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# Exchange rate dynamics and monetary policy – Evidence from a non-linear DSGE-VAR approach

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# Exchange rate dynamics and monetary policy

## Evidence from a non-linear DSGE-VAR approach

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**Abstract**

In this paper, we reconsider the question how monetary policy influences exchange rate dynamics. To this end, a vector autoregressive (VAR) model is combined with a two-country dynamic stochastic general equilibrium (DSGE) model. Instead of focusing exclusively on how monetary policy shocks affect the level of exchange rates, we also analyze how they impact exchange rate volatility. Since exchange rate volatility is not observed, we estimate it alongside the remaining quantities in the model. Our findings can be summarized as follows. Contractionary monetary policy shocks lead to an appreciation of the home currency, with exchange rate responses in the short-run typically undershooting their long-run level of appreciation. They also lead to an increase in exchange rate volatility. Historical and forecast error variance decompositions indicate that monetary policy shocks explain an appreciable amount of exchange rate movements and the corresponding volatility.

**Keywords:** Monetary policy, Exchange rate overshooting, stochastic volatility modeling, DSGE priors.

**JEL Codes:** E43, E52, F31.

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# 1 Introduction

This paper reconsiders the effects of monetary policy on exchange rate dynamics. Instead of focusing exclusively on the reactions of the exchange rate, we propose a flexible empirical model that allows also for analyzing how exchange rate volatility changes in response to monetary policy shocks. Our empirical model, a Bayesian vector autoregressive model with stochastic volatility (BVAR-SV) is combined with a two-country dynamic stochastic general equilibrium (DSGE) model that is used to construct a prior distribution as well as to carry out structural inference. We estimate our model on three country pairs: US and Euro Area, US and UK, and US and Japan. We find that US monetary contractions appreciate the dollar, on impact and in the long-run, with the degree of short-run appreciation typically undershooting its long-run counterpart. A monetary tightening also typically leads to increases in exchange rate volatility. Considering historical decompositions, our analysis indicates that monetary policy disturbances have contributed considerably to explaining fluctuations in the level and volatility of exchange rates, especially during the first half of the sample.

Numerous contributions estimate the effects of monetary policy on exchange rates (see, inter alia, Eichenbaum and Evans, 1995; Grilli and Roubini, 1995; Kim and Roubini, 2000a; Kim, 2001; Kim and Roubini, 2000b; Faust and Rogers, 2003a; Bjornland, 2009; Kim, 2005; Scholl and Uhlig, 2008; Bouakez and Normadin, 2010; Kim *et al.*, 2017; Müller *et al.*, 2019; Schmitt-Grohé and Uribe, 2018). These studies, using VAR models, often find dynamic exchange rate responses that are inconsistent with the exchange rate overshooting hypothesis put forth in Dornbusch (1976). Some studies (see, among others, Eichenbaum and Evans, 1995; Kim, 2001; Kim and Roubini, 2000b) find delayed overshooting, implying that immediate exchange rate reactions are weaker but grow in importance over the impulse response horizon in a hump-shaped fashion. These contributions differ in terms of model specification and identification. Most studies use two-country datasets and introduce hard restrictions to alleviate issues related to overfitting. More precisely, instead of estimating separate equations for home and foreign quantities, a common approach is to consider differences between them and thus model relative movements. Using such restrictions in combination with small models can translate into omitted variable biases that do not capture the information set used by the central bank and might be one reason for observing puzzling exchange rate responses.

The second dimension where studies often differ relates to the identitification strategy used to recover the structural impulse responses. Early contributions (Eichenbaum and Evans, 1995; Grilli and Roubini, 1995) introduce timing restrictions that imply that, with respect to domestic monetary policy shocks, foreign economies react with a lag. One issue with such identifying assumptions, however, is that the researcher needs to decide on how the exchange rate reacts to monetary policy. The standard option would be to assume that the exchange rate is allowed to react immediately to interest rate changes. However, this assumption implies that the central bank is not allowed to react to contemporaneous exchange rate movements. More recently, several studies (Faust and Rogers, 2003a; Scholl and Uhlig, 2008; Bouakez and Normadin, 2010; Kim *et al.*, 2017) use sign restrictions to identify monetary policy shocks and trace the resulting exchange rate reactions. While sign restrictions allow for contemporaneous relations between the reaction function of the central bank and the exchange rate, they only allow for set identification and, if coupled with loose identifying restrictions, often lead to inflated confidence bounds surrounding the impulse responses.

In this paper, we depart from the literature in three respects. *First*, we propose a medium-scale BVAR model with SV. In contrast to the existing literature, we assume that prior information arises from a two-country New Keynesian open economy DSGE model that closely follows Lubik and Schorfheide (2005). To assess whether the DSGE model is consistent with the reduced-form information contained in the VAR, we use novel shrinkage priors that endogenously determine how much weight should be placed on the DSGE model.

*Second*, following Del Negro and Schorfheide (2004), we opt for identifying the model using the implied covariance matrix of the structural shocks in the DSGE model. This implies that our identification scheme rules out shortcomings associated with zero and sign restrictions. More importantly, the corresponding deterministic rotation matrix is fully consistent with the underlying DSGE model.

*Third*, the existing literature focuses exclusively on the reactions of the exchange rate. In this paper, we do not only consider how exchange rates move in response to unexpected monetary policy shocks but also assess whether such shocks impact exchange rate volatility (or uncertainty).[1] Since exchange rate volatility is unobserved, we estimate the volatility process alongside the remaining model quantities. The resulting model is closely related to Carriero *et al.* (2018) and Mumtaz and Theodoridis (2019) and assumes that exchange rate volatility is a function of the remaining quantities in the model.

Our findings can be summarized as follows. Consistent with the literature, we find that the exchange rate appreciates in response to a US monetary contraction. In contrast to much of the literature, we, however, do not find evidence for delayed overshooting of the exchange rate.[2] Our findings on the precise shape of the exchange rate responses are somewhat heterogeneous across countries, but can be summarized as follows: for the case of US monetary policy shocks we find that the immediate degree of appreciation falls short of the long-run degree of appreciation, i.e., we find evidence of exchange rate undershooting. For the country pairs of US-Euro Area and US-UK, the amount of undershooting is only mild (so that the extent of exchange rate appreciation is similar quantitatively in the short and long run), in the case of US-Japan, the dollar undershoots its long-run level of appreciation more strongly. For monetary policy shocks originating abroad, our findings partly mirror the ones from US monetary shocks: for the country pairs of US-Euro Area and US-Japan the dollar depreciates, though the degree of depreciation undershoots the depreciation the dollar experiences in the long-run. Only for the country pair of US-UK and the case of a UK

[1] For studies that deal with the determinants of exchange rate volatility, see, e.g., Rose (1996), Rose and Flood (1999), Devereux and Lane (2003).

[2] In a recent contribution, Kim *et al.* (2017) document that delayed overshooting is a result of the Volcker-period, and does not apply to the post-Volcker sample period.

monetary contraction, we observe an exchange rate response more in line with the exchange rate overshooting hypothesis. The findings of exchange rate undershooting from our novel empirical framework are in line with some more recent contributions in the literature. Müller *et al.* (2019), use local projections to study exchange rates behavior in response to monetary shocks and similarly find evidence for exchange rate undershooting. Schmitt-Grohé and Uribe (2018) estimate a state-space model with temporary and permanent monetary policy shocks – in response to both types of shocks, the exchange rate, in their setting, undershoots. A novel aspect of our empirical framework is that it allows studying the consequences for exchange rate volatility in response to the monetary contraction. In most cases, we find that monetary shocks lead to an increase in exchange rate volatility, again, with some heterogeneity across country pairs and source of monetary contraction (home or abroad). In response to US monetary contractions, exchange rate volatility increases significantly, but with a considerable lag of about ten quarters for the country pairs US-Euro Area and US-Japan, while it shows no reaction for the case of US-UK. The lag in the volatility responses stands in contrast to findings of a recent paper by (Mumtaz and Theodoridis, 2019), who find immediate effects on macroeconomic volatility in response to monetary shocks – the setting, however, is rather different, as their framework is a closed economy, and we focus exclusively on the effects on exchange rate volatility, as the exchange rate is at the core of the international monetary transmission mechanism. For monetary shocks originating abroad, we observe increases in exchange volatility that are more immediate, though not always strongly significant. Finally, to gain an understanding for the importance of monetary shocks as a source of fluctuations in the level and volatility of exchange rate fluctuations, we also perform forecast error variance and historical decompositions. While the forecast error variance decompositions indicate that monetary policy shocks explain a small fraction of the variation in the exchange rate and its volatility, the quantitative contributions in terms of historical decompositions are large, especially during the first half of the sample up to and including the ERM crisis and its aftermaths. Focusing on exchange rate volatility, we find that, historically, US monetary policy shocks appear to have stabilizing effects.

The remainder of the paper is structured in the following way. Section 2 outlines the two-country DSGE model that serves as the basis for both prior construction and identification of the empirical model. Section 3 presents the empirical framework, a nonlinear Bayesian VAR model with stochastic volatility while section 4 presents the empirical findings of the paper. Finally, the last section summarizes and concludes the paper.

## 2 A two-country DSGE model

Below we lay out the theoretical dynamic stochastic general equilibrium (DSGE) model that serves as a basis for obtaining priors for the empirical VAR model. We employ a small-scale two country monetary model that allows us to study the transmission of monetary policy shocks, as well as (other) demand and supply shocks. Households in each country have preferences over consumption of domestically and foreign produced goods. The firm sector in each country produces a country-specific good, but is otherwise modeled in the typical fashion of New Keynesian models: a continuum of firms operate under monopolistic competition and are subject to nominal rigidities in their price setting. We largely follow Lubik and Schorfheide (2005), who have estimated a similar-sized model with Bayesian methods and who, in their model choice, carefully evaluate the trade-offs of potential misspecification of small and stylized models with potential identification and computational costs of larger-scale models. Despite being a small-scale New Keynesian model at the individual country level, our two-country version estimates roughly the same number of structural parameters and fits the model to the same number of time series as a medium-scale estimated model like Smets and Wouters (2007).

We deviate from the model setup of Lubik and Schorfheide (2005) in only two ways. One, we explicitly introduce country size into the model, as we estimate the model also on country pairs that are very unequal in terms of economic size. Two, a difference arising from the technical constraint to keep the number of shocks equal to the number of observed time series is that we abstract from a (common across countries) permanent productivity shock.[3]

## 2.1 Sketch of the model

The world economy consists of a Home country (H) and a Foreign country (F), each of which is specialized in the production of one type of tradable good. Households and firms are defined over a continuum of unit mass. Home and Foreign households are indexed by $j \in [0, n]$ and $j^* \in (n, 1]$, respectively. Foreign variables carry an asterisk. Unless necessary otherwise, in the following we only discuss the problem of Home agents, with an understanding that Foreign agents face an equivalent problem.

### 2.1.1 Households

The domestic household $j$ maximizes her lifetime expected utility, given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left( \frac{\left[ \left( C_t\left(j\right) - hC_{t-1}\left(j\right) \right) / Z_t \right]^{1-\sigma_c}}{1-\sigma_c} \right) + \psi \frac{L_t\left(j\right)^{1+\sigma_l}}{1+\sigma_l} \right\}, \tag{1}$$

where $C_t(j)$ is household $j$'s aggregate consumption, $L_t(j)$ is labor supply. $Z_t$ is the nonstationary level of technology, where we define $\gamma_{z,t} = Z_t/Z_{t-1} = \gamma_z$, where $\gamma_z$ is the gross quarterly growth rate, assumed to be identical across countries. Parameters $\beta$, $\sigma_c$, $\sigma_l$, and $h$ stand for the discount factor, the coefficient of risk aversion, the inverse Frisch elasticity of labor supply and the habit

[3]We also explored a third deviation from Lubik and Schorfheide (2005), allowing for incomplete financial markets, which a recent literature has emphasized to be important in helping to match exchange rate related empirical stylized facts. See the next section for a brief discussion of this avenue.

parameter, respectively. The aggregate consumption index of household $j$ is defined as

$$C_t(j) = \left[ \gamma_c^{\frac{1}{\epsilon}} C_{H,t}^{\frac{\epsilon-1}{\epsilon}}(j) + (1-\gamma_c)^{\frac{1}{\epsilon}} C_{F,t}^{\frac{\epsilon-1}{\epsilon}}(j) \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

with $\gamma_c$ denoting the degree of home bias in consumption, and $\epsilon$ is the elasticity of substitution (or trade elasticity) between domestic, $C_{H,t}(j)$, and foreign goods, $C_{F,t}(j)$. The household's consumption of domestically (foreign) produced goods, $C_{H,t}(j)$ ($C_{F,t}(j)$), is, again, modeled as a basket of the individual varieties of home (foreign) goods

$$C_{H,t}(j) = \left[ \left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n c_t(h,j)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, C_{F,t}(j) = \left[ \left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 c_t(f,j)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

whereby $\theta$ is the elasticity of substitution between varieties, $c_t(h,j)$ ($c_t(f,j)$). Given this setup, the household's demand functions can be derived as:

$$\begin{aligned} c_t(h,j) &= \frac{1}{n}\left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} C_{H,t}(j), c_t(f,j) = \frac{1}{1-n}\left(\frac{p_t(f)}{P_{F,t}}\right)^{-\theta} C_{F,t}(j), \\ C_{H,t}(j) &= \gamma_c \left(\frac{P_{H,t}}{P_t}\right)^{-\epsilon} C_t(j), C_{F,t}(j) = (1-\gamma_c)\left(\frac{P_{F,t}}{P_t}\right)^{-\epsilon} C_t(j). \end{aligned}$$

Household $j$ maximizes equation (1) subject to the budget constraint. Each period household j receives (nominal) wage income, $W_t L_t(j)$, and (nominal) dividends from the monopolistic firms they own, $D_t(j)$, and has to finance lump-sum tax payments, $T_t(j)$, and consumption expenditure $P_t C_t(j)$.

