COMPUTABLE GENERAL EQUILIBRIUM MODELING

Numerical Simulations in a 2-Country Monetary General Equilibrium Model

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Abstract:

This paper presents the concept of numerical CGE modeling with the help of a 2-country general equilibrium model. In the framework of this simple dynamic monetary model the effects of a (unilateral) monetary and fiscal expansion are simulated. The exchange rate of the home vis-à-vis the foreign currency depreciates in response to both types of shocks. The monetary expansion leads to an increase in home relative to foreign private consumption and to a sharp increase in relative home output in the short run, while in the long run output increases in the foreign country and decreases in the home country. The unilateral fiscal expansion, on the other hand, results in a fall of private consumption in the home relative to the foreign country, and in an increase in relative home output in the short as well as in the long run. The world real interest rate falls quite substantially in response to both shocks.

I. Introduction

Besides various theoretical refinements of general equilibrium modeling, the development of computable general equilibrium models has had a long tradition in empirical economics. While especially in the 1970s theoretical propositions on the existence, uniqueness, optimality and stability of solutions to general equilibrium models were explored and further developed, the first full-fledged computable general equilibrium (CGE) model was that of Johansen which dates back to 1960[1]. Since then CGE models have been used as a powerful technique for quantitative analysis of a great variety of questions. These include analyzing the effects on - for instance - industries, regions, the labour market, income and welfare induced by changes in, e.g., government policies including taxes, trade restrictions, and/or technology. More recently also dynamic CGE models have been put forth, enabling researchers to address also multi-period (dynamic) problems by means of numerical general equilibrium analysis.

At this point it seems useful to define what is meant by computable general equilibrium models. The term "general equilibrium" refers to an analytical approach where the economy is regarded as a complete system of interdependent components (different markets, industries, households, etc.) and all decisions are taken according to fully optimizing behavior. Economic shocks affecting any one of theses components may produce repercussions throughout the whole system. Assessing the effects of the shocks can be done by means of simulation, i.e. by measuring the repercussions that are triggered by shocking the system in various ways. The models are called "computable" in the sense that they should produce numerical results that are applicable to particular situations in particular countries. To do so, the coefficients and parameters (elasticities) of the model have to be estimated by making use of real world data.

This paper is organized as follows: Section 2 summarizes and discusses some of the mathematical methods that are used to solve CGE models and to carry out simulations. This theoretical part serves as a methodological introduction to our practical exercise of solving and simulating a specific 2-country general equilibrium model, which is then done in section 3. This paper is intended to present the concept of numerical CGE modeling by means of an example. The specific example model that is considered in the simulation part is a simple dynamic (multi-period) monetary general equilibrium model, which has been introduced by Obstfeld and Rogoff (1995). Finally, section 4 concludes by summarizing again the main results of the foregoing simulation process. To my knowledge, this paper represents the first attempt to solve and simulate a monetary international general equilibrium model with numerical methods. A special general

[1] In their excellent review article on computable general equilibrium modeling Dixon and Parmenter (1996) consider the Johansen model with its 20 cost-minimizing industries and one utility-maximizing household sector the first true CGE model. The input-output models of Leontief and others in the 1930s can be regarded as vital forerunners of CGE models.

equilibrium software, called GEMPACK, is used to compute the numerical results of certain policy simulations in our model, which are then presented in the form of graphs and tables in the Appendix.

II. The Theory of CGE Modeling

Broadly speaking, there are two main approaches to solving CGE models, namely the non-linear programming approach and the derivative approach. The non-linear programming approach, which is not used as frequently as the derivative approach, is based on the idea that the solution to a CGE model can be deduced from the solution to an optimization problem. Although this method has been applied successfully in solving CGE models of various types, the experience of most researches in the CGE field suggests that the derivative approach is more convenient and flexible and, therefore, also better suitable for practical policy analysis and forecasting. For this reason we do not explore the programming approach any further but instead switch to the derivative approach immediately. An important aspect - especially for this paper - within the framework of the derivative approach is the solution of multi-period or intertemporal models along with the need to construct an initial steady-state solution. This special problem will be dealt with in the next subsection.

II.1. The derivative approach - alternatively called Johansen/Euler method

This is the method first used by Johansen when he was solving his pioneering CGE model in 1960. Linearizations and thus derivatives play a key role in this approach, hence the name derivative approach[2].

[2] There seems to be some confusion in the literature about the difference, if there is any, between what is called the Johansen and what is called the Euler method. Some authors, e.g. Harrison and Pearson (1996), associate one-step solutions with the Johansen method and refer to all multi-step solutions (see below) as the Euler method, while other authors use the two expressions interchangeably. Others again - like Dixon et al. (1992) - refer to multi-step solutions as the "extended Johansen method". In fact, it seems logical to refer to the method specifically described by Johansen (1960), which is very similar to - i.e. a percentage change version of - the computations made in equations (5) and (6) herein, as the Johansen method, and, on the other hand, reserve the term Euler method to the general and more technical derivation of solutions to CGE models.

To introduce the main idea of this method let's consider a model in which equilibrium is described by a vector, V, of length n satisfying a system of equations of the form

$$F(V) = 0 \tag{1}$$

where F is a vector function of length m. We further assume that F is (twice continuously) differentiable and that the number of variables, n, exceeds the number of equations, m, by a number (n-m) that is equal to the number of exogenously given variables, while the remaining m variables are kept endogenous. The components of the vector V typically represent demands for and supplies of commodities or factors, policy variables, technological coefficients and other economic variables. The function F imposes relations such as demands equal supplies and prices equal costs, while preferences and technologies are represented by differentiable utility and production functions in system (1). Finally, let's assume that we know an initial solution to the model, V^I, which implies that we have a known vector V^I satisfying

$$F(V^I) = 0\,. \tag{2}$$

Now, to describe the Euler method, we start by rewriting (1) as

$$F(V_1, V_2) = 0 \tag{3}$$

where V_1 is a vector of length m containing all endogenous variables and V_2 is the vector of length n-m of exogenous variables. By totally differentiating (3) we recognize that the deviations dV_1 and dV_2 from our known solution V^I must, to an approximation, satisfy

$$F_1(V^I)dV_1 + F_2(V^I)dV_2 = 0 \tag{4}$$

where F_1 and F_2 are matrices of partial derivatives of F evaluated at V^I. Now, solving for the impact of movements in the exogenous variables, dV_2, on the endogenous variables, dV_1, in the vicinity of V^I, i.e. calculating a one-step Euler or Johansen approximation, gives

$$dV_1 = B(V^I)dV_2 \tag{5}$$

where $B(V^I) = -F_1^{-1}(V^I)F_2(V^I)$ (6)

provided that we can evaluate $B(V^I)$ which directly hinges on the invertibility or equivalently non-singularity of $F_1(V^I)$[3]. The mx$(n$-$m)$ matrix $B(V^I)$ shows the partial derivatives, evaluated at V^I, of the endogenous variables V_1 with respect to the

[3] For a formal discussion of this point see Dixon and Parmenter (1996).

exogenous variables V_2. In other words, $B(.)$ is the Jacobian matrix of the implicit solution function G[4]. Therefore, for a given change in the exogenous variables, equation (5) provides only a first order approximation to the effects on the endogenous variables. If the solution function G is highly non-linear, i.e. if the higher order terms in the associated Taylor's series approximation are not neglegible, then serious approximation errors may arise requiring further refinements to be done.

One way out of this problem is to compute multi-step Johansen/Euler solutions. In this case the change in the exogenous variables dV_2 is split into as many subintervals as is implied by the number of steps, and the initial database V_1 is updated after each step when proceeding from V_2^I to $V_2^I + dV_2$. Thus, in order to reduce the approximation error we can stepwise recompute the numerical Jacobian matrix $B(V^I)$ which represents the slope of the true solution function evaluated at the respective updated solution. If the true solution function can be well approximated by a quadratic function then it can be shown that doubling the number of steps reduces the associated approximation error by one half. This can be illustrated in Figure 1 where a two-step Johansen/Euler solution for the 2 variables case is shown. In this diagram we are concerned with the effects on the endogenous variable V_1 (on the vertical axis) of moving the exogenous variable V_2 from the initial value V_2^I to $V_2^I + dV_2$ (on the horizontal axis). The solution function $V_1 = G(V_2)$ is quadratic in the sense that it is the square root function. Due to the stepwise error reduction as described above, we recognize that simple extrapolation procedures will help us finding an accurate solution[5], i.e. finding point *a* in the diagram.

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[4] We can rewrite (3) as $F(G(V_2), V_2) = 0$ showing the endogenous variables V_1 as a function of the exogenous variables $G(V_2)$ - call it solution function - and by applying the implicit function theorem we finally get equation (6).

[5] One specific extrapolation procedure that is named in Dixon and Parmenter (1996) is the Richardson's extrapolation.

