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Determinants of Fiscal Multipliers Revisited

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Determinants of Fiscal Multipliers Revisited*

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Abstract

We generalize a simple New Keynesian model and show that a flattening of the Phillips curve reduces the size of fiscal multipliers at the zero lower bound (ZLB) on the nominal interest rate. The factors behind the flatting are consistent with micro- and macroeconomic empirical evidence: it is a result of, not a higher level of price rigidity, but an increase in the degree of strategic complementarity in price-setting – invoked by the assumption of a specific instead of an economy-wide labour market, and decreasing instead of constant-returns-to-scale. In normal times, the efficacy of fiscal policy and resulting multipliers tends to be small because negative wealth effects crowd out consumption, and because monetary policy endogenously reacts to fiscally-driven increases in inflation and output by raising rates, offsetting part of the stimulus. In times of a binding ZLB and a fixed nominal rate, an increase in (expected) inflation instead lowers the real rate, leading to larger fiscal multipliers. Conditional on being in a ZLB-environment, under a flatter Phillips curve, increases in expected inflation are lower, so that fiscal multipliers at the ZLB tend to be lower. Finally, we also discuss the role of solution methods in determining the size of fiscal multipliers.

Keywords: Fiscal multipliers, strategic complementarity, Phillips curve, zero lower bound, New Keynesian model

JEL Codes: E52, E62

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1 Introduction

After the introduction of the $750 billion US fiscal stimulus package in 2009 there has been a renewed interest in the effectiveness of fiscal policy in the environment of ultra-low interest rates. Several authors show that the size of fiscal multipliers is significantly higher when the economy is at a zero lower bound (ZLB) of the nominal interest rate (see Eggertsson (2011), Erceg and Linde (2014), Christiano et al. (2011) or Woodford (2011)), making a case for the ability of fiscal policy to curb the adverse effects of financial crisis. The economic consensus on fiscal multipliers in normal times is, that they tend to be small. This is for two reasons: one, increases in government expenditure need to be financed, and thus come with a negative wealth effect, which crowds out consumption and decreases demand; two, a fiscal expansion, increasing inflation and output, triggers an endogenous response of the monetary authority, which raises interest rates, offsetting some of the expansionary effect of fiscal policy. In times when the economy is at the zero lower bound, such endogenous dampening response of monetary policy is absent, as the nominal interest rate stuck at the lower bound and thus constant; in such case, an increase in (expected) inflation, resulting from a fiscal expansion, leads to a drop in the real interest rate, which further stimulates demand and thus increases fiscal multipliers.

This paper extends the New Keynesian model of Eggertsson (2011) and studies the size of various types of fiscal multipliers, in normal times, when the nominal interest rate is positive, and when the economy is at the zero lower bound. We calibrate our model to the US economy and study four different types of fiscal multipliers: a government spending, a payroll tax, a sales tax, and a financial asset tax multiplier. We document that the size of fiscal multipliers at the ZLB crucially depends on the slope of the Phillips curve, with a flatter Phillips curve being associated with smaller multipliers. This is because in the context of the New Keynesian model an, e.g., increase in government spending can raise output owing to a rise in expected inflation which, at the zero lower bound, decreases the real interest rate, stimulating consumption and output. A flatter Phillips curve attenuates the inflation channel and, thus, decreases the value of the multiplier. A sufficiently flat Phillips curve, consistent with recent empirical estimates, delivers a spending multiplier at or below one and a consumption tax cut multiplier that is strictly below one.

The reasons behind the flattening of the Phillips curve that we consider in our model are consistent with both the macroeconomic and microeconomic empirical evidence. In particular, we do not obtain a flatter Phillips curve from employing a higher degree of nominal rigidity; instead, it results from an increase in the degree of strategic complementarity in price-setting, invoked in the model through assumptions of (i) a specific labour market[1] and (ii) decreasing returns-to-scale in production. There is a growing macroeconomic literature suggesting a flattening of the Phillips curve (see, e.g., Blanchard et al., 2015, among others), i.e. a weaker link between economic activity and inflation. The reasons and implications of the flattening of the Phillips curve have been primarily examined for the (lack of) inflation after the crisis or more generally, for monetary policy strategy (Blanchard et al., 2015). We document that this consideration is equally consequential for fiscal multipliers. This macroeconomic literature on the flattening of the Phillips curve is supported by a growing microeconomic literature suggesting that strategic complementarity is an important factor in how firms set prices, and that a high degree of strategic complementarity results in a flat Phillips curve (Coricelli and Horvath (2010), Woodford (2003)). Using micro-level Belgian consumer prices data, Amiti et al. (2019) develop a general theoretical framework and empirical identification strategy to directly estimate firm price responses to changes in prices of their competitors. Their results suggest an elasticity of more than one-third in response to the price changes of its competitors (i.e. strategic complementarity) and an elasticity of nearly two-thirds in response to its own cost shocks. Interestingly, this 'strategic complementarity' elasticity increases to one-half for large firms.[2]

Our results suggest that the empirically relevant reasons for a flattening of the Phillips curve, that we incorporate in our model, lead to smaller fiscal multipliers at the ZLB. More generally, we present detailed results for multipliers for our four types of fiscal instruments, in both normal and ZLB times, and show how they are influenced by the different settings

[1]In general, the labour market can be modeled either as an economy-wide or specific labour market. An economy-wide labour market (one type of labour for all firms) implies strategic substitutability in price-setting i.e. an individual firm which observes a rise in the prices of goods of the other firms will lower the price of its own good. In contrast, a specific factor market leads to the synchronisation of prices across firms which implies a case of strategic complementarity.

[2]In addition, based on a survey conducted for nearly 11 000 firms in the Euro Area, Fabiani et al. (2006) find that the prices of around 30 percent of Euro Area firms are shaped by competitors' prices, while the remaining 70 percent of the firms set prices according to markup (see Alvarez et al., 2006, where this result is discussed, too). Overall, this empirical evidence suggests that strategic complementarity plays an important role for firms' price setting behaviour. Strategic complementarity in price-setting also helps to jointly match the micro-evidence on the frequency of firms' price adjustment and the low estimates on the slope of the New Keynesian Phillips curve (NKPC) (see Linde and Trabandt (2018)). See Nakamura and Steinsson (2008) who estimated a duration of price rigidity is about 2-3 quarters using US micro data. Estimates on the slope of the NKPC vary between 0.009-0.04 (see, e.g., Adolfson et al. (2005), Altig et al. (2011), Gali and Gertler (1999), Woodford (2003)).

of specific versus economy-wise labour market and constant versus decreasing returns to scale.[3] We also present evidence that shows that the level of steady-state government spending-to-GDP ratio affects the size of the resulting multiplier.[4] Finally, we present results from robustness checks in terms of the solution method used to compute fiscal multipliers, considering multipliers that are computed not only from a linear solution method but also from more accurate global solution methods.

Our work is closely related to Boneva et al. (2016) and Ngo (2019), who also study the consequences of a flattening of the Phillips curve for fiscal multipliers, which, however, in their setting is due to an increase in price rigidity parameters. Two further, recently published papers also emphasize the importance of the slope of the Phillips curve for the conduct of monetary policy at the zero lower bound, or for the value of the fiscal multiplier. Belgibayeva and Horvath (2019) explore how the degree of strategic complementarity in price-setting affects optimal monetary policy in a New Keynesian model with wage and price setting frictions. Linde and Trabandt (2018) find that strategic complementarity, introduced via a Kimball consumption basket instead of the constant-elasticity-of-substitution (CES) aggregator, accounts for the difference between the value of the multiplier calculated from the linear and non-linear solution of the model.

Other related contributions include Miao and Ngo (2019), who find that the multipliers behave differently in the non-linear Calvo and Rotemberg models. Surprisingly, they find that the mutliplier is increasing (decreasing) with the duration of the ZLB in the Calvo (Rotemberg) model. They also find that the spending multiplier is a non-linear function of the persistence of the government spending shock. Eggertsson and Singh (2016) argue that the multipliers do not differ a lot across the linear and non-linear New Keynesian

[3] Our model version with decreasing returns in labour is equivalent to a model with firm-specific fixed capital (and variable input labour), which Altig et al. (2011) consider important in reconciling the micro-evidence on the frequency of price changes with the macro evidence on the slope of the Phillips curve. The decreasing returns to scale of technology implies a flatter Phillips curve, again giving rise to smaller multipliers compared to the constant-returns-to-scale assumption of Eggertsson (2011).

[4] Many influential papers, such as Eggertsson (2011) and Woodford (2011), assume a zero government spending-to-GDP ratio when calculating fiscal multipliers. However, US post-war data show that the government spending-to-GDP ratio ranges between 17-20 per cent. Not accounting for a positive government spending-to-GDP ratio distorts the correct size of the private consumption-to-GDP ratio based on the aggregate resource constraint and has an impact on the effective value of the elasticity of intertemporal substitution (IES). Using our model, we show that allowing for positive government spending-to-GDP ratio has non-negligible effects on the size of the government spending multiplier. Interestingly, this issue is largely overlooked in the empirical literature. For example, the existing meta-analyses on the fiscal multipliers do not mention the possible effect of government spending-to-GDP ratio on the size of multiplier (Gechert (2015) and Gechert and Rannenberg (2018)).

models (with either Calvo or Rotemberg pricing) as long as we consider empirically realistic calibration of the models. Boneva et al. (2016) also show the sign and size of the multipliers with respect to the slope of the NKPC and the duration of the zero lower bound using the linear and non-linear New Keynesian model with Rotemberg pricing. Importantly, they show that the labour tax cut multiplier is negative for empirically realistic durations of the zero lower bound in the linear as well as the non-linear New Keynesian model. Ngo (2019) uses US data to calculate the unconditional probability of hitting the zero lower bound and calibrates a model with occasionally binding zero lower bound constraint. He finds a government spending multiplier of around 1.25, which is larger than the one in the model without occasionally binding constraint or transient government spending shocks. He also confirms the finding of Miao and Ngo (2019) regarding the nonlinearity of the multiplier with respect to the persistence of the government spending shock. The focus of our paper differentiates us from the previous papers. In particular, we explore how the recent flattening of the Phillips curve as resulting from a higher degree of strategic complementarity, and show that this affects the size of fiscal multipliers significantly.

Hills and Nakata (2018) show that the government spending multiplier is very sensitive to the inclusion of interest rate smoothing in the Taylor rule. Once one allows for inertia in the interest rate rule, the multiplier decreases from 1.9 to 0.5. Leeper et al. (2017) estimate fiscal multipliers using Bayesian methods on US data. With several combinations of model specifications and different priors they find impact multipliers of about 1.4. Further, they find that multipliers are much higher in a regime with passive monetary and active fiscal policy relative to a regime with active monetary and passive fiscal policy.

The paper proceeds as follows. Section 2 lays out our modelling framework, while section 3 describes the equilibrium of the model. Section 4 discusses intuition and economic channels at play to help interpret fiscal multipliers. Section 5 focuses on the calibration of the model. Section 6 contains the numerical results as well as an explanation of the sign and magnitude of fiscal multipliers. Section 7 presents results from a non-linear solution method to verify robustness of our results. Section 8 provides concluding remarks. An Appendix with the model derivations can be found at the end of the paper.

2 The log-linear model

We log-linearise a basic New Keynesian model as in Eggertsson (2011) around its non-stochastic zero inflation steady state. The New-Keynesian IS curve along with the log-linear aggregate resource constraint, $\hat{Y}_t = (1-g)\hat{C}_t + \hat{G}_t$, yields the aggregate demand curve:

$$\hat{Y}_t - E_t\hat{Y}_{t+1} = \hat{G}_t - E_t\hat{G}_{t+1} - \check{\sigma}\left(i_t - E_t\pi_{t+1} - r_t^e\right) + \check{\sigma}\chi^S\left[E_t\hat{\tau}_{t+1}^S - \hat{\tau}_t^S\right] + \check{\sigma}\chi^A\hat{\tau}_t^A. \quad (1)$$

In the expression above, $g \equiv 1 - \bar{C}/\bar{Y} = \bar{G}/\bar{Y} > 0$ is the steady state government spending-to-GDP ratio. Parameter $\sigma \equiv -\frac{\bar{u}_c}{\bar{u}_{cc}\bar{C}}$ is the IES of consumption. $\check{\sigma} \equiv \sigma(1-g)$ is the IES re-scaled by the government spending-to-GDP ratio.

Variables with a hat are defined as: $\hat{Y}_t \equiv \log(Y_t/\bar{Y})$, $\hat{C}_t \equiv \log(C_t/\bar{C})$, $\hat{G}_t \equiv (G_t - \bar{G})/\bar{Y}$, $\hat{\tau}_t^i \equiv \tau_t^i - \bar{\tau}^i$, $i \in \{A, S, W\}$ and $r_t^e \equiv \log\beta^{-1} + E_t(\hat{\xi}_t - \hat{\xi}_{t+1})$ where $\hat{\xi}_t \equiv \log(\xi_t/\bar{\xi})$.[5] The $\chi^S \equiv \frac{1}{1+\bar{\tau}^S}$, $\chi^A \equiv \frac{1-\beta}{1-\bar{\tau}^A}$ are constants scaling the sales and capital taxes.

The NKPC (or aggregate supply—AS curve) is given by:

$$\pi_t = \kappa\hat{Y}_t + \kappa\psi(\chi^W\hat{\tau}_t^W + \chi^S\hat{\tau}_t^S - \check{\sigma}^{-1}\hat{G}_t) + \beta E_t\pi_{t+1}, \quad (2)$$

with

$$\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)\vartheta}{\alpha}; \vartheta \equiv \frac{\check{\sigma}^{-1} + \phi(1+\omega) - 1}{1+\omega_y\theta}; \qquad \psi \equiv \frac{1}{\check{\sigma}^{-1} + \phi(1+\omega) - 1};$$
$$\omega_y \equiv \phi(1+\mathcal{I}\omega) - 1; \qquad \omega \equiv \frac{\bar{v}_{ll}\bar{l}}{\bar{v}_l}; \qquad \chi^W \equiv \frac{1}{1-\bar{\tau}^W}.$$

The production function is given by $y_t = l_t^{1/\phi}$.[6] ϕ governs the degree of the returns-to-scale in technology production ($\phi = 1$ is CRS, constant returns-to-scale; $\phi > 1$ is DRS, decreasing returns-to-scale). ω is the elasticity of the marginal disutility of work. ω_y is defined similar to ω but also allows for DRS (for CRS $\omega_y = \omega$). χ^W scales labour taxes. β is the discount factor which is used to discount future utilities and profit streams to the present and θ is the elasticity of substitution among intermediary goods. κ is called the

[5] $\hat{\tau}_t^A$ is defined such that a one percent increase in capital income per year is comparable with the tax on labour income.

