK model. The intuition is clear: with persistent shocks the dispersion of prices across the economy will increase because firms which set their prices infrequently face very different economic conditions. When we employ a parameterization in which shocks are less persistent and have a milder impact on the real economy, as well as for a CGG version with trend-stationary shocks, we find that model dynamics remain within the standard range.

[11]See Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2018) for ways how to tackle this issue.

with Rotemberg adjustment costs (instead of Calvo pricing), where, in column RS7, we use a linear production function instead of decreasing return to scale (DRS), or where, in columns RS8 and RS9, we remove the effects of trend inflation by inflation indexation to either steady-state inflation or last period inflation.[12] (As before, Appendix B shows a set of parallel results for the CGG New Keynesian model.) The model features just discussed, which fix the problems with exploding moments documented in Table 1, have a common mechanism: they mitigate the dispersion of prices in the economy. The following subsection describes the mechanisms at play in more depth.

Table 2: Empirical and Model-Based Unconditional Moments

Unconditional Moment	RS2 $\bar{\pi} = 1.6\%$	RS6 Rotemberg	RS7 $Y_t = A_t N_t$	RS8 $\iota = 0$	RS9 $\iota = 1$
SD(dC)	8.29	0.43	0.45	0.71	0.49
SD(C)	12.88	0.49	0.53	0.89	0.68
SD(N)	38.16	1.45	1.39	2.50	1.85
Mean(i)	0.46	3.16	4.80	5.73	4.83
SD(i)	49.39	2.09	2.46	3.43	3.07
Mean(π)	-2.12	-0.48	1.05	2.22	1.42
SD(π)	40.84	2.14	2.33	2.98	2.58
SD($i^{(40)}$)	31.25	1.54	1.54	2.37	1.84
Mean($NTP^{(40)}$)	2.50	0.83	0.64	1.08	1.23
SD($NTP^{(40)}$)	7.21	0.36	0.10	0.55	0.03
Mean($R^{(40)} - R$)	2.72	0.84	0.61	1.11	1.27
SD($R^{(40)} - R$)	26.57	1.03	1.13	1.61	1.59
Mean(S^{-1})	1.05	–	1.00	0.99	1.00
SD(S^{-1})	0.82	–	0.00	0.01	0.00

Note: All variables are quarterly values expressed in percent. Inflation, interest rates and the term premium are expressed at an annual rate. Unlike in Table 1, there are no observations of the inverse price dispersion in violation of the economically feasible range. ***RS2****: equal to RS2 from Table 1.* ***RS6****: as in RS2, but with Rotemberg adjustment costs instead of Calvo pricing.* ***RS7****: as in RS2, but with a labor-only-CRS production function, $Y_t = A_t N_t$.* ***RS8****: as in RS2, but with indexation to steady-state inflation ($\iota = 0$).* ***RS9****: as in RS2, but with indexation to last-period inflation ($\iota = 1$).*

[12] There is little empirical support for firm price indexation as well as for Calvo pricing mechanism. We show that in the presence of Calvo pricing the economic costs of positive trend inflation can be largely mitigated by price indexation.

2.3 Trend inflation and price dispersion – channels

A well-known feature of the Calvo assumption is that in each period only a fraction of firms is allowed to re-set their prices optimally, which means that firms with many different prices co-exist in the economy, captured by the measure of price dispersion, S_t, equation (14). As first brought to light in a paper by Ascari (2004), and further contributions by the same author that are summarized in Ascari and Sbordone (2014), price dispersion raises the resource cost of production by introducing a wedge between aggregate output and the amount of inputs[13] needed to produce this level of output, $Y_t = S_t^{-1} A_t K^{\theta} N_t^{1-\theta}$. This wedge becomes significantly amplified in the case of trend inflation. Trend inflation adds a drift into the evolution of prices and, thus, drives the distribution of optimal prices, P_t^*, further apart from the average price index P_t. To better understand the mechanism at play, we lay out three channels through which trend inflation has a key influence on price dispersion and the dynamics of real economic variables: i) a marginal-cost channel and ii) a trend-inflation markup channel[14] and iii) a channel of a price-inflation spiral.

2.3.1 The marginal-cost channel

We show formally here that in the model with Calvo prices the firms which are stuck with their old price will produce more goods, employ more inputs and produce with higher marginal costs than firms which are able to re-set their price in the given period. We further show that this wedge between the price re-setters and price keepers increases with the trend inflation and it its precisely this wedge which amplifies the response of variables to exogenous shocks.

Let, in the following, variables carrying an asterisk denote prices and quantities of a firm that, in period t, is allowed to re-set its price optimally . Let variables without asterisk denote aggregate, economy-wide variables, that include firms that are not allowed to re-set their price in the current period and are stuck with prices from the past.

Lemma 2.1. *In the economy with Calvo contracts and trend inflation, firms which cannot change their price produce more output than price re-setting firms,* $Y_t^* < Y_t$.

[13] Ascari and Sbordone (2014) discuss the steady-state implications of trend inflation, whereas our focus is more on the dynamics, which is crucial for asset pricing.

[14] Our decomposition is somewhat different compared to other contributions in the literature, where the focus is on a trend-inflation markup channel. For example, Ascari and Sbordone (2014) decompose the markup, ϕ_t, into a price adjustment gap, P_t^*/P_t and P_t^*/MC_t to study the implications of trend inflation for the model's deterministic steady-state. Our discussion of the marginal-cost channel is in this sense novel.

The wedge between Y_t^* and Y_t increases with trend inflation.

$$Y_t^* = \phi_{o,t} Y_t \qquad \phi_{o,t|\bar{\pi}>0} < \phi_{o,t|\bar{\pi}=0} \tag{15}$$

Proof. The ratio between output of price resetting firm and aggregate output is determined by the price adjustment gap, $Y_t^* = A_t K_t^{\theta} N_t^{1-\theta} = \left(\frac{P_t^*}{P_t}\right)^{-\epsilon} Y_t$. Which can be re-written using the aggregate price index as $\frac{P_t^*}{P_t} = \left[\frac{1-\zeta(\pi_t)^{\epsilon-1}}{1-\zeta}\right]^{\frac{1}{1-\epsilon}}$. For $Y_t^* < Y_t$ it has to hold that $\phi_{o,t} = \left[\frac{1-\zeta(\Pi_t)^{\epsilon-1}}{1-\zeta}\right]^{\frac{\epsilon}{\epsilon-1}} < 1$.

$$Y_t \begin{cases} = Y_t^* & \text{for } \Pi_t = 1, \phi_{o,t} = 1 \\ > Y_t^* & \text{for } \Pi_t > 1, \phi_{o,t} < 1 \\ < Y_t^* & \text{for } \Pi_t < 1, \phi_{o,t} > 1 \end{cases} \tag{16}$$

In the economy with trend inflation, $\Pi_t > 1$ on average and therefore $Y_t^* < Y_t$ on average. □

We next move to document that, with trend inflation, we also have $N_t^* < N_t$ and $MC_t^* < MC_t$ on average. In particular, Lemma 2.1 implies that the average firm needs to hire more labor units than the price optimizing firm, $N_t > N_t^*$ to satisfy demand Y_t. We develop these results in the baseline setting of the RS model with fixed capital, which is a case of decreasing returns to scale (DRS). The appendix develops a similar set of results for the case of variables capital or constant returns to scale (CRS). We express the ratio of labor inputs of the price re-setting and the average firm, $\frac{N_t^*}{N_t}$, by using the firm production function, $Y_t^* = A_t \bar{K}^{\theta} N_t^{*(1-\theta)}$, the demand for output of the price re-setting firm, $Y_t^* = \left(\frac{P_t^*}{P_t}\right)^{-\epsilon} Y_t$, and the aggregate production function, $Y_t = A_t S_t^{-1} \bar{K}^{\theta} N_t^{1-\theta}$, which yields

$$\frac{N_t^*}{N_t} = \frac{\left(\left(\frac{P_t^*}{P_t}\right)^{-\epsilon} \frac{Y_t}{A_t \bar{K}^{\theta}}\right)^{\frac{1}{1-\theta}}}{\left[\frac{Y_t S_t}{A_t K^{\theta}}\right]^{\frac{1}{1-\theta}}} = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{S_t^{\frac{1}{1-\theta}}} = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]}. \tag{17}$$

Labor demand of the firm re-setting its price at time t, N_t^*, is therefore related to that of the average firm, N_t, by,

$$N_t^* = \phi_{n,t} N_t \qquad \text{where} \qquad \phi_{n,t} = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]}. \tag{18}$$

Under trend inflation, firms that are not able to reset their prices hire, on average, a higher amount of labor, where $\phi_{n,t}$ can be interpreted as a measure of labor market inefficiency[15]. Before we show formally that under trend inflation $\phi_{n,t} < 1$ on average, we demonstrate that, under DRS, firms which cannot re-set their price face also higher marginal costs, $MC_t > MC_t^*$. The dispersion of prices therefore also leads to the dispersion of marginal costs in the economy. If all firms could re-optimize at a given point in time their prices the economy would produce the same amount of output with lower costs. We will show that the dispersion in marginal costs across the economy is the reason why the shocks into the firms productivity unrealistically amplify the model dynamics. Lemma 2.2 shows that average marginal costs are proportional to marginal costs of price re-setting firm and this proportion is determined by the ratio of two price indexes, the price adjustment gap and price dispersion.

Lemma 2.2.

$$MC_t^* = \phi_{mc,t} MC_t \qquad \textit{where} \qquad \phi_{mc,t} = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\theta\epsilon}{1-\theta}}}{S_t^{\frac{1}{1-\theta}}}. \tag{19}$$

Proof. Marginal costs for the price re-setting firm can be written using the production function of price re-setting firms as,

$$MC_t^* = \frac{W_t}{(1-\theta)A_t K^\theta N_t^{*-\theta}}, \tag{20}$$

Aggregate marginal cost in case of DRS are derived from aggregate production function expressed for N_t, which is $N_t = \left[\frac{Y_t}{A_t K^\theta}\right]^{\frac{1}{1-\theta}} S_t^{\frac{1}{1-\theta}}$, and by taking $\frac{\partial W_t N_t}{\partial Y_t}$ we arrive at,

$$MC_t = S_t \frac{W_t}{(1-\theta)\left(A_t K^\theta N_t^{-\theta}\right)}. \tag{21}$$

Multiplying equation (20) by $\frac{N_t^{-\theta}}{N_t^{-\theta}}$ and using equation (17) delivers after rearrang-

[15] It should be noted that already Ascari (2004) discusses a related effect by looking at the production function, and pointing to the fact that the relationship between employment and output is proportional to the price adjustment gap.

ing,

$$MC_t^* = MC_t \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\theta\epsilon}{1-\theta}}}{S_t^{\frac{1}{1-\theta}}} \tag{22}$$

□

Next, in proposition 2.1 we show that the ratio of price adjustment gap to price dispersion, $\phi_{n,t}$ and $\phi_{mc,t}$ is, on average, lower than one in the economy with trend inflation.