The availability of any assets of domestic household $j$ depends on the assumptions of the structure of international financial markets, where we follow the original model of Lubik and Schorfheide (2005) in assuming complete markets: in this case the household has access to a full set of state-contingent (Arrow-Debreu) securities. Let $Q(s_{t+1}|s_t)$ denote the price of one unit of Home currency delivered in period $t+1$ contingent on the state of nature at $t+1$ being $s_{t+1}$. With complete markets, $Q(s_{t+1}|s_t)$ is the same for all individuals. Let $B_{H,t}(j, s_{t+1})$ denote the claim to $B_{H,t}$ units of Home currency at time $t+1$ in the state of nature $s_{t+1}$, that household $j$ buys at time $t$ and brings into time $t+1$. The nominal interest rate can be expressed as $R_t = 1/\sum_{s_{t+1}} Q(s_{t+1}|s_t)$. The budget constraint under complete markets is then given by:

$$\sum_{s_{t+1}} Q(s_{t+1}|s_t) B_{H,t}(j, s_{t+1}) \leq B_{Ht-1}(j, s_t) + W_t L_t(j) + D_t(j) - T_t(j) - P_t C_t(j). \tag{4}$$

We should note that an integral part of coming up with a suitable two-country model to base the priors of our empirical model on was to explore the role of international financial markets for the ability to explain exchange rate behavior. Several recent contributions, most prominently Corsetti *et al.* (2008), as well as Heathcote and Perri (2002), Enders and Mueller (2009), and Rabitsch (2012; 2016) emphasize the importance of a low trade elasticity, especially when coupled with incomplete international financial markets, as key in addressing exchange rate empirical stylized facts, such as its high observed volatility, or the Backus-Smith puzzle (cf. Backus and Smith, 1993). We therefore explored estimating an alternative model version also with incomplete financial markets, assuming that both countries can engage in financial trade only through trade in one-period nominal bonds. While advantageous from a theoretical perspective, we (in the end) stick to the complete-markets version as the baseline model mostly for technical complications in the estimation of the incomplete-markets version.[4]

The households first order conditions with respect to consumption and labor are,

$$\Lambda_t\left(j\right) = \left(\frac{C_t\left(j\right) - h\gamma_z C_{t-1}\left(j\right)}{Z_t}\right)^{-\sigma_c}\frac{1}{Z_t} + \beta\left(-\frac{h\gamma_z}{Z_{t+1}}\right)\left(\frac{C_{t+1}\left(j\right) - h\gamma_z C_t}{Z_{t+1}}\right), \tag{5}$$

$$\psi L_t\left(j\right)^{\sigma_l} = \Lambda_t\left(j\right)\frac{W_t(h,j)}{P_t}.$$

The optimality condition with respect to the internationally traded Arrow-Debreu securities can be written, for the domestic and the foreign country (who faces a symmetric problem)

$$Q(s_{t+1}|s_t) = \beta E_t\left\{\frac{\Lambda_{t+1}\left(j\right)}{\Lambda_t\left(j\right)}\frac{P_t}{P_{t+1}}\right\}, Q(s_{t+1}|s_t) = \left\{\frac{\Lambda^*_{t+1}\left(j\right)}{\Lambda^*_t\left(j\right)}\frac{P^*_t}{P^*_{t+1}}\frac{\varepsilon_t}{\varepsilon_{t+1}}\right\}, \tag{6}$$

where $\varepsilon_t$ denotes the nominal exchange rate, expressed as units of Home currency per unit of Foreign currency (so that an increase in $\varepsilon_t$ implies a Home (dollar) depreciation). The above domestic and foreign optimality condition can be combined and forward-solved to obtain the risk sharing condition: full risk-sharing at the international level equates the ratio of marginal utilities across countries to the real exchange rate, up to a constant $\kappa$ which depends on initial conditions:

$$\frac{\varepsilon_t P^*_t}{P_t} = \kappa\frac{\Lambda^*_t\left(j\right)}{\Lambda_t\left(j\right)}. \tag{7}$$

### 2.1.2 Firms

Home and Foreign goods come in many varieties, each produced by a single firm, indexed by $h \in [0, n]$ for domestic and $f \in (n, 1]$ for foreign firms. Each domestic firm $h$ specializes in one variety producing according to a Cobb-Douglas production function:

$$Y_t\left(h\right) = A_t Z_t L_t\left(h\right). \tag{8}$$

[4]In particular, the incomplete-markets setup gives rise to a discontinuity in the model likelihood, related to a model behavior that is discontinuous in the parameter of the trade elasticity, $\varepsilon$.

Firms operate under monopolistic competition and each set a common price (in their own currency) in the local and foreign market so as to maximize profits, taking as given the households' demand for that good, $a_t(h)$,

$$a_t(h) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta}\left(C_{H,t} + \frac{1-n}{n}C^*_{H,t} + G_t\right) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} Y_t.$$

The firm is subject to quadratic price adjustment costs in re-setting the price, with price adjustment cost parameter $\alpha_H$.[5] Defining (nominal) marginal cost, $MC_t(h) = \frac{W_t}{A_t Z_t}$, the firm's problem is therefore, choosing $p_t(h)$, to maximize:

$$E_t \sum_{t=0}^{\infty} \Omega_{0,t}\left\{\left[\frac{p_t(h)}{P_{Ht}}\tau - \frac{MC_t(h)}{P_{Ht}}\right]\left[\left(\frac{p_t(h)}{P_{H,t}}\right)^{-\theta} Y_t\right] - \frac{\alpha_H}{2}\left(\frac{p_t(h)}{p_{t-1}(h)} - \pi_H\right)^2 Y_t\right\}. \tag{9}$$

The firm's optimal price setting, imposing symmetry across firms results in:

$$\left[\theta\frac{MC_t}{P_{Ht}} - (\theta-1)\tau\right] = \alpha_H(\pi_{Ht} - \pi_H)\pi_{Ht} - \Omega_{t,t+1}\alpha_H(\pi_{Ht+1} - \pi_H)\pi_{Ht+1}\frac{Y_{t+1}}{Y_t}, \tag{10}$$

with $\pi_{H,t}$ being producer price inflation, defined as $\pi_{H,t} = P_{H,t}/P_{H,t-1}$, $Y_t$ denoting aggregate output, and $\tau$ is a production subsidy that offsets the markup from monopolistic competition.

Following Lubik and Schorfheide (2005) and Monacelli (2005), we assume that endogenous deviations from purchasing power parity (PPP) arise due to the existence of monopolistically competitive importers. In particular, domestic consumers cannot purchase foreign-produced goods directly from foreign firms, but only from monopolistically competitive importers. Importers buy foreign goods at world-market prices $\varepsilon_t p^*_t(f)$, and sell these goods, charging a markup over their cost, to domestic consumers at price $p_t(f)$.[6] Their optimization problem reads:

$$\max_{p_t(f)} E_t \sum_{t=0}^{\infty} \Omega_{0,t}\left\{\left[\frac{p_t(f)}{P_{Ft}}\tau - \varepsilon_t p^*_t(f)\right]\left[\left(\frac{p_t(f)}{P_{F,t}}\right)^{-\theta}\frac{nC_{F,t}}{1-n}\right] - \frac{\alpha_F}{2}\left(\frac{p_t(f)}{p_{t-1}(f)} - \pi_F\right)^2 \frac{nC_{F,t}}{1-n}\right\} \tag{11}$$

resulting in a law of motion for $\pi_{F,t} = P_{F,t}/P_{F,t-1}$ similar to equation (10),

$$\left[\theta\frac{\varepsilon_t P^*_{Ft}}{P_{Ft}} - (\theta-1)\tau\right] = \alpha_F(\pi_{Ft} - \pi_F)\pi_{Ft} - \Omega_{t,t+1}\alpha_F(\pi_{Ft+1} - \pi_F)\pi_{Ft+1}\frac{C_{F,t+1}}{C_{F,t}}. \tag{12}$$

[5]In the estimation process, we do not estimate parameter $\alpha_H$ directly, but a parameter $\xi_H$ in a parameter transformation that has the interpretation of an equivalent Calvo setup, $\alpha_H = \frac{\xi_H[(\theta-1)\tau]}{(1-\beta\xi_H)(1-\xi_H)}$.

[6]Alternatively, one may have assumed that a share of firms $\omega_{LCP}$ set the price in the foreign market in not their own (producer) currency, but in the currency of the local market (local currency pricing), which would result a similar non-linear Phillips-curve relationship as equation (10), together with an equation that models $\pi_{F,t}$ as a weighted average of the law of motion implied by that additional Phillips curve (with weight $\omega_{LCP}$) and the dynamics that would be implied by the law of one price (with weight $(1-\omega_{LCP})$), allowing for intermediate degrees of exchange rate pass through. As this alternative setup requires a higher number of estimated model parameters to capture the same dynamics, we preferred the modeling strategy reported in the main text.

### 2.1.3 The government

The monetary authority is assumed to apply a Taylor-style interest-feedback rule. The interest rate is related to CPI inflation and output, in deviation from their long-run value, and exchange rate changes according to:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\overline{\pi}}\right)^{\rho_\pi} \left(\frac{Y_t}{\overline{Y}}\right)^{\rho_Y} (\Delta\varepsilon_t)^{\rho_{\Delta\varepsilon}}\right]^{1-\rho_R} e^{\epsilon_{R,t}}. \tag{13}$$

The fiscal authority keeps a balanced budget and finances its government expenditure entirely through lump-sum taxes each period, i.e. $G_t = T_t$.

### 2.1.4 Market clearing and exogenous processes

Arrow-Debreu securities are in zero net supply:

$$\int_0^n B_{H,t}(j)dj + \int_n^1 B^*_{H,t}(j^*)dj^* = 0, \int_n^1 B^*_{F,t}(j^*)dj^* = 0. \tag{14}$$

Equilibrium in the factor markets requires:

$$L_t = \int_0^n L_t\,(h)\,dh, L^*_t = \int_n^1 L^*_t\,(f)\,df. \tag{15}$$

Goods market clearing gives:

$$A_t Z_t L_t = \left[C_{H,t} + G_t + \tfrac{1-n}{n} C^*_{H,t}\right], \quad A^*_t Z^*_t L^*_t = \left[\tfrac{n}{1-n} C_{F,t} + C^*_{F,t} + G^*_t\right]. \tag{16}$$

The model is subject to seven exogenous disturbances. A domestic and foreign monetary shock, $\epsilon_{R,t}$ and $\epsilon^*_{R,t}$ affecting the short-term nominal interest rates in the Taylor rule, equation (13). Country specific technology levels and government expenditures follow:

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_{A,t}, \quad \log A^*_t = \rho^*_A \log A^*_{t-1} + \epsilon^*_{A,t}, \tag{17}$$

$$g_t = \rho_g g_{t-1} + (1-\rho_g)\, g + \epsilon_{g,t}, \quad g^*_t = \rho^*_g g^*_{t-1} + \left(1-\rho^*_g\right) g + \epsilon^*_{g,t}. \tag{18}$$

Here $g_t = 1/\left(1-\xi_t\right)$, and government expenditure, $G_t$, is a fraction $\xi_t$ of output, $G_t = \xi_t Y_t$. Finally, we follow Lubik and Schorfheide (2005) in specifying the final shock as a non-structural shock capturing deviations in purchasing power parity (PPP) that are not otherwise captured in the model, denoted $\epsilon_{PPP,t}$.

### 2.1.5 Observables

The model is estimated on quarterly data for the period of 1970:Q2 to 2016:Q4. We use the same seven variables as observables to estimate the DSGE model that we will carry in our VAR model of section 3: the quarterly growth rates of log GDP at home and abroad, $\Delta Y_t^{obs}$ and $\Delta Y_t^{*obs}$, annualized inflation rates, $\pi_t^{obs}$ and $\pi_t^{*obs}$, annualized nominal interest rates, $R_t^{obs}$ and $R_t^{obs}$, and the change in the bilateral nominal exchange rate, $\Delta\varepsilon_t^{obs}$. Our observables are related to DSGE-model variables as:

$$
\begin{array}{ll}
\Delta Y_t^{obs} = \gamma_z^q + \widehat{y}_t - \widehat{y}_{t-1}, & \Delta Y_t^{*obs} = \gamma_z^q + \widehat{y}_t^* - \widehat{y}_{t-1}^*, \\
R_t^{obs} = \pi^{ss} + r^{ss} + 4\gamma_z^q + 4\widehat{R}_t, & R_t^{*obs} = \pi^{ss} + r^{ss} + 4\gamma_z^q + 4\widehat{R}_t^*, \\
\pi_t^{obs} = \pi^{ss} + \widehat{\pi}_t, & \pi_t^{*obs} = \pi^{ss} + \widehat{\pi}_t^*, \\
\Delta\varepsilon_t^{obs} = \widehat{\varepsilon}_t - \widehat{\varepsilon}_{t-1} + \epsilon_{PPP,t}, &
\end{array}
\tag{19}
$$

where a variable with a hat indicates percentage deviations of that variable from its steady state, i.e. for any variable $X_t$, $\widehat{X}_t = \log(X_t/X)$. Parameter $\gamma_z^q$ denotes the quarterly growth rate of GDP in percent, so that $\gamma_z = 1 + \frac{\gamma_z^q}{100}$; parameters $\pi^{ss}$ and $r^{ss}$ denote the annualized steady state inflation rate and the real interest rate in percent, respectively, which, together with $\gamma_z^q$ determine the model's discount factor $\beta = \left(1 + \frac{r^{ss}}{400}\right)^{-1}$. Appendix A provides detailed information on all data sources and transformations.

## 2.2 DSGE parameters and estimations results

This section briefly discusses DSGE parameters for the different country-pairs we estimate our model on. In particular, we combine data on the Euro Area, UK, and Japan with US data. In each case, a country pair is assumed to make up the world economy, that is, a 'rest of the world' is not explicitly modeled.

A small subset of model parameters is not estimated, but calibrated. Country size, $n$, is computed from a country's share of real GDP at PPP in 'world' GDP over the 1970-2016 sample. That is, for the estimation of country pair US and Euro Area, $n = Y^{US}/(Y^{US} + Y^{EA}) = 0.56$. For the other country pairs, US-UK $n = 0.87$, and for US-Japan $n = 0.76$. The parameters related to the home bias preference parameter, the weight of the Home and the Foreign country on their own goods in their respective consumption baskets are calibrated as one minus the long-term averages of the import share, which is obtained as $Imp/(GDP - Exp + Imp)$, found to be 0.1034 and 0.1166 for the US economy and for the Euro Area, respectively.[7] The import shares for the other countries are equal to 0.26 for the UK and 0.11 for Japan. The elasticity of substitution between varieties is set to 6, implying that prices are set at a markup of 20% above marginal costs on average. Finally, the ratio of government expenditure to GDP is 0.2. All other model parameters are estimated.

[7]For the Euro Area, we considered Extra Euro Area imports and exports, since we are interested in the openness with respect to outside of the region.

Table B.1 in Appendix B.2 presents a summary table of prior distributions, prior means and standard estimations, and estimated posterior means together with the 16% and 84% HPD intervals for all country pairs.

For simplicity, we employ the same prior assumptions irrespective of the country pair. We do not comment in detail on the estimated parameter values, but only highlight a couple of key findings. Like Lubik and Schorfheide (2005) we find, for most country pairs, (extremely) low values of the trade elasticity, $\epsilon$; we do not find this surprising, as in a setting of internationally complete markets exchange rate volatility is a decreasing function of the trade elasticity (cf. Corsetti *et al.*, 2008; Rabitsch, 2012; 2016). Finding low estimated values of the trade elasticity thus can be interpreted as the model's attempt to explain as much exchange rate variation as possible endogenously in the model. Nonetheless, and again similar to the findings in Lubik and Schorfheide (2005), we find a large standard deviation of the PPP shock, reflecting the difficulty of theoretical two-country models in generating sizeable fluctuations in exchange rates, so that a large remainder of this volatility is captured by the exogenous PPP shock.[8] In contrast to Lubik and Schorfheide, we find quite sizable values of the standard deviations of the productivity shocks, $\sigma_A$ and $\sigma_A^*$. We explain this through the fact that we (needed to) abstract from a (common across countries) permanent productivity shock to keep the number of shocks equal to the number of observed time series, in which case a lot of variation is attributed to the two countries' temporary technology disturbances.[9] For most other parameters the estimation results are within a reasonable range.