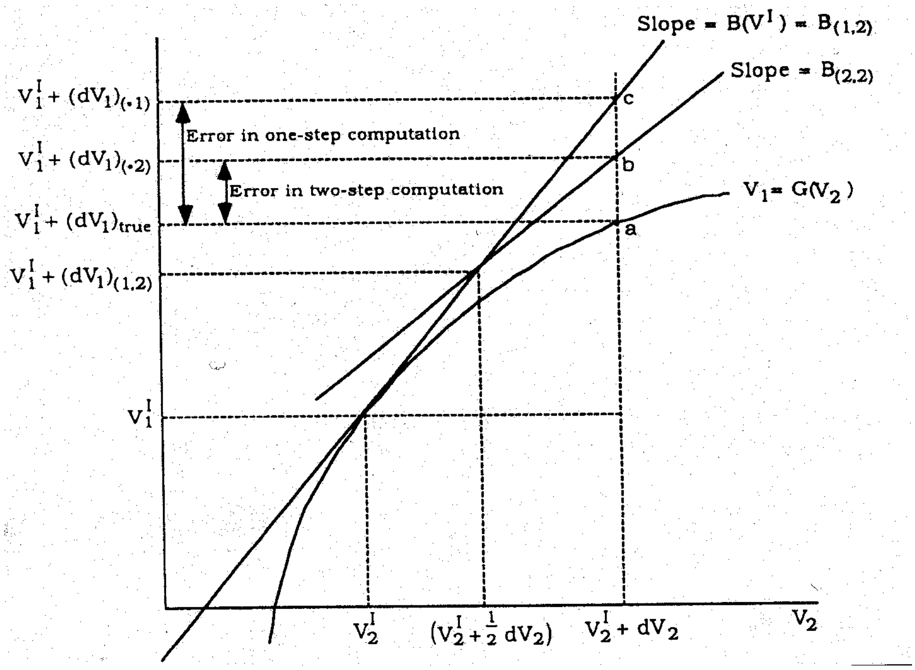

Figure 1: Two-step Johansen/Euler solutions

Moreover, for most models it seems to be more convenient to use variables in percentage change form rather than in change form[6] - as implied by (4) - when solving the system. In this case, instead of (5), the solution of the model is

$$v_1 = B^*\left(V^I\right)v_2 \tag{7}$$

where v_1 and v_2 are vectors of percentage changes in the variables V_1 and V_2, respectively, and $B^*(V^I)$ is as before and additionally contains the diagonal matrices formed from V^I_1 and V^I_2.

There are, however, two more methods of how to calculate multi-step solutions: the Gragg method and the midpoint method. These methods, which are equivalent except that Gragg's method does one extra pass at the end of a multi-step simulation to further refine the result, differ from the Euler method mainly in the way how the intermediate updates are computed. More specifically, the difference is that, after each update and after the computation of the slope - the matrix $B(V^u)$ (u for update) - with which to proceed, the Gragg and the midpoint methods proceed from the previous point and not from the current point, as is done in the Euler method. Therefore, for a large class of solution functions the resulting consecutive update is nearer to the true solution function

[6] For variables which may be, both, positive or negative or take the value zero the percentage change form is not appropriate, whereas for variables which are either always positive or always negative the percentage change form is preferable.

than an update computed via the Euler method, i.e. to say that, in general, the Gragg and the midpoint methods produce more accurate results than the Euler method[7].

II.2. Solving multi-period models and problems concerning the initial database

A multi-period or dynamic model is a model in which some variables at time *t* are determined by their own values or the values of other variables at periods different from *t*[8]. The variables of such a model are connected intertemporally with each other, hence the name intertemporal models.

Difficulties arise when the model - like most sophisticated intertemporal models - cannot be solved recursively. In this case we can nevertheless apply the Johansen/Euler approach with all equations in all periods being solved simultaneously. For large models this might involve handling linear systems containing millions of equations and variables, which modern CGE computer packages such as GEMPACK are easily capable of. To describe this approach let's consider a multi-period model of the form

$$F\big(V(1),V(2),...,V(T)\big)=0 \qquad (8)$$

where $V(t)$ is the vector of (endogenous and exogenous) variables applying to period *t* and function *F* includes intratemporal as well as intertemporal equations. Further assuming that we have an initial solution for any point in time

$$V^I=\big(V^I(1),V^I(2),...,V^I(T)\big), \qquad (9)$$

we can rewrite (8) in the percentage change form

$$F^*\big(V^I\big)v=0 \qquad (10)$$

where F^* is the Jacobian matrix of *F* evaluated at V^I (and multiplied by V^I) and *v* is the vector of the variable's percentage deviations from their initial values. From here on we can proceed as before to eventually obtain a solution paralleling the expression in equation (7), with matrices and vectors being much more extensive, though.

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7 In the unusual case of a highly non-linear solution function, however, applying Gragg's or the midpoint methods may cause the updated results to diverge more rapidly than when Euler's method is used. In this case Gragg and midpoint become unfeasible; see Harrison and Pearson (1994).

8 This includes forward and backward looking equations, of which formulations of rational expectations and adaptive expectations are respective examples.

The problem with non-recursive multi-period CGE models is not so much a technical one but rather how to find an initial solution, V^I. As equation (9) indicates, one has to find an ”initial” solution of the model for all periods, i.e. initial here does not mean just for the first period. Contrary to this, finding an initial solution of one-period CGE models is quite easy: Depending on the respective structure of the model one can simply read a solution from official input-output tables - in the case of a sectoral model - or from other data sources that are publicly available. For most multi-period models, however, it is possible to find - at least analytically - a steady state or balanced-growth path solution. That is, starting from an initial steady state we may calculate the future path of all variables by applying constant growth rates to, eventually, find

$$V^I = \left(V^I(1), gV^I(1), g^2V^I(1), \ldots, g^{T-1}V^I(1)\right) \tag{11}$$

where g is a diagonal matrix (possibly the identity matrix) of growth factors. If a solution of this form is a realistic solution, around which one would want to compute deviations, remains an open question, though. There are, however, multi-period models for which no solution of the above type can be found. For this class of models, more sophisticated methods of finding an initial solution have been developed, a few of which are summarized in Dixon and Parmenter (1996).

III. Simulation of a 2-Country General Equilibrium Model

III.1. The Model

The model which is used to conduct the simulation is a perfect-foresight two-country monetary general equilibrium model with fixed nominal output prices in the short run, additionally featuring monopolistic competition. Let the world be inhabited by a continuum of infinitely-lived producer-consumers, indexed by $z \in [0,1]$, each of whom specializes in the production of a single differentiated good, also indexed by z. The home country consists of producers on the interval [0,n], while the remaining producers (n,1] reside in the foreign country. All individuals in both countries have the same preferences, dependent upon a real consumption index (first expression in the square brackets in (12)), real money balances (second expression) and disutility from work (third expression). Thus, the intertemporal utility function of a representative home individual is given by

$$U_t = \sum_{s=t}^{\infty} b^{\,s-t} \left[\log C_s + c \, \log\left(\frac{M_s}{P_s}\right) - \frac{k}{2} y_s(z)^2 \right] \tag{12}$$

where β is a discount factor (0 < β < 1), χ is a factor determining the importance of real balances in the utility function, and the real consumption index, C, (as well as the real government spending index, G, below) is given by a CES specification

$$C = \left[\int_0^1 c(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \tag{13}$$

where c(*z*) denotes the representative home individual's consumption of good *z* and θ is the elasticity of substitution with θ > 1. (The foreign consumption index C^* as well as the utility function of a representative foreign individual U^* are defined analogously with asterisks always denoting foreign variables.) The price deflator for nominal money balances is the consumption-based money price index of the home country which is given by

$$P = \left[\int_0^1 p(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}} \tag{14}$$

where p(*z*) is the home-currency price of product *z*. Furthermore, it is assumed that there are no impediments to trade, so that the law of one price holds for each individual good in both countries:

$$p(z) = Ep^*(z) \tag{15}$$

where E is the nominal exchange rate, defined as the price of foreign currency measured in units of the home country's currency. Due to the symmetry of preferences in both countries, PPP holds also for the consumption-based price indices, thus

$$P = EP^* \tag{16}$$

When maximizing the utility function (12) the representative home individual faces the following period budget constraint:

$$P_t B_{t+1} + M_t = P_t(1+r_t)B_t + M_{t-1} + p_t(z)y_t(z) - P_t C_t - P_t \tau_t \tag{17}$$

where r_t denotes the real interest rate earned on bonds between t-1 and t, M_t and B_{t+1} are the demands for nominal money balances and real bonds[9] in period t, $y_t(z)$ is the output produced by agent *z* at time t and sold at the price $p_t(z)$ of good *z*, and τ_t denotes real

[9] The only internationally traded asset is a riskless real bond denominated in terms of the composite consumption good.

lump-sum taxes. Assuming that, on the other hand, the government runs a balanced budget each period, the government budget constraint specifies that the real government (per capita) purchases, G, are fully financed by taxes and seignorage revenues, i.e.

$$G_t = t_t + \frac{M_t - M_{t-1}}{P_t} \tag{18}$$

By combining the private budget constraint (17) and the government budget constraint (18), we get an expression for the resource constraint of a home individual:

$$B_{t+1} = (1+r_t)B_t + \frac{p_t(z)y_t(z)}{P_t} - C_t - G_t \tag{19}$$

This equation actually shows the current account imbalance of the home country, since it is the change in the net foreign asset position from one period to the next, $B_{t+1} - B_t$, which represents a current account surplus or deficit in that period. Given equations (13) and (14), a home individual's consumption demand for good *z* at date t can be shown to be

$$c_t(z) = \left(\frac{p_t(z)}{P_t}\right)^{-q} C_t \tag{20}$$

where it turns out that θ, being the elasticity of substitution in (13), now represents also the price elasticity of demand. Adding up the private and government demand (which is derived in a similar manner as (20)) for a particular good *z* we get the world demand for that good

$$y_t^d(z) = \left(\frac{p_t(z)}{P_t}\right)^{-q} \left(C_t^W + G_t^W\right) \tag{21}$$

where $C^W = nC + (1-n)C^*$ and $G^W = nG + (1-n)G^*$ are population-weighted averages of home and foreign consumption and government spending indices, respectively. Next, by substituting (21) into the individual's intertemporal utility function (12) and maximizing the resulting expression with respect to B_{t+1}, M_t and $y_t(z)$, taking the aggregate price level, P_t, as well as C_t^W and G_t^W as given, we can find the following first-order-conditions[10]:

[10] In order to fully characterize the equilibrium, the so-called transversality condition, in addition to the first-order-conditions (21) - (23) and the period budget constraint (17), is required:

$$C_{t+1} = b\,(1+r_{t+1})C_t \tag{22}$$

$$\frac{M_t}{P_t} = c\,\frac{P_{t+1}(1+r_{t+1})}{P_{t+1}(1+r_{t+1})-P_t}C_t \tag{23}$$

$$y_t(z)^{\frac{q+1}{q}} = \left(\frac{q-1}{qk}\right)C_t^{-1}\left(C_t^W + G_t^W\right)^{\frac{1}{q}} \tag{24}$$

Analogous equations, of course, apply to foreign variables. In order to define a global general equilibrium, i.e. a state in which all markets of the global economy – the money market, the asset market and the goods market – are in equilibrium, three market-clearing conditions for the three markets have to be formulated:

- Aggregate money demand must equal money supply in each country, since there is no currency substitution in this model.
- World net foreign asset holdings must be zero:

$$nB_t + (1-n)B_t^* = 0\,, \quad \forall \mathrm{t} \tag{25}$$

- The output-market-clearing condition states that global private and government real consumption equals global real income:

$$C_t^W + G_t^W = n\frac{p_t(h)y_t(h)}{P_t} + (1-n)\frac{p_t^*(f)y_t^*(f)}{P_t^*} \tag{26}$$

where y(*h*) and p(*h*) [$y^*(f)$ and $p^*(f)$] are output and price – measured in the respective domestic currency – of a representative home [foreign] good.