[6] More generally the production function of firm i can be written as $y_t(i) = k_t(i)f(l_t(i)/k_t(i))$ where f is an increasing and concave function. We abstract from total factor productivity, as it is not in the focus of the present paper. Index i reflects the fact that either capital or labour can be firm-specific in our setup. In line with Woodford (2003, 2005, 2011) we make two assumptions. First, in the case of a specific labour market there exists a rental market for capital while the rental market does not exist in the case of an economy-wide labour market with firm-specific capital. Second, capital is normalised to one in the case of a specific labour market.

slope of the NKPC.

The slope of the Phillips curve is governed by the assumption of the factor market.[7] It can be shown (see, e.g. Woodford (2003) and below) that the slope of the NKPC is smaller with a higher degree of strategic complementarity—firms adjust quantities more than prices in response to shocks. Consequently, the impact of fiscal measures, which alter the marginal cost in the NKPC, on inflation and expected inflation is also smaller.

An economy-wide factor market (one type of factor for all firms) implies strategic substitutability in price-setting (or, equivalently, a steeper Phillips curve) i.e. an individual firm which experiences a rise in the prices of goods of the other firms will decrease the price of its own good. On the other hand, a specific factor market leads to the synchronization of prices across firms which implies a case of strategic complementarity. Strategic complementarity represents an important factor in how firms set prices (see empirical evidence for the US by Amiti et al. (2019) and for Europe by Fabiani (2006)). An economy-wide factor market implies a steeper Phillips curve than a firm-specific one.

Let $\mathcal{I}$ be an indicator variable which takes the value of one when we assume strategic complementarity, owing to a specific labour market. The case of $\mathcal{I} = 0$ corresponds to the setup with an economy-wide labour market. $\vartheta < 1$ means that there is some degree of strategic complementarity which is supported by empirical evidence (see, Woodford (2003)). The case of strategic substitutability, $\vartheta > 1$, is not covered here because it is not supported by data.

For $\phi = 1$, $g = 0$, $\mathcal{I} = 1$ the Eggertsson (2011) setup is derived. Note that only the content of parameters $\check{\sigma}$, κ, ϑ and ψ changes when we generalise Eggertsson (2011) for positive long-run government spending and DRS. Table (1) provides an overview how the slope of NKPC (κ) changes due to the various assumptions (economy-wide versus specific labour market and CRS versus DRS): estimates for the slope of New Keynesian Phillips curve vary between 0.0076-0.1999 (see e.g. Linde and Trabandt (2018) for a collection of estimates for the US). We make the following observations. First, we do not consider the economy-wide labour market with CRS to calculate fiscal multipliers because the slope of the NKPC in that case is out of range of the empirical estimates. Second, DRS is a substantial source of strategic complementarity even in the case of an economy-wide labour market. Third, a specific labour market implies a substantial degree of strategic

[7] Factor market means labour market in this paper. However, instead of assuming a firm-specific labour market we can arrive at similar results under the alternative assumption of a homogeneous (or economy-wide) labour market with firm-specific (fixed) capital and decreasing returns in production.

Table 1: The effect of various labour market assumptions (economy-wide/specific or, equivalently, steeper/flatter Phillips curve) and production technology (constant or decreasing returns-to-scale) assumptions on the value of the slope of the New Keynesian Phillips curve.

	Economy-wide	Specific
	$\mathcal{I} = 0$	$\mathcal{I} = 1$
CRS ($\phi = 1$)	0.1999	0.0095
DRS ($\phi = 1.5$)	0.0386	0.0076

complementarity with either CRS or DRS. It is important to note that the flattening of the Phillips curve could, alternatively, occur due to a rise in price rigidity parameter as analyzed in Boneva et al. (2016) and Ngo (2019).

Monetary policy follows Taylor rule, generalized to allow for the case of a zero lower bound:

$$i_t = \max\{0, r_t^e + \phi_\pi \pi_t + \phi_Y \hat{Y}_t\}, \tag{3}$$

where $\phi_\pi > 1$ and $\phi_Y > 0$ and the max operator refers to the zero lower bound on the nominal interest rate.

3 Description of the equilibrium

We analyse a short-run and a long-run equilibrium. Initially, we are in steady state ($t = 0$). Then, from time $t = 1$, for some interval, $0 < t < T$, which we can call the *short-run* (see subscript S), a shock hits the economy. That is, when $t < T$ the shock is described by an exogenous decrease in $r_t^e = r_S^e < 0$ with T denoting the stochastic date at which the shock vanishes.

In period t, the shock persists with probability μ or dies out with $1-\mu$ for all $t < T$. In the short-run, the zero lower bound on nominal interest can be either binding ($i_t = i_S = 0$) or not binding ($i_t = i_S > 0$). In the non-binding case, the nominal interest is governed by the Taylor rule. For time, $t \geq T$, variables take on their *long-run* steady-state values. We proceed to describe the equilibria under positive and zero nominal interest rates.

Positive Interest rate. We assume that inflation and output are linear functions of the fiscal variables, $\hat{F}_S = \{\hat{G}_S, \hat{\tau}_S^W, \hat{\tau}_S^S, \hat{\tau}_S^A\}$:

$$\pi_S = A_\pi \hat{F}_S, \tag{4}$$

$$\hat{Y}_S = A_Y \hat{F}_S, \tag{5}$$

where A_π and A_Y are coefficients to be determined.

The fiscal instrument F follows an AR(1) process:

$$F_{t+1} = F_t^\rho \exp(\varepsilon_{t+1}) \tag{6}$$

where ρ measures persistence and ε is an *i.i.d.* shock with zero mean and constant variance.

The fiscal multipliers are computed separately, e.g., a sales tax cut is computed under the assumption of no change in other fiscal instruments. Also, we assume that changes in spending (or taxes) are offset by present or future lump-sum taxes/transfers, i.e. the Ricardian evidence holds.

Zero nominal interest rate. In period t and $t+1$ variable $\hat{X}_i = \{\hat{F}_i, \hat{Y}_i, \pi_i\}$ with $\hat{F}_i = \{\hat{G}_i, \hat{\tau}_i^W, \hat{\tau}_i^S, \hat{\tau}_i^A\}$ for $i \in \{t, t+1\}$ are taking, respectively, the following values:

$$\hat{X}_t = \begin{cases} \hat{X}_t = \hat{X}_S, \quad 0 < t < T, \text{ zero bound binding,} \\ \hat{X}_t = 0, \quad t \geq T, \text{ zero bound not binding,} \end{cases}$$

and

$$\hat{X}_{t+1} = \begin{cases} (1-\mu)\hat{X}_S = 0, \quad \text{with probability } 1-\mu, \ \hat{X}_{t+1} \text{ reverts to steady state,} \\ \mu\hat{X}_S, \quad \text{with probability } \mu \text{ zero bound continues to bind.} \end{cases}$$

It is necessary to formulate conditions under which the zero bound binds. Condition *C1* ensures that the shock in r_S is large enough to make the zero bound binding even with an expansionary fiscal policy:[8]

$$r_t^e < -\frac{\kappa\breve{\sigma}^{-1}(1-\mu)(\breve{\sigma}-\psi)\phi_\pi + [(1-\mu)(1-\beta\mu)-\kappa\psi\mu]\phi_Y}{\kappa\mu(\phi_\pi-\mu)+[1+\breve{\sigma}\phi_Y-\mu](1-\beta\mu)}[\hat{G}_t - \breve{\sigma}\chi^S\hat{\tau}_t^S]$$
$$-\frac{(1-\mu)\kappa\psi\phi_\pi + \breve{\sigma}\mu\kappa\psi\phi_Y}{\kappa\breve{\sigma}(\phi_\pi-\mu)+[1+\breve{\sigma}\phi_Y-\mu](1-\beta\mu)}\chi^W\hat{\tau}_t^W$$
$$-\frac{\phi_\pi\kappa + (1-\beta\mu)\phi_Y}{\kappa\breve{\sigma}(\phi_\pi-\mu)+[1+\breve{\sigma}\phi_Y-\mu](1-\beta\mu)}\breve{\sigma}\chi^A\hat{\tau}_t^A$$

while condition *C2* makes sure that the crises do not last for too long[9]:

$$L(\mu) \equiv (1-\mu)(1-\beta\mu) - \breve{\sigma}\mu\kappa > 0. \tag{7}$$

Proposition 1 *In the short-run when $i_S > 0$ and C1 does not hold, the equilibrium π_S, $\hat{Y}_S$ and i_S are described, respectively, by:*[10]

$$\pi_S = \mathcal{A}\hat{G}_S + \mathcal{B}\hat{\tau}_S^S + \mathcal{C}\hat{\tau}_S^W + \mathcal{D}\hat{\tau}_S^A, \qquad \mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D} > 0 \text{ (constants)}, \tag{8}$$

$$\begin{aligned}\hat{Y}_S = & \frac{\kappa\psi[\phi_\pi-\rho]+(1-\rho)(1-\beta\rho)}{[1+\breve{\sigma}\phi_Y-\rho](1-\beta\rho)+\kappa\breve{\sigma}[\phi_\pi-\rho]}\hat{G}_S \\ & - \frac{\kappa\psi(\phi_\pi-\rho)+(1-\rho)(1-\beta\rho)}{[1+\breve{\sigma}\phi_Y-\rho](1-\beta\rho)+\kappa\breve{\sigma}(\phi_\pi-\rho)}\breve{\sigma}\chi^S\hat{\tau}_S^S \\ & - \frac{\kappa\psi\chi^W\breve{\sigma}(\phi_\pi-\rho)}{[1+\breve{\sigma}\phi_Y-\rho](1-\beta\rho)+\kappa\breve{\sigma}(\phi_\pi-\rho)}\hat{\tau}_S^W \\ & + \frac{\breve{\sigma}\chi^A(1-\beta\rho)}{[1+\breve{\sigma}\phi_Y-\rho](1-\beta\rho)+\breve{\sigma}\kappa(\phi_\pi-\rho)}\hat{\tau}_S^A\end{aligned} \tag{9}$$

and

$$i_S = i_S^e + \phi_\pi\pi_S + \phi_Y\hat{Y}_S. \tag{10}$$

[8]This condition can be derived by substituting equations (8) and (9) into the Taylor rule, equation (A.6).

[9]Condition *C2* also facilitates i) the avoidance of the deflationary black hole which would arise at $\bar{\mu}$ that satisfies $L(\bar{\mu}) = 0$ and ii) ensures that the coefficient on r_t^e in equation (11) is positive so that $r_t^e < 0$ is satisfied.

[10]In the interest of space we do not report coefficients $\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D}$ ($\check{\mathcal{A}},\check{\mathcal{B}},\check{\mathcal{C}},\check{\mathcal{D}},\check{\mathcal{E}}$). To derive the fiscal multipliers it is sufficient to have the expressions for output.

Similarly, in the short-run when $i = 0$, C1 and C2 hold, the equilibrium is as follows:

$$\pi_S = \check{\mathcal{A}}\hat{G}_S + \check{\mathcal{B}}\hat{\tau}_S^S + \check{\mathcal{C}}\hat{\tau}_S^W + \check{\mathcal{D}}\hat{\tau}_S^A + \check{\mathcal{E}}r_S^e, \quad \check{\mathcal{A}}, \check{\mathcal{B}}, \check{\mathcal{C}}, \check{\mathcal{D}}, \check{\mathcal{E}} > 0 \text{ (constants)},$$

$$\begin{aligned} \hat{Y}_S = & \frac{(1-\mu)(1-\beta\mu) - \mu\kappa\psi}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}\hat{G}_S + \frac{\check{\sigma}\mu\kappa\psi\chi^W}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}\hat{\tau}_S^W \\ & - \frac{[(1-\mu)(1-\beta\mu) - \mu\kappa\psi]\chi^S}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}\check{\sigma}\hat{\tau}_S^S + \frac{(1-\beta\mu)\chi^A}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}\hat{\tau}_S^A \\ & + \frac{\check{\sigma}(1-\beta\mu)}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}r_S^e \end{aligned} \tag{11}$$

and

$$i_S = 0.$$

For the proof, we use the method of undetermined coefficients. In particular, we derive equation 9 through the combination of equations 1, 2 and 10. Equation 11 can be obtained using equations 1, 2 and $i_S = 0$. A similar procedure can be used to generate the expressions for inflation for both $i > 0$ and $i = 0$.

Note that the fiscal multiplier can be derived as $d\hat{Y}_S/d\hat{F}_S$ with $\hat{F}_S = \{\hat{G}_S, \hat{\tau}_S^W, \hat{\tau}_S^S, \hat{\tau}_S^A\})$ using equations (9) and (11) for $i > 0$ and $i = 0$ cases, respectively. We follow Eggertsson (2011) in assuming that the persistence parameters for the exogenous processes of fiscal instruments equal the parameter of the probability of remaining in a ZLB scenario, $\rho = \mu$. An approximate equilibrium that is correct up to the first order is a collection of stochastic processes for $\{\hat{Y}_t, \pi_t, i_t, r_t^e\}$ that solves equations (1)-(A.6) given paths for fiscal policy, $\{\hat{G}_t, \hat{\tau}_t^W, \hat{\tau}_t^S, \hat{\tau}_t^A\}$.

4 Intuition for the multipliers

This section provides an illustration of the main mechanisms in our model to develop intuition for the section 6, where we present results on the values of multipliers for our four fiscal instruments, based on the calibration of our model reported in section 5.

We start by discussing why the labour demand is upward-sloping at the peculiar environment of the zero lower bound. We then elaborate on the effects of the degree of strategic complementarity on the slope of the labour demand.

4.1 Upward-sloping labour demand at the zero lower bound

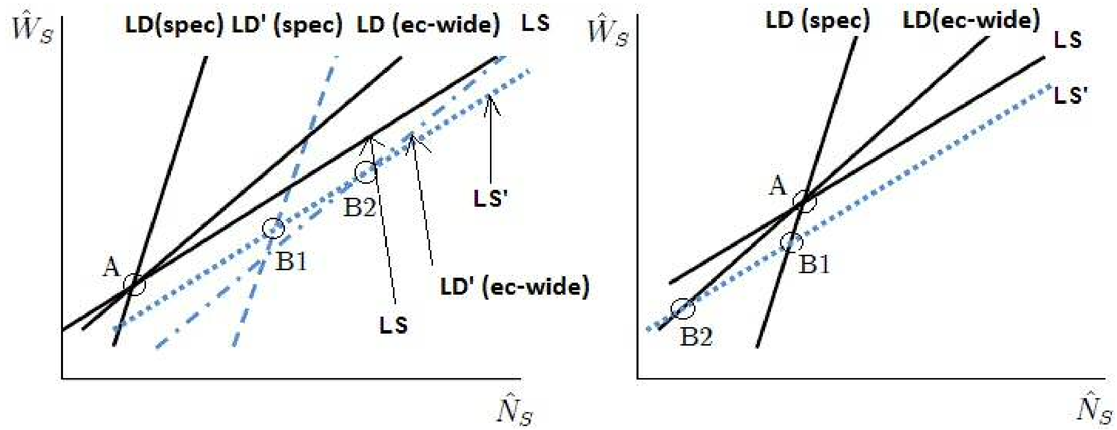
We build upon the intuition from Eggertsson (2011). To better understand the argument in case of the zero lower bound, it is useful to start with describing normal times, i.e. when the nominal interest rate is positive and is determined through an interest rate rule. In this case, labour demand is downward-sloping relationship in the real wage-labour system. The story could, alternatively, also be told in terms of aggregate demand (AD), which is a downward-sloping relationship in an inflation-output system. In such setting, a decrease in inflation implies that the nominal interest is cut more than the fall in inflation, in line with the logic of the Taylor rule (the coefficient on inflation is higher than one, $\phi_\pi > 1$, see the equation A.6). A lower nominal interest rate thus results in a lower real interest rate, stimulating aggregate demand. Thus, the labour demand or AD has a negative slope.