### 2.3 DSGE-based impulse responses: building some intuition

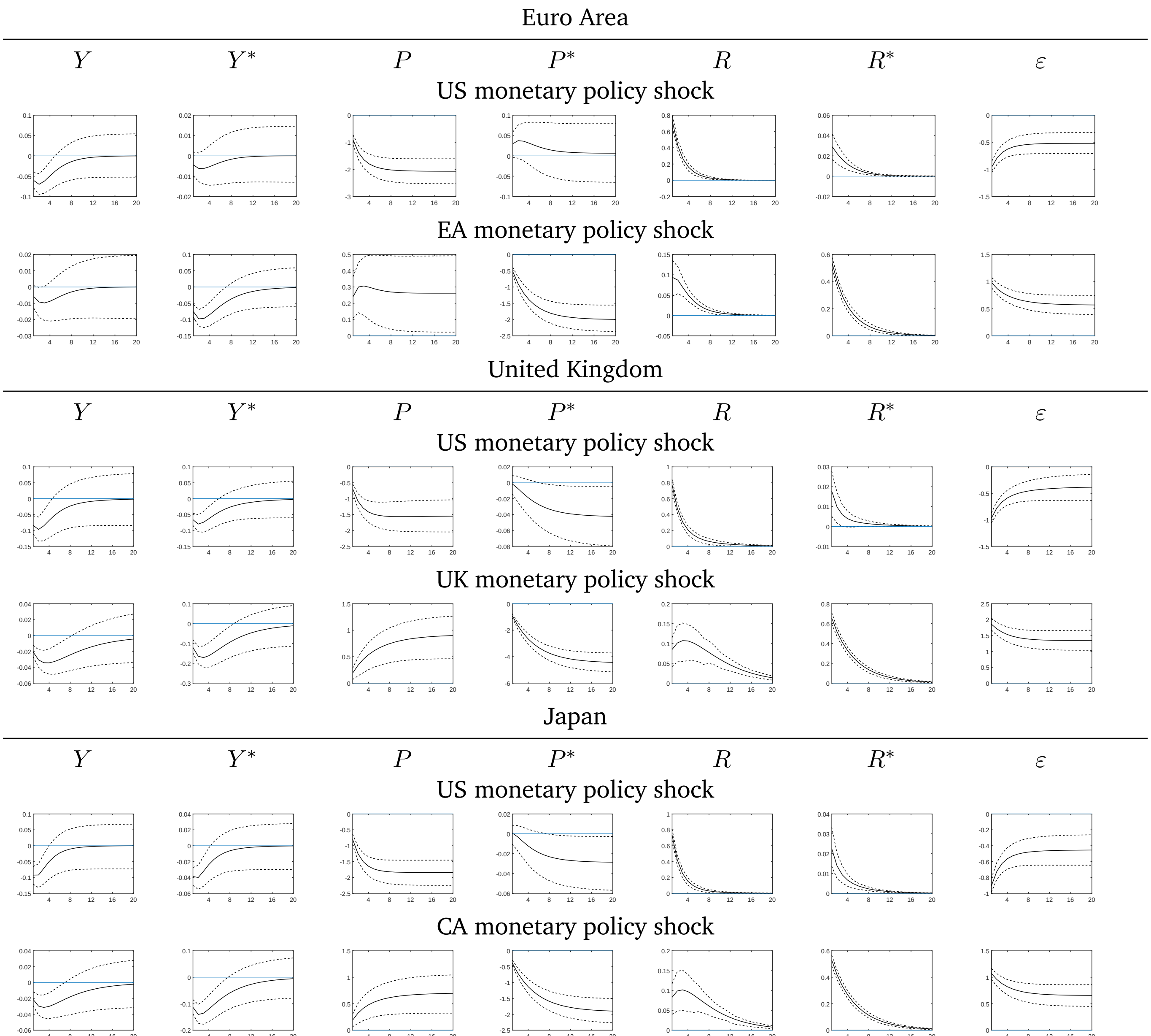
Figure 1 presents impulse responses to monetary policy shocks arising from the DSGE model across our three country pairs. For each country pair, the first row depicts the responses to a domestic (US) contractionary shock, while the second row depicts the responses to a contractionary monetary shock in the foreign country (Euro Area, United Kingdom or Japan, respectively).

To lay out the economic intuition, let us start by focusing on the US-based monetary policy shock in the US-Euro Area country pair, i.e. on row 1 of Figure 1. The responses of US macroeconomic variables ($Y$, $P$, $R$) follow the standard logic of New Keynesian macroeconomic models. The unanticipated monetary contraction raises the nominal interest rate, which, because of nominal rigidities translates into an increase of the real rate. The higher real interest rate incentivizes households to delay their consumption and increase their savings. In response to the resulting decrease

[8] As mentioned, we explored avenues that recent contributions emphasize can generate high exchange rate volatility (cf. Corsetti *et al.*, 2008; Rabitsch, 2012; 2016), such as an incomplete financial markets setting coupled with a low trade elasticity. Estimating (on, e.g. the US-EA country pair) a model version with incomplete financial markets we, however, obtained a similarly high value of $\sigma_{ppp}$, suggesting that exogenous factors of variations remain necessary to similar degrees. Since the behavior of the exchange rate under incomplete financial markets is known to switch sign at an asymptotic threshold value of the trade elasticity, the incomplete markets model version, unfortunately, also gives rise to a discontinuous model likelihood. We therefore prefer to stick to the complete markets case.

[9] In fact, in a version with the permanent productivity shock, estimated on US-Euro Area for the sample used in Lubik and Schorfheide (2005), we obtain virtually identical results.

**Fig. 1:** Impulse responses to monetary policy shocks in the US and the foreign economy from the estimated DSGE model: Euro area, UK, and Japan




**Notes**: This figure presents impulse responses at the posterior mean and the 16% and 84% interval for: US output ($Y$), foreign output ($Y^*$), US inflation ($\pi$), foreign inflation ($\pi^*$), US nominal interest rate ($R$), foreign nominal interest rate ($R^*$), and US-foreign nominal exchange rate, ($\varepsilon$).

in consumption demand, firms respond by lowering prices, but because of nominal rigidities only a fraction of firms is able to do so, implying that on average, prices stay too high.[10] Output thus is demand determined and drops as well.

Next, let us turn to discussing responses of macroeconomic variables in the foreign economy ($Y^*$, $P^*$, $R^*$), the Euro Area, together with the exchange rate response ($\varepsilon$). In response to the US monetary contraction, the exchange rate experiences an appreciation. Since exchange rate dynamics are at the core of this paper, we will discuss the mechanisms at work in driving the appreciation in detail below. For now, we focus on the effect that such exchange rate appreciation has on foreign macroeconomic variables. In particular, the response of output in the foreign country is a result of different forces at play. In a model version without nominal rigidities at the importer level ($\alpha_F = \alpha_H^* = 0$) our theoretical setup would correspond to a standard complete markets economy with producer currency pricing, such as widely studied in the literature (cf. Obstfeld and Rogoff, 2002; Corsetti and Pesenti, 2005; Corsetti *et al.*, 2010). In this case, the law of one price would hold and an exchange rate appreciation would be fully passed through into import prices at the consumer level, and would lead to expenditure switching effects, i.e., a switching of both home and foreign demand away from home goods toward foreign goods,[11] leading foreign output and inflation to expand. However, the assumption of a Calvo price setting mechanism of monopolistic importers implies that pass-through of exchange rate movements into import prices is complete only at the border level, but does not, generally, hold at the consumer level. Since the law of one price does no longer hold, expenditure switching effects are absent – in fact, both countries' demands for both home and foreign goods contract – $C_H$ and $C_H^*$ but also $C_F$ and $C_F^*$ fall (not reported in Figure 1). As a result, the output response abroad, $Y^*$, is slightly negative as well (panel 2, row 1 of Figure 1), so that the contraction of the US economy spills over to the foreign economy, and the contraction in Euro Area output also decreases the Euro Area price level, $P^*$ (panel 4, row 1 of Figure 1).

Exchange rate movements in our model are determined by the same forces emphasized originally in Dornbusch (1976). Long-run behavior is determined by purchasing parity, together with short-run variations being driven by uncovered interest rate parity. The latter condition holds because of the assumption of complete financial markets made in our model economy. More specifically, in the long run, once nominal rigidities play no role in affecting the macroeconomy and all prices have adjusted, the US monetary contraction leads to a lower US price level. The exchange rate in the long run thus appreciates in order to restore purchasing power parity. In the short run, as domestic interest rates exceed foreign rates after the home monetary contraction, investors expect

[10] Note that the price level and the level of the nominal exchange rate are not pinned down in the DSGE model, only their respective rates of change; Figure 1 nonetheless plots the impulse response in terms of level variables (by cumulating up impulse responses of $\pi_t$ or $\Delta\varepsilon_t$) as it is customary in the literature studying international transmission of monetary shocks and exchange rate behavior.

[11] That is, $C_H$ and $C_H^*$ would fall, and $C_F$ and $C_F^*$ would rise.

a depreciation, as dictated by the uncovered interest rate parity condition, $(R_t/R_t^*) = E_t\varepsilon_{t+1}/\varepsilon_t$.[12] Expectations of an exchange appreciation in the long-run are consistent with expectations of a depreciation in the short-run only if the exchange rate overshoots its long-run level of appreciation initially. This famous exchange rate overshooting result was originally suggested in Dornbusch (1976) as the first explanation of the large variability of exchange rates observed in the data.

The exchange rate behavior in response to monetary shocks has since been an extensive topic of research. Early influential empirical studies following Dornbusch, by Eichenbaum and Evans (1995) and Grilli and Roubini (1995), found evidence of exchange rate overshooting, however, not on impact of the shock, but appearing with a lag of several years after the shock, i.e. delayed overshooting. This finding was rationalized mostly through deviations in uncovered interest rate parity for which there is wide empirical evidence (literature reviews include the contributions by Froot and Thaler, 1990; Engel, 1996; Bacchetta, 2013).

The results on delayed overshooting were challenged by a number of authors (cf. Kim and Roubini, 2000a; Faust and Rogers, 2003a; Bjornland, 2009), partly because of the recursive identification assumptions of early contributions. However, evidence for delayed overshooting has been presented across various alternative specifications and identification assumptions since, such as, e.g. in Kim (2005); Scholl and Uhlig (2008); Bouakez and Normadin (2010)). In a more recent study Kim *et al.* (2017) document that the delayed overshooting result is a result primarily of the Volcker era and is no longer found in the more recent time period post-Volcker. They also provide evidence that in the post-Volcker era uncovered interest rate parity holds conditional on monetary shocks (while it continues to fail unconditionally). Finally, Müller *et al.* (2019), and similarly Schmitt-Grohé and Uribe (2018), find not only no evidence for a delay in overshooting, but actually a lack of overshooting entirely. In their empirical setups, the exchange rate appreciates in response to a US contractionary monetary shock, but the short-run degree of appreciation actually falls short of its long-run appreciation, i.e. they find evidence for 'exchange rate undershooting'.

Given the widely differing results on the precise dynamic responses of the exchange rate to a contractionary monetary shock – overshooting, delayed overshooting, or undershooting– our paper pursues the strategy of drawing inference based on a well grounded theoretical model. While our model –with complete markets and UIP holding perfectly– is stylized, generating overshooting dynamics along the lines suggested by Dornbusch (1976), we believe that it nonetheless provides a reasonable basis for forming a Bayesian prior for the empirical model. On the one hand, some recent evidence by, e.g. Kim *et al.* (2017) points against a delayed overshooting exchange rate response and towards UIP holding conditionally on monetary shocks. On the other hand, our prior, described in detail in subsection 3.2, is flexible enough to let the data speak and thus, if necessary, depart from the theoretically implied parameter restrictions.

[12] The uncovered interest rate parity condition can formally be derived by combining the foreign country's Euler condition with respect to the Home country's Arrow-Debreu security, with the foreign country's Euler condition with respect to its own Arrow-Debreu security.

## 3 Empirical framework

This section starts by discussing the proposed empirical model, a non-linear vector autoregression, in subsection 3.1 while subsection 3.2 discusses the Bayesian prior setup.

### 3.1 A non-linear vector autoregressive model

To investigate how macroeconomic shocks determine exchange rate volatility, we propose a novel macroeconometric framework that is similar in spirit to Carriero *et al.* (2018). In the previous section, we assumed that the latent states $\boldsymbol{z}_t$ in the underlying DSGE model drive a set of $m$ observed quantities in $\boldsymbol{y}_t$. Here, we assume that $\boldsymbol{y}_t$ follows a VAR process of order $p$. In what follows, we assume that the exchange rate equation is ordered last, implying that $y_{mt} = \Delta\varepsilon_t^{obs}$.

Before describing how our VAR model is defined, we introduce a law of motion that links lags of $\boldsymbol{y}_t$ with the logarithm of exchange rate volatility, defined as $h_t$. Specifically, $h_t$ follows,

$$h_t = \phi_1 h_{t-1} + \cdots + \phi_p h_{t-p} + \boldsymbol{\beta}_1' \boldsymbol{y}_{t-1} + \ldots, + \boldsymbol{\beta}_p' \boldsymbol{y}_{t-p} + v_t, \tag{20}$$

whereby $\phi_j$ $(j = 1, \ldots, p)$ are autoregressive parameters associated with lags of $h_t$ while $\boldsymbol{\beta}_j$ denote $m$-dimensional vectors of regression coefficients that link $h_t$ to the lags of $\boldsymbol{y}_t$. The shocks $v_t$ are normally distributed with zero mean and variance $\sigma_h^2$.

Consistent with the structural model outlined in Section 2, the reduced-form representation of the VAR model for $\boldsymbol{y}_t$ is given by,

$$\boldsymbol{y}_t = \boldsymbol{A}_1 \boldsymbol{y}_{t-1} + \cdots + \boldsymbol{A}_p \boldsymbol{y}_{t-p} + \boldsymbol{\gamma}_1 h_{t-1} + \ldots \boldsymbol{\gamma}_p h_{t-p} + \boldsymbol{\eta}_t. \tag{21}$$

Here, we let $\boldsymbol{A}_j$ $(j = 1, \ldots, p)$ denote $m \times m$ matrices of VAR coefficients associated with lagged values of $\boldsymbol{y}_t$ and $\boldsymbol{\gamma}_j$ are $m$-dimensional coefficient vectors that relate the lags of $h_t$ to $\boldsymbol{y}_t$. Moreover, we assume that $\boldsymbol{\eta}_t \sim \mathcal{N}(\mathbf{0}_m, \boldsymbol{\Sigma}_t)$ is a vector white noise process with $\boldsymbol{\Sigma}_t$ being a time-varying variance-covariance matrix that can be decomposed as

$$\boldsymbol{\Sigma}_t = \boldsymbol{U}\boldsymbol{\Omega}_t\boldsymbol{U}', \tag{22}$$

with $\boldsymbol{U}$ denoting a lower triangular matrix with diag$(\boldsymbol{U}) = \boldsymbol{\iota}_m$ and $\iota_m$ being an $m$-dimensional vector of ones while $\boldsymbol{\Omega}_t = \text{diag}(e^{\omega_{1t}}, \ldots, e^{\omega_{mt}})$ is a diagonal matrix that varies over time.

The $\omega_{jt}$'s obey the following law of motion,

$$\omega_{jt} = \begin{cases} \mu_j + \rho_j(\omega_{jt-1} - \mu_j) + u_{jt} \text{ for } j = 1, \ldots, m-1, \\ h_t \text{ if } j = m, \end{cases} \tag{23}$$

with $\mu_j$ $(j = 1, \ldots, m-1)$ denoting the long-run unconditional mean, $\rho_j$ the autoregressive parameter (with $|\rho_j| < 1$), and $u_{jt} \sim \mathcal{N}(0, \sigma_u^2)$ is a white noise shock with innovation variance $\sigma_u^2$.

Equation (23) implies that for the elements in $\boldsymbol{y}_t$ other than the exchange rate equation, the error variances feature a standard stochastic volatility specification. For the exchange rate equation, we assume that the volatility evolves according to the law of motion described in Eq. (20).

One important feature of our proposed model is that Eq. (20) can be viewed as an additional equation in the VAR model. In order to allow for contemporaneous relations across the observed quantities in $\boldsymbol{y}_t$ and the log-volatility of the exchange rate $h_t$, we assume that the shocks to the exchange rate volatility equation and the reduced form innovations in the VAR are correlated. More specifically, this implies that

$$\underbrace{\begin{pmatrix} v_t \\ \boldsymbol{\eta}_t \end{pmatrix}}_{\boldsymbol{w}_t} \sim \mathcal{N}(\mathbf{0}_{m+1}, \Xi_t), \tag{24}$$

$$\Xi_t = \begin{pmatrix} \sigma_h^2 & \boldsymbol{\xi}' \\ \boldsymbol{\xi} & \boldsymbol{\Sigma}_t \end{pmatrix}, \tag{25}$$

where $\mathbf{0}_{m+1}$ denotes an $(m+1)$-dimensional vector of zeros, $\Xi_t = \boldsymbol{Q}\boldsymbol{R}_t\boldsymbol{Q}'$ represents a time-varying variance-covariance matrix and $\boldsymbol{\xi}$ is a $m$-dimensional vector that stores the covariances between $v_t$ and $\boldsymbol{\eta}_t$. The lower uni-triangular matrix $\boldsymbol{Q}$ is the lower Cholesky factor of $\Xi_t$ and $\boldsymbol{R}_t$ is a diagonal matrix. Multiplication of $\boldsymbol{w}_t$, the vector of reduced form shocks, with $\boldsymbol{Q}^{-1}$ from the left yields a set of uncorrelated shocks $\tilde{\boldsymbol{w}}_t \sim \mathcal{N}(0, \boldsymbol{R}_t)$.