Proposition 2.1. *The ratio of price indexes,* $\phi_n = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]} \gtreqless 1$ *and* $\phi_{mc,t} = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\theta\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]} \gtreqless 1$ *, for*

$$\phi_n \begin{cases} < 1 & \textit{for } \bar{\pi} = 0 \quad \& \quad \hat{\pi}_t > 0, \\ & \textit{for } \bar{\pi} > 0 \quad \& \quad \hat{\pi}_t > -\bar{\pi}, \\ = 1 & \textit{for } \bar{P} = P_t^* = P_t(j) = P_t, \\ > 1 & \textit{for } \bar{\pi} = 0 \quad \& \quad \hat{\pi}_t < 0, \\ & \textit{for } \bar{\pi} > 0 \quad \& \quad \hat{\pi}_t < -\bar{\pi}, \end{cases} \tag{23}$$

where $\bar{P}$ *is the deterministic steady state of the price level and* $\hat{\pi}_t$ *is the deviation of inflation from its steady state.*

Proof. The ratio $\phi_n < 1$ if $\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} < \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di$. From Proposition A.1 in Appendix A, $\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di \geq 1$. Thus, it must be true that if $\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} \leq 1$ then $\phi_n \leq 1$. This will hold for all cases when $P_t^* \geq P_t$. We have already shown in proof 2.3.1 that $\frac{P_t^*}{P_t} = \left[\frac{1-\zeta(\Pi_t)^{\epsilon-1}}{1-\zeta}\right]^{\frac{1}{1-\epsilon}} \geq 1$ for $\Pi_t \geq 1$. In case of positive steady state inflation, $\bar{\pi} > 0$, the deviation of inflation from its steady state can reach $\hat{\pi}_t > -\bar{\pi}$ for $\phi_n \leq 1$. □

Proposition 2.1 shows that in a setting *without trend inflation* ($\bar{\pi} = 0$), the ratio of the price adjustment gap to price dispersion will be smaller than one, $\phi_{n,t} < 1$, in states of the economy with positive inflation realizations, $\pi_t > 0$; equivalently it is bigger than one, $\phi_{n,t} > 1$, in states in which $\pi_t < 0$. In the case of *positive trend inflation*, ($\bar{\pi} > 0$), the probability of realizations of deflationary states of the world decreases, because current inflation needs to fall not only below its steady-state value of $\bar{\pi} > 0$ but below zero. Equivalently, the likelihood of observing states of nature where $\pi_t > 0$ increases. For this reason, the value of $\phi_{n,t}$ will be less than one, $\phi_{n,t} < 1$ for most of the states of the world and accordingly also moves the average value $\phi_{n,t}$ below one. Positive trend inflation thus amplifies the inefficiency coming from the price stickiness.

Analogically the mean of $\phi_{mc,t}$, by Proposition 2.1, will be less than one with trend inflation, $E\{\phi_{mc,t}\} < 1$. The quantitative impact of the higher cost of production in a stochastic steady-state on the economic dynamics is substantial. The explanation is straightforward. The average firm needs to employ more factor inputs to meet the higher demand for its goods (given by its lower prices), and, as its moves along the concave production function to the right, its marginal costs rise with the level of production. The fact that the average firm will produce at a higher marginal cost than the price optimizing firm at time t adds an additional inefficiency in the production and amplifies the real costs of price dispersion in the economy.

Summarizing, we have shown that trend inflation increases the dispersion of prices in the economy which leads to the higher dispersion of marginal costs across the economy. As firms produce under different cost conditions some of them are better equipped to accommodate exogenous (productivity) shocks. Equation (21) shows that shifts in TFP are magnified by S_t as compared to marginal costs of price re-setting firm in equation (20). This magnification is the reason we observe the excess dynamics of the model nominal and real variables in the table 1.

In the case of constant returns to scale[16], all firms face the same marginal costs (equation (A.16)), and this channel is muted. However, in case of CRS, the average firm will still produce more output and thus employ more factor inputs[17] (see appendix).

2.3.2 Trend-inflation markup channel

The presence of trend inflation leads firms to set their price at an additional markup over (current and future expected) marginal costs, which we call the trend-inflation markup: a markup implied by sticky prices and elevated by trend inflation that

[16]Especially in case of a linear production function, $\theta = 0$.

[17]In the model with capital the wedge between capital hired by the average and the price re-setting firm will further amplify the effects of price dispersion.

occurs over and above the traditional markup from monopolistic competition. Trend inflation enters the firm price decision problem, and therefore the first order condition for the optimal price represents another important channel. The price re-setting firm is forward-looking, it can foresee trend inflation and will therefore, on average, set its price above the aggregate price level (which includes non-resetting firms' prices from the past), $P_t^* > P_t$. It is because the optimal price has to equate the present value of future marginal revenues with marginal costs,

$$\sum_{k=0}^{\infty} \zeta^k E_t Q_{t,t+k} \Pi_{t+k}^{\epsilon-1} Y_{t+k} \left(\frac{P_t^*}{P_t}\right)^{1+\frac{\varepsilon\theta}{1-\theta}} = \frac{\epsilon}{\epsilon-1} \sum_{k=0}^{\infty} \zeta^k E_t Q_{t,t+k} \Pi_{t+k}^{\frac{\epsilon}{1-\theta}} Y_{t+k} MC_{t+k}^r(j), \tag{24}$$

The trend growth in prices increases both the firms' costs of production and the revenues from the sold output. Nevertheless, nominal marginal costs (the expression in the infinite sum on the right hand side of equation (24)) grow at a faster rate than nominal revenues (the left hand side of equation (24)). So, to keep the equality of marginal revenues with marginal cost in present value terms, the price setting firm must set P_t^* above P_t.[18] The difference between the rate of growth in marginal cost and marginal revenue shapes the firm's markup over (present and future) marginal costs. Equation (25) defines the price adjustment gap that depends on the weighted average of the firm's current and expected future real marginal costs.

$$\left(\frac{P_t^*}{P_t}\right)^{1+\frac{\varepsilon\theta}{1-\theta}} = \frac{\epsilon}{\epsilon-1} E_t \sum_{k=0}^{\infty} \phi_{t+k} MC_{t+k}(j) \qquad \text{where} \qquad \phi_{t+k} = \frac{m_{t+k}\Pi_{t+k}^{\frac{\epsilon}{1-\theta}}}{\sum_{k=0}^{\infty} m_{t+k}\Pi_{t+k}^{\epsilon-1}}, \tag{25}$$

where $m_{t+k} = \zeta^k E_t Q_{t,t+k} Y_{t+k}$. Ascari and Sbordone (2014) show that the markup, ϕ_t, increases with inflation[19]– and, thus, as trend inflation increases, the firm's trend-inflation markup amplifies the distortion implied by monopolistic competition.

The rise in ϕ_t means that firms put more weight on marginal costs far in the future compared to current marginal costs[20]. Future marginal costs are discounted

[18]Note that trend-inflation markup channel is enforced through the parameter θ which further widens the gap between costs and revenues. Thus, strictly speaking there is a another interaction channel.

[19]As Π goes up, the numerator grows faster – at rate $\Pi_{t+k}^{\frac{\epsilon}{1-\theta}}$ – than the denominator –which grows by $\Pi_t^{\epsilon-1}$

[20]Ascari and Sbordone (2014) shows that overly forward looking agents de-anchor inflation expectations and decrease the determinacy region. This fact also applies to our model as the model solution is indeterminate for $\bar{\pi} > 1.6\%$.

by the model's implied yield curve with maturity k, where $Q_{t,t+k}(1/\Pi_{t+k})$ is the nominal price of the bond with maturity k. In the model with an upward sloping yield curve and high inflation risks, firms will discount the future relatively more.

A decrease in the wedge between marginal costs and revenues can mitigate this channel, which can be done by introducing a (full) inflation indexation. Another option is to increase the monopolistic mark-up (decrease ϵ): having a larger mark-up allows the firm to accommodate bigger deviations from the optimal price. Note, also, that θ and ϵ increase the non-linearity of model equilibrium conditions, which, as we later show, substantially increases approximation errors.

2.4 Price-inflation spiral and approximation accuracy of price dispersion

A lesser known feature implied by Calvo pricing is an endogenous upper bound on inflation. This upper bound on inflation is effective only under the presence of positive trend inflation or at higher orders of approximation. As most models in the field are linearized up to the first order, typically around zero inflation steady state, these issues have never been of much of concern. Lemma 2.3 implies that due to the upper bound on inflation and given a concave profit function, a firm could maximize profits by not producing at all whenever inflation is beyond the upper bound.

Lemma 2.3. *The deterministic steady state of the model and measure of price dispersion is defined only when*

$$\bar{\Pi} < \Pi^{upper} \qquad \Pi_t < \Pi^{upper}. \tag{26}$$

Proof. The model with fixed capital and production function $Y_t(i) = A_t \bar{K}^{\theta} N_t^{1-\theta}(i)$ delivers the following equation for price dispersion:

$$S_t^{\frac{1}{1-\theta}} = (1-\zeta)\,(p_t^*)^{-\frac{1+\lambda}{(1-\theta)\lambda}} + \zeta\,(S_{t-1})^{\frac{1}{1-\theta}}\,\Pi_t^{\frac{1+\lambda}{(1-\theta)\lambda}} \tag{27}$$

The reset price, p_t^*, can be written as a function of CPI inflation, π_t, by using the definition of aggregate price index which yields

$$S_t^{\frac{1}{1-\theta}} = (1-\theta)\left[\frac{1-\zeta\,(\Pi_t)^{\epsilon-1}}{1-\zeta}\right]^{\frac{\epsilon}{(\epsilon-1)(1-\theta)}} + \theta(\Pi_t)^{\frac{\epsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}}, \tag{28}$$

where the power term over the square brackets in the equation (28) implies that the real value for S_t exists only if $\left(\left[\frac{1-\zeta(\Pi_t)^{\epsilon-1}}{1-\zeta}\right]\right) > 0$ which means that $\pi_t < \left(\frac{1}{\zeta}\right)^{\frac{1}{\epsilon-1}}$. □

The upper bound on inflation implies a kink in the policy function which we visualize in figure 2. Andreasen and Kronborg (2017) demonstrate that this non-linearity is poorly captured by standard perturbation methods. The fact that the approximation of the policy function does not reflect the upper bound on inflation generates a 'price-inflation spiral'. The importance of this channel (non-linearity) increases with trend inflation. This is because trend inflation increases the average level of gross inflation, Π_t. Because local approximation-based solution methods ignore the kink in agent's policy function implied by the upper bound on inflation, agents in the model do not reflect this maximum admissible level of inflation in their expectations when setting their prices. Figure 2 shows policy functions for inflation and inverse price dispersion, expressed as a function of the state variable capturing the inverse of dispersion of prices in the economy. In the figure, state variables other than price dispersion are held constant at their respective steady state values. The figure demonstrates that both linearized and even third order approximations cannot account for the kink in policy function implied by the upper bound on inflation and inverse price dispersion.