Woodford (2011) provides an alternative explanation for why, at positive interest rates, the government spending multiplier is equal to at most exactly one or below one at positive interest rates in the sticky-price model. The intuition for this proceeds as follows. The multiplier is exactly one as long as the real interest rate is fixed because consumption will not change through the Euler equation (the negative wealth effect of higher government spending on private consumption is eliminated). Then the spending multiplier can be simply derived from the aggregate resource constraint and takes on the value of one. When the real interest rate is allowed to change then higher spending will trigger a higher nominal and, thus, through the Taylor rule, real interest rate, crowding out private consumption. In this case, the multiplier is typically lower than one, as long as consumption and hours worked are separable in the utility function implying that they are substitutes[11].

The previous intuition changes at the zero lower bound: a reduction in inflation is no longer counteracted by the Taylor rule. When the nominal interest rate is fixed, a deflationary policy implies higher real interest rates, depressing labour demand and aggregate demand. Figure 1 provides a graphical illustration of the effects of higher government purchases and lower taxes on labour demand and supply at the zero lower bound. The left (right) panel of Figure 1 shows the effects of higher government purchases (lower labour taxes) on the labour demand and supply. The initial situation is denoted by solid lines. The labour tax-cut does not have an effect on the labour demand (or AD) equation while government purchases affects both LD and LS.

[11] Complementarity between consumption and hours worked can imply a multiplier of one or slightly higher than one with positive interest rates, see the discussion of Christiano et al. (2011).

Figure 1: Labour demand and supply at the zero lower bound



Notes: Left panel: an increase in government spending. Right panel: a decrease in labour tax. In both panels LD refers to labour demand while LS is labour supply. Spec. refers to specific while ec-wide refers to economy-wide labour market. An increase in government spending shifts both LS and LD to the right while the labour tax-cut shifts only the labour supply. The higher is strategic complementarity in price-setting (the case of specific labour market relative to economy-wide labour market) the steeper is the labour demand and the flatter is the Phillips curve.

A labour tax-cut which reduces marginal costs[12], shifts labour supply to the right, and is thus deflationary. Contrary to the conventional wisdom of New Keynesian models in normal times, the model predicts that cuts in the payroll tax are contractionary at the zero lower bound.

Next, we proceed to study the effects of higher government expenditure which affects both LD and LS. Higher government spending has a strong negative wealth effect, making the representative household reduce consumption and leisure, as both of them are normal goods. The decrease in leisure automatically leads to a rise hours worked, as the time endowment is fixed. In other words, the household wants to insure against the negative wealth effect by working more (LS shifts to the right). Despite crowding out consumption, the higher government spending raises aggregate demand overall, which would induce firms to raise their prices in a flexible price environment. However, because firms face nominal

[12]In our setup there is no technology shock, and production is a function of labour input only, so the real wage equals real marginal costs.

rigidities in their price setting, output is demand determined, and firms respond to higher aggregate demand by producing more: they demand more labour, so that LD shifts to the right.

4.2 The degree of strategic complementarity and the size of multipliers

To highlight the importance of the degree of strategic complementarity for the size of fiscal multipliers we study the labour market equilibrium analytically and graphically. Combining the log-linear Euler equation, the NKPC and market clearing equations, we obtain the inverse labour demand curve:

$$\hat{W}_S = \Lambda \mu \phi^{-1} \hat{N}_S - \Lambda(1-\mu)^{-1} r^e_S - \Lambda \check{\sigma}^{-1} \hat{G}_S + \Lambda \chi^S \hat{\tau}^S_S - \Lambda \chi^A (1-\mu)^{-1} \hat{\tau}^A_S \tag{12}$$

where $\Lambda \equiv \frac{1-\beta(1-\mu)}{\kappa\psi}$. Equation 12 shows that the slope of the labour demand is influenced by the degree of strategic complementarity in price setting. In particular, higher strategic complementarity lowers κ, i.e., flattens the Phillips curve, which raises the slope of the labour demand, Λ. labour demand is affected by the discount factor shock (see the r^e_S term in equation 12) while labour supply (see equation 13) is not. Government spending, $\hat{G}_S$, labour taxes, $\hat{\tau}^W_S$, and consumption taxes, $\hat{\tau}^S_S$ appear in both labour demand and supply equations while the tax rate on bonds, $\hat{\tau}^A_S$ shows up only in the labour demand equation.

Similarly, let us substitute the log-linear market clearing for consumption into the log-linear intratemporal condition to arrive at the inverse labour supply:

$$\hat{W}_S = \left[\frac{\omega\phi + \check{\sigma}^{-1}}{\phi}\right] \hat{N}_S + \chi^W \hat{\tau}^W_S + \chi^S \hat{\tau}^S_S - \check{\sigma}^{-1} \hat{G}_S. \tag{13}$$

Equation 13 shows that the value of κ does not influence labour supply. However, it enters labour demand through Λ. For the rest of this sub-section we assume that there is DRS in both types of labour market. It remains true that strategic complementarity is higher with firm-specific labour market. Formally, this means that the value of κ in case of an economy-wide labour market –denoted as κ^{ew}– is higher than the κ under firm-specific labour market – denoted κ^{sp}):

$$\kappa^{\text{sp}} < \kappa^{ew}. \tag{14}$$

To see why inequality (14) is true one can recall the definitions of κ^{sp} and κ^{ew}:

$$\kappa^{\text{sp}} \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{\phi(1+\omega)-1+\check{\sigma}^{-1}}{1+\omega_y\theta}; \quad \kappa^{ew} \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{\phi(1+\omega)-1+\check{\sigma}^{-1}}{1+(\phi-1)\theta}, \tag{15}$$

where the difference between κ^{sp} and κ^{ew} lies in their denominator:

$$\omega_y \equiv \phi(1+\omega)-1 > (\phi-1). \tag{16}$$

The latter is always satisfied because $\omega > 0$. It follows that $\Lambda^{ew} < \Lambda^{\text{sp}}$ holds, and the slope of the labour demand under a firm-specific factor market is higher than the one with an economy-wide labour market:

$$\left(\frac{\partial \hat{W}_S}{\partial \hat{N}_S}\right)_{ew} < \left(\frac{\partial \hat{W}_S}{\partial \hat{N}_S}\right)_{\text{sp}}.$$

Looking at the labour demand (LD) and supply (LS) equations (12) and (13) we find:

$$(\check{\sigma}^{-1})^{LS} < (\Lambda\check{\sigma}^{-1})^{LD-ew} < (\Lambda\check{\sigma}^{-1})^{LD-\text{sp}}. \tag{17}$$

Let us return to the case of an increase in government spending, which is depicted on the left panel of Figure (1). The relation in equation (17) tells us that a rise in government purchases will change wages more, ceteris paribus, through the labour demand (see the constant terms multiplying $\hat{G}_S$ in equation (12)) in the specific labor market case relative to the economy wide case. Hence, the labour demand curve in the firm-specific labour market setting ($LD-\text{sp}$) shifts to the right by more than the labour demand under the economy-wide factor market ($LD-ew$ see dashed-dotted line). The relation in equation (17) also indicates that labor supply shifts less than labor demands (with either economy-wide or firm-specific labor markets).

Figure (1) also shows that labour expands more under the economy-wide factor market as the labour demand curve in the economy wide case is flatter than the firm-specific one (see equilibrium points B2 and B1, respectively). Overall, we conclude that the rise in labour demand and supply due to higher government purchases leads to higher output produced under economy-wide labour market relative to the firm-specific labour market. Intuitively, the higher is the slope of the NKPC, the higher is the rise in inflation, resulting from increases in the marginal cost (through the NKPC) and, thus, the lower is the real interest rate stimulating private spending at the ZLB.

The right panel of Figure (1) displays the effects of a cut in labour taxes. The labour tax rate appears only in the labour supply equation, so that labour demand is not affected. Due to the fact that labour demand in the economy-wide case is flatter, the rightward shift of the labour leads to larger recession (see equilibrium point B2) relative to the specific factor market outcome (B1).[13] Alternatively, this can be explained as follows. The labour tax cut decreases marginal costs, and thus leads to a drop in inflation. This drop is larger in case of a steeper Phillips curve, so that it causes a deeper recession in the case of the economy-wide labour market. Note that the sales tax cut works similar to the increase in government spending, but has smaller positive effects. Capital tax cuts are deflationary, similar to labour tax cuts, but lead to multipliers close to zero.

4.3 The effects of the returns-to-scale on the value of the multiplier

The assumption of either CRS or DRS technology is equivalent to assuming a lower or higher degree of strategic complementarity, respectively, and the previous arguments apply. Returns to scale are governed by parameter ϕ, where $1/\phi$ is the coefficient on labour in the production function, $y_t = l_t^{1/\phi}$. Having previously defined parameter $\omega_y \equiv \phi(1+\mathcal{I}\omega)-1$, one can shown that under CRS, with $\phi = 1$, $\omega_y \equiv 0$ for the case of an economy-wide labour market ($\mathcal{I} = 0$), and, $\omega_y \equiv \omega$, for the case of a specific labour market ($\mathcal{I} = 1$). Instead, under DRS, with $\phi > 1$, $\omega_y \equiv \phi - 1$ for the case of an economy-wide labour market ($\mathcal{I} = 0$), and, $\omega_y \equiv \phi(1+\omega)-1$, for the case of a specific labour market ($\mathcal{I} = 1$). It can thus be seen that, ω_y is, for each labour market assumption, larger in the case of DRS compared to CRS, so that according to equation (15) the Phillips curve slope is smaller. This is also summarised in Table 2. It is important to note that the economy wide labour market with DRS delivers a lower degree of strategic complementarity than specific labour market with CRS.

[13] Note that, at the zero lower bound, the response of labour to a payroll tax decrease is undoubtedly negative for the case of the linear solution described here (due to the omission of labour contracts from the model, i.e. lack of a downward nominal wage rigidity). This is also the case in the linear solution of Eggertsson (2011). More generally, however, this may not be the case in the exact nonlinear environment. Boneva et al. (2016) show that, when using a fully nonlinear solution, a payroll tax cut leads to an *increase* in employment. We confirm the results of Boneva et al. (2016) using our global solution: based on the scenario computed in Table 5, we find that employment indeed slightly increases in response to the payroll tax decrease at the zero lower bound. Section 7 discusses our robustness checks and implied results on fiscal multipliers from the global method in detail.

4.4 Introducing positive government purchases-to-GDP ratio

Instead of assuming zero government spending-to-GDP ratio as in previous papers (see e.g. Eggertsson (2011) and Woodford (2011)) we introduce a positive 20 per cent g ratio, which is in line with post-war US data. This also helps us to have a more reasonable calibration for the steady-state consumption-to-GDP ratio. The introduction of a positive government purchases-to-GDP ratio ($g > 0$) modifies the slopes of the demand and the supply of labour as well as re-scales the size of the government spending. In the numerical exercises below (cf. Table 4), we find that the introduction of $g > 0$ has an only minor quantitative effects on the multipliers in case of positive nominal interest rate. However, in the case of constant nominal interest rate the multipliers are smaller in (absolute) value when $g > 0$, because positive g reduces the intertemporal elasticity of substitution (IES) and the representative agent responds less to changes in the real interest rate by changing its consumption. One can notice that higher g would raise the slope of the NKPC as well as the multiplier. So there are two opposing effects. The total of effect of higher g on the multiplier is negative, however. $\check{\sigma}$ governs the strength of the wealth effect of the government spending shock on consumption.
To see this more clearly recall the log-linear aggregate resource constraint, $\hat{Y}_t = (1-g)\hat{C}_t + \hat{G}_t$ and differentiate it with respect to $\hat{G}_t$. We obtain the government spending multiplier and it is apparent that it depends negatively on g:

$$\frac{d\hat{Y}_t}{d\hat{G}_t} = 1 + (1-g)\frac{d\hat{C}_t}{d\hat{G}_t}$$

The previous formula shows that the consumption multiplier, $\frac{d\hat{C}_t}{d\hat{G}_t}$, is scaled by g. Christiano et al. (2011) explain that lower values of $\check{\sigma}$ lead to lower government spending multipliers. In total, it seems that the second effect (wealth effect) dominates in the case of introducing $g > 0$.

5 Calibration

We follow Eggertsson (2011) who estimated the linearised model to match a 30 percent drop in output and a 10 percent drop in inflation, as experienced during the Great Depression. The values are summarised in Table 2:

Table 2: Parameterisation of the model

β	σ	ω	ρ	ϕ_π	ϕ_Y	$1/\phi$
0.9970	0.86	1.5692	0.9030	1.5	0.5/4	2/3
α	μ	g	$\bar{\tau}^S$	$\bar{\tau}^A$	$\bar{\tau}^W$	θ
0.7747	0.9030	0.2	0.05	0	0.2	12.7721

Notes: g is from Christiano et al. (2011). ϕ is from Woodford (2003).

In addition to the 'Great Depression'-scenario, Eggertsson and Singh (2016) also consider an additional empirically relevant calibration scenario, which is the 'Great Recession'-scenario, whereby US output and inflation dropped about -10 percent and -2 percent, respectively. In Table 5 we provide results based on a fully non-linear solution, for such a 'Great Recession'-sized output drop.[14] In the non-linear solution of the model one needs to assign values to the size of the fiscal shocks, which we set in the range of $[0.001(1-\beta), 0.01]$, which is consistent with the Bayesian estimates of Zubairy (2014) on post-war US data.

6 Results

Based on the calibration just outlined, we compute fiscal multipliers for a number of comparison scenarios, summarised in Tables 3 and 4. Four main results emerge.