The assumption of correlated shocks implies that our stochastic volatility specification for the exchange rate closely resembles a multivariate stochastic volatility model with leverage (Yu, 2005; Omori *et al.*, 2007). Notice, however, that in contrast to standard stochastic volatility models with leverage, the proposed specification also assumes that lagged values of $h_t$ enter the observation equations across the system.

## 3.2 Prior setup

The model described in subsection 3.1 is heavily parameterized and the involved time series are quite short. This calls for Bayesian techniques to carry out estimation and inference. Conducting a Bayesian analysis requires the specification of suitable priors on the parameters of the model. This joint prior distribution is then combined with the likelihood function to obtain a joint posterior distribution over the parameters and latent quantities in the model. Before proceeding to the specific prior setup, it turns out to be convenient to rewrite the empirical model as follows:

$$\boldsymbol{z}_t = \boldsymbol{B}\boldsymbol{d}_t + \boldsymbol{w}_t, \tag{26}$$

where $\boldsymbol{z}_t = (h_t, \boldsymbol{y}_t')'$, $\boldsymbol{B} = (\boldsymbol{\beta}, \boldsymbol{A})'$, $\boldsymbol{\beta} = (\boldsymbol{\beta}_1', \dots, \boldsymbol{\beta}_p', \phi_1, \dots, \phi_p)'$, $\boldsymbol{A} = (\boldsymbol{A}_1, \dots, \boldsymbol{A}_p, \boldsymbol{\gamma}, \dots, \boldsymbol{\gamma}_p)'$ and $\boldsymbol{d}_t = (\boldsymbol{y}_{t-1}', \dots, \boldsymbol{y}_{t-p}', h_{t-1}, \dots, h_{t-p})'$.

Using theoretical models as prior distributions for VAR models dates at least back to the seminal contributions of Ingram and Whiteman (1994) and Del Negro and Schorfheide (2004). The key advantage of this approach is that information arising from some structural model can easily be combined with data information to improve estimation accuracy and softly introduce theoretical restrictions on the parameter space. In its original form, the DSGE-prior of Del Negro and Schorfheide (2004) is conjugate, implying that the functional form of the prior and likelihood are identical, translating into a closed-form joint posterior distribution that also features the same distributional form. One key disadvantage, however, is that using DSGE models to form a conjugate prior can be potentially restrictive because the likelihood function of the VAR model features a Kronecker structure that speeds up computation but also implies that shrinkage towards the theoretically inspired restrictions. To ease this restriction, De Luigi and Huber (2018) propose using only the DSGE-implied prior mean while estimating the prior variance-covariance matrix under a suitable shrinkage prior. In this paper, we adopt a similar approach that is based on using a hierarchical modeling strategy.

The prior distribution on each of the $k(= (m+1)^2 p)$ elements of $\boldsymbol{b} = \text{vec}(\boldsymbol{B})$, denoted by $b_j$ $(j = 1, \ldots, k)$, follows a conditionally Gaussian distribution,

$$b_j|\delta_j \sim \mathcal{N}(b_{0j}, \tau_j), \tag{27}$$

where $b_{0j}$ denotes the $j$th element of a prior mean vector $\boldsymbol{b}_0 = \text{vec}(\boldsymbol{B}_0)$ and $\tau_j$ is a prior scaling parameter. Due to the fact that the DSGE model does not feature a separate equation for exchange rate volatility, we need to construct the prior mean matrix $\boldsymbol{B}_0$ that centers the coefficients in Eq. (23) on zero. The $K \times (m+1)$-dimensional matrix $\boldsymbol{B}_0$ (with $K = (m+1)p$) is constructed as follows:

$$\boldsymbol{B}_0 = \begin{pmatrix} 0 & \mathbf{0}_m' & 0 & \ldots & \mathbf{0}_m' & 0 \\ \mathbf{0}_m & \boldsymbol{\Psi}_1 & \mathbf{0}_m & \ldots & \boldsymbol{\Psi}_p & \mathbf{0}_m \end{pmatrix}' \tag{28}$$

with $\mathbf{0}_m$ denoting an $m$-dimensional vector of zeros, $\boldsymbol{\Psi}_j$ selects the coefficients associated with the $j$th lag from the DSGE-implied prior mean matrix $\boldsymbol{\Psi}$.

The prior scaling parameter $\tau_j$ plays a crucial role since it determines the weight placed on the DSGE-implied moments embodied in $b_{0j}$. In what follows, we estimate this weight by assuming that $\tau_j$ depends on a set of binary indicators $\delta_j$ such that

$$\tau_j = s_{0j}\delta_j + s_{1j}(1 - \delta_j), \tag{29}$$

with $s_{0j}$ being close to zero and $s_{1j}$ taking a large positive value such that $s_{0j} \ll s_{1j}$. Hence, if $\delta_j = 1$, we strongly push the posterior estimates $a_j$ towards the DSGE-implied prior mean while in the case that $\delta_j = 0$, we place little weight on the structural model. Since the parameters $s_{0j}$ and $s_{1j}$ play a crucial role and depend on the scaling of the data, we follow Ishwaran and Rao (2005) and assume that $s_{0j} = \zeta s_{1j}$, with $\zeta = 1/10^4$ being a known constant close to zero. On $s_{1j}^{-1}$, we place a Gamma prior,

$$s_{1j}^{-1} \sim \mathcal{G}(d_0, d_1). \tag{30}$$

Here, we let $d_0$ and $d_1$ denote hyperparameters that we specify to be only weakly informative. More specifically, we set $d_0 = d_1 = 0.01$, translating into a prior mean of $1$ and a prior variance of $0.01/0.01^2$.

To complete the prior on the VAR coefficients, we need to specify the prior restriction probability that a given estimate is forced towards the DSGE model, i.e. the probability that $d_j = 1$. One potential option would be to set $Prob(\delta_j = 1) = 1/2$. However, here we follow a different approach and assume that $Prob(\delta_j = 1) = p^{(n)}$, where $p^{(n)}$ denotes the restriction probability related to the $n$th equation in the system. Hence, if the $j$th element of $\boldsymbol{a}$ is associated with equation $n$, a different restriction probability is adopted, effectively allowing for different restriction patterns across the system of equations. Since prior information on $p^{(n)}$ is generally not available, we use a Beta distributed prior, $p_j^n \sim \mathcal{B}(1, 1)$, and consequently infer $p^{(n)}$ from the data. This captures the notion that, a priori, all restrictions are equally likely.

The priors on the remaining quantities are standard. On the elements in $\boldsymbol{\xi}$, we use Gaussian priors centered on zero with prior variance $10$. The same prior is adopted on the unconditional mean of the log-volatility $\mu_j$ $(j = 1, \ldots, m-1)$. On $\rho_j$, we use a Beta prior $\frac{\rho_j+1}{2} \sim \mathcal{B}(25, 5)$ while $\sigma_u^2$ arises from a Gamma prior, $\sigma_u^2 \sim \mathcal{G}(1/2, 1/2)$. Finally, we use an inverted Gamma prior on $\sigma_h^2 \sim \mathcal{G}^{-1}(0.01, 0.01)$.

Inference is carried out using the algorithm discussed in Appendix C.1. Here, it suffices to note that we repeat this algorithm 30,000 times and discard the first 15,000 draws as burn-in. Apart from the draws of $h_t$, the remaining parameters and states mix quite well, with inefficiency factors below 15 in most cases. For $h_t$, inefficiency factors are quite high but still acceptable.

# 4 Empirical results

This section starts by briefly summarizing identification issues. We then discuss selected reduced-form results of our model in subsection 4.2 before presenting the structural responses of macroeconomic quantities other than the exchange rate to monetary policy shocks in subsection 4.3. In subsection 4.4, we discuss the responses of the exchange rate and its volatility to monetary policy shocks while subsection 4.5 shows historical and forecast error variance decompositions, respectively.

## 4.1 Identification

The reduced-form nature of the model discussed in section 3 implies that structural inference can only be carried out after identifying the structural shocks of interest. The empirical literature on the effects of monetary policy on exchange rates typically exploits identification strategies based on zero (cf. Eichenbaum and Evans, 1995; Grilli and Roubini, 1995)) or sign (cf. Faust and Rogers, 2003a; Scholl and Uhlig, 2008; Bouakez and Normadin, 2010; Kim *et al.*, 2017)) restrictions. Most of these studies, however, find several empirical puzzles like the delayed exchange rate overshooting puzzle. As a potential reason for this, the literature identifies either omitted variables (Sims, 1992a) or identification issues. In both cases, the resulting impulse responses will be severely biased and thus misleading. In this paper, we adopt a novel approach outlined in Del Negro and Schorfheide (2004) and utilize the DSGE model to recover the structural representation of the empirical model.

The main identification issues stem from the relationship between the reduced-form and structural shocks given by:

$$\boldsymbol{w}_t = \boldsymbol{Q}\tilde{\boldsymbol{\Omega}}\boldsymbol{v}_t, \quad \boldsymbol{v}_t \sim \mathcal{N}(0, \boldsymbol{R}_t). \tag{31}$$

Hereby, $\tilde{\boldsymbol{\Omega}}$ denotes an $(m+1) \times (m+1)$-dimensional orthonormal matrix and $\boldsymbol{v}_t$ are the structural shocks. The identification issues stems from the fact that the reduced-form covariance matrix $\boldsymbol{\Xi}_t$ can be decomposed as $\boldsymbol{\Xi}_t = \boldsymbol{Q}\tilde{\boldsymbol{\Omega}}\tilde{\boldsymbol{\Omega}}'\boldsymbol{Q}'$ for any rotation matrix $\tilde{\boldsymbol{\Omega}}$. Notice that if $\tilde{\boldsymbol{\Omega}} = \boldsymbol{I}_{m+1}$, the model is identified and we obtain a standard Cholesky-type identification scheme. In what follows, we reorder the elements in $\boldsymbol{z}_t$ as follows:

$$\boldsymbol{z}_t = (\Delta Y_t^{obs}, \Delta Y_t^{*obs}, h_t, R_t^{obs}, R_t^{*obs}, \pi_t^{obs}, \pi_t^{*obs}, \Delta\varepsilon_t^{obs})'.$$

In general, if sign restrictions are adopted, the resulting impulse responses will be invariant with respect to reordering the elements in $\boldsymbol{z}_t$. Our identification strategy is only partially order invariant since $h_t$ is not included in the DSGE model. Hence, we need to introduce additional restrictions to fully identify the model.

Following Del Negro and Schorfheide (2004), the impact responses of the observed quantities to the structural disturbances in the DSGE model are given by

$$\left(\frac{\partial \boldsymbol{y}_t}{\partial \boldsymbol{v}_t'}\right)_{\text{DSGE}} = \boldsymbol{Q}^*\boldsymbol{\Omega}^*, \tag{32}$$

where $\boldsymbol{Q}^*$ is a $m \times m$-dimensional lower triangular matrix and $\boldsymbol{\Omega}^*$ denotes an orthonormal matrix, also of dimension $m \times m$. Del Negro and Schorfheide (2004) suggest using $\boldsymbol{\Omega}^*$ to identify the reduced-form VAR model. However, since our DSGE model features no separate equation for $h_t$, we need to modify $\boldsymbol{\Omega}^*$ accordingly. This is achieved by setting

$$\tilde{\boldsymbol{\Omega}} = \begin{pmatrix} \boldsymbol{Q}^* & \boldsymbol{0}_m \\ \boldsymbol{0}_m' & 1 \end{pmatrix}. \tag{33}$$

The main implication is that monetary policy shocks are identified by using information arising from the structural model while the shock to $h_t$ is identified recursively. This recursive identification implies that $h_t$ reacts contemporaneously to all shocks in the system while output growth at home and abroad reacts sluggishly with respect to a shock to exchange rate volatility. However, since we are exclusively interested in the effects of domestic and international monetary policy shocks, this additional restriction plays only a minor role in the subsequent discussion. Using this DSGE-based identification scheme is novel in the literature that deals with the nexus between monetary policy shocks and exchange rate fluctuations. One key advantage is that, as opposed to sign restrictions, this identification strategy yields a rotation matrix $\tilde{\mathbf{\Omega}}$ that is deterministic (conditional on an estimated DSGE model), translating into a point identified model.

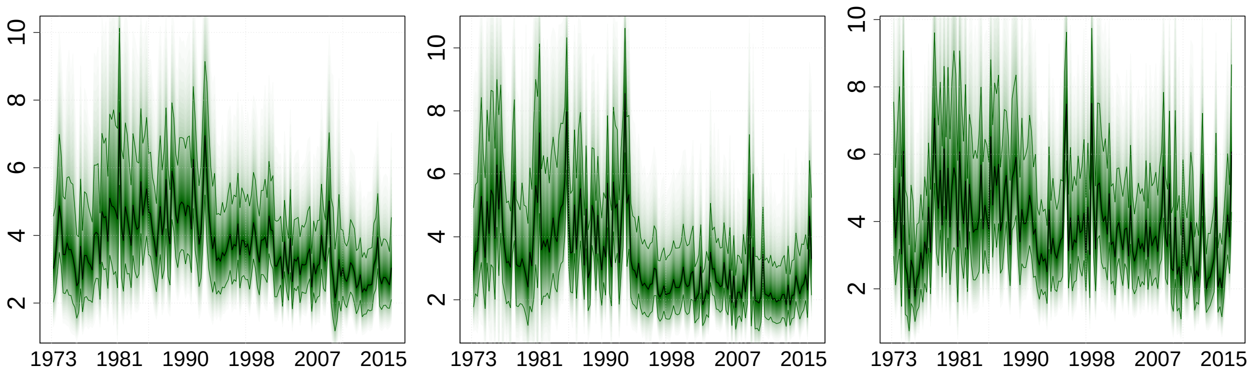
## 4.2 Reduced-form results

In this section, we briefly discuss selected reduced-form model features. Figure 2 shows the posterior distribution of $h_t$ across the three country pairs considered. The solid light green lines refer to the 16th and 84th percentiles of the posterior distribution, respectively. Moreover, the solid dark green line denotes the posterior median of $h_t$.

The figure indicates that the different estimates of $h_t$ display some similarities (especially so for the Euro and the Pound). This suggests that the volatility of exchange rates vis-á-vis the US dollar exhibit a pronounced degree of comovement. This finding is consistent with the literature (see, among many others, Diebold and Nerlove, 1989; Aguilar and West, 2000; Engel *et al.*, 2015) that reports strong comovements across the change and the volatility of several exchange rates against the dollar. Across all country pairs under consideration, we observe that exchange rate volatility peaks during the Volcker disinflation, the first gulf war and the crisis of the European exchange rate mechanism (ERM) in the beginning of the 1990s and in the global financial crisis in 2008/2009. Notice that we observe an appreciable increase in the volatility of the Pound surrounding the Brexit referendum towards the end of the estimation sample. One interesting pattern is the secular decline in the overall level of exchange rate volatility for the Euro and the Pound that started in the beginning of the 1990s.

The next model feature we consider is the posterior distribution of the equation-specific weights $p^{(n)}$. These weights illustrate how much weight is placed on the DSGE model, with values of $p^{(n)}$ close to unity indicating that a lot of mass is placed on the DSGE-implied restrictions associated with equation $n$'s coefficients. We find only little differences across country pairs, indicating that there seems to be no systematic difference in the way the structural model fits the coefficients for the selected economies. Considering differences across equations shows that, according to the posterior median of $p^{(n)}$, the exchange rate equation receives the largest overall weight of around 0.4. This implies that on average, the theoretical moments of the structural model are introduced with a probability of 40 percent across all coefficients. Equation (A.7), however, shows that $p^{(n)}$ determines the variable-specific restriction probabilities, suggesting that there might be important idiosyncratic deviations from this pattern. For the exchange rate equation, the corresponding DSGE-implied moments translate into a prior mean that is sparse, centering most coefficients around zero. The two exceptions are the coefficients associated with lagged interest rates at home and abroad. For these two covariates, the algorithm strongly centers the final estimates around the

**Fig. 2:** Posterior distribution of exchange rate volatility




**Notes:** The figure shows the posterior distribution of $e^{h_t/2}$ across the three exchange rate pairs considered. The dark green line represents the posterior median while the light green lines denote the 16th/84th percentiles of the posterior distribution.

prior moments whereas for the remaining covariates, we allow for non-zero regression coefficients. For the remaining equations, we find weights that range from approximately 20 (in the case of $R_t$) to 35 percent (in the case of $h_t$, $\Delta P_t$ and $\Delta P_t^*$).

