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We augment a model of endogenous credit cycles by Matsuyama et al. (2016) with human capital to study the impact of human capital on the stability of central economic aggregates. Thus we offer a linkage between human capital formation and credit market instability on a macro-level combined with an analysis of functional income distribution. Human capital is modelled as pure external effect of production following a learning-by-producing approach. Agents have access to two different investment projects, which differ substantially in their next generations spillover effects. Some generate pecuniary externalities and technological spillovers through human capital formation whereas others fail to do so and are subject to financial frictions. Due to this endogenous credit cycles occur and a pattern of boom and bust cycles can be observed. We explore the impact of human capital on the stability of the system by numerical simulations which indicate that human capital has an ambiguous effect on the evolution of the output. Depending on the strength of the financial friction and the output share of human capital it either amplifies or mitigates output fluctuations. This analysis shows that human capital is an essential factor for economic stability and sustainable growth as a high human capital share tends to make the system's stability robust against shocks.

Keywords: Human capital, Learning-by-producing, Credit cycles, Financial instability.

JEL Codes: C61, E32, E24, J24.

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1 Introduction

As the complexity of our financial system emerged distortions in credit markets seem to experience a rising intrigue for economists. Especially when it comes to business cycle models frictions on those markets become essential for a broad range of dynamics and fundamental instability. To certain extent those fluctuations can be explained by exogenous shocks amplified or mitigated by frictions on financial markets (see, i.e., Bernanke & Gertler (1989) or Kiyotaki & Moore (1997)). However, endogenous explanations for persistent credit cycles appear to be scarce. Recent contributions are in the spirit of the Kindleberger-Minsky hypothesis (Kindleberger (1996) and Minsky (1982)) which explain instability as endogenous phenomenon. However, the role of human capital in such a setting remains unclear even though it is an essential factor in long-run economic development[1]. After the seminal work of Mankiw et al. (1992) and Barro (2001), a vast branch of publications focus on the role of human capital when studying business cycles and economic growth issues especially in an empirical context. Most of the empirical literature works on on education, schooling and varieties of skill achievement and its impact on economic well being (for an overview, see Krueger & Lindahl (2001) or Acemoglu & Autor (2012)). Briefly summarized most studies find that human capital, in general, enhances productivity and thus increases growth. But little is known about the effects in a nonlinear dynamic setting, especially when it comes to models where credit markets are involved and irregular cyclicity is an issue. Therefore we suggest to include human capital as production factor in a credit cycle model. Although there are recent contributions on micro level (see, eg., Andolfatto & Gervais (2006), Cunha et al. (2010), Lochner & Monge-Naranjo (2011) or Abbott et al. (2013)) where human capital accumulation is analysed in a setting where education is costly and financed with credit, we want to study it in a more aggregate level and its impact on stability.

Thus, a good starting point to cover those facts is Matsuyama (2013) and Matsuyama et al. (2016) who propose a credit cycle model where financial market frictions cause irregular cycles on an aggregate level. Due to its nonlinear (regime switching) set-up it is able to generate irregular boom and bust cycles without any exogenous shocks, recapitulating the Kindleberger-Minsky hypothesis (Kindleberger (1996) and Minsky (1982)) in a formal economic framework with fully rational agents. Moreover, the results are in line with state-of-the-art empirical rese-

[1]See contributions in growth theory, i.e. Uzawa (1965), Romer (1986, 1990), augmented with human capital in Lucas (1988) or Benhabib & Perli (1994)