Result 1. Table 3 documents, that under positive nominal interest ($i_t > 0$), the government spending and sales tax multipliers are higher the flatter the Phillips curve in the underlying model, or, respectively, the higher the degree of strategic complementarity. In particular, the government spending multiplier and the sales tax multiplier in Table 4 are given by 0.6772 and 0.4448 respectively, for the case of a high degree of strategic complementarity and a flat Phillips curve, coming from the assumption of a specific labour market ($\mathcal{I} = 1$). In contrast, for the low degree of strategic complementarity and steeper Phillips

[14] The output drop of 10 percent is achieved by choosing the size of the shock that puts the economy into a ZLB scenario, accordingly. Since we keep all parameters constant to the ones of Eggertsson (2011), reported in Table 2, and only vary one parameter (the size of the ZLB-shock), the inflation drop is not fully matched.

curve, coming from the assumption of an economy-wide labour market, the resulting multipliers are lower, 0.6108 and 0.4012, respectively. This is in line with the basic intuition on how the monetary authority reacts to the state of the economy, as described by the Taylor rule. Under a steep Phillips curve, when an expansionary fiscal policy shifts out the AD curve, the resulting inflation increase is relatively large. The central bank reacts to this increase in inflation with a relatively strong increase in the nominal interest rate, which (because this translates into an increase in the real interest rate in a world of sticky prices) contracts output and offsets part of the fiscally-driven expansion – because of the strong response of the monetary authority, the implied multipliers are relatively small. In contrast, when the Phillips curve is flat, inflation rises only little in response to the fiscal expansion, and the offsetting effect from monetary policy are mild – the implied multipliers are larger. It should be noted, however, that, while intuitive, there is no guarantee that the government spending or the sales tax multiplier are always larger under a flatter Phillips curve. E.g., Linnemann and Schabert (2003) show that for very persistent government spending increases, labour supply shifts out strongly, due to the negative wealth effect of the government spending shock (leisure decreases, so one has to work more). Recall from Figure 1 that the economy-wide labour market (the steep PC scenario) implied a flat LD curve. If the outward shift in labour supply is large because of a large negative wealth effect, it may actually be the case that the real wage, and, in consequence, marginal cost and inflation, all decrease. In this case, the endogenous response of monetary policy implies that the multiplier is larger for a steeper Phillips curve. Miao and Ngo (2019) and Ngo (2019) similarly document the described nonlinearities of the multiplier with respect to the persistence of the government spending shock. Even if we have now discussed various reasons for the directions in which fiscal multipliers differ across steep versus flat Phillips curve slopes, we want to emphasize that, overall, our results from Tables 3 and 4 indicate, that, in normal times, at positive interest rates, fiscal multipliers are similar across scenarios; the quantitative differences in the various multipliers in normal times are minor.

Result 2. When the zero lower bound on nominal interest becomes binding, the government spending, and the sales tax cut multipliers are higher in the case of a steeper slope of the Phillips curve, or, equivalently with a lower degree of strategic complementarity. Table 3 shows this to be the case for the economy-wide labour markets ($\mathcal{I} = 0$, steep PC, low degree of strategic complementarity): the spending multiplier equals 1.7350, the labour tax cut multiplier -0.3219, and the sales tax cut multiplier 1.1396. For the case of the firm-specific ($\mathcal{I} = 1$, flat PC, high degree of strategic complementarity) the resulting

Table 3: Fiscal multipliers with high ($\mathcal{I} = 1$: specific labour market; flat Phillips curve) and low ($\mathcal{I} = 0$: economy-wide labour market; steep Phillips curve) degree of strategic complementarity

	outside ZLB, DRS		ZLB, DRS	
	Strategic complementarity:			
Multipliers	High degree ($\mathcal{I} = 1$) (flat PC)	Low degree ($\mathcal{I} = 0$) (steep PC)	High degree ($\mathcal{I} = 1$) (flat PC)	Low degree ($\mathcal{I} = 0$) (steep PC)
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}, g > 0$	0.6772	0.6108	1.0767	1.7350
Payroll tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^W}$	0.0173	0.0706	-0.0336	-0.3219
Sales tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^S}$	0.4448	0.4012	0.7073	1.1396
Capital tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^A}$	-0.0068	-0.0055	-0.0115	-0.0218

Notes: For the estimated value of μ in Eggertsson (2011) (our baseline calibration), condition C2 is not satisfied in case of a lower degree of strategic complementarity. Hence, the comparison is accomplished using a lower value of $\mu = .8$ from Christiano et al. (2011). The comparison is made for the case of DRS because C2 in the case of CRS and a lower degree of strategic complementarity is satisfied for the maximum of $\mu = .69$ which may be empirically implausible.

Table 4: The effect of constant-returns-to-scale (CRS, $\phi = 1$: steep Phillips curve) versus decreasing-returns-to-scale (DRS, $\phi = 1.5$: flat Phillips curve), and the effect of positive government spending-to-GDP ratio on the multipliers

	Constant Returns (steep PC)		Decreasing Returns (flat PC)	
Multipliers	no ZLB	ZLB	no ZLB	ZLB
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}, g = 0$	0.4650	2.2858	0.4447	1.9464
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}, g > 0$	0.5208	1.8182	0.5013	1.6366
Payroll tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^W}$	0.0815	-1.0242	0.0472	-0.4145
Sales tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^S}$	0.3818	1.8768	0.3659	1.5982
Capital tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^A}$	-0.0104	-0.0863	-0.0107	-0.0622

Notes: Grey cells contain the values computed from the fiscal multiplier formulas of Eggertsson (2011). Each multiplier is calculated under the assumption of a specific labour market.

multipliers are 1.0767, -0.0336 and 0.7073, respectively.[15] This exercise implies that, in both cases, a unit of government purchases brings more than one unit of GDP, but more so when strategic complementarity is low. Whereas, the case of high degree of strategic complementarity leads to an only mild multiplier effects (the multiplier is slightly higher than one).

Further, the payroll tax-cut multiplier is less negative in the case of a lower degree of strategic complementarity (see -0.03 in the same Table). The latter is consistent with Christiano (2011), who finds in a model similar to ours but containing wage rigidities, that the payroll tax-cut multiplier may be slightly negative or close to zero.

The empirical SVAR literature finds, however, labour tax cuts to have positive effects on the economy. Using the SVAR models with different identifying assumptions regarding tax shocks based on US data, Mertens and Ravn (2012) and Romer and Romer (2010) find that tax-cuts are stimulative. The model in our paper does not address the problem of the negative payroll tax-cut multiplier. Kaszab (2016) modifies the basic New Keynesian model by adding non-Ricardian households and wage rigidity and finds that this model extension changes the sign of the payroll tax-cut multiplier from negative to positive. Wieland (2019) provides empirical evidence on the contractionary effects of negative supply shocks, such as rises in oil prices and the Great East Japan earthquake at the zero lower bound. The standard New Keynesian model predicts the opposite: negative supply shocks are expansionary. Wieland (2019) argues that the inclusion of financial frictions in the New Keynesian model leads to the results in line with the empirical evidence.

Result 3. When the government spending-to-output ratio is positive ($g > 0$), multipliers are higher than with $g = 0$, in the case of positive interest rates for both CRS and DRS. At zero nominal interest rate the government spending multiplier is higher with CRS relative to DRS (irrespective of a positive or zero choice for g). In the case of zero nominal interest rate, the difference is larger between the size of government spending multipliers across CRS and DRS with $g = 0$ than with $g > 0$.

The comparison of the multipliers with positive or zero government spending-to-output

[15] Multipliers with either low or high degrees of strategic complementarity in the case of DRS are not directly comparable with $\mu = 0.903$ (the estimated value of Eggertsson (2011) and our baseline parametrisation) when $i_t = 0$ because C2 is not satisfied. Table 3, instead, uses a value of $\mu = 0.80$, under which C2 is satisfied again.

In the absence of a specific factor market ($\mathcal{I} = 0$), $g > 0$ and DRS ($\phi = 3/2$) the maximum value of μ that satisfies condition C2 is 0.85. For $\mu = 0.85$ the multiplier is implausibly large. Hence, we use the somewhat lower but empirically still plausible value of $\mu = 0.8$ of Christiano et al. (2011) for comparison.

ratio can be found in Table 4. This Table makes use of the baseline calibration of μ so that our results are comparable to the ones in Eggertsson (2011). The models of Eggertsson (2011) and Woodford (2011) calculate fiscal multipliers under the assumption of a zero steady-state government spending-to-GDP ratio ($g = 0$). Instead, in this paper we also consider the empirically more realistic case of positive steady-state government purchases-to-GDP ratio and show that $g > 0$ has non-negligible impact on the size of the government spending multiplier. When $g > 0$ the value of IES, $\breve{\sigma} = \sigma(1 - g)$, declines and consumers are less willing to substitute present consumption for future consumption after the positive government spending shock, even if the negative wealth effect forces consumers to do so. Thus, a lower $\breve{\sigma}$ results in a smaller consumption loss and a higher multiplier when $i > 0$.

In contrast, multipliers in the case of $i = 0$ become smaller with $g > 0$. When $i = 0$, expansionary fiscal policy leads to a rise in inflation, which –in the absence of a Taylor rule– implies a decline in the real rate. A smaller real rate serves as an incentive for households to consume more in the present and, thereby, increases the multiplier. However, as our results presented in Table 4 show, this incentive is less strong with smaller a IES ($\breve{\sigma} < \sigma$ due to $g > 0$).

Result 4. Multipliers (in absolute value) in the case of DRS are lower than those for CRS irrespective of whether $i > 0$ or $i = 0$. The presence of DRS in production can itself imply strategic complementarity even in the absence of a specific labour market because DRS reduces κ (see the term, $(\phi-1)\theta$, in the denominator of κ in Equation (2)). Multipliers in case of $i > 0$ do not differ a lot across CRS and DRS. However, for $i = 0$ we observe that the government spending multiplier in case of $g = 0$ with DRS (1.94) is lower than with CRS (2.28) and the largest is the difference for payroll tax cut (-1.02 and -0.41 for CRS and DRS, respectively).

7 Robustness checks – results on fiscal multipliers obtained from non-linear solution method

This section presents results from a robustness exercise with respect to the solution method. So far, the results presented stem from a log-linear approximation, for which a closed-form solution can be derived. A number of authors have computed fiscal multipliers also in a fully non-linear setting[16], with somewhat differing findings. While Eggertsson and Singh

[16] A non-exhaustive list of references includes, Miao and Ngo (2019), Fernandez-Villaverde et al. (2015), Boneva et al. (2016), Eggertsson and Singh (2016), Nakata (2016), Richter and Throckmorton (2016),

(2016) find that multipliers from a linear model are similar to their non-linear counterparts, other contributions have found significant differences, namely, that multipliers tend to be smaller when computed from a non-linear method (see, e.g. Boneva et al. (2016) and Lindé and Trabandt (2018)). As a consequence, we also derive numerical results, equivalent to the ones presented in Table 3 and 4, but computed from a global approximation method. The method used is time iteration (cf. Coleman (1990, 1991), which amounts to computing, given some initial guesses, the solution to the exact non-linear system of first order and equilibrium conditions over a grid of fixed points, and then iterating on the guesses until convergence. We choose 31 gridpoints for the endogenous state variable (price dispersion) and 5 gridpoints for each of the four exogenous state variables, G_t, τ_t^W, τ_t^S, τ_t^A. The exogenous continuous AR(1) processes are discretized using the method of Rouwenhorst (1995). We use linear interpolation for computing the solution in between gridpoints. We iterate on guesses of the conditional expectation appearing in the Euler equation, and in the two auxiliary equations of the Calvo price setting problem. The algorithm is layed out in detail in Rabitsch (2012, 2016). Tables 5 and 6 repeat the exercises of section 6 and present fiscal multipliers for various scenarios from the global solution. As is well known, in a non-linear setting, the size of shocks affects the solution and thus the size of fiscal multipliers. Unless noted otherwise, in the computations below we set $\sigma_{G,t} = 0.01$, $\sigma_{\tau^W,t} = 0.009$, $\sigma_{\tau^S,t} = 0.009$, and $\sigma_{\tau^A,t} = 0.001(1-\beta)$. Else, parameters take on the values summarised in the calibration section, Table 2. Table5 presents the results for the different degrees of strategic complementarity, from the assumptions of either a firm-specific ($\mathcal{I} = 1$) or an economy-wide ($\mathcal{I} = 0$) labour market. The upper part presents multipliers at the zero lower bound for a 'Great Depression' scenario, where the size of the shock that puts the economy into a ZLB is such that output drops by about 30 percent – the table also reports the size of the ZLB-shock, and the implied drops in output and (annualized) inflation in percent. Unfortunately, for the 'Great Depression' scenario, the solution for the economy-wide labour market ($\mathcal{I} = 0$) cannot be obtained at the given set of parameters. We do not find this surprising, as, in fact, because of the steepness of the Phillips curve in the economy-wide labor market setting under the given set of parameters, the implied changes in inflation that accompany a 30 percent drop in output, would be enormous.[17] Table 5 thus proceeds in two steps. The upper part, presenting the 'Great

Lindé and Trabandt (2018), and Belgibayeva and Horvath (2019).

[17]To make this point more precisely: we also computed the sets of multipliers from a quasi-nonlinear solution, 'Occbin', of Guerrieri and Iacoviello (2015). In this case, a solution can be obtained, and it provides

Depression'-scenario results for the case of $\mathcal{I} = 1$, allows contrasting the global results for this case to the multipliers obtained from the linear method (summarised in Table 3). The lower part compares multipliers from scenarios $\mathcal{I} = 1$ and $\mathcal{I} = 0$, for a setting where they can be computed in both cases (a 'Great Recession' scenario, of a ZLB-shock sized such that a drop of output of 10 percent results; in addition, the shock sizes are scaled by one-half their regular size). The latter scenario allows a direct comparison between the cases of flat versus steep Phillips curves (respectively, high versus low degrees of strategic complementarity) in the global solution. Finally, Table 6 presents the parallel set of results for the cases of CRS versus DRS – always under a 'Great Depression' scenario.

The following set of results emerges: multipliers in normal times, when the nominal interest rate is positive, are roughly similar in size compared to the multipliers obtained from the linear solution; when the interest rate is at a ZLB, the multipliers are typically substantially smaller than under the linear method throughout. We thus confirm the insights from Boneva et al. (2016) or Lindé and Trabandt (2018). Nonetheless, almost all main results established in section 6 for the linear method, as well as the ordering of multipliers across the different scenarios, continue to hold. Table 5 documents that the government spending and sales tax multiplier in normal times is higher under a flat Phillips curve or high degree of strategic complementarity (Result 1). Table 5 and 6 document that, at the zero lower bound, multipliers are larger in absolute magnitude (compared to normal times), because the monetary authority no longer counteract the effects of a fiscal stimulus at fixed nominal interest rates; now, a steeper Phillips curve implies a larger inflation increase in response to a fiscal expansion, so that multipliers are larger in this case (Result 2). We continue to find that government spending multipliers computed for the case of a positive government-spending-to-GDP ratio ($g > 0$) exceed their counterparts when $g = 0$ in normal times, at positive interest rates (Result 3). Unlike in the results based on the linear method, this situation does not change when turning to times of a binding ZLB:

an indication of what may be the source of the difficulties of solving this model-scenario fully non-linearly: in the Occbin-solution of this scenario, an output drop of 30 would be accompanied by a 21 percent drop in inflation. This indicates a clear counterfactual behavior of the economy-wide model version under this set of parameters. One would, in fact, need to re-calibrate this model version, to obtain realistic scenarios of a -30 percent output and -10 percent inflation response. Eggertsson and Singh (2016) follow this strategy, estimating the set of parameters needed to achieve such Great Depression scenario (even though not for a model version of economy-wide labor markets). This is, however, not our main exercise. We are interested in portraying how fiscal multipliers are affected as the slope of the Phillips curve steepens. A re-calibration of the economy-wide model version, so that the inflation response is more in line with the experience in the Great Depression would then require a parameter combination that implies a somewhat less steep Phillips curve again.