To further examine the role of price dispersion in generating explosive dynamics we look into the numerical accuracy of the approximation to the price dispersion equation (14). Alongside the more rigorous study of Andreasen and Kronborg (2017) on numerical accuracy of approximation methods we calculate a more accurate measure of price dispersion, noting that price dispersion can be written recursively as

$$S_t^{\frac{1}{1-\theta}} = (1-\zeta)\left[\frac{1-\zeta\left(\Pi_t\right)^{\epsilon-1}}{1-\zeta}\right]^{\frac{\epsilon}{(\epsilon-1)(1-\theta)}} + \zeta(\Pi_t)^{\frac{\epsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}}. \tag{29}$$

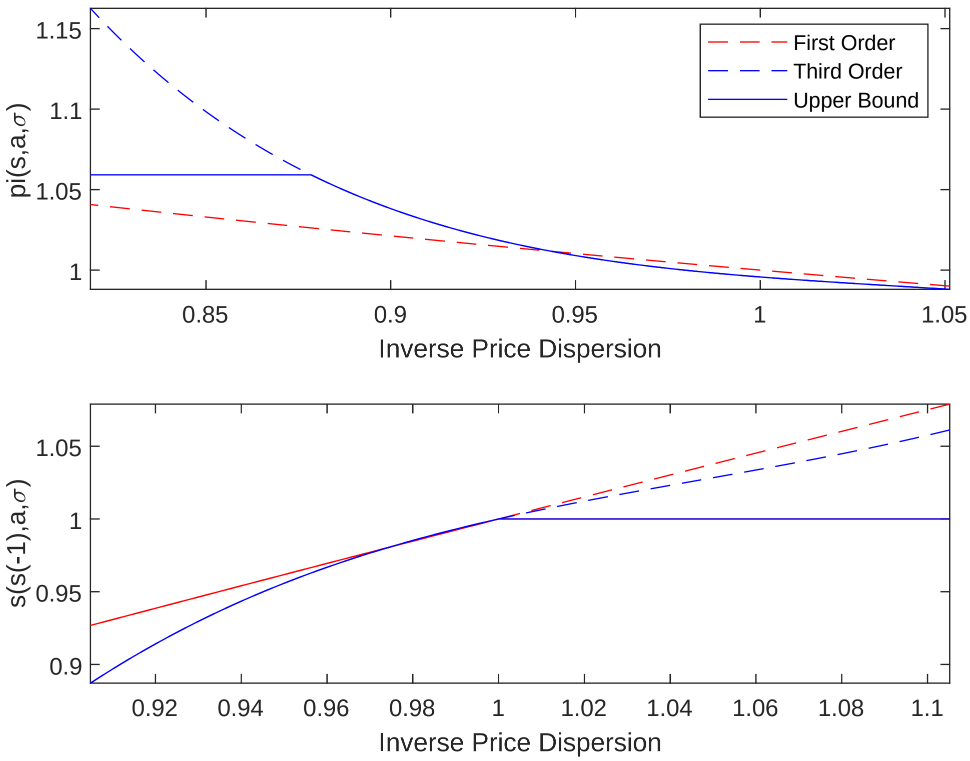
We proceed as follows.[21] First, we use an initial value S_{t-1} from the approximated model as a starting point. Second, we iterate the equation forward to get an exact solution conditional on the model-approximated time path of Π_t. Third, we compare this more exact measure of price dispersion with its counterpart from the third- order approximation[22].

The panels in Figure 3 contrast simulated paths for price dispersion, as computed from a third-order approximation of the model with the 'exact' behavior for price dispersion, using equation (29) of the main text, for several model versions. As can be seen, the third order approximation *a*) deviates sharply from the path of price

[21] We very much thank Larry Christiano for suggesting to look at the problem in this way.

[22] Andreasen and Kronborg (2017) shows that although the conditioning on inflation delivers somewhat different solution compared to the use of more accurate projection methods, the approximation errors of our more exact measure should be small. For this reason, the conditioning on inflation should not harm our argument.

Figure 2: Upper Bound on Inflation and Inverse Price Dispersion



Note: The solid line shows the upper bound on inflation for the true policy function. Blue line shows third order and red line first order approximation of policy function. State variables other than price dispersion are held constant at their respective steady state values.

dispersion using the exact formula, and b) includes many infeasible realizations of $S^{-1} > 1$. The problems diminish or disappear when adopting one of the proposed fixes documented in section 2.2.

Subpanel 'RS1' of Figure 3 stresses this finding by showing that the approximation of price dispersion is poor even for the original RS model[23] as the deviations between the third-order and 'exact' solution are large. Perturbation methods do an even poorer job in the case of positive trend inflation. In addition, in the case of positive inflation the third order approximation generates state of the worlds which are economically infeasible as S_t^{-1} exceeds one, which would imply that more resources

[23] Note that the original Rudebusch and Swanson (2012) results are sensitive to the seed of random number generator even for very long simulations.

are spent than produced, $Y_t < C_t + I_t + G_t$. Subpanel 'RS1*' in the second row shows that a first-order approximation delivers smaller approximation errors.

The approximation errors for the cases of indexation to past inflation and constant return to scale in labor are negligible. The RS model with positive steady-state inflation and indexation delivers both small price dispersion and negligible approximation errors, as can be observed by the almost complete overlay of the two simulated series. However, it should be noted that in the case of the linear production function, the more exact measure of price distortion is still large. There are states of the world when price dispersion implies an almost 10% quarterly output loss, which is at odds with empirical evidence (see, for example, Nakamura, Steinsson, Sun, and Villar (2018)). Andreasen and Kronborg (2017) conjecture that these explosive dynamics in price dispersion come from the price-inflation spiral generated by the fact that the perturbation methods up to third order fail to account for an upper bound on inflation.

A natural question which arises in the context of the numerical inaccuracies is whether the bad performance of the New Keynesian model with trend inflation is simply driven by the highly inaccurate third-order perturbation approximation to known discontinuities in the true policy functions or whether most of the excess dynamics come from the economic channels we discussed earlier in this section.

To better isolate the effects of the marginal cost and trend inflation markup channel we implement the extended perturbation solution method of Andreasen and Kronborg (2017)[24]. The idea behind this exercise is to eliminate the price-inflation spiral channel so that we can see how much of the amplification in simulated moments persists and can be attributed to the marginal cost and trend inflation markup channels. Table C.4 in appendix C shows that even if we internalize the discontinuities implied by the upper bound on inflation and inverse price dispersion (see figure 2) by solving the model more accurately the excessive dynamics of the model remain[25]. The application of the extended perturbation demonstrates that the upper bound on inflation is not what causes the excessive dynamics of the model. Instead, they are caused by the higher-order terms (particularly, by the higher-order risk-

[24]Andreasen and Kronborg (2017) show that extended perturbation improves accuracy of the perturbation solution especially in the case of non-linear presence of Calvo pricing. Appendix C provides more detailed discussion of the method and our application

[25]We are nevertheless aware of the limitations of this exercise. These limitation comes from the fact that the extended perturbation improves the accuracy only of certainty equivalent part of the solution, whereas the stochastic part of the solution remains approximated by standard third order perturbation methods. However, Andreasen and Kronborg (2017) argue that the approximation errors coming from the stochastic part of the solution are usually small. We provide more discussion in appendix C.

adjustments) and thus by the fact that inflation becomes excessively uncertain and risky rather than being too large (in the sense of crossing the upper bound threshold). The fact that the uncertainty related to the price setting of firms is the culprit of the excessive dynamics in the model can be also seen by assuming that agents in the model are risk neutral. In this case the price of risk is zero in the model and the excessive dynamics disappear (see table C.5 in appendix). Another way how to mitigate the inflation uncertainty triggered by the trend inflation in the model while keeping agents risk averse is to increase the weight on inflation in Taylor rule (see table C.2 in the appendix). The increase in ϕ_π disciplines the second moments of model macro variables by reducing the spread of the distribution of prices in the economy. The fact that the central bank strongly fights any deviation in the path of inflation from its target, provides an anchor to the uncertainty of inflation and makes the path of inflation much more predictable. In such case, the Calvo pricing firm, which has to set its price based on its expectations of the development of its future costs and revenues, has a much easier job as the uncertainty about nominal variables is significantly mitigated.

In appendix C we provide a detailed discussion on the sensitivity of our results to a specific model calibration and report a set of results showing that the amplification of moments is mildly sensitive to specific parameter values and persists under different parameters setups.

3 Conclusion

Our paper emphasizes that an attempt to realign the current macro-finance workhorse modeling framework with recent empirical evidence should include incorporating positive trend inflation into such a framework. We document that pricing assets in models that are based on the Calvo price mechanism can lead to extremely counterfactual model dynamics, once trend inflation is present; we then propose a number of directions to overcome such complications. This way, we contribute to providing guidance along the path of finding a new, empirically well-motivated and consistent modeling framework.

References

Andreasen, M. M., J. Fernandez-Villaverde, and J. F. Rubio-Ramirez (2018): "The Pruned State-Space System for Non-Linear DSGE Models: Theory and Empirical Applications," *Review of Economic Studies*, 85(1), 1–49.

Andreasen, M. M., and A. Kronborg (2017): "The Extended Perturbation Method: New Insights on the New Keynesian Model," CREATES Research Papers 2017-14, Department of Economics and Business Economics, Aarhus University.

Ascari, G. (2004): "Staggered Prices and Trend Inflation: Some Nuisances," *Review of Economic Dynamics*, 7(3), 642–667.

Ascari, G., and T. Ropele (2009): "Trend Inflation, Taylor Principle, and Indeterminacy," *Journal of Money, Credit and Banking*, 41(8), 1557–1584.

Ascari, G., and A. M. Sbordone (2014): "The Macroeconomics of Trend Inflation," *Journal of Economic Literature*, 52(3), 679–739.

Backus, D., A. W. Gregory, and S. E. Zin (1989): "Risk Premiums in the Term Structure," *Journal of Monetary Economics*.

Bauer, M. D., and G. D. Rudebusch (2017): "Interest Rates Under Falling Stars," Discussion paper.

Boneva, L. M., R. A. Braun, and Y. Waki (2016): "Some unpleasant properties of loglinearized solutions when the nominal rate is zero," *Journal of Monetary Economics*, 84(C), 216–232.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

Cieslak, A., and P. Povala (2015): "Expected Returns in Treasury Bonds," *Review of Financial Studies*, 28(10), 2859–2901.

Clarida, R., J. Gali, and M. Gertler (1999): "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, 37(4), 1661–1707.

Cochrane, J. H. (2001): *Asset pricing*. Princeton University Press.

Cogley, T., G. E. Primiceri, and T. J. Sargent (2010): "Inflation-Gap Persistence in the US," *American Economic Journal: Macroeconomics*, 2(1), 43–69.

Costain, J., and A. Nakov (2011): "Distributional Dynamics under Smoothly State-Dependent Pricing," *Journal of Monetary Economics*, 58, 646–665.