arch about credit cycles (see, e.g., Schularick & Taylor (2012), Gali (2014), Thakor (2015)). The existing model uses for simulation purposes a simple Cobb-Douglas technology with physical capital and labour in a two-period overlapping generation model. Our aim is to analyse the effects of human capital on both, the economic implications and the features of the dynamical system (long run stability). In fact, we take a production function with labour, physical and human capital as a starting point and assume that human capital is a pure external effect of production and transferred intergenerationally. In an economy where young and old generations coexist (i.e. overlapping generation structure), old agents transfer their knowledge about production processes to the young. On the one hand, this drives up the expected profits (which are assumed to be the realised profits, i.e. perfect foresight is assumed) of projects with positive pecuniary external effects (i.e. "Good" projects) and thus make it easier to compete with non-spillover but more profitable projects (i.e. "Bad" projects)[2]. On the other hand it rises the wage rate of the young which is crucial for following reasons: After their working period, young agents need to allocate their accumulated net worth (marginal product from labour) to maximise their second period consumption. They choose between Good and Bad projects and lending, but as the latter is subject to a borrowing constraint it can only be financed by a collateral. Thus, an increase of the net worth eases the borrowing constraint. Therefore it needs to be studied if one effect dominates the other or, put differently, how they are interlinked. We expect significant contributions to the question of stability features of human capital in an overlapping generations, nonlinear (regime switching) model setting with both, pecuniary and technological externalities.

Subsequently, the model set-up allows us to rigorously analyse the income distribution over the business cycle. Not only since Piketty (2014) the linkages between personal income inequality, functional income distribution and business cycles are prevalent in economic analyses. Most of the empirical studies provide results about (personal) income inequality based on extensive microeconomic survey data throughout a lot of countries and samples. Even though, we will focus on the impact of aggregated credit market imperfections, a very recent contribution by Hai & Heckman (2017) gives, inter alia, a comprehensive survey about models of agents' educational choice under individual credit constraints. In a more business cycle related setting, Castello & Domenech (2002) elaborate on an assessment of inequality of human capital as a factor and compare it with personal income distribution results. In this paper, we provide a formal framework which allows us to analyse

[2]This is taken from Matsuyama et al. (2016) who offers a discussion and a justification of this terminology.

changes of functional income distribution caused by both, credit market frictions and structural changes on a macrolevel. Due to the overlapping two-period generation structure, we can easily trace the income changes across generations. Our main finding points towards human capital not only tends to stabilise output fluctuations but also stabilises the functional income distribution across generations in sense of periodic fluctuations.

Thus our paper provides three major contributions, consistently modelled in a framework with rational and optimising agents: First, we analyse the effect of human capital on the stability of an economic system with credit market frictions. Most interestingly, we find that especially in transition periods when human capital gains importance, instabilities regarding output evolution occur. Nevertheless, human capital drives out the instability generating financial frictions pointing out that human capital in general has a stabilizing effect. Second, we provide an insight in the functional income distribution over different cycle periods and observe again stabilising impacts of human capital. Finally, as the model is continuous piecewise smooth, two dimensional in physical (k_t) and human capital (h_t), with seven parameters, we highlight the importance of regime switching models in economics. We are able to show that dynamic phenomena might have a strong economic interpretation and simply get lost by applying a linearisation.

The rest of this paper is organised as follows: In Section 2 we will briefly sum up the original model and the key mechanisms which lead to fluctuations and instability but omit the detailed derivation, as this can be found in the aforementioned publications. Section 3 continues by describing the human capital extensions and the new dynamical law of motion. Section 4 is dedicated to a simulation exercise where we use numerical simulations to analyse various scenarios and compare it to the original publication. In Section 5 we give a discussion about the income assessment and Section 6 concludes.

2 The Model and the Mechanism

This section provides a concise summary of the original model structure, the extensions and the core mechanism. The basic framework is close to Matsuyama et al. (2016) which uses an overlapping generations model (see Diamond (1965)) with two period lives. Time is discrete and extends from zero to infinity, $t = 1, 2, 3, \dots$. In each period one final good, the numeraire, is produced which can be used for investments or for consumption. As we want to study the dynamic effects of human capital, the final goods sector uses following Cobb-Douglas technology

$$Y_t = AK_t^{\alpha} H_t^{\gamma} L_t^{1-\alpha-\gamma}, \tag{1}$$

where A denotes some exogenous total factor productivity; K_t is physical capital, H_t is human capital and L_t is simple labour at time t; and $(\alpha + \gamma) < 1$ are the production elasticities. Put differently, those parameters can be seen as cost share of the total production. A higher value of the parameter thus signals a higher importance in the production process. Using the notation in "units of labour" and the normalisation $(1 - \alpha)A = 1$

$$\frac{Y_t}{L_t} = y_t = \frac{1}{1-\alpha} k_t^{\alpha} h_t^{\gamma}. \tag{2}$$