Table 5: Results from the global solution: fiscal multipliers with high ($\mathcal{I} = 1$: specific labour market; flat Phillips curve) and low ($\mathcal{I} = 0$: economy-wide labour market; steep Phillips curve) degree of strategic complementarity

	Strategic complementarity:			
Multipliers	High degree ($\mathcal{I} = 1$) (flat PC)	Low degree ($\mathcal{I} = 0$) (steep PC)	High degree ($\mathcal{I} = 1$) (flat PC)	Low degree ($\mathcal{I} = 0$) (steep PC)
			Great Depression scenario	
	outside ZLB, DRS		ZLB, DRS	
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}, g > 0$	0.5764	–	1.3266	–
Payroll tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^W}$	0.0240	–	-0.0706	–
Sales tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^S}$	0.3081	–	0.4220	–
Capital tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^A}$	-0.0098	–	-0.0155	–
Size of ZLB-shock,	–	–	0.1137	–
implied change in Y,	–	–	-30.0154	–
implied change in π	–	–	-5.0099	–
			Great Recession scenario	
	outside ZLB, DRS		ZLB, DRS	
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}, g > 0$	0.5555	0.4928	0.8925	1.0151
Payroll tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^W}$	0.0381	0.0990	-0.0245	-0.1437
Sales tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^S}$	0.2925	0.2626	0.3962	0.4640
Capital tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^A}$	-0.0085	-0.0057	-0.0135	-0.0184
Size of ZLB-shock,	–	–	0.0436	0.0309
implied change in Y,	–	–	-10.0017	-10.0000
implied change in π	–	–	-1.5657	-8.0830

Table 6: Results from the global solution: The effect of constant-returns-to-scale (CRS, $\phi = 1$: steep Phillips curve) versus decreasing-returns-to-scale (DRS, $\phi = 1.5$: flat Phillips curve), and the effect of positive government spending-to-GDP ratio on the multipliers

	Constant Returns (steep PC)		Decreasing Returns (flat PC)	
Multipliers	no ZLB	ZLB	no ZLB	ZLB
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}, g = 0$	0.4679	1.4380	0.4490	1.3929
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}, g > 0$	0.5922	1.5150	0.5761	1.4268
Payroll tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^W}$	0.0665	-0.4269	0.0387	-0.1563
Sales tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^S}$	0.3850	0.7988	0.3700	0.7852
Capital tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^A}$	-0.0109	-0.0366	-0.0111	-0.0294
Size of ZLB-shock,	–	0.0184	–	0.0265
implied change in Y,	–	-30.0046	–	-30.0006
implied change in π	–	-8.3078	–	-5.3877

they continute to be larger for the case of $g > 0$ compared to $g = 0$. Finally, multipliers continue to be lower under DRS than under CRS, irrespective of whether $i > 0$ or $i = 0$ (see Result 4).

8 Concluding remarks

We generalize the New Keynesian model of Eggertsson (2011), calibrate it to US data and show how the size of fiscal multipliers depends on the slope of the Phillips curve. The variations in the slope of the Phillips curve we consider result from differing degrees of strategic complementarity in price setting, from assuming either a firm-specific or an economy-wide labour market, or from considering a constant-returns-to-scale versus a decreasing-returns-to scale production function. Using our extended model, we calibrate two scenarios: a scenario of normal times, with positive interest rates, and a scenario of crisis times, in which a shock moves the economy temporarily into a state of a deep recession, at which the zero lower bound is binding.

The previous literature finds very high fiscal multipliers when the economy is at the zero lower bound. We show that the introduction of strategic complementarity reduces multipliers at the zero lower bound due to the fact that higher strategic complementarity decreases the slope of the Phillips curve and the fiscal stimulus induces less inflation and a smaller reduction in the real interest rate, which is the driver of private spending.

Outside the zero lower bound (in normal times) multipliers are not much different either with high or low degree of strategic complementarity and remain below one. The payroll tax-cut multiplier is also less negative (smaller in absolute value) in case of a higher degree of strategic complementarity at the zero lower bound. Overall, our findings suggest that the size of fiscal multipliers are quite sensitive to degree of strategic complementarity in price setting at the zero lower bound.

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Appendix A Technical Appendix

A.1 Abbreviations

Some notations are here to explain shorthands in this appendix:

$$
\begin{aligned}
\text{AD} &= \text{Aggregate Demand} \\
\text{AS} &= \text{Aggregate Supply} \\
g &\equiv \frac{G}{Y} \text{ steady-state government spending-to-GDP ratio} \\
\text{CRS} &= \text{constant returns to scale technology} \\
\text{DRS} &= \text{decreasing returns to scale technology}
\end{aligned}
$$

A.2 Derivation of the AD curve when $g > 0$

Note that economy-wide or specific labour market will influence the AS curve (derived in detail below) and AD curve is only affected by the choice of g (positive or zero).

The AD curve is the loglinear version of Euler equation based on separable preferences. The consumption Euler equation can be written as:

$$
E_t \left\{ \frac{u'(Y_{t+1} - G^N_{t+1})}{u'(Y_t - G^N_t)} (1 - \tau^A_{t+1}) R^{-1}_{t+1} \right\} = \beta^{-1} E_t \left\{ \frac{\xi_t}{\xi_{t+1}} \frac{(1 + \tau^S_{t+1})}{(1 + \tau^S_t)} \frac{P_{t+1}}{P_t} \right\}.
$$

In the previous equation we substituted in the aggregate resource constraint ($Y_t = C_t + G^N_t$) for consumption (C_t).

The previous equation can be log-linearised around the zero inflation non-stochastic steady-state as:

$$
\widehat{Y}_t = E_t \widehat{Y}_{t+1} - \check{\sigma}(i_t - E_t \pi_{t+1} - r^e_t) + (\widehat{G}_t - E_t \widehat{G}_{t+1}) + \check{\sigma}\chi^S(\widehat{\tau}^S_{t+1} - \widehat{\tau}^S_t) + \check{\sigma}\chi^A \widehat{\tau}^A_t. \quad \text{(A.1)}
$$

In the previous equation the following definitions are applied:

$$
\begin{aligned}
\check{\sigma} &\equiv -\frac{\overline{u}_c}{\overline{u}_{cc}\overline{C}} \frac{\overline{C}}{\overline{Y}} = -\frac{\overline{u}_c}{\overline{u}_{cc}\overline{C}} s_C = -\frac{\overline{u}_c}{\overline{u}_{cc}\overline{C}} (1 - g) = \sigma(1 - g), \\
\chi^S &\equiv \frac{1}{1 + \bar{\tau}^S}, \qquad \chi^W \equiv \frac{1}{1 - \bar{\tau}^W}, \qquad \chi^A \equiv \frac{1 - \beta}{1 - \bar{\tau}^A}.
\end{aligned}
$$

Variables with a hat denote percentage deviation from steady-state: e.g. $\widehat{Y}_t \equiv \log(Y_t/\bar{Y}) \approx (Y_t - \bar{Y})/\bar{Y}$ where the upper bar denotes steady-state. Note that government spending is denifed relative to steady-state GDP as in Eggertsson: $\widehat{G}_t \equiv (G_t - \bar{G})/\bar{Y}$. The tax rates are already in per cent so they are defined as deviation from their steady-states: $\hat{\tau}_t^i \equiv \tau_t^i - \bar{\tau}^i$, $i \in \{A, S, W\}$. The discount factor shock which makes the zero lower bound binding is defined as: $r_t^e \equiv \log \beta^{-1} + E_t(\hat{\xi}_t - \hat{\xi}_{t+1})$ where $\hat{\xi}_t \equiv \log(\xi_t/\bar{\xi})$. Inflation is defined as: $\pi_t = \log(P_t/P_{t-1})$.

Government spending is wasteful spending in our paper (denoted with superscript N in Eggertsson (2011), we simply dropped the superscript N from $\widehat{G}_t$). We can see that the introduction of positive g results in a redefinition of the intertemporal elasticity of substitution (the original IES is σ and the redefined one is $\breve{\sigma} \equiv \sigma(1-g)$):

A.3 Derivation of AS curves

A.3.1 Economy-wide labour market ($g = 0$ and CRS)

It is the same as in Woodford (2011) who sketches the derivation. It can also be found more detailed in Woodford (2003). The AS curve for economy-wide factor market in Eggertsson (2011) is achieved by setting $\omega\theta = 0$ in the definition of κ which can be found in his footnote 13.

A.3.2 Firm-specific labour market ($g = 0$ and CRS)

This is the same as the one in Eggertsson (2011).

A.3.3 Firm specific labour market ($g > 0$ and DRS)

This is the most general case and it is derived here (note that the $g = 0$, CRS and economy-wide labour market are simply parameter restrictions of this more general setup). Let us start from the FOC of intermediary firm i (this is the optimality condition of the Calvo firm which chooses the price p_t^* optimally at time t taking into account with probability α it will stuck with this optimal price for T periods $T > t$:

$$\sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_T}\right)^{-\theta-1} Y_T \left[\frac{p_t^*}{P_T} - \frac{\theta}{\theta - 1} mc_{t,T}(i)\right] = 0,$$

and let us focus on the terms in the square bracket, []:

$$
\begin{aligned}
0 &= \frac{p_t^*}{P_T} - \frac{\theta}{\theta-1}\frac{(W_T/P_T)}{(1/\phi)[l_T(j)]^{1/\phi-1}} \\
&= \frac{p_t^*}{P_T} - \frac{\theta}{\theta-1}\frac{\frac{1+\tau_T^S}{1-\tau_T^W}\frac{v_l(l_T(j))}{u_c(Y_T-G_T)}}{(1/\phi)[l_T(j)]^{(1-\phi)/\phi}} \\
&= \frac{p_t^*}{P_T} - \frac{\theta}{\theta-1}\frac{1+\tau_T^S}{1-\tau_T^W}\frac{v_l\left(l_T(j)\right)}{u_c(Y_T-G_T)}\phi[l_T(j)]^{(\phi-1)/\phi} \\
&= \frac{p_t^*}{P_T} - \frac{\theta}{\theta-1}\frac{1+\tau_T^S}{1-\tau_T^W}\frac{v_l\left(l_T(j)\right)}{u_c(Y_T-G_T)}\phi[Y_T(j)]^{(\phi-1)} \\
&= \frac{p_t^*}{P_T} - \frac{\theta}{\theta-1}\frac{1+\tau_T^S}{1-\tau_T^W}\frac{v_l\left(\left[\left(\frac{p_t^*}{P_T}\right)^{-\theta}Y_T\right]^{\phi}\right)}{u_c(Y_T-G_T)}\phi\left[\left(\frac{p_t^*}{P_T}\right)^{-\theta}Y_T\right]^{(\phi-1)}
\end{aligned}
$$

where in the first line we made us of the definition of the marginal cost: $mc_t = (W_t/P_t)/MPL_t$ with W_t/P_t meaning the real wage and MPL_t denoting the marginal product of labour derived from the DRS production function in the main text. Note that we substituted the intratemporal condition for the real wage in the second row and used the production function in the fourth row. The last row uses the demand curve of variety i.

Next we log-linearise the FOC[18] as follows:

$$
\begin{aligned}
\hat{p}_t^* - \log\left(\prod_{i=1}^{T}\Pi_{t+i}\right) &= \frac{v_{ll}}{v_l}\bar{l}\hat{l}_T - \frac{u_{cc}C}{u_c}\frac{Y}{C}\hat{Y}_T + (\phi-1)\hat{Y}_T + \frac{u_{cc}}{u_c}C\frac{Y}{C}\hat{G}_T \\
&+ \left[-\theta\phi\frac{v_{ll}\bar{l}}{v_l} - \theta(\phi-1)\right]\left[\hat{p}_t^* - \log\left(\prod_{i=1}^{T}\Pi_{t+i}\right)\right] \\
&+ \frac{1}{1+\tau^S}\hat{\tau}_T^S + \frac{1}{1-\tau^W}\hat{\tau}_T^W.
\end{aligned}
$$

where $\hat{p}_t^* \equiv \log(p_t^*/P_t)$, $\hat{l}_T \equiv \frac{l_T-\bar{l}}{\bar{l}}$, $\hat{Y}_T \equiv \frac{Y_T-\bar{Y}}{\bar{Y}}$, $\hat{G}_T \equiv \frac{G_T-\bar{G}}{\bar{Y}}$, $\hat{\tau}_T^i \equiv \tau_T - \bar{\tau}$, $i = \{S, W\}$. Let

[18] Note that it is enough to log-linearise the expression in the square bracket due to the fact that the steady-state in the squared bracket is zero and therefore all loglinear terms outside the bracket would be multiplied by zero.

us re-arrange some terms:

$$\left[1+\theta\phi\frac{v_{ll}\bar{l}}{v_l}+\theta(\phi-1)\right]\hat{p}_t^* = \frac{v_{ll}}{v_l}\bar{l}\hat{l}_T - \frac{u_{cc}C}{u_c}\frac{Y}{C}\hat{Y}_T + (\phi-1)\hat{Y}_T + \frac{u_{cc}}{u_c}C\frac{Y}{C}\hat{G}_T$$
$$+\left[1+\theta\phi\frac{v_{ll}\bar{l}}{v_l}+\theta(\phi-1)\right]\sum_{\tau=t+1}^{T}\pi_\tau + \frac{1}{1+\tau^S}\hat{\tau}_T^S + \frac{1}{1-\tau^W}\hat{\tau}_T^W$$

where $\log\left(\prod_{i=1}^{T}\Pi_{t+i}\right) \equiv \sum_{\tau=t+1}^{T}\pi_\tau$. In the next, we introduce notations for the elasticities:

$$\hat{p}_t^*[1+\theta\phi\omega+\theta(\phi-1)] = \left[\omega\phi+\check{\sigma}^{-1}+(\phi-1)\right]\hat{Y}_T - \check{\sigma}^{-1}\hat{G}_T$$
$$+[1+\theta\phi\omega+\theta(\phi-1)]\sum_{\tau=t+1}^{T}\pi_\tau + \chi^S\hat{\tau}_T^S + \chi^W\hat{\tau}_T^W$$

where $\check{\sigma} \equiv -\frac{\bar{u}_c}{\bar{u}_{cc}C}\frac{C}{Y} = \sigma(1-g)$, $\omega \equiv \frac{v_{ll}\bar{l}}{v_l}$, $\chi^S \equiv \frac{1}{1+\bar{\tau}^S}$, $\chi^W \equiv \frac{1}{1-\bar{\tau}^W}$.