For the purpose of simulating the effects of exogenous shocks on the endogenous variables, we first have to find a well-defined steady state solution of the model. We define the steady state as an equilibrium in which all variables are constant[11]. With consumption being constant in the steady state, the steady state world real interest rate, $\bar{r}$, is determined by the consumption Euler equation (22):

$\lim_{T\to\infty} R_{t,t+T}\left(B_{t+T+1} + \frac{M_{t+T}}{P_{t+T}}\right) = 0$ where $R_{t,s}$ is a discount factor. The transversality condition is imposed to rule out unbounded borrowing.

[11] This is a very common formulation of a steady state.

$$\bar{r} = \frac{1-b}{b} \tag{27}$$

where overbars (here and in the following) indicate steady state values. After imposing constant bond holdings in equation (19) and constant money supply, one can find steady state real per capita consumption:

$$\bar{C} = \bar{r}\bar{B} + \frac{\bar{p}(h)\bar{y}(h)}{\bar{P}} - \bar{G} \tag{28}$$

(An equivalent relation also holds for foreign steady state real per capita consumption, $\bar{C}^*$). The above equations (12) - (28) are all the important expressions that are needed to carry out the simulation process, while the remaining equations defining the initial steady state solution of the model will be explored very shortly in the next section.

III.2. The Simulation

III.2.1. The Initial Solution

Our 2-country general equilibrium model is an intertemporal model, thus the problems of finding an initial solution mentioned in section II.2. apply. For the ease of exposition, we assume the initial steady state to be symmetric, not only with respect to agents within each country, but also symmetric across countries. A steady state of this type does arise when the initial distribution of wealth is equal, such that $\bar{B}_0 = \bar{B}_0^* = 0$, where $\bar{B}_0(^*)$ denotes the net foreign asset holdings of the home (foreign) country in the initial steady state[12]. In this case, the relation $\bar{p}_0(h)/\bar{P}_0 = \bar{p}_0^*(f)/\bar{P}_0^* = 1$ holds, since in a globally symmetric equilibrium any two goods produced anywhere in the world have the same price when measured in the same currency, i.e. $\bar{p}_0^*(h) = \bar{p}_0^*(f)$. This implies symmetric output and consumption levels, hence $\bar{y}_0(h) = \bar{y}_0^*(f) = \bar{C}_0 + \bar{G}_0 = \bar{C}_0^* + \bar{G}_0^* = \bar{C}_0^W + \bar{G}_0^W$, with initial output and consumption being related

$$\bar{y}_0 = \bar{y}_0^* = \left(\frac{q-1}{qk}\right)\bar{C}_0^{-1} \tag{29}$$

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[12] In principle, there is no particular need to assume an equal distribution of wealth, which is not even a very realistic assumption *per se*, but here it serves as a neutral starting point, from which the countries may possibly develop in opposite directions due to macroeconomic policy shocks.

On the other hand, initial real balances are determined by

$$\frac{\overline{M}_0}{\overline{P}_0} = \frac{\overline{M}_0^*}{\overline{P}_0^*} = c\,\frac{\left(1+\bar{r}\right)}{\bar{r}}\,\overline{C}_0 \qquad (30)$$

A very important ingredient of the model is the presence of short-run nominal output price rigidity, which is introduced by assuming that the prices of representative domestic and foreign goods, $p(h)$ and $p^*(f)$, are fixed (predetermined) during the period when the shock occurs and fully adjust to flexible-price levels only in the second period after the shock. Therefore, output becomes demand determined in the short run[13].

One further simplification for the simulation process is the fact that we restrict our attention to only two different time periods when analyzing the dynamics of the system. Thus, we shall assume that the endogenous variables will reach a new long-run steady state just one period after a monetary and/or fiscal policy shock has hit the system, because that is how long nominal prices take to adjust. Hence, the first period after the shock can be interpreted as the short-run disequilibrium response and the second period as the long-run equilibrium response to some exogenous policy shock[14]. This implies that, in order to capture the dynamics, it is sufficient to consider only the first and the second period after the shock, since all subsequent periods are equal to the second period. This simplifies the simulation process in that the intertemporal set of time periods consists of only two elements with the initial solution - as defined above - being the baseline solution for both periods, and the deviation in the second period representing the long-run equilibrium percent change in the respective variable.

When trying to find an initial solution, we have to recognize that our model is highly stylized in the way that the world population and also the number of goods are normalized to 1 and the initial distribution of wealth is normalized to 0. Therefore, the point is not to feed the model with some specific external data, but rather to find an initial solution that is plausible and reflects certain idiosyncracies of the countries under consideration, e.g. country size and exchange rate relations. Moreover, the levels of the initial solution do not matter so much as long as the relations are correct[15], because the simulation results are reported as percent deviations from the initial (=baseline) solution.

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[13] For an interpretation of monopolistic competition with lagged price setting behavior, see Rumler (1998).

[14] This is, admittedly, a rather crude way of modeling the dynamics of an intertemporal model, but it nevertheless serves the purpose of analyzing short-run and long-run responses to policy shocks. Alternatively, one could allow for richer price adjustment mechanisms, like staggered price setting, which would just lead to a longer persistence of nominal shocks without modifying the central results of our analysis. Moreover, depending on the respective values of the time preference rate β, the periods can easily be interpreted as longer intervals of time, such as years or decades instead of quarters.

[15] For instance, if initial bond holdings are zero, private consumption must not be higher than output with non-zero government spending, or *vice versa*, government spending must not be higher than output.

When specifying the initial solution we have to start with determining the values of certain variables at the outset: One country has to serve as the numeraire with respect to the national price level, which in our case is the foreign country[16]. Bonds - as already mentioned - are set to 0, and the exchange rate is set to 7.0[17]. The relative country size, *n*, is set to 0.3, which means that the foreign country is more than double in size compared to the home country. The level of government spending is set to 0.1 in both countries[18]. The values of the remaining variables are calculated by solving the equations (16), (27), (28), (29) and (30). The complete initial solution of the model is reported in Table 1, where, again, it has to be borne in mind that the levels of this solution are only indicative and not crucial for the simulation results.

Variables	Country 1	Country 2
real per capita consumption C	0.529511	0.529511
real per capita income y	0.629511	0.629511
per capita money demand (supply) M	37.0658	5.29511
real per capita government spending G	0.1	0.1
real net-foreign assets B	0	0
price index P	7.0	1.0
individual prices pi	7.0	1.0
world real interest rate r	0.052632	
exchange rate E	7.0	
world real consumption demand C^W	0.529511	
world real government spending G^W	0.1	

Table 1: The initial solution of the model

In addition, the following values are assigned to the parameters of the model:

$\theta = 3.0$
$\kappa = 2.0$
$n = 0.3$
$\beta = 0.95$
$\chi = 0.5$

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16 I.e. the price level of country 2 in our input file is normalized to 1. This implies that also the individual prices, pi, are equal to 1.

17 This is arbitrary although the parallel to the Austrian-German exchange rate doesn't seem purely accidental.

18 With output being equal to 0.6295 (see Table 1) this would imply that the government consumption is of a magnitude of about 16% of the country's GDP, which represents an empirically quite plausible relation. Cf. the empirical values in 1998 for Austria 18.3%, Germany 18.9% and Switzerland 14.7% (source: OECD National Accounts).

While the values of κ and χ do not matter for the simulation, since they both cancel in the linearization of the model, β and θ are the central parameters which drive the simulation results. A value assigned to the time preference rate, β, of 0.95 implies a rate of time preference of 5% per period which is consistent with observed real rates of return at a yearly frequency, thus one period can be interpreted to correspond to one year. The parameter θ, which represents the elasticity of demand and at the same time the elasticity of substitution between goods, is set to 3.0 which is consistent with standard econometric estimates (Woodford 1996)[19]. As $\theta \to \infty$ perfect competition is approximated and the scope for activist monetary and fiscal policy becomes smaller and smaller, as do the real effects of nominal shocks.

III.2.2. The Shocks

After the initial solution is found and the parameter values are assigned, the next step is to specify the closure of the model and to formulate the shocks that we want to simulate. By the closure of the model we understand the correct split of endogenous and exogenous variables, with the requirement that the model contains as many equations as there are endogenous variables. In our case we have a total of 36 variables, 28 of which are endogenous (8 exogenous[20]) and, therefore, the model has to consist of 28 equations (the input file is available upon request).