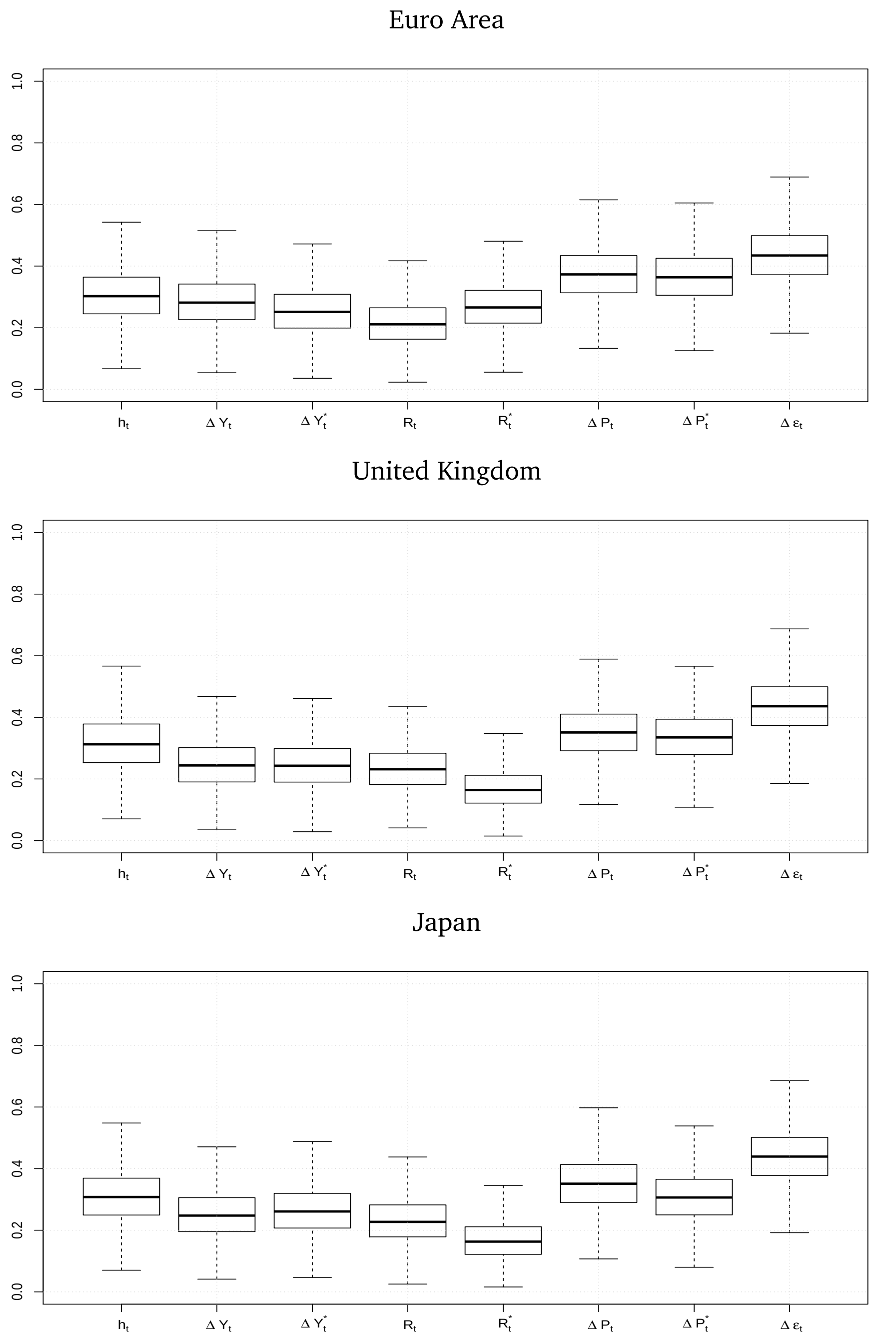
### 4.3 Dynamic responses of the macroeconomic quantities to monetary policy shocks

Before we discuss the responses of the exchange rate and its volatility, we assess how well the reactions of the other macroeconomic quantities compare with existing results reported in the literature (see, for example, Eichenbaum and Evans, 1995; Cushman and Zha, 1997; Kim and Roubini, 2000b; Faust and Rogers, 2003b; Scholl and Uhlig, 2008).

To this end, Fig. 4 shows the impulse responses to a one standard deviation monetary policy shock in the US and in the foreign country (i.e. the Euro Area, the United Kingdom, and Japan). The responses arising from our empirical model can be considered a weighted average of the reactions from an otherwise standard VAR model with stochastic volatility and the DSGE model used to a.) inform the estimates of the VAR coefficients and b.) identify the empirical model. At a first glance, the reactions show similarities with the ones obtained from the theoretical model, displaying some variation across countries.

The reactions of real activity indicate that for all countries under consideration, US output declines to both, a US-based and foreign monetary tightening. By contrast, we observe that foreign output reactions appear to be much weaker and significant posterior mass is located above zero. This is at odds with the responses from the estimated structural model, where output abroad declines as well. However, as the discussion in subsection 2.3 suggested, an increase in foreign output in response to a domestic monetary contraction is not at odds with a theoretical model where devi-

**Fig. 3:** Posterior distribution of equation-specific weights $p^{(n)}$




**Notes:** The figure shows boxplots of the posterior distribution of the equation-specific weights across the three country pairs considered.

ations from the law of one price are zero and where expenditure-switching effects would increase demand for the foreign good.

Across all countries and both types of monetary policy shocks considered, we observe that prices persistently decline in reaction to a monetary tightening. This indicates that our model and identification approach succeeds in avoiding the well-known price puzzle (Sims, 1992b) that is associated with inappropriate identification strategies in combination with small information sets.

Finally, considering the reactions of US short-term interest rates indicates that a US monetary policy shock increases interest rates by around 0.25 percentage points on impact, with a peak reaction of around 0.4 percentage points after around five quarters. This hump-shaped reaction is interesting since it implies that interest rate reactions increase in magnitude over the first few quarters. Considering the international reaction reveals some differences across country pairs. For the Euro Area, the short-term interest rate tends to comove with US short-term interest rates. In reaction to increases in US money market rates, interest rates in the Euro Area also tend to increase with a slight lag. This finding, however, does not carry over to a foreign monetary policy shock. For the case of the UK and Japan, the increase in US interest rates does not lead to spillovers abroad since interest rate responses in the UK and Japan remain insignificant.

### 4.4 Dynamic responses of the exchange rates to monetary policy shocks

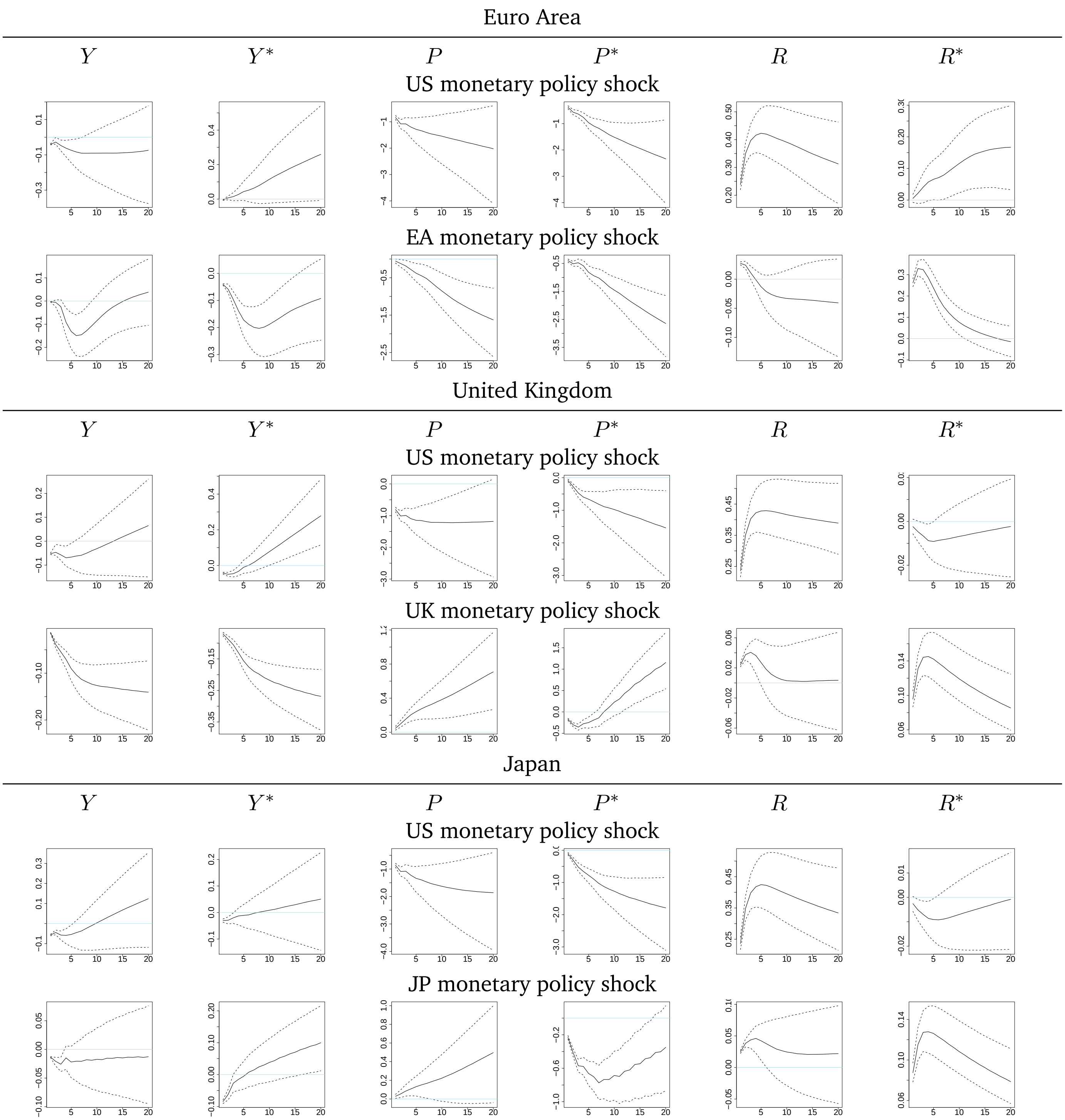
In this section, we focus on how monetary policy shocks impact the exchange rate and its volatility. The upper part of Fig. 5 shows the reaction of the three currency pairs to a US-based monetary policy shock while the lower part presents the reactions to a foreign monetary policy shock.

Starting with the upper panel of Fig. 5, we observe that across all country pairs, the US dollar immediately appreciates. This is consistent with the reactions predicted by the DSGE model. Notice, however, that, for all country pairs, the DSGE model yields exchange rate responses in line with Dornbusch's overshooting hypothesis, so that the degree of appreciation on impact is more pronounced than the long-run degree of appreciation to which the exchange rate depreciates to monotonically.

The exchange rate reactions of the empirical model are more complex, and also differ somewhat across country pairs. For the country pairs of US-Euro Area and the US-UK, the exchange rate responses in the first few quarters appear to be consistent with the overshooting behavior of the theoretical model, in that the degree of appreciation is high initially (in the first two quarters) and then starts to decrease. In particular, the dollar appreciates by around 0.6 percent for the United Kingdom and one percent for the Euro area within the first two quarters after the shock hit the system.

After four to five quarters, this effect becomes smaller for the United Kingdom and the Euro Area, reaching median values of 0.5 and 0.9 percent, respectively. Medium-run reactions appear to be statistically insignificant. However, when we consider the median reactions, the results suggest that exchange rate reactions become quantitatively stronger again over time. This is especially pronounced for Japan, where the Yen persistently depreciates vis-á-vis the US dollar in a statistically significant manner. This at first sounds reminiscent of the delayed overshooting result (cf. Eichen-

**Fig. 4:** Impulse responses to monetary policy shocks in the US and the foreign economy: Euro area, United Kingdom and Japan




**Notes**: This figure presents the posterior distribution of impulse responses alongside the 16th and 84th percentiles(in dashed black) and the median response (in solid black). The columns represent the responses of the macroeconomic quantities (excluding the exchange rate) to monetary policy shock in the US and the foreign country.

baum and Evans, 1995; Grilli and Roubini, 1995)) in that also in this case the degree of exchange rate appreciation in response to the monetary contraction is less pronounced on impact and grows stronger in the medium run. However, unlike in the contributions on delayed overshooting, the exchange rate remains at this high degree of appreciation also in the long run.[13]

Our novel empirical setup and identification approach thus finds that the short-run level of appreciation undershoots its long-run level. These findings are consistent with some more recent contributions. Müller *et al.* (2019), using local projections, also find evidence that exchange rates tend to undershoot. Schmitt-Grohé and Uribe (2018) estimate a state-space model with temporary and permanent monetary policy shocks, – they also find that, in response to both types of shocks, the exchange rate undershoots. Concerning the short-run reactions, the immediate reactions of our model are consistent with other studies (see, among others, Cushman and Zha, 1997; Zettelmeyer, 2004; Bjørnland, 2009).

Next, we consider the reactions of exchange rate volatility to US monetary policy shocks (see the second row of the upper panel in Fig. 5). In principle, the effects of monetary policy (either expansionary or restrictive) on exchange rate variability is ambiguous. On the one hand, appropriate monetary policy measures in reaction to asymmetric business cycle shocks that are accompanied by elevated levels of exchange rate uncertainty could help calming the markets, reducing exchange rate volatility. On the other hand, unexpected movements in interest rates increase, under the assumption that UIP holds, the variability of the exchange rate.

Considering the volatility responses to US monetary policy shocks reveals that immediate reactions of exchange rate volatility tend to be insignificant, with significant posterior mass located below zero across the three currencies considered. For the EUR/USD exchange rate, we observe that exchange rate volatility increases after around ten quarters. For GBP/USD, the reaction of exchange rate volatility is insignificant throughout the impulse response horizon. For the JPY/USD exchange rate, we find that volatility ticks up after around six quarters. Longer-run reactions for this exchange rate pair appear to be persistent and remain significant for around 20 quarters.

These results indicate that it takes some time before volatility reacts in a statistically significant manner. This rather slow reaction of $h_t$ appears to be inconsistent with the recent literature on the relationship between monetary policy and macroeconomic volatility (Mumtaz and Theodoridis, 2019).

The exchange rate reactions with respect to foreign shocks are described in the lower panel of Fig. 5. In response to a foreign monetary policy tightening, the US dollar depreciates across all three country pairs considered, as expected. For two of our country pairs, US-Euro Area and US-Japan, we obtain similar findings regarding the exchange rate reactions in the short-run versus in the long-run as before. Specifically, we observe that the dollar depreciates against the Euro (Yen) on impact, but the degree of dollar depreciation in early periods after the shock undershoots its long-run degree of depreciation. By contrast, the dollar depreciation in response to a UK monetary

[13]While the impulse responses of Figure 5 only report 20 periods, the exchange rate remains at this level of appreciation or continues to appreciate also, e.g., 60 periods after the shock or in the long run.

policy contraction is strongest in the early periods after the shock, in line with the overshooting hypothesis. Our findings thus suggest that the short-run effects of foreign monetary policy shocks are similar across all three country pairs, in the sense that they all amount to a depreciation of the dollar, and that short-run responses differ only quantitatively – with respect to the Euro, the Pound and Yen the dollar depreciates by around 0.8, 0.5 and 0.9 percent on impact, respectively. However, the long-run effects differ strongly. After around 20 quarters, the Euro and Yen appreciate by around 1.5 and 2.5 percent vis-á-vis the dollar, respectively, whereas the GBP/USD exchange rate has reverted back to the level prior to the UK-based monetary contraction.