The production function is inspired by Mankiw et al. (1992). However, our approach substantially differs, since we do not consider a resource requirement for human capital formation. We assume that human capital formation is a pure external effect of production ('learning by producing') and that human capital is transferred from the old to the young generation. Thus, a straightforward law of motion for the accumulation follows

$$h_{t+1} = \sigma_h y_t + (1 - \delta_h) h_t, \tag{3}$$

where σ_h is the strength of the external effect and δ_h a depreciation rate on human capital which we consider to be well below unity.

Factor markets are competitive and the factors are rewarded with

$$\rho_t = \frac{\partial f(k_t, h_t)}{\partial k_t} = \frac{\alpha}{1-\alpha} k_t^{\alpha-1} h_t^{\gamma} \quad \text{and} \quad w_t = f(k_t, h_t) - k_t \frac{\partial f(k_t, h_t)}{\partial k_t} = k_t^{\alpha} h_t^{\gamma}. \tag{4}$$

This formulation implies that workers do not only receive the marginal product of labour, but also the marginal product of human capital (which is similar to Mankiw et al. (1992)).

Agents are born at the beginning of each period and stay active for two periods. Young agents are endowed with one unit of labour and the human capital that they inherited from the previous generation; they work in first period and thereby they accumulate human capital. At the end of first period, i.e. in point of time $t + 1$, they earn the factor reward of labour, save everything (savings rate equals unity) and thus accumulate wealth. At the same time, the young generation becomes the old generation (and a new young generation is born to which the human capital is transferred) and they have to decide how to use wealth (accumulated in form of the numeraire) in order to maximise consumption at the end of second period. Fig. 1 presents a graphical description of the generational set-up. Agents have three

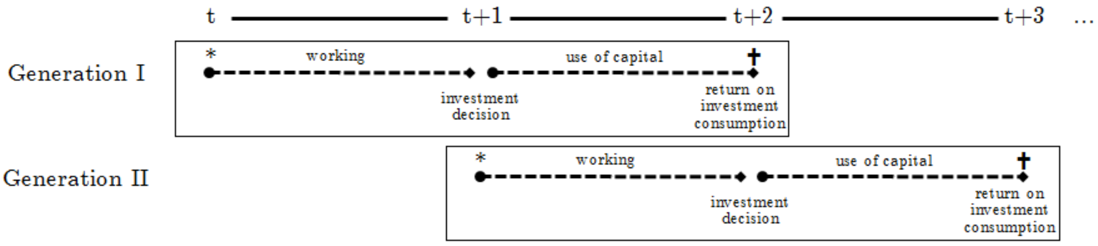


Fig. 1: Overlapping generation structure

possibilities to allocate wealth: They can (1) invest in a "Good" investment project or (2) they can start a "Bad" investment project; in addition, (3) they can lend funds to other same cohort agents. Good projects are investments in the final goods production sector that uses the final good as physical capital input, k_{t+1}. Assuming perfect foresight, the expected return of this investment project type is equal to the marginal product of capital, $\rho_{t+1}^e = \rho_{t+1}$. Thus, Good projects fuel production processes that generate labour income for the next generation and induce human capital formation. Instead, Bad projects do not involve production processes. They can be seen as simple trading or storing activities, and essentially fail to create any positive externalities for the next generations. Those type of projects are assumed to require an indivisible amount of $m > w_t$ units of the final good and to transform it into mB units of the final good in period $t+1$. The known and constant parameter $B > 0$ indicates the profitability of the Bad projects. Implicitly, it is assumed that agents who want to run those projects need to borrow $m - w_t > 0$ at an interest rate r_{t+1}; this interest rate is agreed upon in $t+1$ and has to be paid at the end of period $t+1$. Since Bad projects require credit financing lending is a third option for wealth allocation. Given the possibility of investing in Good projects, the interest rate on credit has to be equal to the expected marginal product of capital $r_{t+1} = \rho_{t+1}^e$ in equilibrium. Now two constraints enter the game: The *profitability* constraint and the *borrowing* constraint. The profitability constraint follows from the consideration that agents only intend to start Bad projects if return is greater or equal to the return of simple lending or investing in Good projects. Thus,