As already mentioned before, the simulation is carried out with the help of the software package GEMPACK (General Equilibrium Modelling PACKage). Gempack is a multi-purpose user-friendly general equilibrium simulation software especially designed to handle intertemporal models[21]. The simulation process in Gempack is organized as follows: First, an input file has to be written by the user, where the sets, the variables and the coefficients are defined, the initial solution is specified and the equations of the model are supplied in algebraic form. A sub-program called ”Tablo” processes the input file, i.e. it calculates the initial solution, (log-)linearizes the levels equations and checks the input file for syntax and semantic errors[22]. Log-linearizing means that all variables are expressed in percentage change form, which is done by numerical differentiation of all the equations. Next, the closure, the solution method[23], and, most importantly, the shocks which we want to simulate have to be specified in a command file. The shocks are usually specified in percent deviations, such that they directly apply to the linearized

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[19] A value of 3.0 for θ reflects a degree of monopoly distortion that causes output to be about 20% lower than it would be in a competitive equilibrium. (A value of 2, however, does not alter the simulation results substantially.)

[20] These are all the components of money, M, and government spending, G, which are also the variables that will be shocked later.

[21] see Harrison and Pearson (1996).

[22] An information file of this ”Tablo”-procedure is available on request.

[23] We may choose among Gragg, Euler or the midpoint method and, in addition, have to specify the number and sequence of approximation steps.

model. The sub-program ”Gemsim” executes the simulation process and yet another program called ”Gempie” delivers the results in a clearly arranged and handsome form.

The simulations, which are reported in this paper, are carried out for two different shocks: A permanent (i.e. short-run and long-run) increase in home money supply by 5% and a 10% permanent increase in home government spending[24]. Of course, also transitory shocks to money supply and government spending have been simulated but the results are less interesting and are not presented here, since the effects on the endogenous variables are much smaller and are confined exclusively to the short run with long-run variables eventually returning to their initial steady state values. The two shocks are analyzed separately, although they could be analyzed jointly, but there is no gain in doing so, because, due to the fact that the simulation is carried out in a linearized system, simply adding the results obtained from the separate simulations is equivalent to conducting a joint simulation. The solution method is Gragg with 4,6,8 approximation steps (see section II.1.)

III.2.3. The Results

The simulation results for the two shocks are reported in detail in the Appendix. Let’s consider the money supply shock first: This shock is modeled by assuming that at some point in time the money supply in country 1 is increased permanently by 5%. In terms of the model this implies that, both, the long-run and the short-run component of M in country 1 is (positively) shocked by 5%[25].

[See the tables and graphs in the Appendix concerning simulation 1.]

As a result of the money shock real per capita consumption is increased in both countries in the short run, but more in the country where the shock occurs. In the long run, however, consumption in the home country is higher than in the initial steady state by about 0.4% and lower in the foreign country by only 0.17%. This is due to the induced current account surplus of 0.038[26] (corresponding to 6% of GDP) in country 1 which allows the individuals in that country to permanently consume more out of the interest income received from increased asset holdings. On the other hand, the citizens of the country which has become a net-debtor - with a current account deficit of 2.6% of GDP - are permanently restricted to consume less, since part of their income is spent for debt and interest repayment. This implies, that on a country level we have a long-run non-

[24] Thus, only asymmetric shocks, i.e. shocks which are stronger in one country than in the other, are analyzed, because it is this type of shocks that is responsible for interesting asymmetric dynamics of the endogenous variables in the two countries. In fact, the asymmetric shocks we consider are extreme in that they appear in one country only, i.e. we analyze purely unilateral shocks.

[25] As the linearizations are valid only for small enough shocks, a rather small shock of 5% is considered.

[26] The current account imbalance is obtained by calculating $B_{t+1} - B_t$.

neutrality of monetary policy, which is due to the permanently higher interest income in one of the countries induced by the international transfer of wealth[27].

The reason for the current account surplus of country 1 lies in the depreciation of the exchange rate of its currency vis-à-vis the foreign currency (by 4.4%[28]), thus, causing domestic income to increase sharply (by 10.7%) compared to foreign income (-2.7%) in the short run which induces the individuals of country 1 temporarily to consume less than they produce (due to consumption smoothing). Furthermore, we may observe that the size of the current account imbalance is dependent on the relative size of the countries, with larger countries' current accounts being affected less than the current accounts of smaller countries.

Interestingly, output falls in the foreign country in the short run (by 2.7%) in response to a unilateral monetary increase in country 1, while output also falls in the long run in the home country (-0.3%). The former of these results indicates that a ”beggar-thy-neighbour” policy is effective only in the short run, particularly since output in country 2 is even increasing by 0.13% in the long run. On the other hand, the output fall in the long run in country 1 can be explained by the fact that the individuals of that country, being permanently wealthier, respond by substituting out of work and into leisure in order to maintain a certain level of utility.

The aggregate price levels in both countries can change in the short run (although the individual prices, pi, are fixed) since the exchange rate - which is contained in both price levels - may change. In the long run also the individual prices adjust, with the effect that the relative prices, i.e. $p(z)/P$, in country 1 have increased (by 0.1%) and decreased in country 2 (by 0.05%) - see the table and graphs corresponding to simulation 1 in the Appendix.

The world real interest rate is free to move only in the short run, because in the long run it is tied to the rate of time preference as specified in equation (27). The short-run decrease of the real interest rate by 30% is quite substantial in response to a 5% monetary shock and it is due to the fact that, when money supply is increased, the interest rate has to fall in order to restore monetary equilibrium (in equation (23)). As a consequence of the fall in the world real interest rate, short-run world consumption demand rises (by 1.51%) pursuant to equation (22) and its foreign analog.

The exchange rate of the home currency vis-à-vis the foreign currency, E, depreciates by 4.4% in response to a 5% money supply increase in the home country. The fact that the reaction of the exchange rate is smaller than the asymmetric monetary shock which

[27] In the long run, however, world output and world consumption (C^W) return to their initial steady state values, thus, establishing long-run monetary neutrality at the level of world aggregates.

[28] With permanent monetary and fiscal shocks, the change in the exchange rate is also permanent (see results of simulation 1 and 2 in the Appendix).

triggered it, indicates that in this model monetary policy shocks have real effects in the short as well as in the long run. In the simulation, however, the long-run real effects turned out to be rather small, with long-run consumption and output deviatons ranging from 0.1% to a maximum of 0.4%. Thus, for reasonable parameter values[29] our simulation establishes nearly a long-run monetary neutrality result - see simulation 1 in the Appendix.

[See the tables and graphs in the Appendix concerning simulation 2.]

Now let us turn to the government spending shock: Paralleling the modeling of the monetary shock, we model the fiscal shock by assuming that government spending in country 1 is increased permanently by 10%. As a result of this asymmetric government spending shock real per capita consumption in country 1 is falling in the short as well as in the long run (by 2.4% and 2.6% respectively). This can be explained by the fact that, according to the individual's budget constraint, permanently higher government spending in the home country has to be borne fully by home residents, whereas the benefits - in the form of higher demand - fall on foreigners as well. Therefore, and because of the international transfer of wealth from country 1 to country 2 - see simulation 2 in the Appendix - real per capita consumption increases in the foreign country also in the long run (by 1% in the short and 0.75% in the long run). In the home country, however, the positive output effect (7.6% in the short and 2% in the long run) is more than offset by the increased tax burden, thereby inducing private consumption to fall. Foreign output, on the other hand, falls in the short as well as in the long run (by 2.6% and 0.5% respectively), since with increased consumption levels the individuals residing in country 2 can afford to substitute out of work and into leisure.

Although country 1 became a net-debtor due to the induced international transfer of wealth, its current account, which denotes the intertemporal change in the net-foreign asset position, shows a surplus (at the rate of 3.4% of GDP). The reason for this surplus is the same as in the case of the monetary shock (depreciation of the exchange rate). The converse, of course, is true for the foreign country, i.e. it runs a current account deficit at the rate of 1.5% of GDP.

Furthermore, lower consumption in country 1 implies lower money demand, requiring a rise in the domestic price level (by 2.4% in the short and 2.6% in the long run) and a permanent depreciation of the home currency vis-à-vis the foreign currency by 3.4%, whereas foreign prices move in the opposite direction. The individual prices, on the other hand, are allowed to move only in the long run with relative prices - $p(z)/P$ - falling in the home country by 0.6% and increasing in the foreign country by 0.25% in the long run - see the results of simulation 2 in the Appendix.

[29] Alternative simulations that were computed for different values of the elasticity of demand, θ, did not produce any substantially different results. Thus, our results seem fairly robust with respect to different values of θ.

As with the monetary shock, the short-run world real interest rate falls in response to a permanent fiscal expansion (by 5.1%), because with lower consumption and fixed money supply, the real interest rate has to fall in order to restore monetary equilibrium. This result is, however, contrary to the result obtained in the standard-textbook Mundell-Fleming model[30]. Another interesting result is that in the long run aggregate world consumption falls by 0.26% in response to a permanent increase in world government spending. Since world private consumption falls by much less than the rise in world government spending - only partial crowding-out of private by public consumption -, long-run world output is increased by about the same rate.

IV. Conclusions

In the previous section the simulation results of a 5% permanent money supply shock and a 10% permanent government spending shock in the home country were presented. The most outstanding result is that the exchange rate of the home vis-à-vis the foreign currency depreciates in response to both types of shocks. The (positive) monetary shock in country 1 leads to an increase in home relative to foreign private consumption in the short as well as in the long run, and to a sharp increase in relative home output in the short run, while in the long run output increases in the foreign country and decreases in the home country. The permanent fiscal expansion in the home country, on the other hand, results in a fall of private consumption in country 1 relative to country 2 in, both, the short as well as in the long run, and - as before - in a sharp increase in relative home output in the short run and, this time, (in a mitigated form) also in the long run. The world real interest rate falls quite substantially in the short run in response to, both, the monetary and the fiscal expansion.