Den-Haan, W. (1995): "The term structure of interest rates in real and monetary economies," *Journal of Economic Dynamics and Control*.

Golosov, M., and R. E. Lucas (2007): "Menu Costs and Phillips Curves," *Journal of Political Economy*, 115, 171–199.

Gurkaynak, R. S., B. Sack, and E. Swanson (2005): "The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models," *American Economic Review*, 95(1), 425–436.

Hordahl, P., O. Tristani, and D. Vestin (2008): "The Yield Curve and Macroeconomic Dynamics," *Economic Journal*, 118(533), 1937–1970.

Ireland, P. N. (2007): "Changes in the Federal Reserve's Inflation Target: Causes and Consequences," *Journal of Money, Credit and Banking*, 39(8), 1851–1882.

Kliem, M., and A. Meyer-Gohde (2017): "(Un)expected Monetary Policy Shocks and Term Premia," Bundesbank Discussion Paper No. 30/2017. 2017-015, Berlin.

Kozicki, S., and P. A. Tinsley (2001): "Shifting endpoints in the term structure of interest rates," *Journal of Monetary Economics*, 47(3), 613–652.

Miao, J., and P. V. Ngo (2019): "Does Calvo Meet Rotemberg at the Zero Lower Bound?," *Macroeconomic Dynamics, forthcoming*.

Midrigan, V. (2011): "Menu Costs, Multiproduct Firms, and Aggregate Fluctuations," *Econometrica*, 79, 1139–1180.

Nakamura, E., J. Steinsson, P. Sun, and D. Villar (2018): "The Elusive Costs of Inflation: Price Dispersion during the U.S. Great Inflation," *The Quarterly Journal of Economics*, 133(4), 1933–1980.

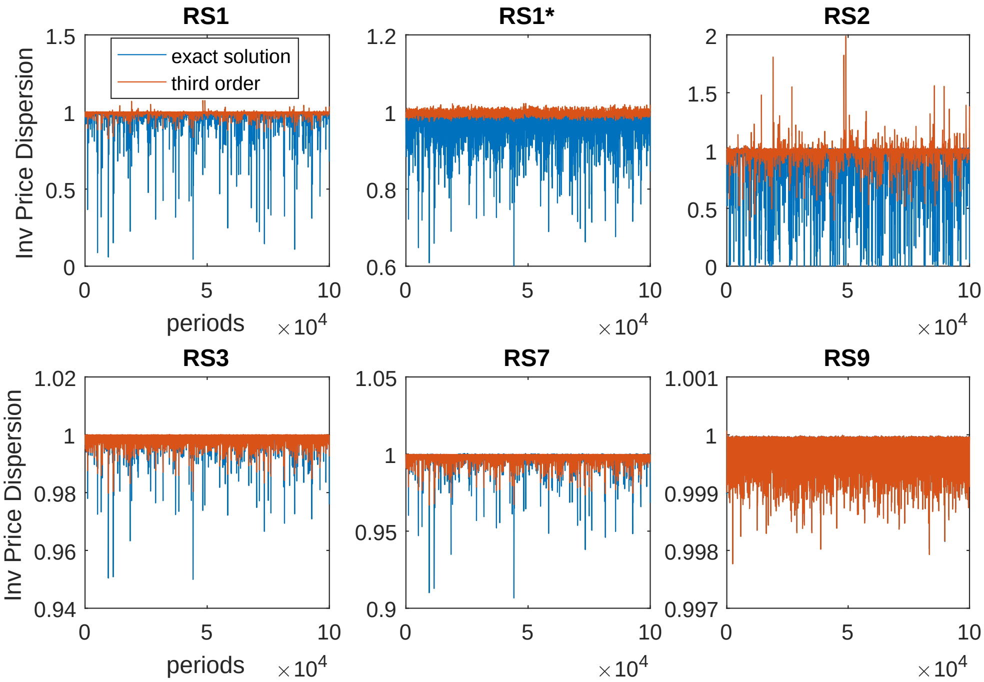
Rudebusch, G. D., and E. T. Swanson (2012): "The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks," *American Economic Journal: Macroeconomics*, 4(1), 105–43.

Smets, F., and R. Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

Stock, J. H., and M. W. Watson (2007): "Why Has U.S. Inflation Become Harder to Forecast?," *Journal of Money, Credit and Banking*, 39(s1), 3–33.

Uribe, M. (2020): "Staggered Price Indexation," Working Paper 27657, National Bureau of Economic Research.

Figure 3: Approximation Errors for Price Dispersion



RS1: original RS model, with the following features: fixed capital $Y_t = A_t \bar{K}^{\theta} N_t^{1-\theta}$, time-varying inflation target, π_t^*, zero trend inflation, $\bar{\pi} = 0\%$.
RS1*: as in RS1, but: approximated only up to first order.
RS2: as in RS1, but: with positive trend inflation $\bar{\pi} = 1\%$.
RS3: as in RS1, but: with labor-only-DRS $Y_t = A_t N_t^{1-\theta}$, trend inflation $\bar{\pi} = 1\%$.
RS7: as in RS2, but: with labor-only-CRS $Y_t = A_t N_t$.
RS9: as in RS2, but: with indexation to last-period inflation ($\iota = 1$).

*Note: The panels contrast simulated paths for price dispersion, as computed from a third-order approximation of the model with the 'exact' behavior for price dispersion, using equation (29), conditioning on the simulated time path of Π from the third-order-approximated model. The level of inverse price dispersion clearly violates the upper bound in on inflation seen in the policy function in figure 2. This is because figure 2 is conditional on one other states being zero. From the non-reported results we can se that when S_{t-1}^{-1} is small, S_t^{-1} is very small irrespective of value of π_t. **RS1**: original RS model, with the following features: fixed capital $Y_t = A_t \bar{K}^{\theta} N_t^{1-\theta}$, time-varying inflation target, π_t^*, zero trend inflation, $\bar{\pi} = 0\%$. **RS1***: as in RS1, but approximated only up to the first order. **RS2**: as in RS1, but with positive trend inflation of $\bar{\pi} = 1\%$. **RS3**: as in RS1, but with trend inflation of $\bar{\pi} = 1\%$ and a labor-only-DRS production function, $Y_t = A_t N_t^{1-\theta}$. **RS7**: as in RS1, but with trend inflation of $\bar{\pi} = 1\%$ and with a labor-only-CRS production function, $Y_t = A_t N_t$. **RS9**: as in RS1, but with trend inflation of $\bar{\pi} = 1\%$ and with indexation to last-period inflation ($\iota = 1$).*

Appendix A Rudebusch and Swanson (RS) Model

This appendix gives a summary of the equilibrium conditions of Rudebusch and Swanson (2012). Table A.1 summarizes the system of equations of the Rudebusch Swanson model in terms of stationary allocations and real (relative) prices (i.e., in term of detrended and deflated variables, denoted by lowercase variables) defined as $c_t = \frac{C_t}{Z_t}$, $y_t = \frac{Y_t}{Z_t}$, $\Pi_t = \frac{P_t}{P_{t-1}}$, $w_t = \frac{W_t}{P_t Z_t}$, $p_t^* = \frac{P_t^*(i)}{P_t}$, $mc_t(i) = \frac{MC_t(i)}{P_t}$, $y_t = \frac{Y_t}{Z_t}$, $\mu_t = \frac{Z_t}{Z_{t-1}}$. The best fit calibration of the RS model based on their Table 3 is summarized in Table A.2. In this setting, model dynamics are driven by three types of shocks, stationary technology shocks, government spending shocks, and inflation target shocks (in particular, there are no trend productivity shocks, so that $\mu_t = \frac{Z_t}{Z_{t-1}} = \mu$ is constant).

A.1 Bond Pricing

The price of a default-free n-period zero coupon bond that pays \$1 at maturity can be described recursively as:

$$p_t^{(n)} = E_t\{Q_{t,t+1} p_{t+1}^{(n-1)}\}$$

where $Q_{t,t+1}$ is the stochastic discount factor; $p_t^{(n)}$ denotes the price of the bond at time t with maturity n, and $p_t^{(0)} \equiv 1$, i.e. the time-t price of \$1 delivered at time t is \$1.

The price of a bond can be decomposed into the risk-neutral price and a term premium. The risk-neutral bond price, $\hat{p}_t^{(n)}$, is defined through the expectations hypothesis of the term structure:

$$\hat{p}_t^{(n)} = e^{-i_t} E_t \hat{p}_{t+1}^{(n-1)} \qquad NTP_{n,t} = i_t^{(n)} - \frac{1}{n}\sum_{j=0}^{n-1} E_t[i_{t+j}] \tag{A.1}$$

where the bond price is discounted by one period rate, i_t. The price of bond reflects, in this case, expectations about inflation and economic activity but abstracts from the uncertainty surrounding the expectations[26]. The continuously compounded yield to maturity of the n-period zero-coupon bond can be written as $i_t^{(n)} = -\frac{1}{n}\log p_t^{(n)}$, (see for instance Cochrane (2001)). The term premium, $NTP_{n,t}$ is defined as the difference between the yield expected by the risk-averse investor ($i_t^{(n)}$) minus the yield awaited by the risk-neutral investor ($\hat{i}_t^{(n)} = \frac{1}{n}\sum_{j=0}^{n-1} E_t[i_{t+j}]$).

[26]This can be understood as the bond price of a 10-year bond expected by the so-called risk-neutral investor who is rolling over a one-period investment for 10 years.