Further, let us work again with the full expression:

$$\begin{aligned}\hat{p}_t^* &= (1-\alpha\beta)\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left[[1+\omega_y\theta]^{-1}\widehat{mc}_T + \sum_{\tau=t+1}^{T}\pi_\tau\right] \\ &= \left(\frac{1-\alpha\beta}{1+\omega_y\theta}\right)\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}E_t\widehat{mc}_T + \sum_{T=t+1}^{\infty}(\alpha\beta)^{T-t}\pi_T \end{aligned}\tag{A.2}$$

where $\omega_y \equiv \phi\omega + (\phi-1) = \phi(1+\omega)-1$ and

$$\widehat{mc}_T = \left[\omega\phi+\check{\sigma}^{-1}+(\phi-1)\right]\hat{Y}_T - \check{\sigma}^{-1}\hat{G}_T + \chi^S\hat{\tau}_T^S + \chi^W\hat{\tau}_T^W.$$

Let us then quasi-difference the equation (A.2) to obtain:

$$\hat{p}_t^* = \left(\frac{1-\alpha\beta}{1+\omega_y\theta}\right)\widehat{mc}_t + \alpha\beta E_t\pi_{t+1} + \alpha\beta E_t\hat{p}_{t+1}^*$$

which together with the log-linear version of the price index,

$$\pi_t = \frac{1-\alpha}{\alpha}\hat{p}_t^*$$

results in what we call NKPC:

$$\pi_t = \kappa \hat{Y}_t + \kappa\psi(\chi^W \hat{\tau}_t^W + \chi^S \hat{\tau}_t^S - \check{\sigma}^{-1}\hat{G}_t) + \beta E_t \pi_{t+1} \tag{A.3}$$

where the parameters for separable preferences are

$$\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)\vartheta}{\alpha}; \vartheta \equiv \frac{\phi(1+\omega) - 1 + \check{\sigma}^{-1}}{1+\omega_y\theta}; \qquad \psi \equiv \frac{1}{\phi(1+\omega)-1+\check{\sigma}^{-1}};$$
$$\omega_y \equiv \phi(1+\mathcal{I}\omega) - 1; \qquad \omega \equiv \frac{\bar{v}_{ll}\bar{l}}{\bar{v}_l}; \qquad \chi^W \equiv \frac{1}{1-\bar{\tau}^W},$$

where $\mathcal{I}$ is an indicator variable which takes on the value of one when we assume specific labour market. DRS in production, $\phi > 1$, can also induce strategic complementarity even under economy-wide labour markets.

For $\phi = 1$, $g = 0$, $\mathcal{I} = 1$ the setup of Eggertsson (2011) is obtained.

For $\phi = 1$, $g = 0$, $\mathcal{I} = 0$ the setup of Woodford (2011) is obtained.

Appendix B Derivation of fiscal multipliers in Tables 3 and 4

B.1 Short run, positive nominal interest, $i > 0$

To derive multipliers under positive nominal interest rate we re-write the AD curve using the method of undetermined coefficients:

$$\pi_S = A_\pi \hat{F}_S, \tag{A.4}$$

$$\hat{Y}_t = A_Y \hat{F}_S, \tag{A.5}$$

for $\hat{F}_S = \{\hat{G}_S, \hat{\tau}_S^W, \hat{\tau}_S^S, \hat{\tau}_S^A\}$ and the Taylor rule

$$i_t = r_t^e + \phi_\pi \pi_t + \phi_Y \hat{Y}_t, \tag{A.6}$$

to express output as a function of the fiscal variable $\hat{F}_S$. The fiscal multiplier is given by A_Y.

B.1.1 Government spending

Let us substitute for π_t and $E_t\hat{Y}_{t+1}$ equations (A.4) and (A.5) and for i_{t+1} the Taylor rule (equation (A.6)) in the AD formula: (see equation (A.1)):

$$\begin{aligned}\widehat{Y}_t &= \rho A_Y \widehat{G}_t + (1-\rho)\widehat{G}_t - \check{\sigma}\left(r_t^e + \phi_\pi A_\pi \widehat{G}_t + \phi_Y \widehat{Y}_t - r_t^e\right)\\ &\quad + \check{\sigma} A_\pi \rho \widehat{G}_t + \check{\sigma}\chi^S(\rho\widehat{\tau}_t^S - \widehat{\tau}_t^S) + \check{\sigma}\chi^A\widehat{\tau}_t^A\end{aligned}$$

where we used the method of undetermined coefficients—described in the main text—when substituting $A_\pi \widehat{G}_t$ for π_t and $\rho A_Y \widehat{G}_t$ for $E_t\widehat{Y}_{t+1}$. The latter also made use of the fact that government spending—similarly to other fiscal instruments—follows an AR(1) process with a persistence parameter ρ. Under positive nominal interest rates the discount factor is not time-varying and does not deviate from its steady-state, $r_t^e = 0$.

In the next we plug in the guess for time $t+1$ variables:

$$\begin{aligned}[1+\check{\sigma}\phi_2]\hat{Y}_t &= A_Y\rho\widehat{G}_t - \check{\sigma}A_\pi\phi_\pi\widehat{G}_t + \check{\sigma}A_\pi\rho\widehat{G}_t + (\widehat{G}_t - \rho\widehat{G}_t)\\ &\quad + \check{\sigma}\chi^S(\rho\widehat{\tau}_t^S - \widehat{\tau}_t^S) + \check{\sigma}\chi^A\widehat{\tau}_t^A\end{aligned}$$

where we set fiscal instruments other than government spending equal to zero ($\widehat{\tau}_t^W = \widehat{\tau}_t^S = \widehat{\tau}_t^A = 0$) and obtain:

$$[1+\check{\sigma}\phi_2]\hat{Y}_t = A_Y\rho\widehat{G}_t - A_\pi\check{\sigma}[\phi_\pi - \rho]\widehat{G}_t + (1-\rho)\widehat{G}_t \tag{A.7}$$

To proceed we need a formula that replaces A_π as a linear function of A_Y. To do so, we need to re-write the NKPC using undetermined coefficients. First, recall NKPC and use equation (A.4) and (A.5) to substitute for $\hat{Y}_t$, π_t and π_{t+1} together with the AR(1) process for the fiscal shock.

$$(1-\beta\rho)A_\pi\widehat{G}_t = [\kappa A_Y - \kappa\psi\check{\sigma}^{-1}]\widehat{G}_t.$$

Then it follows that $A_\pi = \frac{\kappa A_Y - \kappa\psi\check{\sigma}^{-1}}{1-\beta\rho}$ that can be inserted into equation (A.7):

$$\begin{aligned}[1+\check{\sigma}\phi_2]\hat{Y}_t &= A_Y\rho\widehat{G}_t - \frac{(\kappa A_Y - \kappa\psi\check{\sigma}^{-1})}{1-\beta\rho}\check{\sigma}[\phi_\pi-\rho]\widehat{G}_t + (1-\rho)\widehat{G}_t\\ &= \left[A_Y\rho - \frac{(\kappa A_Y - \kappa\psi\check{\sigma}^{-1})}{1-\beta\rho}\check{\sigma}[\phi_\pi-\rho] + (1-\rho)\right]\widehat{G}_t\end{aligned}$$

And

$$A_Y = \frac{A_Y\rho - \frac{(\kappa A_Y - \kappa\psi\breve{\sigma}^{-1})}{1-\beta\rho}\breve{\sigma}[\phi_\pi - \rho] + (1-\rho)}{1+\breve{\sigma}\phi_2}$$

And

$$A_Y\left[1 - \frac{\rho}{1+\breve{\sigma}\phi_2} + \frac{\kappa\breve{\sigma}[\phi_\pi - \rho]}{(1-\beta\rho)(1+\breve{\sigma}\phi_2)}\right] = \frac{\kappa\psi[\phi_\pi - \rho]}{(1-\beta\rho)(1+\breve{\sigma}\phi_2)} + \frac{(1-\rho)}{(1+\breve{\sigma}\phi_2)}$$

Finally

$$\begin{aligned} A_Y &= \frac{\frac{\kappa\psi[\phi_\pi-\rho]}{(1-\beta\rho)(1+\breve{\sigma}\phi_2)} + \frac{(1-\rho)}{(1+\breve{\sigma}\phi_2)}}{1 - \frac{\rho}{1+\breve{\sigma}\phi_2} + \frac{\kappa\breve{\sigma}[\phi_\pi-\rho]}{(1-\beta\rho)(1+\breve{\sigma}\phi_2)}} \\ &= \frac{\kappa\psi[\phi_\pi-\rho] + (1-\rho)(1-\beta\rho)}{(1-\beta\rho)(1+\breve{\sigma}\phi_2) - \rho(1-\beta\rho) + \kappa\breve{\sigma}[\phi_\pi-\rho]} \end{aligned}$$

which is the same as the one reported by Eggertsson (2011). Note that extensions in our paper modify the content of σ and κ. In particular, when allowing for positive g, the σ changes to $\breve{\sigma} \equiv \sigma(1-g)$. Further, the introduction of either DRS or specific labour market leads to lower κ implying higher degree of strategic complementarity in price-setting.

B.1.2 labour tax cut

Recall the AD curve:

$$\begin{aligned} [1+\breve{\sigma}\phi_2]\hat{Y} = {} & A_Y\rho\widehat{G}_t + \breve{\sigma}r_t^e - \breve{\sigma}A_\pi\phi_\pi\widehat{G}_t + \breve{\sigma}A_\pi\rho\widehat{G}_t - \breve{\sigma}r_t^e + (\widehat{G}_t - \rho\widehat{G}_t) \\ & + \breve{\sigma}\chi^S(\rho\widehat{\tau}_t^S - \widehat{\tau}_t^S) + \breve{\sigma}\chi^A\widehat{\tau}_t^A. \end{aligned}$$

As we focus only on $\widehat{\tau}_t^W$only we can set $\widehat{\tau}_t^S = \widehat{\tau}_t^A = \widehat{G}_t = 0$:

$$[1+\breve{\sigma}\phi_2]\hat{Y}_t = A_Y\rho\widehat{\tau}_t^W - \breve{\sigma}A_\pi\phi_\pi\widehat{\tau}_t^W + \breve{\sigma}A_\pi\rho\widehat{\tau}_t^W$$

and use NKPC to obtain $A_\pi = \frac{\kappa}{1-\beta\rho}\left[A_Y + \psi\chi^W\right]$ which can be substituted back to the previous equation to arrive at:

$$[1+\breve{\sigma}\phi_2]\hat{Y}_t = A_Y\rho\widehat{\tau}_t^W - \breve{\sigma}r_t^e - \frac{\kappa}{1-\beta\rho}\left[A_Y + \psi\chi^W\right]\breve{\sigma}(\phi_\pi - \rho)\widehat{\tau}_t^W + \breve{\sigma}r_t^e$$

or

$$[1+\breve{\sigma}\phi_2]\hat{Y}_t = A_Y\rho\widehat{\tau}_t^W - \frac{\kappa}{1-\beta\rho}\left[A_Y + \psi\chi^W\right]\breve{\sigma}(\phi_\pi - \rho)\widehat{\tau}_t^W$$

$$\hat{Y}_t = \left[\frac{A_Y\rho}{1+\breve{\sigma}\phi_2} - \frac{\frac{\kappa}{1-\beta\rho}\left[A_Y + \psi\chi^W\right]\breve{\sigma}(\phi_\pi-\rho)}{1+\breve{\sigma}\phi_2}\right]\widehat{\tau}_t^W$$

$$A_Y = \frac{A_Y\rho}{1+\breve{\sigma}\phi_2} - \frac{\frac{\kappa}{1-\beta\rho}\left[A_Y + \psi\chi^W\right]\breve{\sigma}(\phi_\pi-\rho)}{1+\breve{\sigma}\phi_2}$$

$$\left[1 - \frac{\rho}{1+\breve{\sigma}\phi_2} + \frac{\frac{\kappa}{1-\beta\rho}\breve{\sigma}(\phi_\pi-\rho)}{1+\breve{\sigma}\phi_2}\right]A_Y = -\frac{\frac{\kappa}{1-\beta\rho}\psi\chi^W\breve{\sigma}(\phi_\pi-\rho)}{1+\breve{\sigma}\phi_2}$$

$$\begin{aligned}
A_Y &= \frac{-\frac{\kappa\psi\chi^W\breve{\sigma}(\phi_\pi-\rho)}{(1-\beta\rho)(1+\breve{\sigma}\phi_2)}}{1 - \frac{\rho}{1+\breve{\sigma}\phi_2} + \frac{\kappa\breve{\sigma}(\phi_\pi-\rho)}{(1-\beta\rho)(1+\breve{\sigma}\phi_2)}} \\
&= -\frac{\kappa\psi\chi^W\breve{\sigma}(\phi_\pi-\rho)}{(1-\beta\rho)(1+\breve{\sigma}\phi_2) - \rho(1-\beta\rho) + \kappa\breve{\sigma}(\phi_\pi-\rho)} \\
&= -\frac{\kappa\psi\chi^W\breve{\sigma}(\phi_\pi-\rho)}{(1-\beta\rho)(1-\rho+\breve{\sigma}\phi_2) + \kappa\breve{\sigma}(\phi_\pi-\rho)}
\end{aligned}$$

which is the same as the one reported in the main text.