$$B \geq r_{t+1} = \rho_{t+1}^e \Big(= \frac{\partial f(k_{t+1}, h_{t+1})}{\partial k_{t+1}} \Big) = \rho_{t+1}. \tag{5}$$

The borrowing constraint takes capital market imperfections into account: Due to financial frictions agents can borrow only against a collateral. In addition, only a fraction of the expected project revenue can be pledged for the repayment which reflects the credit market imperfection and is denoted by $0 < \mu < 1$. This minimalistic introduction of a financial friction follows the pledgeability approach proposed by Tirole (2005) and can be justified by some kind of agency problems which may arise in economic transactions. The reader is kindly referred to Matsuyama (2008) for a detailed discussion in a macroeconomic context. As information is complete the borrowing constraint requires that:

$$\mu m B \geq r_{t+1}(m - k_t^\alpha h_t^\gamma) \qquad \text{or} \tag{6}$$

$$\frac{\mu m B}{m - k_t^\alpha h_t^\gamma} \geq r_{t+1}. \tag{7}$$

The lender will only lend up to $\frac{\mu m B}{r_{t+1}}$ which implicitly sets a minimum net worth requirement[3] for agents interested in starting a Bad project. If the financial friction is severe (i.e. $\mu = 0$) the left hand side of Eq. (7) equals zero implying that the net worth of the agents is always too low to start a Bad project. The other extreme case is the absence of a friction (i.e. $\mu = 1$) where agents can fully pledge revenues as a collateral to lenders. The borrowing constraint, Eq. (7), sets a tighter limit for r_{t+1} than the profitability constraint, Eq. (5), if:

$$\frac{\mu m B}{m - k_t^\alpha h_t^\gamma} < B \tag{8}$$

$$k_t < k_\mu = \left(m(1-\mu)h_t^{-\gamma}\right)^{1/\alpha} \tag{9}$$

The critical value, k_μ which is a function of human capital, separates two regions in the $[k, h]$-phase space (see Fig. 2), where either BC or PC is binding. It is strictly decreasing with higher k and lower h.

Analogously to Matsuyama et al. (2016) we define the maximal pledgeable rate of return, $R(k_t, h_t)$, that an agent with the net worth $w_t = k_t^\alpha h_t^\gamma$ can pledge to the lender without violating a constraint:

$$R(k_t, h_t) \equiv B\min\left\{\frac{\mu}{1 - \frac{k_t^\alpha h_t^\gamma}{m}}, 1\right\} = \begin{cases} \dfrac{\mu B}{1 - \frac{k_t^\alpha h_t^\gamma}{m}} & \text{if } k_t \leq k_\mu \quad \text{i.e. if BC is tighter} \\ B & \text{if } k_t \geq k_\mu \quad \text{i.e. if PC is tighter} \end{cases} \tag{10}$$

[3] Recall, that the net worth of young agents at the end of their 'working' period equals $w_t = k_t^\alpha h_t^\gamma$.