It is important to note that these two simulations should be regarded only as a starting point for a more specific and relevant analysis of monetary and fiscal policy shocks under realistic conditions. For this purpose the model would have to be calibrated with respect to observed real-world data, incorporating also the empirical initial distribution of wealth and the price relations between two specific countries. The model could also be extended to a three- or more-country model in order to additionally capture the economic interactions with the rest of the world. Our simulations, however, are merely indicative of the directions of certain effects, with the exact figures being of second-order importance. As already indicated, there is still plenty of scope for future research in the field of computable general equilibrium (CGE) modeling that is applied to simulate the international effects of monetary and fiscal policy interventions.

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[30] In the two-country version of the Mundell-Fleming model a unilateral fiscal expansion in either country is associated with a rising world real interest rate (Dornbusch 1980, Ch. 11).

Appendix

Simulation 1

SHOCKS RELEVANT TO THE PRINT-OUT BELOW

p_M
1 SHOCK = 5.00000
3 SHOCK = 5.00000

THE RESULTS BELOW ARE CUMULATIVE EFFECTS OF ALL SHOCKS ABOVE.

p_C (COUNTRY,TIME) real per capita consumption
p_C(-,sr) results where '-' is in set 'COUNTRY'.

C1	C2
1.917588	1.340582

p_C(-,lr) results where '-' is in set 'COUNTRY'.

C1	C2
0.397917	-0.170575

p_y (COUNTRY,TIME) real per capita output
p_y(-,sr) results where '-' is in set 'COUNTRY'.

C1	C2
10.730491	-2.694521

p_y(-,lr) results where '-' is in set 'COUNTRY'.

C1	C2
-0.298545	0.127907

c_B (COUNTRY,TIME) real net-foreign assets
c_B(-,sr) results where '-' is in set 'COUNTRY'.

C1	C2
0.025904	-0.011101

c_B(-,lr) results where '-' is in set 'COUNTRY'.

C1	C2
0.063839	-0.027359

p_P (COUNTRY,TIME) Price index in country i
p_P(-,sr) results where '-' is in set 'COUNTRY'.

C1	C2
3.022496	-1.323110

p_P(-,lr) results where '-' is in set 'COUNTRY'.

C1	C2
4.582026	0.170546

p_pi (COUNTRY,TIME) individual prices in country i
p_pi(-,sr) results where '-' is in set 'COUNTRY'.

C1	C2
0.000000*	0.000000*

p_pi(-,lr) results where '-' is in set 'COUNTRY'.

C1	C2
4.686528	0.127907

p_r (TIME) world real interest rate

sr	lr
-29.822809	0.000000*

p_E (TIME) nominal exchange rate

sr	lr
4.402853	4.402853

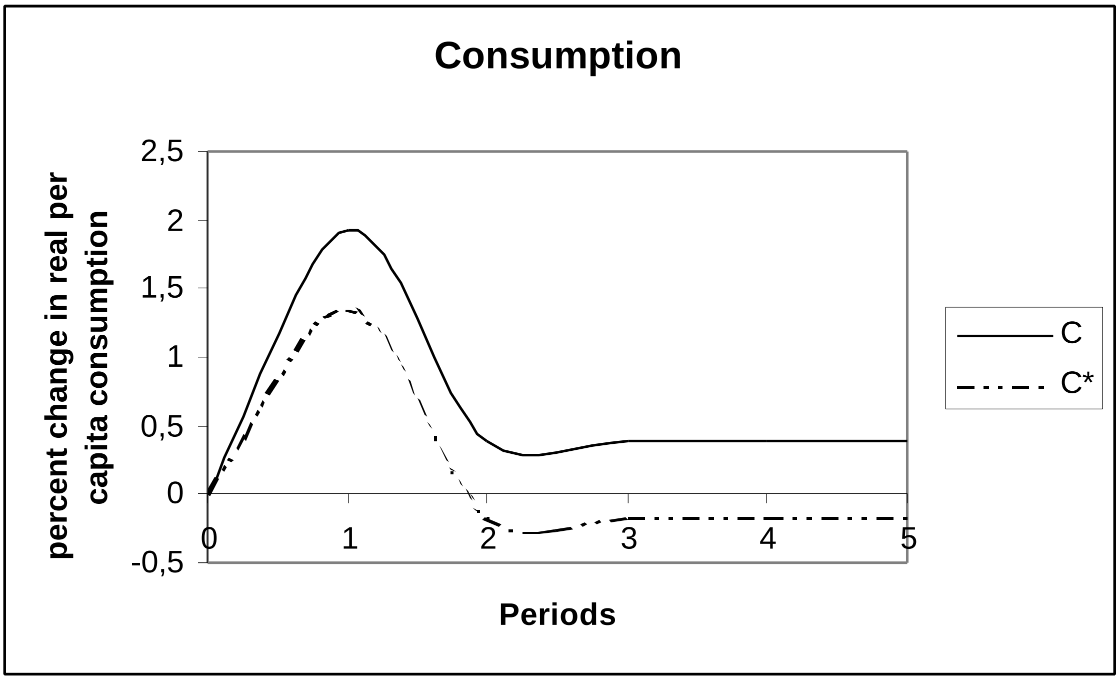
p_CW (TIME) world real consumption demand

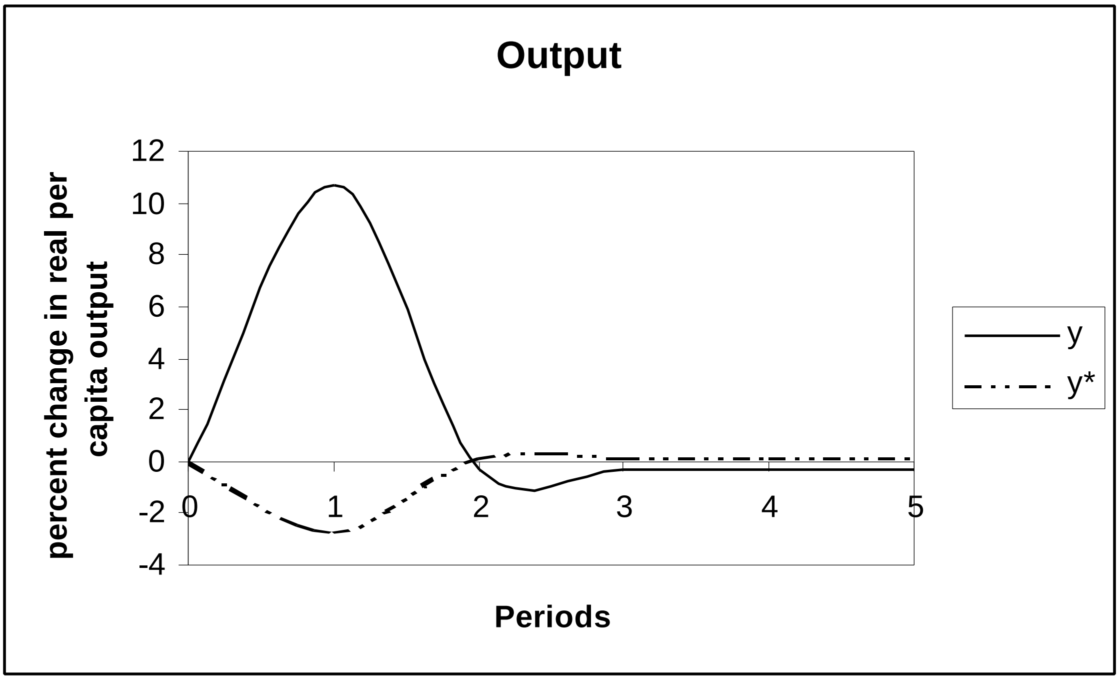
sr	lr
1.513684	-0.000027

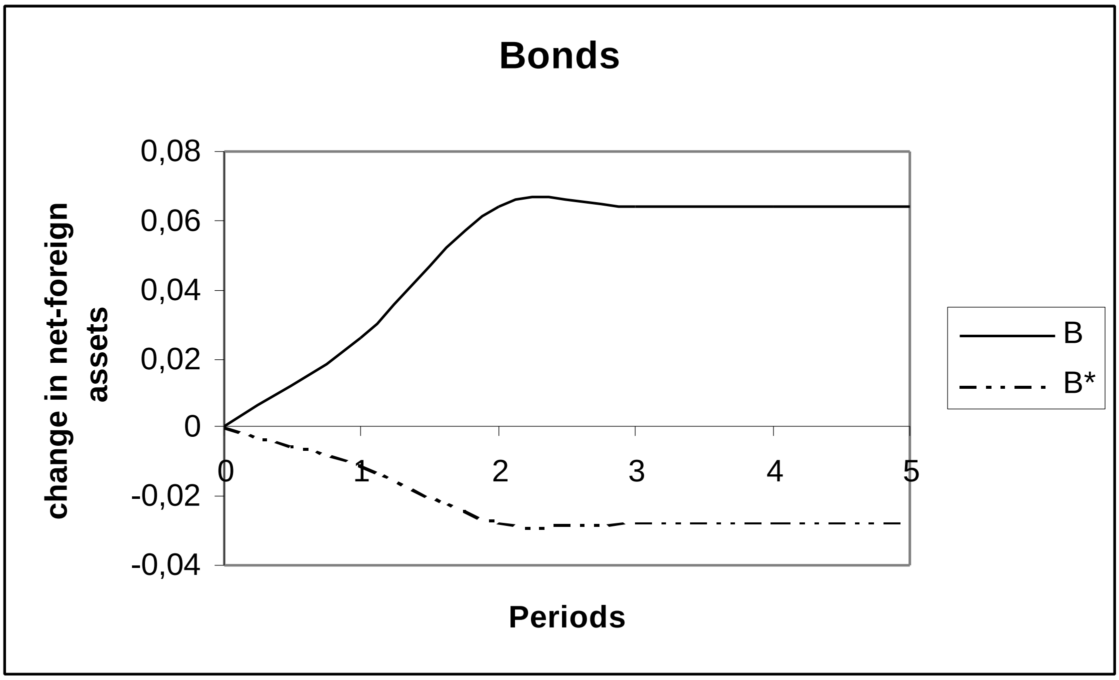
p_GW (TIME) world real government spending