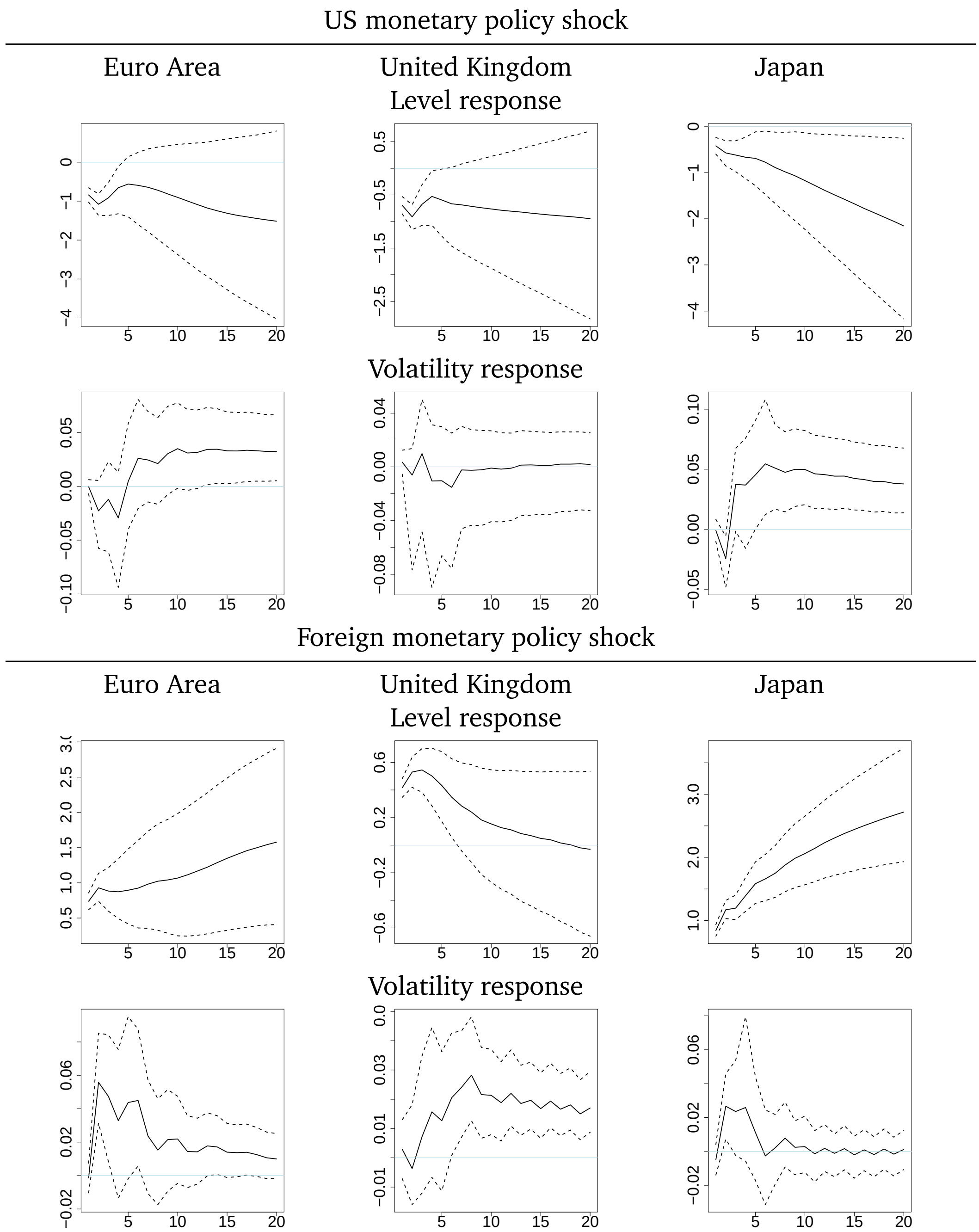
Volatility reactions, again, indicate that exchange rate volatility increases to a non-US monetary policy shock. Compared to the case of the US-based shock, we observe that reactions are somewhat faster, with the volatility of EUR/USD and JPY/USD increasing after around one quarter, but being short-lived. For these two currency pairs, we find that the volatility reaction is only significant within the first year and tends to fade out afterwards. This does not carry over to the case of the Pound. Here we find that volatility increases after six quarters and remains significant over the impulse response horizon.

### 4.5 Explaining exchange rate movements

Next, we assess whether monetary policy shocks determine exchange rate movements. This is achieved by considering forecast error variance decompositions (FEVDs) and historical decompositions (HDs). The FEVDs, shown in Fig. 6, tell a story about the quantitative relevance of US and non-US-based shocks for explaining exchange rate forecast errors. By contrast, we show the posterior mean of the HDs to investigate the role of monetary shocks in explaining exchange rate behavior over time.

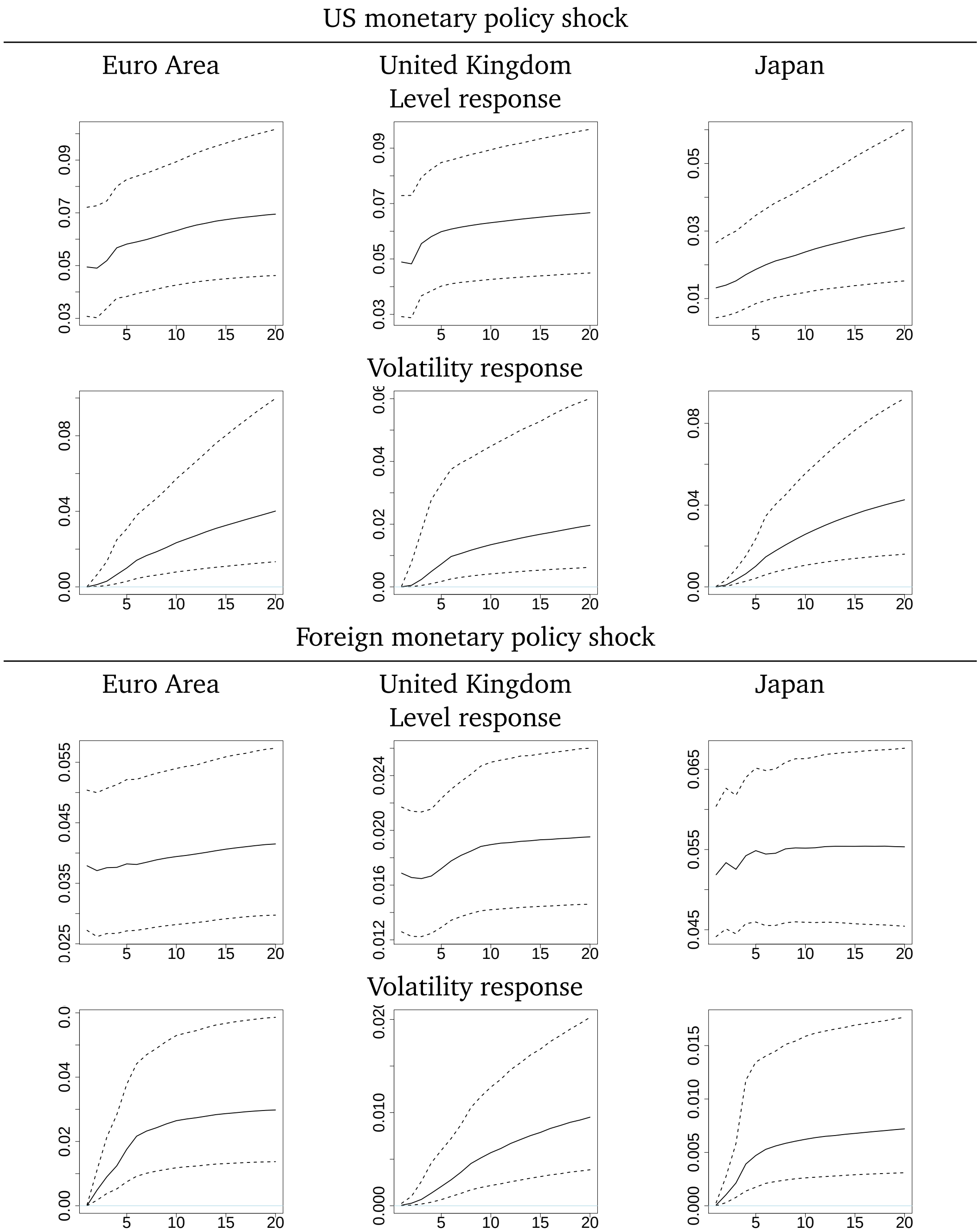
Considering the upper panel of Fig. 6 reveals that US monetary policy shocks explain around five percent of the forecast error variance on impact and in the case of the Euro Area and the United Kingdom while being markedly lower in the case of the Yen. Across all three country pairs, we observe that these shares rise slightly over time, increasing by around one percentage point on average. Inspection of the FEVDs of exchange rate volatility suggests that US monetary shocks explain almost no variation in the short run. This share, however, increases to around four percent after 20 quarters. The lower panel of Fig. 6 shows the FEVDs associated with the foreign monetary policy shock. Similarly to the US monetary policy shock, we find some similarities across countries in terms of the shape of the FEVDs. For all country pairs, the share of forecast error explained by a foreign monetary policy shock increases slightly. In terms of magnitudes, differences in magnitudes across countries are visible. In the case of the United Kingdom, the findings point towards a minor role of non-US monetary shocks while this share is considerably higher for the Yen (reaching around 5 percent in the long run). Inspection of the volatility FEVDs shows that for the United Kingdom and Japan, non-US shocks account for only 0.5 to one percent of the forecast error variance in the longer run. In the Euro Area, this share is slightly higher and reaches around two percent after

**Fig. 5:** Exchange rate responses (level and volatility) to monetary policy shocks in the US and the foreign economy: Euro area, United Kingdom and Japan




**Notes**: This figure presents the posterior distribution of impulse responses alongside the 16th and 84th percentiles(in dashed black) and the median response (in solid black).

**Fig. 6:** Forecast error variance decompositions for the exchange rate in the US and the foreign economy: Contribution of monetary policy shocks




**Notes**: This figure presents the posterior distribution of forecast error variance decompositions alongside the 16th and 84th percentiles (in dashed black) and the median (in solid black).

20 months. Summing up, this discussion shows that, consistent with the literature (see, e.g., Kim

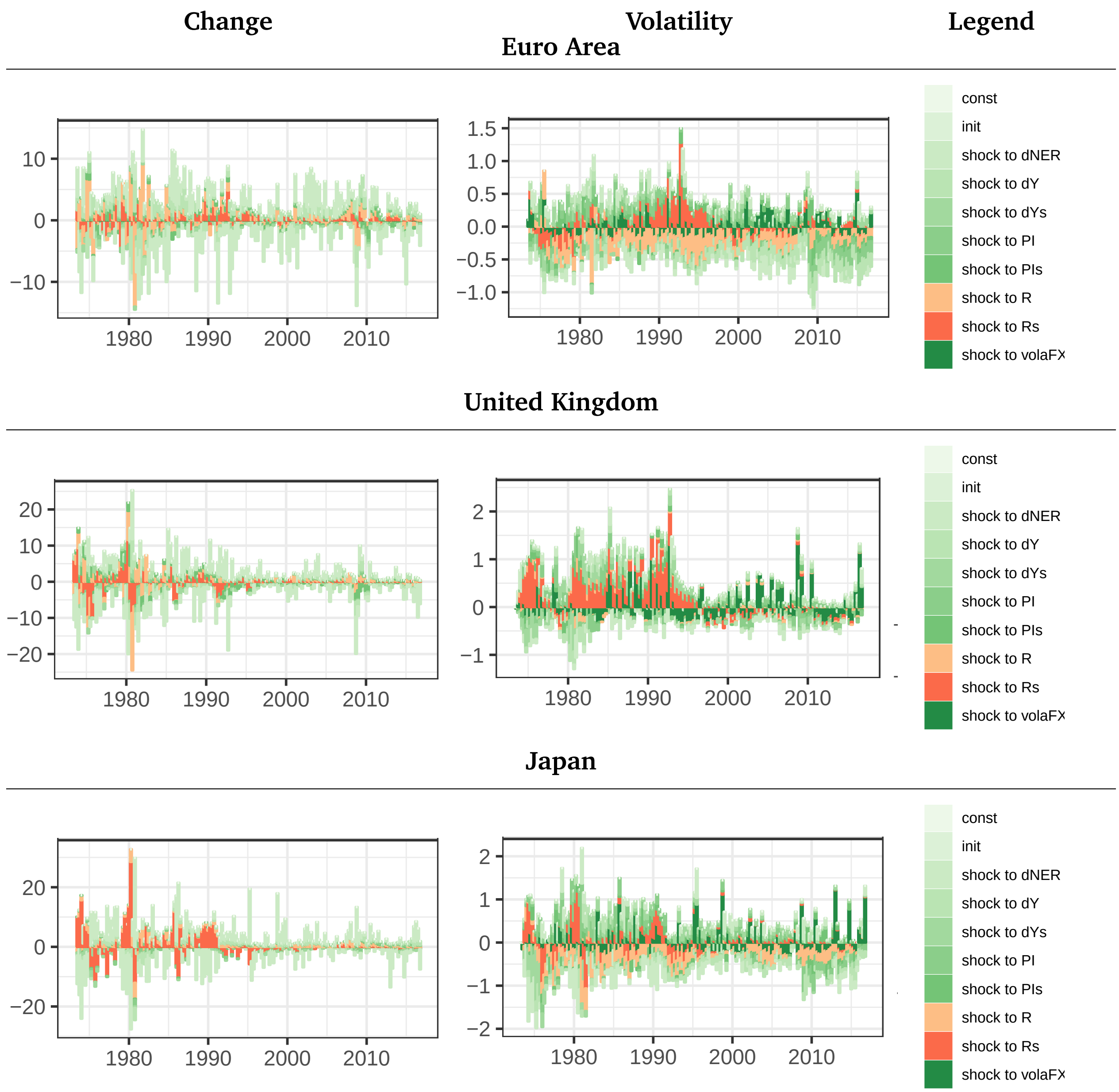



**Fig. 7:** Historical decomposition: Change (left) and volatility (right)

and Roubini, 2000a), monetary policy shocks play a limited role in explaining forecast errors of exchange rates and the corresponding volatility.

Fig. 7 shows the historical decompositions for exchange rate changes (left panel) and its volatility (right panel). The contribution of a US monetary policy shock is marked in light red while the contribution of a non-US monetary shock is shown in dark red. The HDs suggests that across all currency pairs, the role of monetary policy shocks in driving exchange rate fluctuations decreased markedly. Considering the left panel of the figure, we find that the determinants of exchange rate changes shifted over time. Monetary policy shocks (both domestic and foreign) have been the main determinant for all three exchange rates considered. This is especially pronounced during the period of the Volcker disinflation in the end of the 1970s. After the midst of the 1990s, the contribution of monetary policy shocks vanishes across all three exchange rates considered.

Turning to the results for exchange rate volatility, we observe a pronounced degree of heterogeneity across the three currencies considered. Especially for the Euro Area, the HDs point towards a stabilizing role of US monetary policy, with the contributions of US shocks being often negative and thus reducing exchange rate volatility. Notice that Euro Area-based monetary policy shocks contributed sharply to the pronounced increase in volatility during the crisis of the ERM. This pattern can also be found for the GBP/USD exchange rate but with a different role of US monetary policy shocks. Again, we observe that non-US shocks play a particularly strong role in determining exchange rate volatility but, in contrast to the findings for the Euro Area, this also carries over to US-based shocks. Similarly to the case of exchange rate changes, the quantitative contribution of monetary shocks for GBP/USD declines sharply during the midst of the 1990s and remains muted afterwards. For the EUR/USD and JPY/USD exchange rate, US monetary policy shocks generally contributed to a decline in exchange rate volatility.