Table A.1: System of model equations, Rudebusch Swanson model

(RS1):	$V_t = \frac{c_t^{1-\varphi}}{1-\varphi} + \chi_0 \frac{(1-N_t)^{1-\chi}}{1-\chi} + \beta(E_t[(V_{t+1}\mu_{t+1}^{1-\gamma})^{1-\alpha}])^{\frac{1}{1-\alpha}}$
(RS2):	$Q_{t-1,t} = \mu_t^{-\gamma}\left(\frac{(V_t\mu_t^{1-\gamma})}{[E_{t-1}(V_t\mu_t^{1-\gamma})^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right)^{-\alpha}\left(\frac{c_t}{c_{t-1}}\right)^{-\varphi}$
(RS3):	$\chi_0(1-N_t)^{-\chi}c_t^{\varphi} = w_t$
(RS4):	$1 = \beta E_t\left\{Q_{t,t+1}\frac{(1+i_t)}{\Pi_{t+1}}\right\}$
(RS5):	$(p_t^*)^{1+\frac{\theta\epsilon}{1-\theta}} = \frac{aux_{1t}}{aux_{2t}}$
(RS6):	$aux_{1t} = \frac{\epsilon}{\epsilon-1}mc_t y_t + \beta\zeta Q_{t,t+1}\Pi_{t+1}^{\frac{\epsilon}{1-\theta}}aux_{1t+1}$
(RS7):	$aux_{2t} = y_t + \beta\zeta Q_{t,t+1}\Pi_{t+1}^{\epsilon-1}aux_{2t+1}$
(RS8):	$S_t Y_t = A_t\bar{K}^{\theta}(N_t)^{1-\theta}$
(RS9):	$S_t^{\frac{1}{1-\theta}} = (1-\zeta)(p_t^*)^{\frac{-\epsilon}{1-\theta}} + \zeta(\Pi_t)^{\frac{\epsilon}{1-\theta}}S_{t-1}^{\frac{1}{1-\theta}}$
(RS10):	$\Pi_t^{1-\epsilon} = (1-\zeta)(p_t^*\Pi_t)^{1-\epsilon} + \zeta$
(RS11):	$MC_t = \frac{1}{1-\theta}\overline{K}^{\frac{\theta}{1-\theta}}\frac{W_t}{A_t}\left(\frac{y_t}{A_t}\right)^{\frac{\theta}{1-\theta}}$
(RS12):	$y_t = c_t + \overline{I} + g_t$
(RS13):	$4i_t = 4\rho_i i_{t-1} + (1-\rho_i)\left[4(\bar{i}-\bar{\pi}) + (\pi_t^{avg}) + \phi_\pi(4(\pi_t^{avg}) - (\pi_t^*)) + \phi_Y\left(\frac{\mu_t Y_t}{\bar{\mu}\bar{Y}} - 1\right)\right]$
(RS14):	$\pi_t^* = (1-\rho_{\pi^*})\,4\pi_t^{avg} + \rho_{\pi^*}\pi_{t-1}^* + \zeta_{\pi^*}(4\pi_t^{avg} - \pi_t^*) + \sigma_{\pi^*}\varepsilon_{\pi^*,t}$
(RS15):	$\pi_t^{avg} = \theta_{\pi^{avg}}\pi_{t-1}^{avg} + (1-\theta_{\pi^{avg}})\,\pi_t$
(RS16):	$\log A_t = \rho_A \log A_{t-1} + \sigma_A\varepsilon_{A,t}$
(RS17):	$\log(g_t/\bar{g}) = \rho_G\log(g_{t-1}/\bar{g}) + \varepsilon_t^G$

A.2 Aggregation

Here we describe in detail the aggregation across the i-firms in case of decreasing return to scale (i.e., the model version with fixed capital, as in the original model specification of RS), and constant return to scale production function (i.e., with variable capital).

A.2.1 Aggregate Price Index

The aggregate price index $P_t = \left[\int_0^1 P_t^{1-\epsilon}(i)di\right]^{\frac{1}{1-\epsilon}}$ can be written using the Calvo result as,

$$\frac{P_t^*}{P_t} = \left[\frac{1-\zeta\,(\Pi_t)^{\epsilon-1}}{1-\zeta}\right]^{\frac{1}{1-\epsilon}}, \tag{A.2}$$

A.2.2 Aggregation for DRS

The production function of intermediate firm i is given by $Y_t(i) = A_t K^\theta N_t^{1-\theta}(i)$. Using this, plug in for $Y_t(i)$ into the demand for variety i, equation 4, solve for $N_t(i)$ and integrate over all varieties i. Since workers are all the same the aggregation of hours worked is $N_t = \int_0^1 N_t(i)di$. Aggregation thus delivers,

$$N_t = \left(\frac{Y_t}{A_t K^\theta}\right)^{\frac{1}{1-\theta}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} di, \tag{A.3}$$

which can be re-written as

$$Y_t = S_t^{-1} A_t K_t^\theta N_t^{1-\theta}, \tag{A.4}$$

where variable $S_t^{\frac{1}{1-\theta}} = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di$ defines price dispersion.

A.2.3 Re-setting firm vs. aggregate quantities for DRS

The demand function at time $t+k$ for the firm re-setting its price at time t is given by,

$$Y_{t+k}^* = A_{t+k}\bar{K}^\theta N_{t+k}^{*(1-\theta)} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}, \tag{A.5}$$

where P_t^* is the optimal price of firm resetting its price at time t for the horizon k. Factor demand of the price re-setting firm, N_t^* is,

$$N_{t+k}^* = \left(\left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} \frac{Y_{t+k}}{A_{t+k}\bar{K}^\theta} \right)^{\frac{1}{1-\theta}}. \tag{A.6}$$

The ratio of the price re-setting (equation (A.6)) and the aggregate firm's factor demands (A.3)), expressed in terms of time t quantities, is given by

$$\frac{N_t^*}{N_t} = \frac{\left(\left(\frac{P_t^*}{P_t} \right)^{-\epsilon} \frac{Y_t}{A_t\bar{K}^\theta} \right)^{\frac{1}{1-\theta}}}{\left[\frac{Y_t S_t}{A_t K^\theta} \right]^{\frac{1}{1-\theta}}} = \frac{\left(\frac{P_t^*}{P_t} \right)^{-\frac{\epsilon}{1-\theta}}}{S_t^{\frac{1}{1-\theta}}} = \frac{\left(\frac{P_t^*}{P_t} \right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{-\epsilon}{1-\theta}} di \right]}. \tag{A.7}$$

An analogous ratio can be derived for aggregate marginal cost and marginal costs of the price resetting firm. Marginal costs for the price resetting firm are,

$$MC_t^* = \frac{W_t}{(1-\theta)A_t K^\theta N_t^{-\theta}} \frac{N_t^{-\theta}}{N_t^{*-\theta}}, \tag{A.8}$$

Aggregate marginal cost come from $\frac{\partial W_t N_t}{\partial Y_t}$ and, using $N_t = \left[\frac{Y_t}{A_t K^\theta} \right]^{\frac{1}{1-\theta}} S_t^{\frac{1}{1-\theta}}$, delivers,

$$\frac{MC_t}{S_t} = \frac{W_t}{(1-\theta)\left(A_t K^\theta N_t^{-\theta} \right)}. \tag{A.9}$$

Plugging equation (A.7) into (A.8) and rearranging delivers,

$$MC_t^* = MC_t \frac{\left(\frac{P_t^*}{P_t} \right)^{-\frac{\theta\epsilon}{1-\theta}}}{S_t^{\frac{1}{1-\theta}}} \tag{A.10}$$

A.2.4 Aggregation for CRS

For the case of a constant returns to scale production function, where capital is variable, the cost minimization problem is given by

$$\min_{N_t(i)} W_t N_t(i) + R_t^k K_t + MC_t^r(i) \left[Y_t(i) - A_t K_t(i)^\theta N_t^{1-\theta}(i) \right], \tag{A.11}$$

subject to production function, $Y_t(i) = A_t K_t(i)^\theta N_t^{1-\theta}(i)$, and where $MC_t(i)$ is the multiplier associated with the constraint.

The firm's demands for labor and capital are, respectively,

$$W_t = MC_t^r(i)(1-\theta)A_t K_t(i)^\theta N_t^{-\theta}, \tag{A.12}$$

$$R_t^k = MC_t^r(i)A_t\theta K_t(i)^{\theta-1}N_t^{1-\theta}(i), \tag{A.13}$$

Plugging the factor demands into the definition of total costs, $TC_t(i) = W_t N_t(i) + R_t^k K_t(i)$ delivers,

$$TC_t(i) = \left[MC_t^r(i)\right] Y_t(i). \tag{A.14}$$

Marginal costs are defined as a change in total cost when output changes, $\frac{dTC_t(i)}{dY_t(i)} = MC_t^r(i)$, which shows that the Lagrange multiplier equals real marginal costs. From the ratio of equation (A.12) and equation (A.13) we get that,

$$\frac{1-\theta}{\theta} = \frac{W_t N_t(i)}{R_t^k K_t(i)}. \tag{A.15}$$

Since factor prices are common for all the firms, the ratio of $\frac{1-\theta}{\theta}\frac{R_t}{W_t} = \frac{N_t(i)}{K_t(i)}$ is the same for all firms. Plugging factor demands from equation (A.12) and equation (A.13) into production function of firm i we get $Y_t(i) = A_t\left(\frac{MC_t^r(i)\theta Y_t(i)}{R_t^k}\right)^\theta\left(\frac{MC_t^r(i)(1-\theta)Y_t(i)}{W_t}\right)^{1-\theta}$ which after expressing for MC_t^r delivers,

$$MC_t^r = \int_0^1 MC_t^r(i)di = \frac{\left(R_t^k\right)^\theta W_t^{1-\theta}}{A_t\theta^\theta(1-\theta)^{1-\theta}}, \tag{A.16}$$

Marginal costs are therefore the same for all firms, both of price setters and firms with staggered prices.

A.2.5 Re-setting firm vs. aggregate quantities for CRS

From the relationship $Y_t^* = A_t K_t^{*\theta} N_t^{*1-\theta} = \left(\frac{P_t^*}{P_t}\right)^{-\epsilon} Y_t$ we can express for amount of labor input hired by the price re-setting firm as

$$N_t^* = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}}di\right]}\left(\frac{K_t}{K_t^*}\right)^{\frac{\theta}{1-\theta}} N_t. \tag{A.17}$$

The ratio of capital demand equations for the price resetting firm and the aggregate firm delivers,

$$\frac{K_t}{K_t^*} = \frac{Y_t}{Y_t^*}, \tag{A.18}$$

We have shown in Lemma 2.1 that $Y_t^* \leq Y_t$ in the presence of trend inflation. Therefore, as $\frac{K_t}{K_t^*} = \frac{Y}{Y_t^*}$, then $K_t^* \leq K_t$ and $N_t^* \leq N_t$.

A.3 Proofs and Propositions

Proposition A.1. *Price dispersion is bounded by one, $S_t \geq 1$.*

Proof. The aggregate price index is $P_t = \left[\int_0^1 P_t^{1-\epsilon}(i)\right]^{\frac{1}{1-\epsilon}}$. Dividing by P_t gives $1 = \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$. Defining $v_{i,t} = \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon}$ we get that $\left[\int_0^1 v_{i,t}\right]^{\frac{1}{1-\epsilon}} = 1$. Writing price dispersion, $S_t = \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di\right]^{1-\theta}$, in terms of $v_{i,t}$ yields $v_{i,}^{\frac{-\epsilon}{1-\epsilon}\frac{1}{1-\theta}} = \left[\left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon}\right]^{\frac{-\epsilon}{1-\epsilon}\frac{1}{1-\theta}}$ Thus, price dispersion can be written in terms of variable v as, $S_t^{\frac{1}{1-\theta}} = \int_0^1 v_{i,t}^{\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta}}$ And as $\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta} > 1$, Jensen's inequality implies that

$$1 = \left[\int_0^1 v_{i,t}\right]^{\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta}} \leq \int_0^1 v_{i,t}^{\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta}} = S_t^{\frac{1}{1-\theta}}. \tag{A.19}$$

□

A.4 Calibration

Appendix B Nonlinear version of basic New Keynesian (CGG) Model

This section of the appendix outlines the nonlinear version of a basic New Keynesian model and presents results analogous to the ones in the main text. The model closely follows the sticky price model of **?**, with two exceptions: one, we use a production function that is assumed to be of the DRS-labor-only type as our baseline, as in the RS model. Two, we assume that productivity shocks are difference-stationary

Table A.2: Calibration of the RS table 3 (best fit) model

Symbol	Variable	Value
β	Discount factor	0.99
$CRRA$	Risk aversion	110
IES	Intertemporal elasticity	0.09
ϵ	Elasticity of substitution	6
$Frisch$	Frisch elasticity	0.28
ϕ_π	Response to inflation	0.53
ϕ_y	Response to output	0.93
ρ_i	i_t smoothing	0.73
ζ	Price adjustment	0.76
$\bar{G}/\bar{Y}$	Government spending on output	0.17
ρ_G	Autocorrelation Government spending shock	0.95
σ_G	Volatility of Government spending shock	0.004
ρ_A	Autocorrelation of TFP shock	0.95
σ_A	Volatility of TFP shock	0.005
$\theta_{\rho_{pi^*}}$	Inflation target shock persistence	0.995
σ_{π^*}	Volatility of inflation target shock	0.0007
ζ_{π^*}	Inflation target adjustment	0.003
θ	Capital share of output	1/3
$\bar{\Pi}$	Steady state inflation	1.004
δ	Capital depreciation	0.02

(in the case of trend-stationary shocks the channels leading to high levels and poor approximation of price dispersion are quantitatively inconsequential).