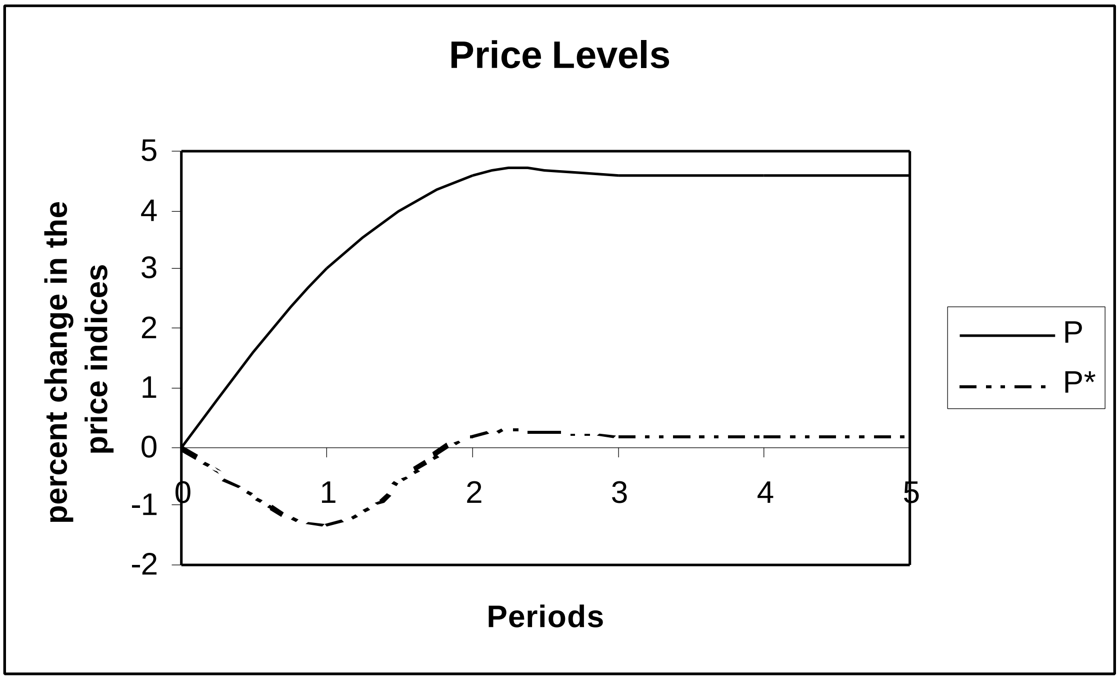
sr	lr
0.000000*	0.000000*

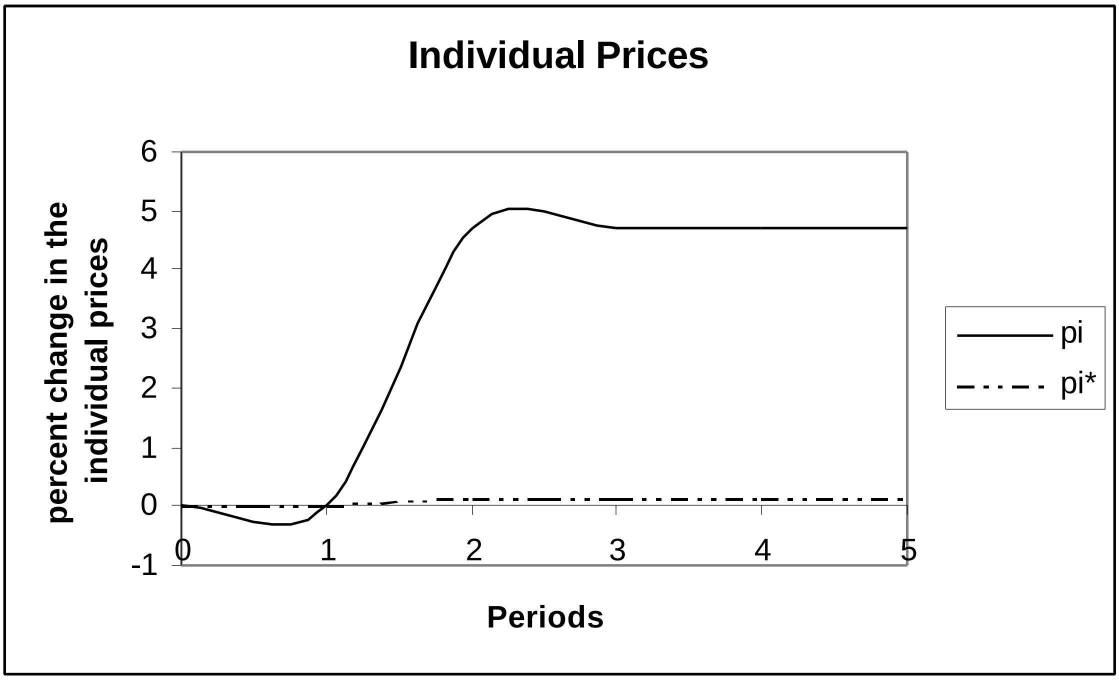
Note: In order to better illustrate the dynamic effects of the shocks, the option ”smooth lines” has been chosen for drawing the following graphs. Therefore, the small movements we observe in the third period are only seeming and not present in reality.






Bonds
0,08
0,06
0,04
0,02
0
-0,02
-0,04
0
1
2
3
4
5
change in net-foreign assets
Periods
B
B*


Price Levels
5
4
3
2
1
0
-1
-2
0
1
2
3
4
5
percent change in the price indices
Periods
P
P*



Simulation 2

SHOCKS RELEVANT TO THE PRINT-OUT BELOW

p_G
1 SHOCK = 10.0000
3 SHOCK = 10.0000

THE RESULTS BELOW ARE CUMULATIVE EFFECTS OF ALL SHOCKS ABOVE.

p_C (COUNTRY,TIME) real per capita consumption
p_C(-,sr) results where '-' is in set 'COUNTRY'.

C1	C2
-2.368470	1.012700

p_C(-,lr) results where '-' is in set 'COUNTRY'.

C1	C2
-2.619622	0.752804

p_y (COUNTRY,TIME) real per capita output
p_y(-,sr) results where '-' is in set 'COUNTRY'.

C1	C2
7.598936	-2.580234

p_y(-,lr) results where '-' is in set 'COUNTRY'.

C1	C2
2.037413	-0.503655

c_B (COUNTRY,TIME) real net-foreign assets
c_B(-,sr) results where '-' is in set 'COUNTRY'.

C1	C2
-0.268400	0.115028

c_B(-,lr) results where '-' is in set 'COUNTRY'.

C1	C2
-0.246348	0.105578

p_P (COUNTRY,TIME) Price index in country i
p_P(-,sr) results where '-' is in set 'COUNTRY'.

C1	C2
2.365896	-1.013168

p_P(-,lr) results where '-' is in set 'COUNTRY'.

C1	C2
2.629973	-0.757854

p_pi (COUNTRY,TIME) individual prices in country i
p_pi(-,sr) results where '-' is in set 'COUNTRY'.

C1	C2
0.000000*	0.000000*

p_pi(-,lr) results where '-' is in set 'COUNTRY'.