B.1.3 Sales tax cut

Recall the AD curve:

$$\begin{aligned}
[1+\breve{\sigma}\phi_2]\hat{Y} &= A_Y\rho\widehat{\tau}_t^S - \breve{\sigma}r_t^e - \breve{\sigma}A_\pi\phi_\pi\widehat{\tau}_t^S + \breve{\sigma}A_\pi\rho\widehat{\tau}_t^S + \breve{\sigma}r_t^e + \breve{\sigma}\chi^S(\rho\widehat{\tau}_t^S - \widehat{\tau}_t^S) \\
&= A_Y\rho\widehat{\tau}_t^S - \breve{\sigma}A_\pi(\phi_\pi-\rho)\widehat{\tau}_t^S + \breve{\sigma}\chi^S(\rho-1)\widehat{\tau}_t^S \\
&= \left[A_Y\rho - \breve{\sigma}A_\pi(\phi_\pi-\rho) + \breve{\sigma}\chi^S(\rho-1)\right]\widehat{\tau}_t^S
\end{aligned}$$

Using the NKPC $A_\pi = \frac{\kappa}{1-\beta\rho}\left[A_Y + \psi\chi^S\right]$:

$$[1+\breve{\sigma}\phi_2]\hat{Y} = \left[A_Y\rho - \breve{\sigma}\frac{\kappa}{1-\beta\rho}\left[A_Y+\psi\chi^S\right](\phi_\pi-\rho) + \breve{\sigma}\chi^S(\rho-1)\right]\widehat{\tau}_t^S$$

Now we can express for A_Y by collecting terms on both RHS and LHS:

$$A_Y = \frac{A_Y \rho}{1+\breve{\sigma}\phi_2} - \breve{\sigma}\frac{\kappa}{(1+\breve{\sigma}\phi_2)(1-\beta\rho)}\left[A_Y + \psi\chi^S\right](\phi_\pi - \rho) + \frac{\breve{\sigma}\chi^S(\rho-1)}{1+\breve{\sigma}\phi_2}$$

$$A_Y\left[1 - \frac{\rho}{1+\breve{\sigma}\phi_2} + \frac{\kappa\breve{\sigma}(\phi_\pi-\rho)}{(1+\breve{\sigma}\phi_2)(1-\beta\rho)}\right] = -\frac{\kappa\breve{\sigma}\psi\chi^S(\phi_\pi-\rho)}{(1+\breve{\sigma}\phi_2)(1-\beta\rho)} + \frac{\breve{\sigma}\chi^S(\rho-1)}{1+\breve{\sigma}\phi_2}$$

$$\begin{aligned} A_Y &= -\frac{\frac{\kappa\breve{\sigma}\psi\chi^S(\phi_\pi-\rho)}{(1+\breve{\sigma}\phi_2)(1-\beta\rho)} + \frac{\breve{\sigma}\chi^S(\rho-1)}{1+\breve{\sigma}\phi_2}}{1 - \frac{\rho}{1+\breve{\sigma}\phi_2} + \frac{\kappa\breve{\sigma}(\phi_\pi-\rho)}{(1+\breve{\sigma}\phi_2)(1-\beta\rho)}} \\ &= \frac{-\kappa\breve{\sigma}\psi\chi^S(\phi_\pi-\rho) - \breve{\sigma}\chi^S(1-\rho)(1-\beta\rho)}{(1+\breve{\sigma}\phi_2)(1-\beta\rho) - \rho(1-\beta\rho) + \kappa\breve{\sigma}(\phi_\pi-\rho)} \\ &= \frac{-[\kappa\psi(\phi_\pi-\rho) + (1-\rho)(1-\beta\rho)]\breve{\sigma}\chi^S}{(1+\breve{\sigma}\phi_2-\rho)(1-\beta\rho) + \kappa\breve{\sigma}(\phi_\pi-\rho)} \end{aligned}$$

which is the same as the one reported in the main text.

B.1.4 Capital tax cut

Recall AD curve:

$$[1+\breve{\sigma}\phi_2]\hat{Y}_t = A_Y\rho\widehat{\tau}_t^A - \breve{\sigma}r_t^e - \breve{\sigma}A_\pi\phi_\pi\widehat{\tau}_t^A + \breve{\sigma}A_\pi\rho\widehat{\tau}_t^A + \breve{\sigma}r_t^e + \breve{\sigma}\chi^A\widehat{\tau}_t^A$$

or

$$[1+\breve{\sigma}\phi_2]\hat{Y}_t = A_Y\rho\widehat{\tau}_t^A - \breve{\sigma}A_\pi\phi_\pi\widehat{\tau}_t^A + \breve{\sigma}A_\pi\rho\widehat{\tau}_t^A + \breve{\sigma}\chi^A\widehat{\tau}_t^A$$

or

$$\begin{aligned} [1+\breve{\sigma}\phi_2]\hat{Y}_t &= \left[A_Y\rho - \breve{\sigma}A_\pi(\phi_\pi-\rho) + \breve{\sigma}\chi^A\right]\widehat{\tau}_t^A \\ \hat{Y}_t &= \left[\frac{A_Y\rho}{1+\breve{\sigma}\phi_2} - \frac{\breve{\sigma}A_\pi(\phi_\pi-\rho)}{1+\breve{\sigma}\phi_2} + \frac{\breve{\sigma}\chi^A}{1+\breve{\sigma}\phi_2}\right]\widehat{\tau}_t^A \end{aligned}$$

and using NKPC, $A_\pi = \frac{\kappa A_Y}{1-\beta\rho}$:

$$\hat{Y}_t = \left[\frac{A_Y\rho}{1+\breve{\sigma}\phi_2} - \frac{\breve{\sigma}\frac{\kappa A_Y}{1-\beta\rho}(\phi_\pi-\rho)}{1+\breve{\sigma}\phi_2} + \frac{\breve{\sigma}\chi^A}{1+\breve{\sigma}\phi_2}\right]\widehat{\tau}_t^A$$

or

$$\hat{Y}_t = \left[\frac{A_Y\rho}{1+\check{\sigma}\phi_2} - \frac{\check{\sigma}\kappa A_Y(\phi_\pi - \rho)}{(1+\check{\sigma}\phi_2)(1-\beta\rho)} + \frac{\check{\sigma}\chi^A}{1+\check{\sigma}\phi_2}\right]\hat{\tau}_t^A$$

And we can express for A_Y:

$$A_Y\left[1 - \frac{\rho}{1+\check{\sigma}\phi_2} + \frac{\check{\sigma}\kappa(\phi_\pi - \rho)}{(1+\check{\sigma}\phi_2)(1-\beta\rho)}\right] = \frac{\check{\sigma}\chi^A}{1+\check{\sigma}\phi_2}$$

Finally,

$$\begin{aligned}
A_Y &= \frac{\frac{\check{\sigma}\chi^A}{1+\check{\sigma}\phi_2}}{1 - \frac{\rho}{1+\check{\sigma}\phi_2} + \frac{\check{\sigma}\kappa(\phi_\pi-\rho)}{(1+\check{\sigma}\phi_2)(1-\beta\rho)}} \\
&= \frac{\check{\sigma}\chi^A(1-\beta\rho)}{(1+\check{\sigma}\phi_2)(1-\beta\rho) - \rho(1-\beta\rho) + \check{\sigma}\kappa(\phi_\pi - \rho)} \\
&= \frac{\check{\sigma}\chi^A(1-\beta\rho)}{(1-\rho+\check{\sigma}\phi_2)(1-\beta\rho) + \check{\sigma}\kappa(\phi_\pi - \rho)}
\end{aligned}$$

which is the same as the one reported in the main text.

B.2 Short run, zero nominal interest, $i = 0$

Fiscal policy is activated (e.g. government spending is higher than its steady-state $\widehat{G}_S > 0$) as long as the zero lower bound on the nominal interest rate is binding:

$$\widehat{G}_t = \hat{F}_S > 0 \text{ for } 0 < t < T^e,$$

$$\widehat{G}_t = 0 \text{ for } t \geq T^e,$$

where $\hat{F}_S = \{\hat{G}_S, \hat{\tau}_S^W, \hat{\tau}_S^S, \hat{\tau}_S^A\}$.

Different from the case of positive interest rate the discount factor in this section is the source of the deflationary shock that makes the zero lower bound on the nominal interest rate binding and is negative, $r_t^e < 0$.

Recall that the NKPC is given by

$$\pi_t = \kappa\hat{Y}_t + \kappa\psi(\chi^W\hat{\tau}_t^W + \chi^S\hat{\tau}_t^S - \check{\sigma}^{-1}\hat{G}_t) + \beta E_t\pi_{t+1},$$

and the AD is written as:

$$[\hat{Y}_t - E_t\hat{Y}_{t+1}] = [\hat{G}_t - E_t\hat{G}_{t+1}] - \check{\sigma}\left(i_t - E_t\pi_{t+1} - r_t^e\right) + \chi^S\check{\sigma}E_t\left[\hat{\tau}_{t+1}^S - \hat{\tau}_t^S\right] + \chi^A\check{\sigma}\hat{\tau}_t^A.$$

B.2.1 Government spending

The short-run AD and AS equations when the zero bound binds can be written as (ignoring taxes):

$$\widehat{Y}_S = \mu\widehat{Y}_S + \check{\sigma}\mu\pi_S + \check{\sigma}r_S^e + (1-\mu)\widehat{G}_S$$

$$\pi_S = \kappa\widehat{Y}_S + \beta\mu\pi_S - \kappa\psi\check{\sigma}^{-1}\widehat{G}_S$$

which latter can be expressed for inflation as:

$$\pi_S = \frac{\kappa\widehat{Y}_S - \kappa\psi\check{\sigma}^{-1}\widehat{G}_S}{1-\beta\mu}$$

that can be put back into the AD equation:

$$(1-\mu)\widehat{Y}_S = \check{\sigma}\mu\left[\frac{\kappa\widehat{Y}_S - \kappa\psi\check{\sigma}^{-1}\widehat{G}_S}{1-\beta\mu}\right] + \check{\sigma}r_S^e + (1-\mu)\widehat{G}_S$$

or

$$(1-\mu)\widehat{Y}_S - \frac{\check{\sigma}\mu\kappa\widehat{Y}_S}{1-\beta\mu} = -\frac{\mu\kappa\psi\widehat{G}_S}{1-\beta\mu} + \check{\sigma}r_S^e + (1-\mu)\widehat{G}_S$$

or

$$[(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa]\,\widehat{Y}_S = [(1-\mu)(1-\beta\mu) - \mu\kappa\psi]\,\widehat{G}_S + (1-\beta\mu)\check{\sigma}r_S^e$$

Then, the government spending multiplier is given by:

$$\frac{\Delta\widehat{Y}_S}{\Delta\widehat{G}_S} = \frac{(1-\mu)(1-\beta\mu) - \mu\kappa\psi}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa},$$

which is the same as the one reported in the main text.

B.2.2 labour tax cut

Recall AS

$$\pi_t = \kappa\hat{Y}_t + \kappa\psi(\chi^W\hat{\tau}_t^W + \chi^S\hat{\tau}_t^S - \check{\sigma}^{-1}\hat{G}_t) + \beta E_t\pi_{t+1}$$

and the AD is

$$[\hat{Y}_t - E_t\hat{Y}_{t+1}] = [\hat{G}_t - E_t\hat{G}_{t+1}] - \check{\sigma}\left(i_t - E_t\pi_{t+1} - r_t^e\right) + \chi^S\check{\sigma}\left[\hat{\tau}_{t+1}^S - \hat{\tau}_t^S\right] + \chi^A\check{\sigma}\hat{\tau}_t^A$$

The AD and AS equations when the zero bound binds can be written as:

$$\widehat{Y}_S = \mu\widehat{Y}_S + \check{\sigma}\mu\pi_S + \check{\sigma}r_S^e$$

$$\pi_S = \kappa\widehat{Y}_S + \beta\mu\pi_S + \kappa\psi\chi^W\hat{\tau}_S^W$$

which latter can be expressed for inflation as:

$$\pi_S = \frac{\kappa\widehat{Y}_S + \kappa\psi\chi^W\hat{\tau}_S^W}{1-\beta\mu}$$

that can be put back into the AD equation:

$$(1-\mu)\widehat{Y}_S = \check{\sigma}\mu\left[\frac{\kappa\widehat{Y}_S + \kappa\psi\chi^W\hat{\tau}_S^W}{1-\beta\mu}\right] + \check{\sigma}r_S^e$$

After collecting terms we obtain:

$$\left[(1-\mu) - \frac{\check{\sigma}\mu\kappa}{1-\beta\mu}\right]\widehat{Y}_S = \frac{\check{\sigma}\mu\kappa\psi\chi^W}{1-\beta\mu}\hat{\tau}_S^W + \check{\sigma}r_S^e$$

or

$$\begin{aligned}
\widehat{Y}_S &= \frac{\frac{\check{\sigma}\mu\kappa\psi\chi^W}{1-\beta\mu}}{(1-\mu) - \frac{\check{\sigma}\mu\kappa}{1-\beta\mu}}\hat{\tau}_S^W + \frac{\check{\sigma}}{(1-\mu) - \frac{\check{\sigma}\mu\kappa}{1-\beta\mu}}r_S^e \\
&= \frac{\check{\sigma}\mu\kappa\psi\chi^W}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}\hat{\tau}_S^W + \frac{\check{\sigma}(1-\beta\mu)}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}r_S^e
\end{aligned}$$

Then the labor tax cut multiplier is given by:

$$\frac{\Delta\widehat{Y}_S}{\Delta\hat{\tau}_S^W} = \frac{\check{\sigma}\mu\kappa\psi\chi^W}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa},$$

which is the same as the one reported in the main text.

B.2.3 Sales tax cut (short run, zero nominal interest, $i = 0$)

The AD is:

$$[\hat{Y}_t - E_t\hat{Y}_{t+1}] = [\hat{G}_t - E_t\hat{G}_{t+1}] - \check{\sigma}\left(i_t - E_t\pi_{t+1} - r_t^e\right) + \chi^S\check{\sigma}\left[\hat{\tau}_{t+1}^S - \hat{\tau}_t^S\right] + \chi^A\check{\sigma}\hat{\tau}_t^A$$

The AD and AS equations when the zero bound binds can be written as:

$$\widehat{Y}_S = \mu\widehat{Y}_S + \check{\sigma}\mu\pi_S + \check{\sigma}r_S^e + \chi^S\check{\sigma}(\mu-1)\hat{\tau}_S^S$$

$$\pi_S = \kappa\widehat{Y}_S + \beta\mu\pi_S + \kappa\psi\chi^S\hat{\tau}_S^S$$

which latter can be expressed for inflation as:

$$\pi_S = \frac{\kappa\widehat{Y}_S + \kappa\psi\chi^S\hat{\tau}_S^S}{1-\beta\mu}$$

that can be put back into the AD equation:

$$(1-\mu)\widehat{Y}_S = \check{\sigma}\mu\left[\frac{\kappa\widehat{Y}_S + \kappa\psi\chi^S\hat{\tau}_S^S}{1-\beta\mu}\right] + \check{\sigma}r_S^e + \chi^S\check{\sigma}(\mu-1)\hat{\tau}_S^S$$

And

$$\left[(1-\mu) - \frac{\check{\sigma}\mu\kappa}{1-\beta\mu}\right]\widehat{Y}_S = \left[\frac{\check{\sigma}\mu\kappa\psi\chi^S}{1-\beta\mu} + \chi^S\check{\sigma}(\mu-1)\right]\hat{\tau}_S^S + \check{\sigma}r_S^e$$

and

$$\begin{aligned}
\widehat{Y}_S &= \frac{\frac{\check{\sigma}\mu\kappa\psi\chi^S}{1-\beta\mu} + \chi^S\check{\sigma}(\mu-1)}{(1-\mu) - \frac{\check{\sigma}\mu\kappa}{1-\beta\mu}}\hat{\tau}_S^S + \frac{\check{\sigma}}{(1-\mu) - \frac{\check{\sigma}\mu\kappa}{1-\beta\mu}}r_S^e \\
&= \frac{\check{\sigma}\mu\kappa\psi\chi^S + \chi^S\check{\sigma}(\mu-1)(1-\beta\mu)}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}\hat{\tau}_S^S + \frac{\check{\sigma}(1-\beta\mu)}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}r_S^e \\
&= \frac{[\mu\kappa\psi - (1-\mu)(1-\beta\mu)]\chi^S\check{\sigma}}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}\hat{\tau}_S^S + \frac{\check{\sigma}(1-\beta\mu)}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}r_S^e
\end{aligned}$$

The sales tax cut multiplier is given by:

$$\frac{\Delta\widehat{Y}_S}{\Delta\hat{\tau}_S^S} = -\frac{[(1-\mu)(1-\beta\mu) - \mu\kappa\psi]\chi^S\check{\sigma}}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}$$

which is the same as the one reported in the main text.