Otherwise, the model features are standard, firms are monopolistically competitive, face nominal rigidities à la Calvo, and the monetary authority follows a standard Taylor rule. Below we provide a sketch of the model and a list of first order and equilibrium conditions.

B.1 Model sketch, CGG model

B.1.1 Households

A representative household has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left(\frac{C_t}{Z_t}\right)^{1-\tau}}{1-\tau} - \xi_t \frac{N_t^{1+\varphi}}{1+\varphi} \right\}, \tag{A.20}$$

where utility from consumption is divided by the (growing) level of technology, such as to have a well-defined balanced growth path. The household maximizes the above preferences subject to its budget constraint:

$$P_t C_t + B_t \leq B_{t-1} R_{t-1} + W_t N_t + T_t. \tag{A.21}$$

B.1.2 Final good firms

Final good firms have production technology

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{A.22}$$

where $Y_t(i)$ are differentiated types of intermediate goods used as production inputs. The final good firm maximizes profits by selling Y_t at P_t and buying $Y_t(i)$ at prices $P_t(i)$.

B.1.3 Intermediate goods firms

An intermediate good firm's problem can be split into a (static) cost minimization and a (dynamic) profit maximization problem. The cost minimization problem reads

$$\min_{N_t(i)} \left\{ W_t N_t(i) + MC_t(i) \left[Y_t(i) - Z_t N_t(i)^{1-\alpha} \right] \right\},$$

from which an expression for the firm's marginal cost $MC_t(i)$ can be derived. The firm's profit maximization problem, taking as given the demand function the firm faces for its product, is then given by:

$$\max_{P_t(i)} E_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} \left\{ \left[P_t(i) - MC_t(i) \right] Y_t(i) \right\}. \tag{A.23}$$

Table B.1: System of model equations, New Keynesian model

(NK1):	$\xi_t N_t^{\varphi} c_t^{\tau} = w_t$
(NK2):	$c_t^{-\tau} = \beta E_t c_{t+1}^{-\tau} \frac{1}{\mu_{t+1}} \frac{R_t}{\Pi_{t+1}}$
(NK3):	$p_t^{*\left(1+\frac{\varepsilon\alpha}{1-\alpha}\right)} = (1-\nu) \frac{\varepsilon}{(\varepsilon-1)} \frac{aux_{1,t}}{aux_{2,t}}$
(NK4):	$aux_{1t} = mc_t y_t + E_t \beta\theta \Pi_{t+1}^{\frac{\varepsilon}{1-\alpha}} \frac{c_{t+1}^{-\tau}}{c_t^{-\tau}} aux_{1t+1}$
(NK5):	$aux_{2t} = y_t + E_t \beta\theta \Pi_{t+1}^{\varepsilon-1} \frac{c_{t+1}^{-\tau}}{c_t^{-\tau}} aux_{2t+1}$
(NK6):	$S_t y_t = N_t^{1-\alpha}$
(NK7):	$S_t^{\frac{1}{1-\alpha}} \equiv (1-\theta)\left(p_t^*\right)^{\frac{-\varepsilon}{1-\alpha}} + \theta\left(\Pi_t\right)^{\frac{\varepsilon}{1-\alpha}} \Delta_{t-1}^{\frac{1}{1-\alpha}}$
(NK8):	$p_t^* = \left[\frac{1-\theta\Pi_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}}$
(NK9):	$mc_t = \frac{1}{1-\alpha} w_t y_t^{\frac{\alpha}{1-\alpha}}$
(NK10):	$c_t = y_t$
(NK11):	$\frac{R_t}{R} = \left(\frac{R_t}{R}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\rho_\Pi} \left(\frac{y_t}{y_t^{flex}}\right)^{\rho_y}\right]^{1-\rho_R} e^{\varepsilon_{R,t}}$
(NK12):	$y_t^{flex} = \left[(1-\nu)\frac{\varepsilon}{(\varepsilon-1)}\frac{1}{\xi_t}\right]^{\frac{1-\alpha}{\varphi+\tau(1-\alpha)}}$
(NK13):	$\log\left(\mu_t\right) = \rho_\mu \log\left(\mu_{t-1}\right) + \left(1-\rho_\mu\right)\mu + \varepsilon_{\mu,t}$
(NK14):	$\log\left(\xi_t\right) = \rho_\xi \log\left(\xi_{t-1}\right) + \left(1-\rho_\xi\right)\xi + \varepsilon_{\xi,t}$

B.1.4 System of model equations

Table B.1 summarizes the system of equations of the New Keynesian model in terms of stationary allocations and real (relative) prices (i.e., in term of detrended and deflated variables, denoted by lowercase variables), defined as $c_t = \frac{C_t}{Z_t}$, $y_t = \frac{Y_t}{Z_t}$, $\Pi_t = \frac{P_t}{P_{t-1}}$, $w_t = \frac{W_t}{P_t Z_t}$, $b_t = \frac{B_t}{P_t Z_t}$, $t_t = \frac{T_t}{P_t Z_t}$, $p_t^* = \frac{P_t^*(i)}{P_t}$, $mc_t(i) = \frac{MC_t(i)}{P_t}$, $y_t = \frac{Y_t}{Z_t}$, $\mu_t = \frac{Z_t}{Z_{t-1}}$, and where price dispersion is defined as $S_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\varepsilon}{1-\alpha}} dj$. Table B.2 summarizes parameter values used in model simulations.

B.2 Results

This section of the appendix lays out results from model simulations for the New Keynesian model. Table B.3 and B.4 mirror the model versions and results for the RS

Table B.2: Calibration of the CGG New Keynesian model

Symbol	Variable	Value
β	Discount factor	0.99
τ	Coefficient of relative risk aversion	1
ϵ	Elasticity of substitution betw. varieties	9
φ	Inverse Frisch elasticity	5
ρ_π	Coefficient on inflation, Taylor rule	1.5
ϕ_y	Coefficient on output gap, Taylor rule	0
ρ_R	Interest rate smoothing, Taylor rule	0.75
θ	Calvo parameter	0.75
α	1-alfa is the weight on labor in prod. fct.	1/4
ρ_ξ	Autocorrelation, preference shock	0.95
σ_ξ	Volatility, preference shock	0.01
ρ_μ	Autocorrelation, TFP growth shock	0.85
σ_μ	Volatility, TFP growth shock	0.005

model in the main text. Table B.3 reports model moments for the baseline model with zero trend inflation (NK1), the version with positive trend inflation (NK2) and the version with positive trend inflation and variable capital (NK3). As with the RS model, the trend-inflation augmented model version gives rise to problems of inflated model moments and counterfactual regions over which price dispersion travels, as witnessed in particular by the maximum values of S^{-1} observed over the simulation. As stressed already in the main text, whether or not the NK model is susceptible to counterfactual levels of price dispersion and the resulting problems of unreasonable model moments is ultimately a quantitative question. Simply changing the persistence parameter ρ_μ from the reported value in table B.2 to 0.5 implies that none of the model versions, also not NK2 or NK3, give rise to any problems and display well-behaved regions for price dispersion, with MAX(S^{-1}) strictly smaller than one and MIN(S^{-1}) not lower than 0.98. Similarly, we never encounter any signs of elevated levels of price dispersion in a model version with trend-stationary shocks.

Table B.4 reports model moments for the model versions that feature one of the modeling devices that keep the behavior of price dispersion contained and therefore provide a fix to the problems of inflated moments, paralleling table 2 of the main text. In particular, NK4 considers the case of Rotemberg adjustment costs, NK5 is the model version with a linear-in-labor production function, and NK6 and NK7 are the model versions with inflation indexation, either with respect to steady state

Table B.3: Empirical and Model-Based Unconditional Moments

Unconditional Moment	USdata 1961-2007	NK1 $\bar{\pi}=0$	NK2 $\bar{\pi}=2.0\%$	NK3 $Y_t = Z_t K_t^{\theta} N_t^{1-\theta}$
SD(C)	0.83	1.59	11.31	6.30
SD(N)	1.71	2.92	4.62	0.97
Mean(π)	3.50	-0.38	-0.41	-0.57
SD(π)	2.52	3.25	6.11	2.23
MEAN(i)	5.72	-0.57	-0.63	-0.87
SD(i)	2.71	3.40	7.99	2.80
MEAN(S^{-1})	0.00	0.98	0.96	1.00
SD(S^{-1})	0.00	0.02	0.13	0.00
MIN(S^{-1})	0.00	0.88	0.21	0.97
MAX(S^{-1})	0.00	1.00	1.35	1.00

Note: Model moments are calculated from the simulated series. ***NK1****: model with labor-only-DRS production function $Y_t = Z_t N_t^{1-\alpha}$, zero trend inflation, $\bar{\pi} = 0\%$.* ***NK2****: as in NK1, but: with positive trend inflation $\bar{\pi} = 2\%$.* ***NK3****: as in NK1, but: with positive trend inflation $\bar{\pi} = 2\%$, with variable capital $Y_t = Z_t K_t^{\theta} N_t^{1-\theta}$.*

inflation or with respect to past quarter inflation.

Appendix C Non-linearities and solution methods

A natural question which arises in the context of the numerical inaccuracies reported in section 2.4 is whether the bad performance of the RS New Keynesian model with trend inflation is simply driven by the highly inaccurate third-order perturbation approximation to known discontinuities in the true policy functions –of an upper and lower bound on price dispersion– or whether most of the excess dynamics come from the economic channels discussed in the paper.