As compared to FEVDs, the analysis based on HDs reveals that monetary policy shocks contributed significantly to exchange rate fluctuations and movements in volatility during the first part of the sample (up to the mid of the 1990s). For exchange rate volatility associated with the Euro and the Yen, we also observe that US monetary policy disturbances lower the level of volatility, suggesting a stabilizing role of US monetary policy.

# 5 Closing remarks

A large body of literature points towards a strong relationship between monetary policy shocks and exchange rate dynamics. The effect of monetary policy on exchange rate volatility, however, has received relatively little attention in the literature. In this paper, we have developed a VAR model with stochastic volatility to analyze the nexus between monetary policy shocks in the US and abroad on three currencies relative to the US dollar. The model allows for investigating what determines exchange rate volatility and allows for dynamic feedback effects between the change in the exchange rate and its volatility. Our findings indicate that exchange rates tend to appreciate with respect to a domestic monetary tightening. The shape of the exchange rate responses has generated a vivid discussion in the past with a large literature pointing towards a behavior of delayed overshooting. For the country pairs considered, our novel empirical framework cannot confirm this, suggesting instead the immediate exchange rates responses tend to undershoot their level of long-run appreciation, consistent with other recent contributions (c.f. Müller *et al.*, 2019; Schmitt-Grohé and Uribe, 2018). We moreover find that exchange rate volatility tends to increase with respect to monetary policy shocks. Finally, our analysis indicates that while monetary policy shocks explain a small fraction of the forecast error variance of the exchange rate and its volatility, the quantitative contribution in terms of historical decompositions are large, especially during the first half of the sample.

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## Appendix A Data

Below we report in detail all data sources and transformations applied.

- Data US (all series obtained from the FRED database):
  - Real GDP per capita (A939RX0Q048SBEA)
  - Consumer prices index (CPIAUCSL)
  - Federal funds rate (FEDFUNDS)
  - Shares of gross domestic product: Imports of goods and services (B021RE1Q156NBEA)
  - US inflation obtained by 400*diff(log(CPIAUCSL)))
  - US GDP per capita growth obtained by 100*diff(log(GDP/Capita))
- Population growth rates (World Bank: World Development Indicators) on a yearly basis (interpolated with cubic spline to quarterly data) for UK and JP
- Data UK/JP (all series obtained from OECD except exchange rates (IMF) ):
  - Real GDP growth (total, percentage change, previous period)
  - Consumer price index (2015 base year)
  - Short-term interest rate
  - Trade in goods and services Imports, % of GDP
  - Bilateral exchange rates (U.S.) obtained from IMF
  - Inflation obtained by 400*diff(log(CPI)))
  - Exchange rate returns with 100*(diff(log(EXR))
  - Real GDP per capita growth rate is obtained by subtracting the population growth rate from real GDP growth, adjust scale (x100)
- Euro Area (all series obtained from AWM except Euro Area population growth and exchange rate)
  - YER (GDP), HICPSYA (CPI seasonal adjusted), STN (STIR), XTR (EX), MTR (IM)
  - Import share is defined as IMP = MTR/(YER-XTR+MTR)*100)
  - Inflation is obtained by 400*diff(log(HICP))
  - Real GDP per capita growth rate is obtained by subtracting the population growth rate from real GDP growth (diff(log(GDP))), adjust scale (x100)
  - Population growth for EA obtained from ECB data warehouse on a yearly basis, also interpolated with cubic spline to quarterly data

- Bilateral exchange rate (U.S) obtained from IMF for BE, DE, ES, FR, IE, IT, LU, NL, AT, PT, FI, GR up to 1999 and Euro/USD afterwards
  - Apply same method as Lubik and Schorfheide (2005) by using the weights from their paper to construct EA exchange rate before 1999: BE=0.036; DE=0.283; ES=0.111; FR=0.201; IE=0.015; IT=0.195; LU=0.003; NL=0.060; AT=0.030; PT=0.024; FI=0.017; GR=0.025
  - Adjust the level of the two exchange rate series (prior/post 1999) by match the level of the year 1998 and 1999 (see also Lubik and Schorfheide (2005))
  - For Euro/USD exchange rate for periods 1999-2016: two different options:
    - EXR from AWM database
    - Exchange rate from IMF database
  - Exchange rate returns with 100*(diff(log(EXR))

# Appendix B The two-country DSGE model

## B.1 Summary of the DSGE model

Below we list the DSGE first order and equilibrium equations, as put into code, that is, prices are re-expressed in terms of real terms (deflated), and allocations are in terms of stationarized variables, denoted by lowercase variables. In particular, we define $c_t = \frac{C_t}{Z_t}$, $c_{H,t} = \frac{C_{H,t}}{Z_t}$, $c_{F,t} = \frac{C_{F,t}}{Z_t}$, $gc_t = \frac{G_t}{Z_t}$, $c_t^* = \frac{C_t^*}{Z_t}$, $c_{H,t}^* = \frac{C_{H,t}^*}{Z_t}$, $c_{F,t}^* = \frac{C_{F,t}^*}{Z_t}$, $gc_t^* = \frac{G_t^*}{Z_t}$, $w_t = \frac{W_t}{Z_t P_t}$, $w_t^* = \frac{W_t^*}{Z_t P_t^*}$, $y_t = \frac{Y_t}{Z_t}$, $\pi_t = \frac{P_t}{P_{t-1}}$, $\pi_t^* = \frac{P_t^*}{P_{t-1}^*}$, $\pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}}$, $\pi_{H,t}^* = \frac{P_{H,t}^*}{P_{H,t-1}^*}$, $\pi_{F,t} = \frac{P_{F,t}}{P_{F,t-1}}$, $\pi_{F,t}^* = \frac{P_{F,t}^*}{P_{F,t-1}^*}$, $rer_t = \frac{\varepsilon_t P_t^*}{P_t}$, $tot_t = \frac{P_{F,t}}{\varepsilon_t P_{H,t}^*}$, $tot_t^* = \frac{\varepsilon_t P_{H,t}^*}{P_{F,t}}$,

$p_{H,t} = \frac{P_{H,t}}{P_t}$, $p_{F,t} = \frac{P_{F,t}}{P_t}$, $p_{H,t}^* = \frac{P_{H,t}^*}{P_t^*}$, $p_{F,t}^* = \frac{P_{F,t}^*}{P_t^*}$, $\lambda_t = \Lambda_t Z_t$, $b_{Ht} = \frac{B_{Ht}}{Z_t P_t}$, $\lambda_t^* = \Lambda_t^* Z_t^*$, $b_{Ht}^* = \frac{B_{Ht}^*}{Z_t P_t}$, $\frac{Z_t}{Z_{t-1}} = \gamma_z$.

Households' intratemporal optimality conditions: demand functions, CES-based prices, labor supply
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$$\boxed{c_{H,t}} \qquad c_{H,t} = \gamma_c \left(p_{H,t}\right)^{-\epsilon} c_t$$

$$\boxed{c_{F,t}} \qquad c_{F,t} = (1-\gamma_c)\left(p_{F,t}\right)^{-\epsilon} c_t$$

$$\boxed{c_t} \qquad p_{H,t} = \left[\gamma_c + (1-\gamma_c)\left(\frac{tot_t}{gap_t}\right)^{1-\epsilon}\right]^{-\frac{1}{1-\epsilon}}$$

$$\boxed{tot_t} \qquad p_{F,t} = \left[\gamma_c \left(\frac{gap_t^*}{tot_t}\right)^{1-\epsilon} + (1-\gamma_c)\right]^{-\frac{1}{1-\epsilon}}$$

Foreign

$$\boxed{c_{H,t}^*} \qquad c_{H,t}^* = \gamma_c^* \left(p_{H,t}^*\right)^{-\epsilon^*} c_t^*$$

$$\boxed{c_{F,t}^*} \qquad c_{F,t}^* = (1-\gamma_c^*)\left(p_{F,t}^*\right)^{-\epsilon^*} c_t^*$$

$\boxed{c_t^*}$ $$p_{F,t}^* = \left[\gamma_c^* \left(\frac{tot_t^*}{gap_t^*}\right)^{1-\epsilon^*} + (1-\gamma_c^*)\right]^{-\frac{1}{1-\epsilon^*}}$$

$\boxed{tot_t^*}$ $$p_{H,t}^* = \left[\gamma_c^* + (1-\gamma_c^*)\left(\frac{1}{tot_t^* gap_t^*}\right)^{1-\epsilon^*}\right]^{-\frac{1}{1-\epsilon^*}}$$

Firms' optimality conditions

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$\boxed{L_t}$ $$y_t = A_t L_t$$

$\boxed{mc_t}$ $$mc_t = \frac{w_t}{A_t}$$

$\boxed{\pi_{H,t}}$ $$\left[\theta \frac{mc_t}{p_{Ht}} - (\theta-1)\tau\right] = \alpha_H (\pi_{Ht} - \pi_H)\pi_{Ht} - \beta\alpha_H \frac{\lambda_{t+1}}{\lambda_t}(\pi_{Ht+1} - \pi_H)\pi_{Ht+1}\frac{y_{t+1}}{y_t}$$

$\boxed{\pi_{F,t}}$ $$\left[\theta \frac{rer_t p_{Ft}^*}{p_{Ft}} - (\theta-1)\tau\right] = \alpha_F (\pi_{Ft} - \pi_F)\pi_{Ft} - \beta\alpha_F \frac{\lambda_{t+1}}{\lambda_t}(\pi_{Ft+1} - \pi_F)\pi_{Ft+1}\frac{c_{F,t+1}}{c_{F,t}}$$

$\boxed{p_{H,t}}$ $$\pi_{H,t} = \frac{p_{H,t}}{p_{H,t-1}}\pi_t$$

$\boxed{p_{F,t}}$ $$\pi_{F,t} = \frac{p_{F,t}}{p_{F,t-1}}\pi_t$$

$\boxed{gap_t}$ $$gap_t = \frac{p_{F,t}^* rer_t}{p_{F,t}}$$

Foreign

$\boxed{L_t^*}$ $$y_t^* = A_t^* L_t^*$$

$\boxed{mc_t^*}$ $$mc_t^* = \frac{w_t^*}{A_t^*}$$

$\boxed{\pi_{H,t}^*}$ $$\left[\theta \frac{p_{Ht}}{rer_t p_{Ht}^*} - (\theta-1)\tau\right] = \alpha_H^* (\pi_{Ht}^* - \pi_H^*)\pi_{Ht}^* - \beta\alpha_H^* \frac{\lambda_{t+1}^*}{\lambda_t^*}(\pi_{Ht+1}^* - \pi_H^*)\pi_{Ht+1}^*\frac{c_{H,t+1}^*}{c_{H,t}^*}$$

$\boxed{\pi_{F,t}^*}$ $$\left[\theta \frac{mc_t^*}{p_{Ft}^*} - (\theta-1)\tau\right] = \alpha_F^* (\pi_{Ft}^* - \pi_F^*)\pi_{Ft}^* - \beta\alpha_F^* \frac{\lambda_{t+1}^*}{\lambda_t^*}\left(\pi_{Ft+1}^* - \pi_F^*\right)\pi_{Ft+1}^*\frac{y_{t+1}^*}{y_t^*}$$

$\boxed{p_{F,t}^*}$ $$\pi_{F,t}^* = \frac{p_{F,t}^*}{p_{F,t-1}^*}\pi_t^*$$

$\boxed{p_{H,t}^*}$ $$\pi_{H,t}^* = \frac{p_{H,t}^*}{p_{H,t-1}^*}\pi_t^*$$

$\boxed{gap_t^*}$ $$gap_t^* = \frac{p_{H,t}}{p_{H,t}^* rer_t}$$

Market clearing

$\boxed{y_t}$ $$y_t = \left[c_{H,t} + gc_t + \frac{1-n}{n}c_{H,t}^*\right]$$

$\boxed{y_t^*}$ $$y_t^* = \left[\frac{n}{1-n}c_{F,t} + c_{F,t}^* + gc_t^*\right]$$

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$\boxed{\lambda_t}$ $$\lambda_t = \left(c_t - h\frac{c_{t-1}}{\gamma_z}\right)^{-\sigma_c} \quad \text{(or: } \lambda_t = \left(c_t - h\frac{c_{t-1}}{\gamma_z}\right)^{-\sigma_c} - \beta h\left(\frac{1}{\gamma_z}\right)\left(c_{t+1} - h\frac{c_t}{\gamma_z}\right)\text{)}$$

$\boxed{w_t}$ $$\psi L_t^{\sigma_l} = \lambda_t w_t$$

$\boxed{\pi_t}$ $$\lambda_t = \beta E_t\left\{\lambda_{t+1}\frac{R_t}{\pi_{t+1}}\frac{1}{\gamma_z}\right\}$$

$\boxed{b_{Ht}}$ $$\frac{b_{Ht}}{R_t} = b_{Ht-1}\frac{1}{\gamma_z}\frac{1}{\pi_t} + p_{Ht}y_t - c_t - p_{Ht}gc_t$$

Foreign

$\boxed{\lambda_t^*}$ $$\lambda_t^* = \left(c_t^* - h^*\frac{c_{t-1}^*}{\gamma_z}\right)^{-\sigma_c^*} \text{ (or: } \lambda_t^* = \left(c_t^* - h^*\frac{c_{t-1}^*}{\gamma_z}\right)^{-\sigma_c^*} - \beta h^*\left(\frac{1}{\gamma_z}\right)\left(c_{t+1}^* - h^*\frac{c_t^*}{\gamma_z}\right)\text{)}$$

$\boxed{w_t^*}$ $$\psi^* L_t^{*\sigma_l^*} = \lambda_t^* w_t^*$$

$\boxed{\pi_t^*}$ $$\lambda_t^* = \beta E_t\left\{\lambda_{t+1}^*\frac{R_t^*}{\pi_{t+1}^*}\frac{1}{\gamma_z}\right\}$$

$\boxed{rer_t}$ $$\lambda_t^* = \beta E_t\left\{\lambda_{t+1}^*\frac{R_t}{\pi_{t+1}}\frac{rer_t}{rer_{t+1}}\frac{1}{\gamma_z}\right\}$$

Monetary policy

$\boxed{R_t}$ $\quad \frac{R_t}{R} = \left(\frac{R_{t-1}}{1+\bar{i}}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\rho_\pi} \left(\frac{y_t}{\bar{y}}\right)^{\rho_Y} (\Delta\varepsilon_t)^{\rho_{\Delta\varepsilon}}\right]^{1-\rho_R} \exp\left(\epsilon_{R,t}\right), \epsilon_{R,t} \sim N\left(0, \sigma_R^2\right)$

$\boxed{R_t^*}$ $\quad \frac{R_t^*}{R^*} = \left(\frac{R_{t-1}^*}{R^*}\right)^{\rho_R} \left[\left(\frac{\pi_t^*}{\overline{\pi^*}}\right)^{\rho_\pi} \left(\frac{y_t^*}{\overline{y^*}}\right)^{\rho_Y} \left(\frac{1}{\Delta\varepsilon_t}\right)^{\rho_{\Delta\varepsilon}^*}\right]^{1-\rho_R} \exp\left(\epsilon_{R,t}^*\right), \epsilon_{R,t}^* \sim N\left(0, \sigma_R^{*2}\right)$

Exogenous processes

$\boxed{A_t}$ $\quad \log A_t = \rho_A \log A_{t-1} + \epsilon_{A,t}, \epsilon_{A,t} \sim N\left(0, \sigma_A^2\right)$

$\boxed{A_t^*}$ $\quad \log A_t^* = \rho_A^* \log A_{t-1}^* + \epsilon_{A,t}^*, \epsilon_{A,t}^* \sim N\left(0, \sigma_A^{*2}\right)$

$\boxed{g_t}$ $\quad \log g_t = \rho_g \log g_{t-1} + (1-\rho_g) \log g + \epsilon_{g,t}, \epsilon_{g,t} \sim N\left(0, \sigma_g^2\right)$

$\boxed{g_t^*}$ $\quad \log_t^* = \rho_g^* \log g_{t-1}^* + \left(1-\rho_g^*\right) \log g^* + \epsilon_{g,t}^*, \epsilon_{g,t}^* \sim N\left(0, \sigma_g^{*2}\right)$

Change in nominal exchange rate

$\boxed{\Delta ner_t}$ $\quad \Delta ner_t = \frac{rer_t}{rer_{t-1}} \frac{\pi_t}{\pi_t^*} \exp\left(\epsilon_{PPP,t}\right), \epsilon_{PPP,t} \sim N\left(0, \sigma_{PPP}^2\right)$

Observables, given by equation 19 in the main text.