C1	C2
2.037413	-0.503655

p_r (TIME) world real interest rate

sr	lr
-5.145544	0.000000*

p_E (TIME) nominal exchange rate

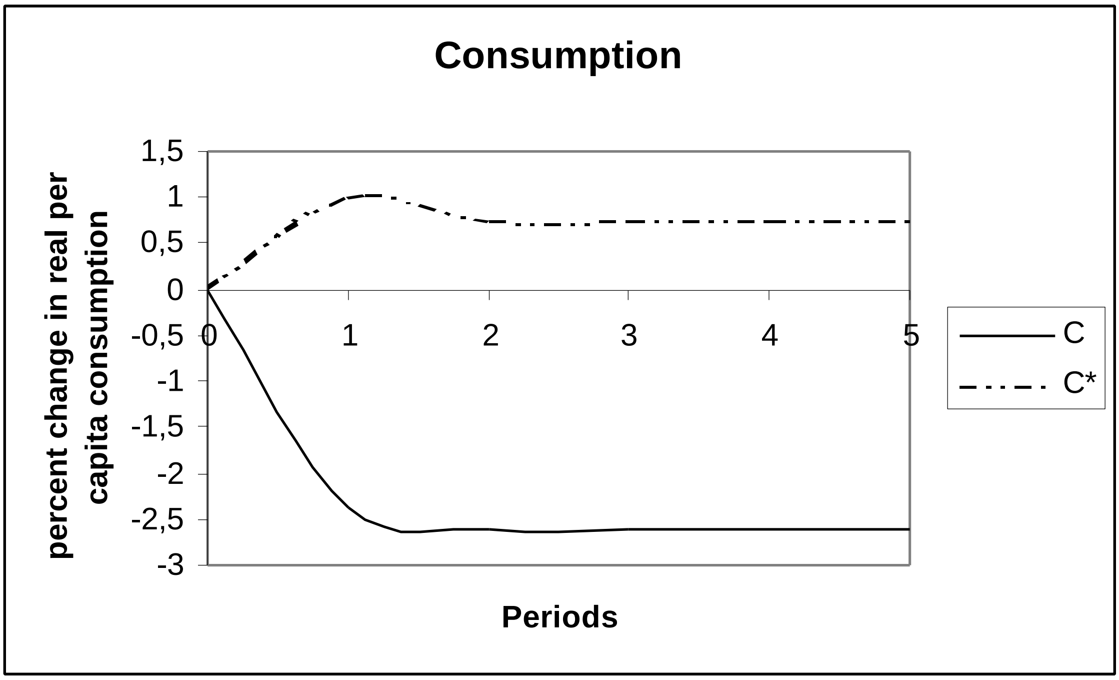
sr	lr
3.377497	3.377497

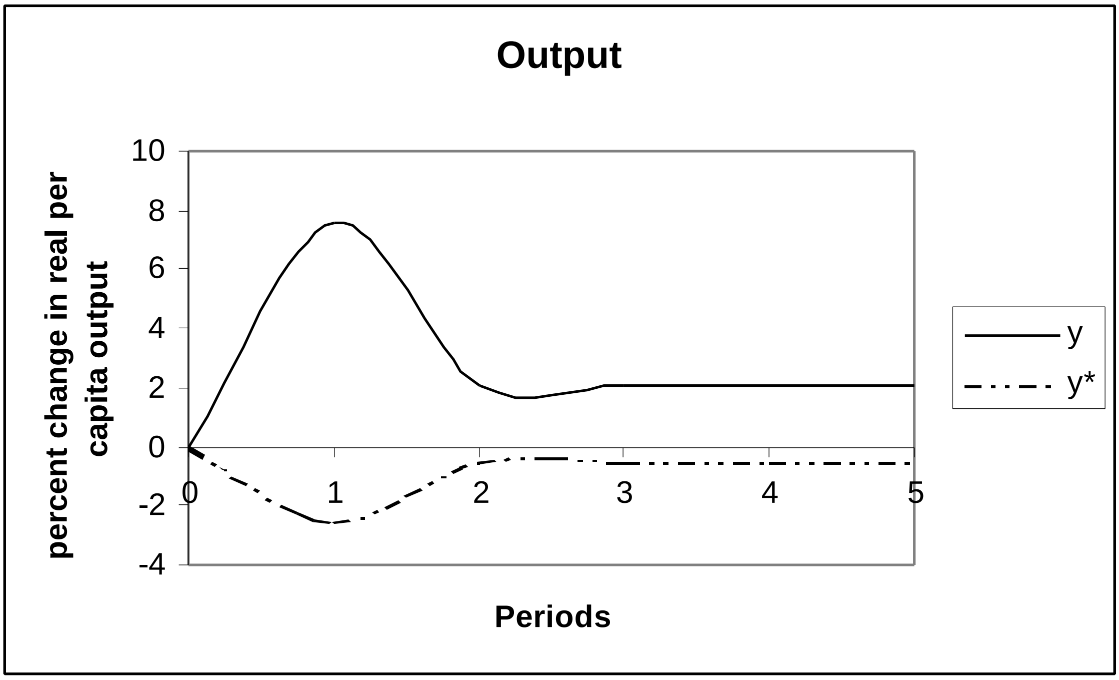
p_CW (TIME) world real consumption demand

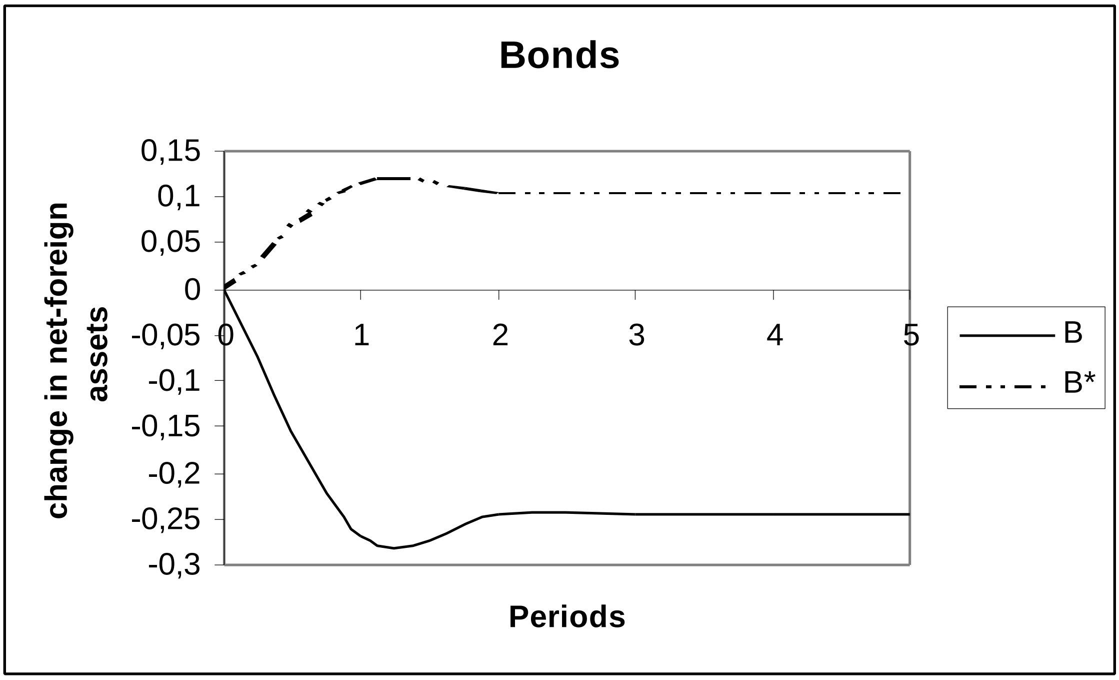
sr	lr
-0.001651	-0.258924

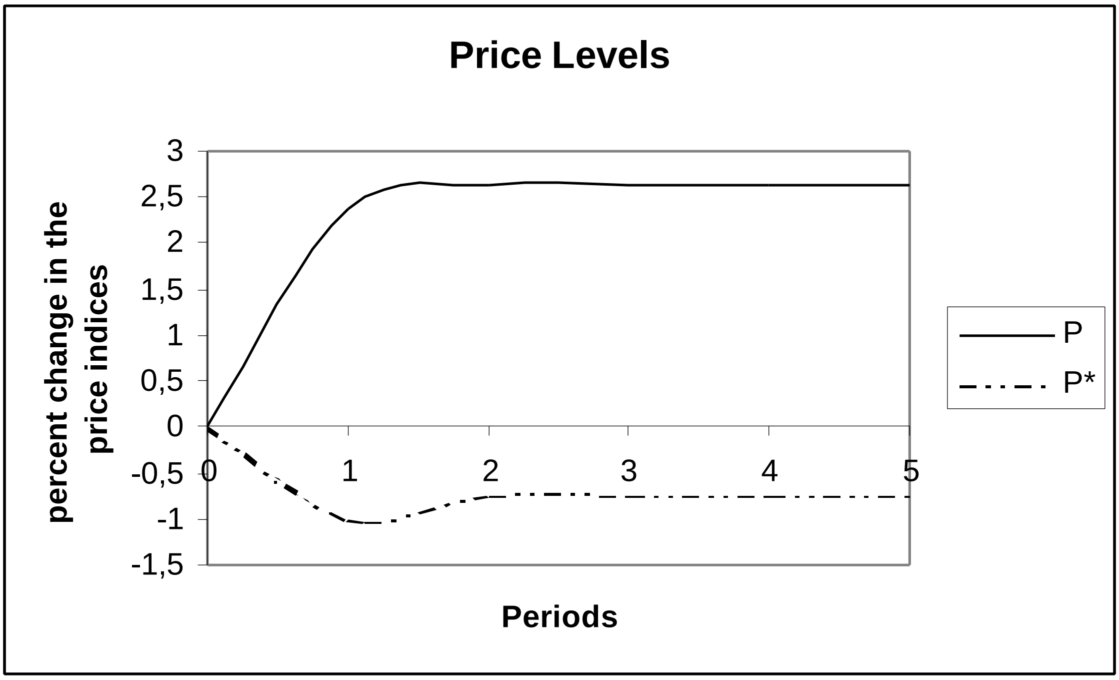
p_GW (TIME) world real government spending