To better isolate the effects of the marginal cost and trend inflation markup channels we implement the Andreasen and Kronborg (2017) extended perturbation solution method. This method decomposes the policy functions into a certainty equivalent and a stochastic part of the solution. To be precise, Andreasen and Kronborg's method uses the extended path algorithm of Fair and Taylor (1983) to remove approximation errors from the certainty equivalent part of the solution, while keeping the stochastic part as being approximated by the perturbation method. This means that the improved accuracy of the overall solution relates to the certainty equivalent part.

Table B.4: Empirical and Model-Based Unconditional Moments

Unconditional Moment	NK2 $\pi = 2.0$	NK4 Rotemberg	NK5 $Y_t = Z_t N_t$	NK6 $\iota = 0$	NK7 $\iota = 1$
SD(C)	1.59	1.51	1.18	1.59	1.85
SD(N)	2.92	4.42	1.40	2.92	2.82
Mean(π)	-0.38	-0.38	-0.36	-0.38	-0.38
SD(π)	3.25	3.62	4.06	3.25	2.92
MEAN(i)	-0.57	-0.57	-0.54	-0.57	-0.58
SD(i)	3.40	3.40	3.88	3.40	3.27
MEAN(S^{-1})	0.98	1.00	0.99	0.98	1.00
SD(S^{-1})	0.02	0.00	0.01	0.02	0.00
MIN(S^{-1})	0.88	0.98	0.94	0.88	0.98
MAX(S^{-1})	1.00	1.00	1.00	1.00	1.00

Note: Model moments are calculated from the simulated series. ***NK2****: equal to NK2 from Table B.3.* ***NK4****: as in NK2, but: with Rotemberg adjustment costs instead of Calvo pricing.* ***NK5****: as in NK2, but: with labor-only-CRS $Y_t = Z_t N_t$.* ***NK6****: as in NK2, but: with indexation to steady state inflation.* ***NK7****: as in NK2, but: with indexation to last-period inflation.*

The important sources of non-linearities of the New Keynesian model with Calvo prices lie in the fact that the policy functions, which map the dispersion of prices to control variables, are subject to discontinuities at threshold values of the inverse price dispersion index, S_t^{-1}. These threshold values for S_t^{-1} reflect, i), the upper bound on inflation and an implied minimum level of S_t^{-1} associated with the inflation-upper bound, and, ii), a maximum admissible value of $S_t^{-1} = 1$ when there is no dispersion of prices at all (see figure 2 and 1).[27]Andreasen and Kronborg (2017) show that after using their method, as opposed to standard perturbation methods with pruning, the approximated policy functions reflect the kinks implied by the respective lower and upper bounds.

Based on the application of the extended perturbation method to our baseline model we find that the reason for the excessive dynamics lies in the stochastic component of the higher-order part of the solution. For this reason, even if we internalize the upper bound on inflation through solving accurately for the certainty equivalent part of the solution, the excessive dynamics of the model remain (see table C.4). The stochastic part of the perturbation solution is also the reason why we get into the impossibility region of inverse price dispersion, reflected in simulated values for S_t^{-1}

[27]Formal proofs for the existence of these bounds can be found for instance in Schmitt-Grohe and Uribe (2007) and Ascari and Sbordone (2014)

higher than one. The numerical algorithm solving for the extended path of the non-stochastic part of the true nonlinear system delivers inverse price dispersion strictly below and respects the upper bound on inflation, but after adding the stochastic part of the solution (which continues to be third-order-perturbation based) inverse price dispersion shifts up and crosses the upper bound.

The application of the extended perturbation is yet another way of how we document that the upper bound on inflation is not what causes the excessive dynamics of the model. Instead, they are caused by the higher-order terms (particularly, of the higher-order risk-adjusments) and thus by the fact that inflation becomes excessively uncertain and risky rather than being too large. We further support this claim by performing two exercises, outlined below.

First, table C.5 reports simulated moments of the RS model with trend inflation for different orders of approximation. The moments of model variables get excessively high only with the higher approximation order when the dispersion of prices has effects on the macro variables (recall that up to a first order approximation $S_t = 1$, and that price dispersion does not affect the first-order part of policy functions).

Second, we show that one of the fixes of the model is to increase the weight on inflation in Taylor rule.[28] The ability of ϕ_π to dampen the extreme dynamics of the model triggered by the presence of trend inflation is documented in table C.2. Why this is the case can be seen from the values of price dispersion which are below but close to one. The increase in ϕ_π disciplines the second moments of model macro variables by reducing the spread of the distribution of prices in the economy. The fact that the central bank strongly fights any deviation in the path of inflation from its target, provides an anchor to the uncertainty of inflation and makes the path of inflation more predictable. In such case, the Calvo pricing firm, which has to set its price based on its expectations of the development of its future costs and revenues, has a much easier job as the uncertainty about nominal variables is significantly mitigated.

The ability of Taylor-rule coefficient ϕ_π to stabilize the model raises the question of what reasonable values of ϕ_π and ϕ_y are. Table C.1 summarizes some of the most prominent Taylor rule estimates for the US. In RS the Taylor rule is calibrated based on Gürkaynak, Sack and Swanson (2005) with the reference to Rudebusch (2002). Rudebusch (2002) estimates $\phi_\pi = 1.523$ and $\phi_y = 0.93$ on data for the period of 1987Q4–1999Q4. For all estimates, it is common that they are derived from the time period when average inflation has been far from zero and the inflation target was

[28] The fact that increasing the weight on inflation in the Taylor rule helps to stabilize the model hints to issues with indeterminacy. We therefore report in figure C.2 that our calibration is safely within the wide region of determinacy.

way above 2%.

Study	Period	ϕ_{pi}	ϕ_y
Taylor (1996)	1987 - 1997	1.53	0.77
Judd and Rudebush (1998)	1987 - 1997	1.54	0.99
Clarida, Gali and Gertler (2000)	1979 - 1996	2.15	0.93
Orphanides (2003)	1979 - 1995	1.89	0.18

Table C.1: Taylor rule estimates for US

The impact of a higher weight on inflation for the NTP is also significant. By increasing the weight on inflation 5 times the NTP drops by about 40 bps. Table C.3 reports moments for versions of the RS model when we adopt their 'benchmark calibration', which is based on their table 2 in Rudebusch and Swanson (2012) (and which includes switching off the time-varying inflation target), instead of using the best-fit calibration of their table 3, which we have adopted throughout this paper.

In what follows, we focus our analysis further on the dispersion of prices in the economy. As the model parameters of RS were calibrated to match moments for the case of $\bar{\pi} = 0$, it may be argued that the model might be not well calibrated, when simply allowing for $\bar{\pi} > 0$ but leaving other model parameters unchanged. We first confirm that the patterns documented in Table 1 hold across a wide range of parameter values.

Figure C.3 shows how the mean of the inverse price dispersion changes over different ranges of other model parameter values and orders of approximation. The first set of panels shows the sensitivity of mean simulated price dispersion to changes in key model parameters for the case of zero trend inflation, for different orders of approximation (first, second, third-order approximations). Pink diamonds reflect the case of the 'RS Table 3'-baseline parameterization. Whereas the mean simulated price dispersion is affected strongly by varying trend inflation (panel 1), including pushing S^{-1} to the infeasible region bigger than one[29], varying other model parameters does not affect the simulated mean price dispersion drastically (and never pushes S^{-1} to an infeasible region). Other than variations in trend inflation, only regions of relatively high elasticities of substitution or high price rigidities lead to large costs from price dispersion (of, e.g. more than 1%, reflected in S^{-1} falling below 0.99). The second set of panels presents comparable figures for the case of positive trend inflation. Pink diamonds reflect the 'RS Table 3'-baseline parameterization, apart for steady-state inflation, which now is $\bar{\pi} = 1\%$. Since the accuracy of the mean

[29] S^{-1} is bounded from above by one. See Proposition A.1

	US data	RS	RS	RS	RS B	RS C
		$\pi = 1$	$\pi = 1$	$\pi = 1.004$,	$\pi = 1$	$\pi = 1.004$
		$\phi_\pi = 0.5$	$\phi_\pi = 2.5$	$\phi_\pi = 2.5$	$\phi_\pi = 2.5$	$\phi_\pi = 2.5$
SD(dC)	2.69	0.75	0.73	0.81	0.72	0.76
SD(C)	0.83	0.91	0.68	0.78	0.68	0.75
SD(N)	1.71	2.59	1.75	1.98	1.72	1.89
Mean(i)	5.72	2.95	2.69	3.86	3.26	4.38
SD(i)	2.71	3.49	1.68	1.82	1.07	1.74
Mean(π)	3.50	-0.55	-0.34	0.93	-0.58	1.30
SD(π)	2.52	3.07	0.94	0.98	0.94	1.00
SD($i^{(40)}$)	2.41	2.40	1.07	1.18	0.09	1.16
Mean($NTP^{(40)}$)	1.06	1.26	0.87	0.91	0.85	0.85
SD($NTP^{(40)}$)	0.54	0.51	0.09	0.18	0.89	0.15
Mean($R^{(40)} - R$)	1.43	1.16	0.81	0.90	0.39	0.80
SD($R^{(40)} - R$)	1.33	1.65	0.83	0.93	0.58	0.85
Mean(S^{-1})	1.00	0.99	1.00	0.99	1.00	0.99
Min(S^{-1})	1.00	0.83	0.99	0.97	0.99	0.96
Max(S^{-1})	1.00	1.07	1.0003	0.99746	1.0003	0.99719

Table C.2: Model simulated moments from re-calibrated Taylor rule. RS is the Rudebusch and Swanson (2012) model, RS B is RS solved with Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2018) pruning. RS C is the RS model without capital.

price dispersion is already somewhat compromised at $\bar{\pi} = 1\%$, regions of relatively high elasticities of substitution or high price rigidities quickly lead to problems (flat lines in the last two reported panels represent cases with indeterminate solutions). Variations in other key parameters continue to leave mean price dispersion mostly unaffected.

	US data	RS $\pi = 1$	RS $\pi = 1.004$	RS $\theta = 0,$	RS Andr $\pi = 1.004$
SD(dC)	2.69	0.42	2.96	0.34	0.85
SD(C)	0.83	0.65	7.27	0.56	1.50
SD(N)	1.71	0.75	6.87	0.69	1.58
Mean(i)	5.72	4.16	10.57	4.20	6.89
SD(i)	2.71	0.86	10.59	0.86	1.62
Mean(π)	3.50	0.15	6.59	0.18	2.69
SD(π)	2.52	0.93	10.89	0.97	2.22
SD($i^{(40)}$)	2.41	0.51	9.05	0.51	0.07
Mean($NTP^{(40)}$)	1.06	0.09	0.82	0.07	0.20
SD($NTP^{(40)}$)	0.54	0.01	0.58	0.01	0.81
Mean($R^{(40)} - R$)	1.43	0.09	0.76	0.08	3.39
SD($R^{(40)} - R$)	1.33	0.40	2.94	0.40	5.06
Mean(S^{-1})	1.00	1.00	0.97	1.00	0.99
Min(S^{-1})	1.00	0.99	0.53	0.98	0.81
Max(S^{-1})	1.00	1.00	1.62	1.00	1.06

Table C.3: Moments based on the RS benchmark calibration as opposed to the best fit calibration.