3 The dynamic equations and the phase space

We are now ready to derive the law of motions for physical and human capital. In equilibrium following equation must hold with equality:

$$\rho_{t+1}^{e}=\rho_{t+1}=r_{t+1}=\frac{\partial f(k_{t+1},h_{t+1})}{\partial k_{t+1}}=\frac{\alpha}{1-\alpha}k_{t+1}^{\alpha-1}h_{t+1}^{\gamma}\geq R(k_t,h_t) \tag{11}$$

Otherwise, if $\rho_{t+1}=r_{t+1}<R(k_t,h_t)$ would hold with strict inequality, agents always want to start Bad projects (higher returns) but nobody would provide the required credit as the rate of return of lending is to low, which is a contradiction and therefore not possible. In the case of $\rho_{t+1}>R(k_t,h_t)$ agents would never run Bad projects due to a violation of the profitability or the borrowing constraint. Following Matsuyama et al. (2016), we also differentiate a *non-distortionary* and a *distortionary* case (see Fig. 2 for the phase space representation):

3.1 The non-distortionary case

The *non-distortionary case* in which the borrowing constraint is never binding and aggregate credit is thus allocated efficiently, occurs if $\left.\frac{\partial f(k_{t+1},h_{t+1})}{\partial k_{t+1}}\right|_{k_t=k_\mu}>B$. In that case, for low k_t and thus a high return on Good projects, $\frac{\partial f(k_{t+1},h_{t+1})}{\partial k_{t+1}}>B$ and Bad projects will not be started as they are less profitable than the Good. All available credit flows into the Good projects and the corresponding dynamic equation is

$$k_{t+1}=\Psi_L=k_t^{\alpha}h_t^{\gamma}. \tag{12}$$

Increasing k_t reduces the return on Good projects, until reaching a threshold $k_B>k_\mu$, defined by $k_{t+1}=k_t^{\alpha}h_t^{\gamma}\,(=w_t)$ and $\frac{\partial f(k_{t+1},h_{t+1})}{\partial k_{t+1}}=B$; the profitability of Good and Bad projects is equal. Beyond that point any additional credit flows in Bad projects and investment in Good projects k_{t+1} is determined by the profitability constraint, i.e. by $\frac{\partial f(k_{t+1},h_{t+1})}{\partial k_{t+1}}=B$. The corresponding dynamic equation is[4]

$$k_{t+1}=\Psi_R=\left(\frac{1}{B}\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\alpha}}(h_{t+1})^{\frac{\gamma}{1-\alpha}}. \tag{13}$$

Note that in this case the financial frictions parameter μ does not occur in the dynamics; thus, this case is indeed *non- distortionary*. The boundary condition for

[4]For notational purpose, we refer to the regime $\Psi_i\in\{L,M,R\}$ according to its location, either left, middle or right, in the phase space.

this case is $\left.\frac{\partial f(k_{t+1},h_{t+1})}{\partial k_t+1}\right|_{k_t=k_\mu} = B$. Observe that for $k_t = k_\mu$ the net worth is given as $w_t = (1-\mu)\,m$ and the output as $y_t = \frac{1}{1-\alpha}(1-\mu)\,m$. Using $k_{t+1} = w_t$ and $h_{t+1} = \sigma_h y_t + (1-\delta_h)h_t$ allows to determine the threshold explicitly as

$$\overline{h} = \frac{1}{1-\delta_h}\left[\left(\frac{1}{B}\frac{\alpha}{1-\alpha}\right)^{-\frac{1}{\gamma}}((1-\mu)\,m)^{\frac{1-\alpha}{\gamma}} - \frac{\sigma_h}{1-\alpha}(1-\mu)\,m\right]. \tag{14}$$

The *non-distortionary case* occurs for $h_t > \overline{h}$ (above the dashed line in Fig. 2).

3.2 The distortionary case

The *distortionary case*, in which the borrowing constraint impinges upon the dynamics and financial frictions play a role, occurs if $\frac{\partial f(k_{t+1},h_{t+1})}{\partial k_{t+1}}|_{k_t=k_\mu} < B$, or if $h_t < \overline{h}$ (see below the dashed line in Fig. 2). Again, for low k_t agents do not intend to start Bad projects because of the high profitability of Good projects. Increasing k_t reduces the profitability of Good projects and at some $k_B < k_\mu$ the profitability of both investment types will be equal and agents start to prefer Bad projects. However, since the wage rate and thus the net worth is still low, the maximum pledgeable rate of return for credit is lower than the return on Good investments and agents cannot obtain the required credit – the borrowing constraint is still binding and agents continue to invest only in the Good projects. In that region of the phase space, the law of motion is given by:

$$k_{t+1} = \Psi_L = k_t^\alpha h_t^\gamma. \tag{15}$$

Further increasing k_t raises the agents' wage rate and thus net worth, which increases the maximum pledgeable rate of return on credit and eases the borrowing constraint. At a threshold k_c, the borrowing constraint is satisfied with equality (while the entire net worth is still invested in Good projects) and the maximum pledgeable rate of return on credit is equal to the profitability of Good projects. k_c is thus implicitly defined by $\frac{\partial f(k_{t+1},h_{t+1})}{\partial k_{t+1}} = \frac{\alpha}{1-\alpha}k_{t+1}^{\alpha-1}h_{t+1}^\gamma = R(k_t,h_t) = \dfrac{\mu m B}{m - k_t^\alpha h_t^\gamma}$, in which $k_{t+1} = k_t^\alpha h_t^\gamma\,(= w_t)$. Beyond this threshold, for $k_t > k_c$ (but $k_t < k_\mu$) credit starts to flow into Bad projects. Investment in Good projects is determined by $\frac{\partial f(k_{t+1},h_{t+1})}{\partial k_{t+1}} = \frac{\alpha}{1-\alpha}k_{t+1}^{\alpha-1}h_{t+1}^\gamma = R(k_t,h_t) = \dfrac{\mu m B}{m - k_t^\alpha h_t^\gamma}$. Solving for k_{t+1}, the law of motion for that region of the phase space is determined by

$$k_{t+1} = \Psi_M = \left(\frac{1}{\mu B}\frac{\alpha}{1-\alpha}\left(1-\frac{k_t^\alpha h_t^\gamma}{m}\right)\right)^{\frac{1}{1-\alpha}} (h_{t+1})^{\frac{\gamma}{1-\alpha}}. \tag{16}$$

After crossing the next threshold, $k_t > k_\mu$, the borrowing constraint is not binding any more, and investment in Good projects k_{t+1} is determined by the profitability constraint, i.e. by $\frac{\partial f(k_{t+1},h_{t+1})}{\partial k_{t+1}} = B$. All additional credit flows into Bad projects and the dynamics follow again Ψ_R,

$$k_{t+1} = \Psi_R = \left(\frac{1}{B}\frac{\alpha}{1-\alpha}\right)^{\frac{1}{1-\alpha}} (h_{t+1})^{\frac{\gamma}{1-\alpha}}. \tag{17}$$