B.2.4 Capital tax cut (short run, zero nominal interest, $i = 0$)

Recall the expression of AD:

$$[\hat{Y}_t - E_t\hat{Y}_{t+1}] = [\hat{G}_t - E_t\hat{G}_{t+1}] - \check{\sigma}\left(i_t - E_t\pi_{t+1} - r_t^e\right) + \chi^S\check{\sigma}\left[\hat{\tau}_{t+1}^S - \hat{\tau}_t^S\right] + \chi^A\check{\sigma}\hat{\tau}_t^A.$$

The AD and AS equations can be written, at the zero bound bind, as:

$$\widehat{Y}_S = \mu\widehat{Y}_S + \check{\sigma}\mu\pi_S + \check{\sigma}r_S^e + \chi^A\check{\sigma}\hat{\tau}_S^A$$

Recall the NKPC:

$$\pi_S = \kappa\widehat{Y}_S + \beta\mu\pi_S$$

which latter can be expressed for inflation as:

$$\pi_S = \frac{\kappa\widehat{Y}_S}{1 - \beta\mu}$$

that can be put back into the AD equation:

$$(1-\mu)\widehat{Y}_S = \check{\sigma}\mu\frac{\kappa\widehat{Y}_S}{1-\beta\mu} + \check{\sigma}r_S^e + \chi^A\check{\sigma}\hat{\tau}_S^A$$

And

$$\left[(1-\mu) - \frac{\check{\sigma}\mu\kappa}{1-\beta\mu}\right]\widehat{Y}_S = \chi^A\check{\sigma}\hat{\tau}_S^A + \check{\sigma}r_S^e$$

And

$$\left[(1-\mu) - \frac{\check{\sigma}\mu\kappa}{1-\beta\mu}\right]\widehat{Y}_S = \chi^A\check{\sigma}\hat{\tau}_S^A + \check{\sigma}r_S^e$$

And

$$\begin{aligned}\widehat{Y}_S &= \frac{\chi^A\check{\sigma}}{(1-\mu) - \frac{\check{\sigma}\mu\kappa}{1-\beta\mu}}\hat{\tau}_S^A + \frac{\check{\sigma}}{(1-\mu) - \frac{\check{\sigma}\mu\kappa}{1-\beta\mu}}r_S^e \\ &= \frac{\chi^A\check{\sigma}(1-\beta\mu)}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}\hat{\tau}_S^A + \frac{\check{\sigma}(1-\beta\mu)}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}r_S^e.\end{aligned}$$

The capital tax cut multiplier is given by:

$$\frac{\Delta \widehat{Y}_S}{\Delta \hat{\tau}_S^A} = \frac{\chi^A \check{\sigma}(1-\beta\mu)}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa},$$

which is the same as the one reported in the main text.

Appendix C Nonlinear Model

C.1 Calvo price setting

Recall the first-order condition of the firm:

$$\frac{p_t^*}{P_t} = \frac{\Sigma_{T=t}^{\infty}(\beta\alpha)^{T-t}\lambda_T \left(\frac{p_t^*}{P_T}\right)^{-\theta-1} Y_T \frac{\theta}{\theta-1} MC_T^{\text{real}}(i)}{\Sigma_{T=t}^{\infty}(\beta\alpha)^{T-t}\lambda_T \left(\frac{p_t^*}{P_T}\right)^{-\theta-1} Y_T \frac{P_t}{P_T}}$$

To manipulate the previous equation further we need to establish connection between firm-specific ($MC_t^{\text{real}}(i)$) and average real marginal costs (MC_t^{real}). Note that in our paper we depart from Eggertsson and Singh and allow for DRS in production with the functional form $Y_t = N_t^{\frac{1}{\phi}}$ where $\phi > 1$; $\phi = 1$ is the case of CRS):

$$
\begin{aligned}
MC_T^{\text{real}}(i) &= \frac{W_T/P_T}{MPL_T(i)} = \frac{v_l(N_t(i))/u_c(.)}{MPL_t(i)} \\
&= \frac{v_l(N_T)/u_c(.)}{MPL_T}\frac{v_l(N_t(i))}{v_l(N_T)}\frac{MPL_T}{MPL_t(i)} \\
&= MC_T^{\text{real}}\frac{v_l(N_t(i))}{v_l(N_T)}\frac{MPL_T}{MPL_t(i)} \\
&= MC_T^{\text{real}}\left(\frac{N_t(i)}{N_T}\right)^{\omega}\left(\frac{Y_T}{Y_t(i)}\right)^{\phi-1} \\
&= MC_T^{\text{real}}\left(\frac{Y_t(i)}{Y_T}\right)^{\phi\omega}\left(\frac{Y_T}{Y_t(i)}\right)^{\phi-1} \\
&= MC_T^{\text{real}}\left(\frac{p_t^*}{P_T}\right)^{-\theta\phi\omega}\left[\left(\frac{p_t^*}{P_T}\right)^{-\theta}\right]^{\phi-1} \\
&= MC_T^{\text{real}}\left(\frac{p_t^*}{P_T}\right)^{-\theta\phi\omega-\theta(\phi-1)} \\
&= MC_T^{\text{real}}\left(\frac{p_t^*}{P_T}\right)^{-\theta\omega_y},
\end{aligned}
$$

Row 2 shows that the marginal cost has two 'specific labor' parts: one part is related to the disutility of labour and the other part is the specific marginal product of labour. Note that the specific labour market assumption does not require wage to be firm-specific. Row 3 defines the average marginal cost $MC_t = \frac{v_l(N_t)/u_c(C_t)}{MPL_t}$. In the last but one row $\theta\phi\omega$ appears only in case of specific labour market. With economy-wide labour market $\theta\phi\omega = 0$. Note that the case of CRS production function ($\phi = 1$) delivers the specific labour model of Eggertsson and Singh (2016). Row 4 used the relative demand for good i.

The last row marks a simple change in notation. In particular, the composite parameter $\omega_y \equiv \phi(1+\omega) - 1$ shows that the labour curvature parameter (ω) is rescaled after the introduction of DRS in technology. Note that when $\phi = 1$ we have $\omega_y = \omega$.

Recall the first-order condition of the firm from the appendix of our paper:

$$
\Sigma_{T=t}^{\infty}(\alpha\beta)^{T-t}\lambda_T\left(\frac{p_t^*}{P_t}\frac{P_t}{P_T}\right)^{-\theta-1}\frac{p_t^*}{P_t}\frac{P_t}{p_T} = \Sigma_{T=t}^{\infty}(\alpha\beta)^{T-t}\lambda_T\left(\frac{p_t^*}{P_t}\frac{P_t}{P_T}\right)^{-\theta-1}Y_T MC_T^{\text{real}}(i)
$$

or

$$\frac{p_t^*}{P_t} = \frac{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_t}\frac{P_t}{P_T}\right)^{-\theta-1} Y_T MC_T^{\text{real}}(i)}{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_t}\right)^{-\theta} \left(\frac{P_t}{P_T}\right)^{-\theta} Y_T}$$

which can be further written using the connection between firm-specific and average marginal cost as:

$$\left(\frac{p_t^*}{P_t}\right)^{1+\theta\omega_y} = \frac{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_T}\right)^{-\theta-1} Y_T \left(\frac{\theta}{\theta-1} MC_T^{\text{real}} \left[\frac{P_t}{P_T}\right]^{-\theta\omega_y}\right)}{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_T}\right)^{-\theta} Y_T \left[\frac{P_t}{P_T}\right]^{-\theta}}$$

which can also be written as:

$$\left(\frac{p_t^*}{P_t}\right)^{1+\theta\omega_y} = \frac{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_t}\frac{P_t}{P_T}\right)^{-\theta} Y_T \left(\frac{\theta}{\theta-1} MC_T^{\text{real}} \left[\frac{P_t}{P_T}\right]^{-\theta\omega_y}\right)}{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_t}\frac{1}{\Pi_T}\right)^{-\theta} Y_T \left[\frac{P_t}{P_T}\right]}$$

and let us multiply both nominator and denominator by $\frac{P_t}{P_T}$:

$$\begin{aligned}\left(\frac{p_t^*}{P_t}\right)^{1+\theta\omega_y} &= \frac{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_t}\frac{1}{\Pi_T}\right)^{-\theta} Y_T \left(\frac{\theta}{\theta-1} MC_T^{\text{real}} \left[\frac{1}{\Pi_T}\right]^{-\theta\omega_y}\right)}{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_t}\frac{1}{\Pi_T}\right)^{-\theta-1} Y_T} \\ &= \frac{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_t}\right)^{-\theta-1} Y_T \left(\frac{\theta}{\theta-1} MC_T^{\text{real}} \Pi_T^{\theta(1+\omega_y)}\right)}{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_t}\right)^{-\theta} Y_T \Pi_T^{\theta-1}}\end{aligned}$$

which is the same as the expression in Eggertsson and Singh.

In the previous equation the average real marginal cost is defined as:

$$\begin{aligned}MC_T^{\text{real}} = \frac{W_T/P_T}{MPL_T} &= \frac{\frac{N_T^{\omega}}{(Y_T-G_T)^{-\check{\sigma}}}}{(1/\phi)N_T^{(1/\phi-1)}} \\ &= \frac{\frac{N_T^{\omega}}{(Y_T-G_T)^{-\check{\sigma}}}}{(1/\phi)N_T^{(1/\phi-1)}} = \frac{\phi Y_T^{\phi(\omega+1)-1}}{(Y_T-G_T)^{-\check{\sigma}}} = \frac{\phi Y_T^{\omega_y}}{(Y_T-G_T)^{-\check{\sigma}}}.\end{aligned}$$

The AS curve (the recursive NK Phillips curve) can be expressed as:

$$
\begin{aligned}
K_t &= \frac{\theta}{\theta-1}\frac{1+\tau_t^S}{1-\tau_t^W}\lambda\xi_t\phi Y_t^{1+\omega_y} + \alpha\beta E_t\left[\Pi_{t+1}^{\theta(1+\omega_y)}K_{t+1}\right] \\
F_t &= \xi_t C_t^{-\frac{1}{\sigma}} Y_t + \alpha\beta E_t\left[\Pi_{t+1}^{\theta-1}F_{t+1}\right] \\
\frac{K_t}{F_t} &= \left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{1+\theta\omega_y}{1-\theta}}
\end{aligned}
$$

C.2 Aggregation

The production function for firm j is given by:

$$Y_t(j) = N_t^{1/\phi}(j)$$

where we abstract from technology shocks.

One derives the aggregate production function by integrating over the $j-$goods.

$$(Y_t(j))^{\phi} = N_t(j)$$

Since the workers are all the same the sum is simply, $N_t = \int_0^1 N_t(j)dj$. Plugging in from the demand function

$$\left(\left(\frac{P_t(j)}{P_t}\right)^{-\theta} Y_t\right)^{\phi} = N_t(j)$$

Integrating over $j-$goods

$$N_t = \int_0^1 \left[\left(\frac{P_t(j)}{P_t}\right)^{-\theta} Y_t\right]^{\phi} dj$$

Taking variables independent from j out of the integral,

$$N_t = (Y_t)^{\phi} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta\phi} dj$$

Now expressing this equation for Y_t,

$$N_t = Y_t^{\phi} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta\phi} dj$$
$$N_t^{\frac{1}{\phi}} = Y_t \left[\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta\phi} dj\right]^{\frac{1}{\phi}}$$

C.3 Price dispersion

Lets define price dispersion, S_t:

$$S_t^{\phi} \equiv \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta\phi} dj$$

where $1/\phi$ is the labor's share in output and θ is the elasticity of substitution between differentiated good j. Next, using the 'Calvo result' (proportion of firms changing its price), we can write price dispersion recursively as:

$$\begin{aligned}
S_t^{\phi} &\equiv \int_0^1 \left(\frac{P_t\,(j)}{P_t}\right)^{-\theta\phi} dj = (1-\alpha)\left(\frac{P_t^*\,(j)}{P_t}\right)^{-\theta\phi} + \alpha(1-\alpha)\left(\frac{P_{t-1}^*\,(j)}{P_t}\right)^{-\theta\phi} + \alpha^2(1-\alpha)\left(\frac{P_{t-2}^*\,(j)}{P_t}\right)^{-\theta\phi} + ... \\
&= (1-\alpha)\left(\frac{P_t^*\,(j)}{P_t}\right)^{-\theta\phi} + \alpha\left(\frac{P_{t-1}}{P_t}\right)^{-\theta\phi}\left[(1-\alpha)\left(\frac{P_{t-1}^*\,(j)}{P_{t-1}}\right)^{-\theta\phi} + \alpha(1-\alpha)\left(\frac{P_{t-2}^*\,(j)}{P_{t-1}}\right)^{-\theta\phi} + ...\right] \\
S_t^{\phi} &\equiv (1-\alpha)\left(\frac{P_t^*\,(j)}{P_t}\right)^{-\theta\phi} + \alpha\left(\frac{P_{t-1}}{P_t}\right)^{-\theta\phi} S_{t-1}^{\phi} \\
S_t^{\phi} &\equiv (1-\alpha)\,(p_t^*)^{-\theta\phi} + \alpha\,(\pi_t)^{\theta\phi}\,S_{t-1}^{\phi} \qquad \text{(A.8)}
\end{aligned}$$

where $(1-\alpha)$ is the probability that the firm will be able to change price. Price dispersion can be written recursively as

$$S_t^{\phi} = (1-\alpha)\left(\frac{P_t^*\,(j)}{P_t}\right)^{-\theta\phi} + \alpha(\pi_t)^{\theta\phi} S_{t-1}^{\phi}$$

Thus, we can write the aggregate production function as,

$$N_t^{1/\phi} = Y_t S_t$$