### B.2 DSGE estimation results

**Table B.1:** Estimated DSGE model parameters

| param. | prior distr. | prior mean | prior std. | US-EA post. mean | US-EA intervals 16% | US-EA intervals 84% | US-UK post. mean | US-UK intervals 16% | US-UK intervals 84% | US-JP post. mean | US-JP intervals 16% | US-JP intervals 84% |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\xi_H$ | beta | 0.50 | 0.15 | 0.74 | 0.71 | 0.80 | 0.66 | 0.59 | 0.74 | 0.66 | 0.60 | 0.73 |
| $\xi_F^*$ | beta | 0.70 | 0.15 | 0.83 | 0.80 | 0.86 | 0.85 | 0.83 | 0.87 | 0.88 | 0.86 | 0.89 |
| $\xi_F$ | beta | 0.50 | 0.15 | 0.46 | 0.25 | 0.65 | 0.86 | 0.81 | 0.92 | 0.79 | 0.71 | 0.88 |
| $\xi_H^*$ | beta | 0.70 | 0.15 | 0.73 | 0.59 | 0.89 | 0.84 | 0.72 | 0.99 | 0.76 | 0.60 | 0.96 |
| $\sigma$ | gamma | 2.00 | 0.30 | 3.77 | 3.26 | 4.25 | 2.79 | 2.35 | 3.20 | 3.38 | 2.90 | 3.87 |
| $h$ | beta | 0.30 | 0.05 | 0.57 | 0.51 | 0.64 | 0.53 | 0.46 | 0.64 | 0.42 | 0.32 | 0.52 |
| $\epsilon$ | gamma | 1.00 | 0.50 | 0.17 | 0.05 | 0.24 | 0.16 | 0.10 | 0.21 | 0.08 | 0.05 | 0.10 |
| $\rho_\pi$ | gamma | 1.60 | 0.10 | 1.35 | 1.23 | 1.47 | 1.25 | 1.15 | 1.34 | 1.33 | 1.23 | 1.44 |
| $\rho_\pi^*$ | gamma | 1.60 | 0.10 | 1.31 | 1.19 | 1.43 | 1.27 | 1.16 | 1.38 | 1.19 | 1.10 | 1.27 |
| $\rho_{\Delta Y}$ | gamma | 0.25 | 0.05 | 0.34 | 0.25 | 0.42 | 0.42 | 0.33 | 0.52 | 0.39 | 0.30 | 0.48 |
| $\rho_{\Delta Y}^*$ | gamma | 0.25 | 0.05 | 0.35 | 0.26 | 0.44 | 0.25 | 0.18 | 0.31 | 0.22 | 0.17 | 0.27 |
| $\rho_{\Delta\varepsilon}$ | gamma | 0.05 | 0.03 | 0.03 | 0.01 | 0.04 | 0.02 | 0.01 | 0.03 | 0.03 | 0.01 | 0.04 |
| $\rho_{\Delta\varepsilon}^*$ | gamma | 0.05 | 0.03 | 0.05 | 0.02 | 0.06 | 0.07 | 0.04 | 0.10 | 0.06 | 0.04 | 0.08 |
| $\rho_R$ | beta | 0.50 | 0.20 | 0.78 | 0.75 | 0.81 | 0.76 | 0.74 | 0.79 | 0.76 | 0.73 | 0.79 |
| $\rho_R^*$ | beta | 0.50 | 0.20 | 0.86 | 0.84 | 0.88 | 0.91 | 0.90 | 0.92 | 0.88 | 0.86 | 0.89 |
| $\rho_A$ | beta | 0.70 | 0.20 | 0.80 | 0.75 | 0.86 | 0.80 | 0.73 | 0.87 | 0.82 | 0.76 | 0.88 |
| $\rho_A^*$ | beta | 0.70 | 0.20 | 0.90 | 0.88 | 0.93 | 0.95 | 0.94 | 0.96 | 0.95 | 0.93 | 0.96 |
| $\rho_g$ | beta | 0.80 | 0.10 | 0.96 | 0.94 | 0.98 | 0.94 | 0.92 | 0.96 | 0.94 | 0.93 | 0.96 |
| $\rho_g^*$ | beta | 0.80 | 0.10 | 0.98 | 0.97 | 0.99 | 0.97 | 0.95 | 0.99 | 0.99 | 0.98 | 1.00 |
| $\sigma_A$ | inv.gam. | 1.50 | 4.00 | 4.02 | 3.30 | 5.00 | 3.87 | 3.09 | 5.00 | 3.62 | 2.65 | 4.76 |
| $\sigma_A^*$ | inv.gam. | 1.50 | 4.00 | 3.82 | 3.04 | 4.95 | 4.79 | 4.60 | 5.00 | 4.58 | 4.23 | 5.00 |
| $\sigma_g$ | inv.gam. | 1.00 | 4.00 | 0.86 | 0.79 | 0.92 | 0.86 | 0.79 | 0.93 | 0.89 | 0.82 | 0.95 |
| $\sigma_g^*$ | inv.gam. | 1.00 | 4.00 | 0.74 | 0.68 | 0.80 | 0.89 | 0.82 | 0.96 | 1.09 | 1.01 | 1.17 |
| $\sigma_R$ | inv.gam. | 0.40 | 4.00 | 0.26 | 0.23 | 0.28 | 0.26 | 0.23 | 0.28 | 0.26 | 0.24 | 0.28 |
| $\sigma_R^*$ | inv.gam. | 0.40 | 4.00 | 0.17 | 0.15 | 0.18 | 0.20 | 0.19 | 0.22 | 0.16 | 0.14 | 0.17 |
| $\sigma_{ppp}$ | inv.gam. | 3.50 | 4.00 | 4.56 | 4.26 | 4.86 | 4.77 | 4.60 | 5.00 | 4.90 | 4.82 | 5.00 |
| $r^{ss}$ | gamma | 0.50 | 0.25 | 0.28 | 0.10 | 0.39 | 0.23 | 0.10 | 0.32 | 0.24 | 0.10 | 0.34 |
| $\pi^{ss}$ | normal | 7.00 | 2.00 | 4.19 | 3.64 | 4.76 | 6.28 | 5.12 | 7.33 | 5.29 | 4.40 | 6.09 |
| $\gamma_z^q$ | gamma | 0.40 | 0.20 | 0.43 | 0.41 | 0.46 | 0.43 | 0.41 | 0.45 | 0.44 | 0.42 | 0.47 |

# Appendix C Empirical model

## C.1 Full conditional posterior simulation

Unfortunately, the joint distribution of the latent quantities and coefficients in our model takes no well known form. This calls for simulation-based techniques. To simulate from the joint posterior, we resort to Markov chain Monte Carlo (MCMC) methods to carry out posterior inference.

It is noteworthy that the model in Eq. (26) is a standard VAR model with the first equation being the law of motion of exchange rate volatility. The presence of stochastic volatility and the non-linear interaction between the change and the (log) volatility of the exchange rate implies that our model is a non-linear and non-Gaussian state space model. Additional complications arise from the fact that the shocks to $\boldsymbol{z}_t$ are correlated, leading to a model closely related to a multivariate stochastic volatility with leverage specification. These issues imply that estimating the full history of $h_t$ is difficult and standard techniques based on Gibbs sampling in state space models (such as the methods proposed in Kim *et al.*, 1998) can not be used.

Fortunately, conditional on the full history of $\omega_{jt}$, the remaining parameters can be simulated by using relatively standard Gibbs updating steps. The corresponding MCMC algorithm cycles between the following blocks:

- We simulate from the posterior of $\boldsymbol{B}$, conditional on the full history of $\omega_{jt}$ (that includes $h_t$) and $\sigma_h^2$, by using the algorithm proposed in Carriero *et al.* (2019). This implies that we perform equation-by-equation estimation, where each equation in $\boldsymbol{z}_t$ is augmented with the shocks of the preceding equations. After cycling through all equations of the system, this yields a draw of $\boldsymbol{B}$, $\boldsymbol{\xi}$ and the free elements in $\boldsymbol{U}$. For each equation $j = 1, \dots, m$, the corresponding conditional posterior is Gaussian and takes a standard form:

$$\boldsymbol{B}_{j-}|\bullet \sim \mathcal{N}(\overline{\boldsymbol{B}}_{j-}, \overline{\boldsymbol{V}_{j-}}). \tag{A.1}$$

  Here, the notation $\boldsymbol{B}_{j-}$ selects the $j$th row of a matrix $\boldsymbol{B}$, while $\underline{\boldsymbol{B}}_{j-}, \overline{\boldsymbol{V}_{j-}}$ denote the posterior mean and variance, respectively. The posterior moments are given by

$$\overline{\boldsymbol{V}}_{j-} = \left(\tilde{\boldsymbol{X}}_j' \tilde{\boldsymbol{X}}_j + \underline{\boldsymbol{V}}_{j-}^{-1}\right)^{-1}, \tag{A.2}$$

$$\overline{\boldsymbol{B}}_{j-} = \overline{\boldsymbol{V}}_{j-} \left(\tilde{\boldsymbol{X}}_j' \tilde{\boldsymbol{z}} + \underline{\boldsymbol{V}}_{j-}^{-1} \underline{\boldsymbol{b}}_j\right), \tag{A.3}$$

  whereby $\tilde{\boldsymbol{X}}_j$ and $\tilde{\boldsymbol{z}}$ denote $T \times K_j$ (for $K_j = K + j - 1$) and $T \times 1$ full-data matrices, respectively. Both matrices are rescaled by dividing each row of $\tilde{\boldsymbol{X}}_j$ and $\tilde{\boldsymbol{z}}$ by $e^{\omega_{jt}/2}$ to render the model conditionally homoscedastic. The matrix $\tilde{\boldsymbol{X}}_j$ differs across equations because it consists of of $\boldsymbol{D} = (\boldsymbol{d}_1, \dots, \boldsymbol{d}_T)'$ and the reduced-form shocks of the preceding $j-1$ equations, $\tilde{\boldsymbol{X}}_j = (\boldsymbol{D}, \boldsymbol{w}_1, \dots, \boldsymbol{w}_{j-1})$, with $\boldsymbol{w}_i$ denoting the stacked shocks associated with equation $i$. Finally, we let $\underline{\boldsymbol{V}}_{j-}^{-1} = \text{diag}(\boldsymbol{\tau}_{j-}, 10, \dots, 10)$, with $\boldsymbol{\tau}_{-j}$ denoting the $j$th column of a $K \times m$ matrix that stores the prior scaling parameters $\tau_i$ and $\underline{\boldsymbol{b}}_j = (\boldsymbol{B}'_{0_{-,j}}, 0, \ldots, 0)'$ consisting of the $j$th column of $\boldsymbol{B}_0$ and additional zeroes that capture the prior mean on the corresponding elements in $\boldsymbol{\xi}$. For $j = 1$, the corresponding posterior moments simplify further since $\tilde{\boldsymbol{X}}_j = \tilde{\boldsymbol{D}}$.

- Conditional on all parameters of the model, we obtain the full history of the log-volatilities of $h_t$ by relying on a single-step Metropolis Hastings (MH) algorithm and propose $h_t^*$ from a Gaussian distribution that takes a well-known form (for more information, see, Carlin *et al.*, 1992; Jacquier *et al.*, 2002). Conditional on this proposed value of $h_t^*$, we compute the MH acceptance probability by evaluating the likelihood of the VAR model. One key advantage of our model is that $h_t$ never shows up directly in the mean equation but only indirectly because of $\boldsymbol{\xi}$. This yields the convenient property that the only equation that depends directly on $h_t$ is the exchange rate equation, since $\tilde{w}_{mt}$, the $m^{th}$ element of $\tilde{\boldsymbol{w}}_t$, is uncorrelated with $\tilde{w}_{jt}$ (for $j \neq m$) yielding:
$$p(\tilde{w}_{mt}|\bullet, h_t) = e^{-h_t/2} \times \exp\left(-\frac{\tilde{w}_{mt}^2}{e^{h_t}}\right) \tag{A.4}$$
Hence, conditional on time $t$ and $\boldsymbol{Q}$, the remaining $m-1$ elements in $\tilde{\boldsymbol{w}}_t$ do not depend on $h_t$ but only on its lags. This simplifies the likelihood of the model enormously, implying that the time $t$ acceptance probability reduces to
$$p_{MH}(h_t^*, h_t) = \min\left(\frac{p(\tilde{w}_{mt}|\bullet, h_t^*)}{p(\tilde{w}_{mt}|\bullet, h_t)}, 1\right). \tag{A.5}$$

- The full history of the log-volatilities and the corresponding parameters of the state equation are simulated by means of the algorithm proposed in Kastner and Frühwirth-Schnatter (2014). This is achieved by using the R package stochvol (Kastner, 2016).

- $\sigma_h^2$ is obtained, conditional on $\boldsymbol{\beta}$ and $\{h_t\}_{t=1,\ldots,T}$, from an inverted Gamma distributed posterior distribution that takes a standard form:
$$\sigma_h^2|\bullet \sim \mathcal{G}^{-1}\left(0.01 + \frac{T}{2}, 0.01 + \frac{\sum_{t=1}^{T} v_t^2}{2}\right). \tag{A.6}$$

- The indicators that determine whether the coefficients are pushed towards the prior restrictions (i.e. the DSGE model in the case of all quantities except $h_t$) can be obtained by simulating, one at a time, from a Bernoulli distributed posterior distribution with posterior probability taking a particularly simple form:
$$\delta_j|\bullet \sim \text{Bernoulli}(\overline{p}_j) \tag{A.7}$$

with

$$\overline{p}_j = \frac{\frac{1}{\sqrt{s_{0j}}} \exp\left(-\frac{(b_j - b_{0j})^2}{2s_{0j}}\right) p_j^{(n)}}{\frac{1}{\sqrt{s_{0j}}} \exp\left(-\frac{(b_j - b_{0j})^2}{2s_{0j}}\right) p_j^{(n)} + \frac{1}{\sqrt{s_{1j}}} \exp\left(-\frac{(b_j - b_{0j})^2}{2s_{1j}}\right) (1 - p_j^{(n)})}. \tag{A.8}$$

- The inverse scaling parameters $s_{1j}^{-1}$ are obtained from a Gamma distribution,

$$s_{1j}^{-1}|\bullet \sim \mathcal{G}\left(d_0 + 1/2, d_1 + \frac{(b_j - b_{0j})^2}{2\{d_j\zeta + (1 - d_j)\}}\right). \tag{A.9}$$

- Finally, the equation-specific prior inclusion probability can be simulated from a Beta distribution:

$$p^{(n)}|\bullet \sim \mathcal{B}\left(1 + \sum_{i=1}^{K} \boldsymbol{\delta}_{-,n}, 1 + K - \sum_{i=1}^{K} \boldsymbol{\delta}_{-,n}\right), \tag{A.10}$$

where $\boldsymbol{\delta}_{-,n}$ denotes the $n^{th}$ column of a $K \times m$-dimensional matrix that stores the indicators $\delta_j$ for each equation in its columns.