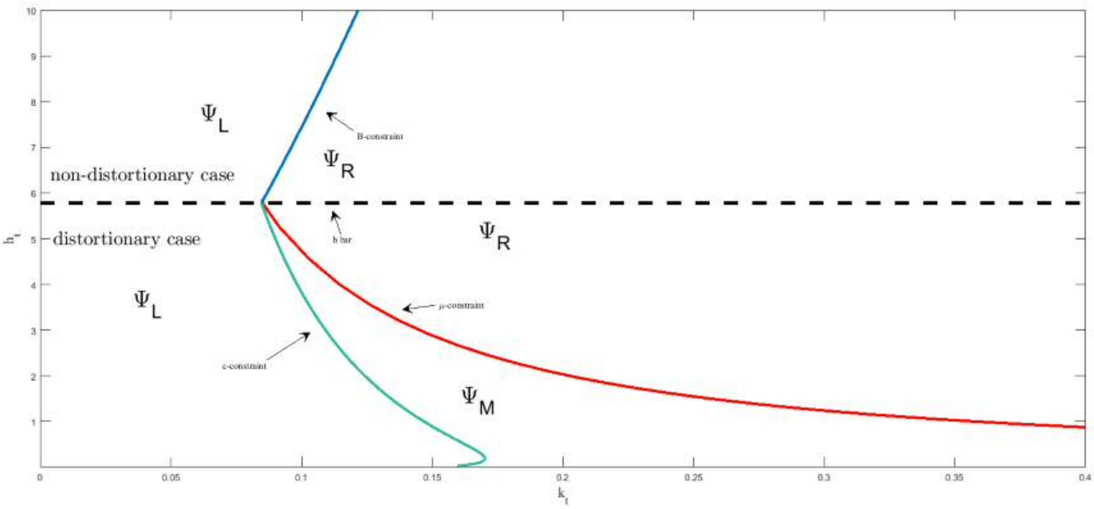


Fig. 2: Phase space

Tab. 1 provides a qualitative and Tab. 2 a formal summary of the relevant thresholds and specifications. Fig. 2 indicates the phase space formed by those thresholds.

Threshold	Meaning
$k_t = k_B$	Good is as profitable as Bad (non-distortionary case)
$k_t = k_c$	BC starts to be binding
$k_t = k_\mu$	BC is not binding anymore

Tab. 1: Threshold values (qualitative)

Threshold	General specification	CD specification
k_B	$\frac{\partial f(k_{t+1},h_{t+1})}{\partial k_{t+1}} = B$ $k_{t+1} = w_t$	$k_B = \left[\frac{1}{B}\frac{\alpha}{1-\alpha}h_{t+1}^{\gamma}\right]^{\frac{1}{\alpha(1-\alpha)}} h_t^{-\frac{\gamma}{\alpha}}$
k_c	$\frac{\partial f(k_{t+1},h_{t+1})}{\partial k_{t+1}} = R(k_t, h_t)$ $k_{t+1} = w_t$	$k_c = \left[\frac{1}{\mu B}\frac{\alpha}{1-\alpha}\left(1 - \frac{k_c^\alpha h_t^\gamma}{m}\right)h_{t+1}^{\gamma}\right]^{\frac{1}{\alpha(1-\alpha)}} h_t^{-\frac{\gamma}{\alpha}}$
k_μ	PC = BC	$k_\mu = \left[m(1-\mu)h_t^{-\gamma}\right]^{1/\alpha}$

Tab. 2: Threshold values (analytical)

Putting above mentioned equations together, we now can construct the dynamical system taking human capital as an external effect into account, for the *non-distortionary* case, if $h_t > \overline{h}$:

$$\Psi : \begin{pmatrix} k_{t+1} \\ h_{t+1} \end{pmatrix} \mapsto \begin{pmatrix} k_{t+1} = \begin{cases} \Psi_L = k_t^\alpha h_t^\gamma & \text{if } k_t \leq k_B \\ \Psi_R = \left(\dfrac{\alpha}{B(1-\alpha)}\right)^{\frac{1}{1-\alpha}} \left[h_{t+1}\right]^{\frac{\gamma}{1-\alpha}} & \text{if } k_t \geq k_B \end{cases} \\ h_{t+1} = \left\{ \frac{\sigma_h}{1-\alpha} k_t^\alpha h_t^\gamma + (1-\delta_h) h_t \right. \qquad \forall k_t \end{pmatrix} \tag{18}$$

and for the *distortionary* case, if $h_t < \overline{h}$:

$$\Psi : \begin{pmatrix} k_{t+1} \\ h_{t+1} \end{pmatrix} \mapsto \begin{pmatrix} k_{t+1} = \begin{cases} \Psi_L = k_t^\alpha h_t^\gamma & \text{if } k_t \leq k_c \\ \Psi_M = \left[\dfrac{1}{\mu B}\dfrac{\alpha}{1-\alpha}\left(1 - \frac{k_t^\alpha h_t^\gamma}{m}\right)\right]^{\frac{1}{1-\alpha}} \left[h_{t+1}\right]^{\frac{\gamma}{1-\alpha}} & \text{if } k_c \leq k_t \leq k_\mu \\ \Psi_R = \left(\dfrac{\alpha}{B(1-\alpha)}\right)^{\frac{1}{1-\alpha}} \left[h_{t+1}\right]^{\frac{\gamma}{1-\alpha}} & \text{if } k_t \geq k_\mu \end{cases} \\ h_{t+1} = \left\{ \frac{\sigma_h}{1-\alpha} k_t^\alpha h_t^\gamma + (1-\delta_h) h_t \right. \qquad \forall k_t \end{pmatrix}. \tag{19}$$