	USdata	RS Table3 $\phi_\pi = 0.53 \times 2$ $\rho_a = 0.005/2$	Extended path $\phi_\pi = 0.53 \times 2$ $\rho_a = 0.005/2$
SD(dC)	2.69	0.33	0.00
SD(C)	0.83	0.34	0.35
SD(N)	1.71	0.97	1.02
Mean(i)	5.72	3.82	4.41
SD(i)	2.71	1.21	1.29
Mean(π)	3.50	-0.01	0.59
SD(π)	2.52	0.98	1.09
SD($i^{(40)}$)	2.41	0.86	0.97
Mean($NTP^{(40)}$)	1.06	0.27	0.27
SD($NTP^{(40)}$)	0.54	0.05	0.04
Mean($R^{(40)} - R$)	1.43	0.22	0.00
SD($R^{(40)} - R$)	1.33	0.54	0.00
Mean(S^{-1})	1.00	0.999	0.99876
Min(S^{-1})	1.00	0.98334	0.98131
Max(S^{-1})	1.00	1.0001	1.0001

Table C.4: This table compares pruned solution using perturbation as implemented in dynare with extended perturbation of Andreasen and Kronborg (2017) for 20 000 periods of simulation

	FirstOrder	SecondOrder	ThirdOrder
SD(dC)	0.85	2.22	11.51
SD(C)	1.22	3.36	16.08
SD(N)	3.68	10.11	46.06
Mean(i)	6.42	-1.75	-0.88
SD(i)	5.20	13.37	59.15
Mean(π)	2.38	-4.00	-2.94
SD(π)	4.57	11.51	49.76
SD($i^{(40)}$)	3.66	9.11	39.12
Mean($NTP^{(40)}$)	-0.00	3.69	3.59
SD($NTP^{(40)}$)	0.00	0.00	9.68

Table C.5: Rudebusch Swanson (2012) with trend inflation.

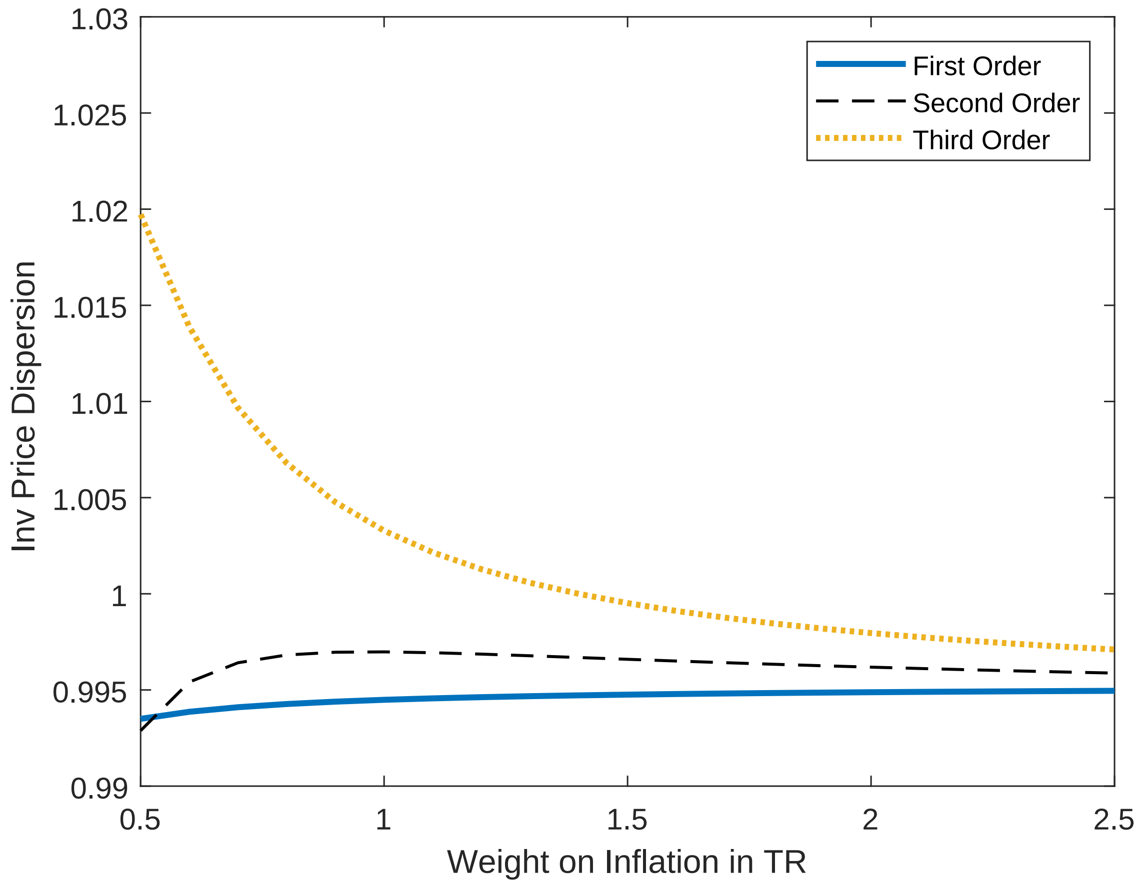


Figure C.1: Simulated mean of inverse of price dispersion (expressed in costs of output) as a function of weight on inflation in TR. Increase in ϕ_π narrows the distribution of prices and S_t approaches

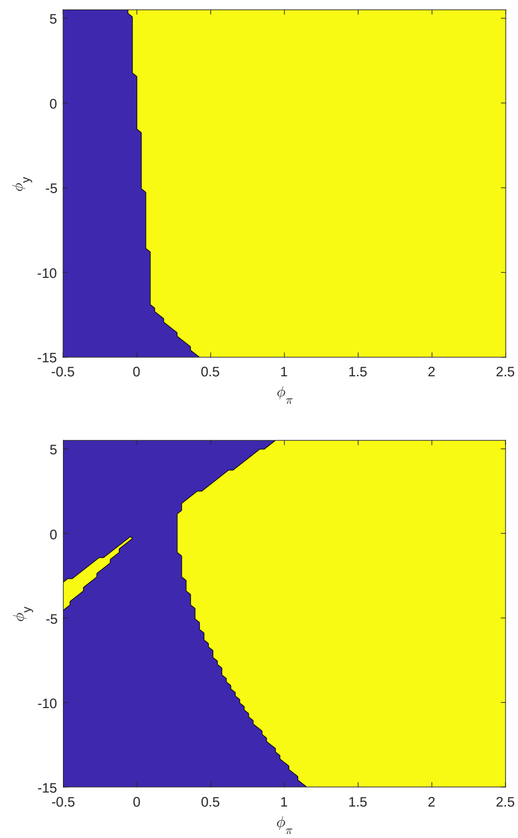
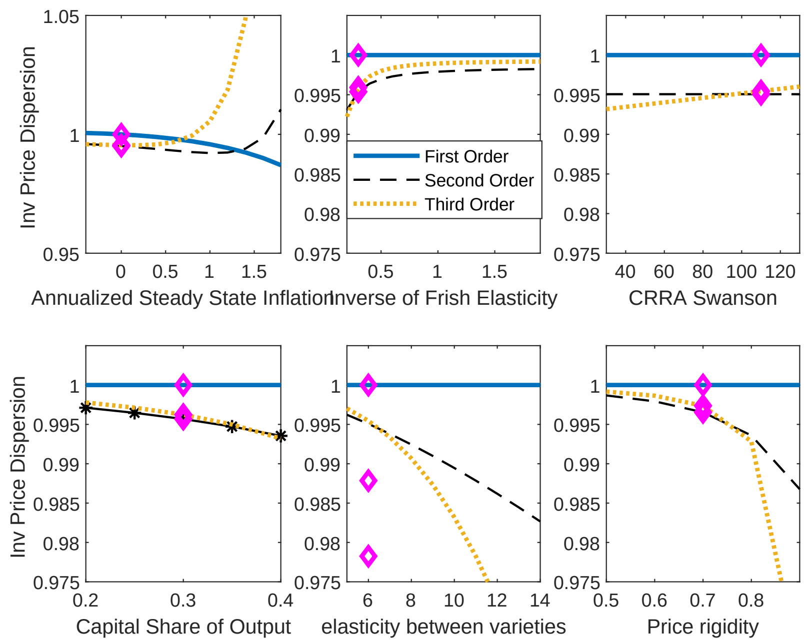


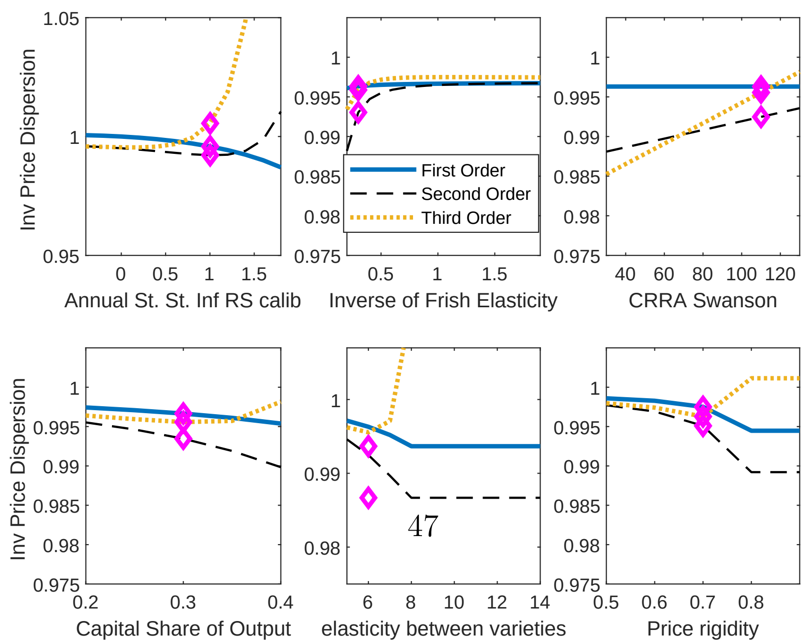
Figure C.2: The blue color depicts the indeterminacy region for ϕ_π and ϕ_y while the yellow surface represents region of determinacy. The first picture shows determinacy region for $\Pi = 1$ and the second panel is for $\Pi = 1.006$

Figure C.3: Parameters sensitivity in the RS (2012) model

Parameter sensitivity of RS (2012) model, case of zero trend inflation, $\bar{\pi} = 0\%$



Parameter sensitivity of RS (2012) model, case of positive trend inflation, $\bar{\pi} = 1\%$



Note: The first set of panels shows the sensitivity of the mean simulated price dispersion to changes in key model parameters for the case of zero trend inflation, for different orders of approximation (first, second and third order approximations). Pink diamonds reflect the case of the 'RS Table 3'-baseline parameterization. The second set of panels presents analogous figures for the case of positive trend inflation. Flat lines in the last two reported panels represent cases with indeterminate