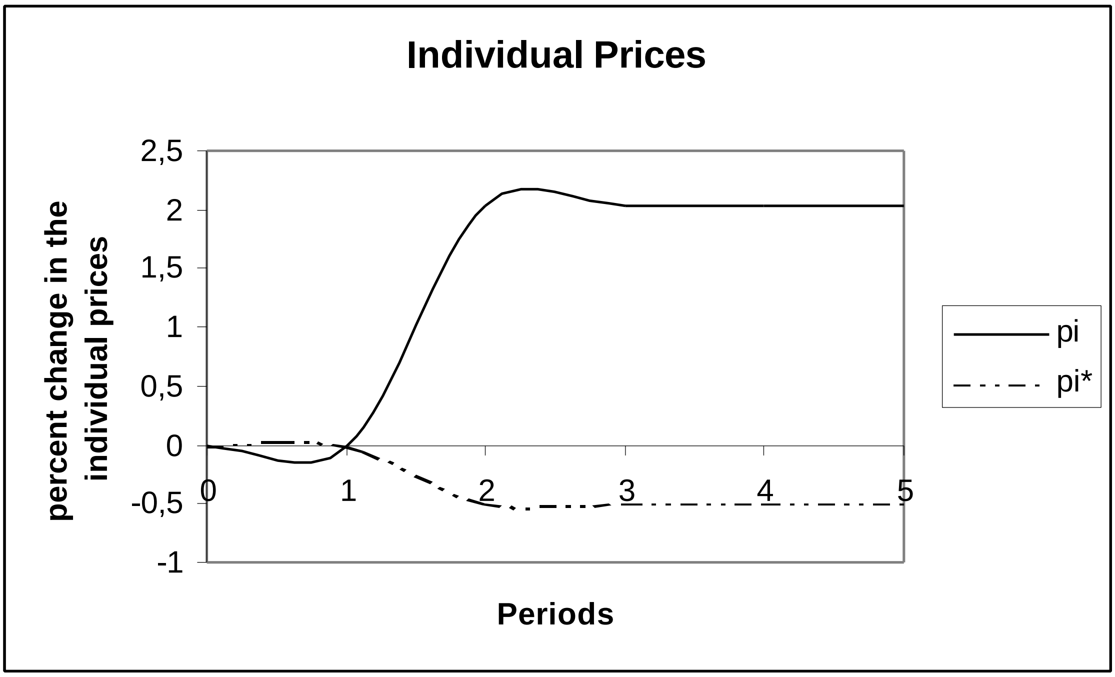
sr	lr
2.999999	2.999999

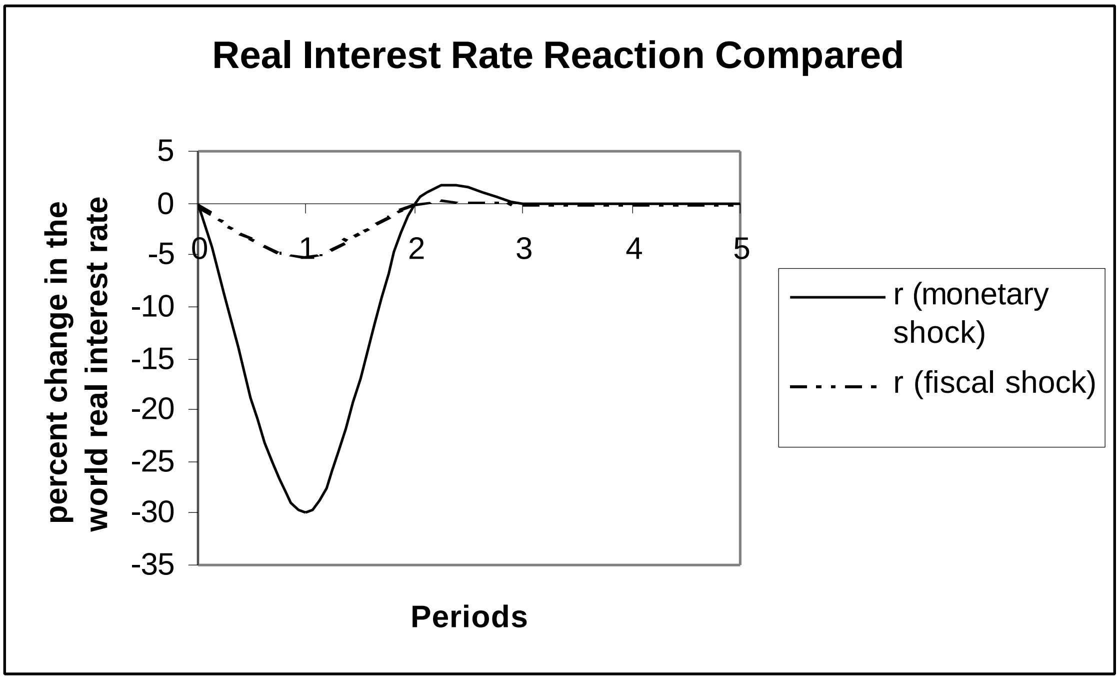


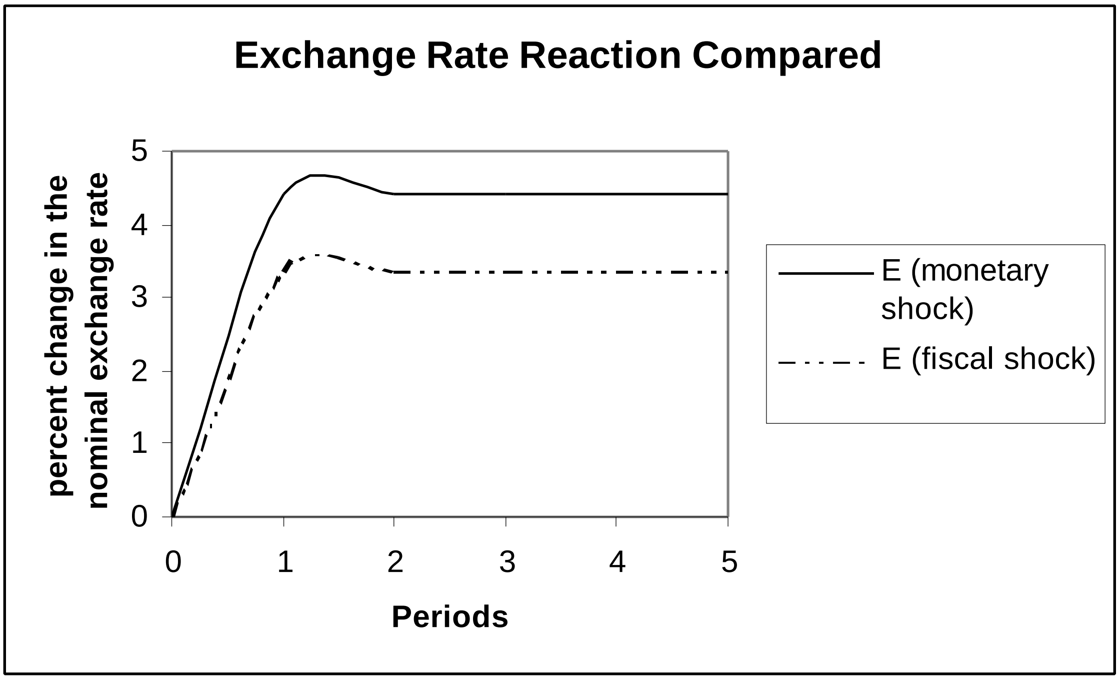

Output
10
8
6
4
2
0
-2
-4
0
1
2
3
4
5
percent change in real per capita output
Periods
y
y*

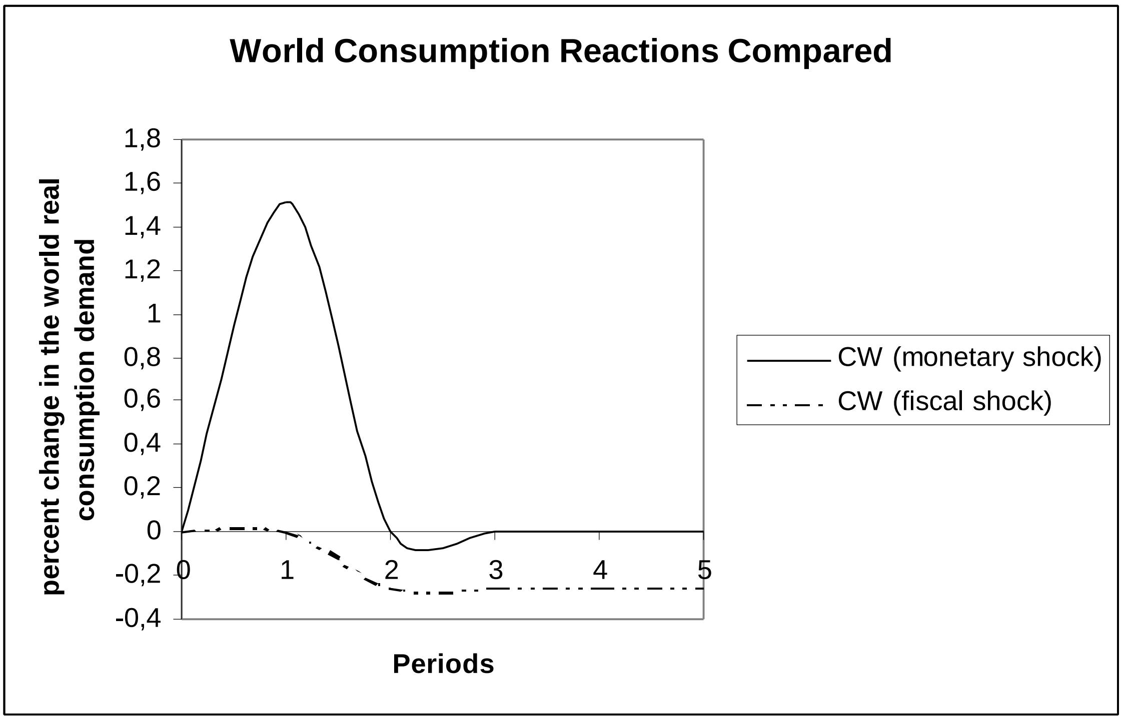

Bonds
0,15
0,1
0,05
0
-0,05
-0,1
-0,15
-0,2
-0,25
-0,3
0
1
2
3
4
5
change in net-foreign assets
Periods
B
B*


Price Levels
percent change in the price indices
3
2,5
2
1,5
1
0,5
0
-0,5
-1
-1,5
0
1
2
3
4
5
Periods
P
P*


Individual Prices
percent change in the individual prices
2,5
2
1,5
1
0,5
0
-0,5
-1
0
1
2
3
4
5
Periods
pi
pi*


Real Interest Rate Reaction Compared
5
0
-5
-10
-15
-20
-25
-30
-35
0
1
2
3
4
5
percent change in the world real interest rate
Periods
r (monetary shock)
r (fiscal shock)


Exchange Rate Reaction Compared
5
4
3
2
1
0
0
1
2
3
4
5
percent change in the nominal exchange rate
Periods
E (monetary shock)
E (fiscal shock)


World Consumption Reactions Compared
percent change in the world real consumption demand
1,8
1,6
1,4
1,2
1
0,8
0,6
0,4
0,2
0
-0,2
-0,4
0
1
2
3
4
5
Periods
CW (monetary shock)
CW (fiscal shock)

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