Thus, the system is continuous piecewise smooth, two dimensional in k and h, with seven parameters, $\alpha, \gamma, \mu, m, B, \sigma_h, \delta_h$. Following restrictions apply: $\alpha + \gamma < 1$, $0 < \mu, \sigma_h, \delta_h < 1$, $B > 0$ and $m > 1$. Similar to the original model, the law of motion for k_t is crucial for the dynamics. But as human capital has an additional positive effect on the next period net worth, it also affects the magnitude of k each period. For the learning-by-producing external effect of human capital, the parameter σ_h indicates the strength of this effect. This knowledge is assumed to be highly persistent therefore the depreciation rate (or rate of forgetfulness) is set far below unity.

Remark: Matsuyama et al. (2016) assume that $W(\bar{K}) = \bar{K}$ (the maximal attainable net worth) is always lower than m, the fixed investment size parameter for Bad projects. Due to the normalisation of the system, the assumption $\bar{K} < m$ holds for $m > 1$. So, young agents who want to start a Bad project always need to borrow as the net worth is necessarily too low. In our model, this assumption might be violated for high values of the production function parameters α and γ. Due to the human capital the available net worth could be much higher than the required investment size, m, such that agents have excess net worth which they can also lend (in such situations, credit becomes negative). But this features does not affect the general stability results as it (under certain parameter configurations, though far away from empirical justifiability) just drives the equilibrium values of physical and human capital, return on labour and income beyond unity. But the agents will not find borrowers for excess net worth as all the other agents have enough to invest. Thus, the excess net worth remains an unproductive (in the sense of no return) residual which will not produce any additional dynamics. Note that in a situation of negative credit, the system will always reach its fixed point, so the dynamics of the central dynamic variables are, after an initial transient phase, converging to an equilibrium value. As we are interested in analysing cyclical characteristics we stick to a parametrisation which ensures that we are in such regions. In general, one can avoid the situation by quasi endogenising the fixed investment size parameter m, as a excess percentage of the maximum attainable net worth.

4 Dynamic analysis

In this version of the model, human capital enhances the profitability of the next generation's Good projects (higher h_t increases $\rho^e_{t+1} = \frac{\partial f(k_{t+1},h_t)}{\partial k_{t+1}} = \frac{\alpha}{1-\alpha} k_{t+1}^{\alpha-1} h_{t+1}^{\gamma}$, the expected reward of starting a Good project in period t). Thus, investing into Bad projects, which neither generate pecuniary nor technological externalities for the next generation, becomes more unattractive. In terms of the model, the profitability constraint tightens up. Through this mechanism human capital is expected to serve as a stabiliser as it creates more incentives (i.e. profit) to start Good projects. On the other hand, it also might boost the general output as the consumption of the old generation (which allocated net worth at the end of period t) at the end of period $t+1$ is now higher, as human capital is also included in the final good, the numeraire.

4.1 Bifurcation phenomena

Before starting the dynamical analysis, we shall point to the fact, that by setting $\gamma = 0$ and $h_0 = 0$, the model collapses in the original Matsuyama et al. (2016) model. We will present the dynamics of k_t, the physical capital stock per unit of labour. As the two dimensional equation which governs the dynamics is of complicated structure we offer numerical simulations to determine stability features. The critical parameters in the original paper were the strength of the credit frictions (μ), the gross return of Bad projects (B) and the fixed investment size (m) of the Bad projects. Our set-up provides new parameters concerning human capital elasticity (γ) which can be seen as component representing the economy's structure, a depreciation rate (δ_h) and the strength of the external effect σ_h. Thus, a comprehensive study of the dynamics might go beyond the scope of this paper and therefore we concentrate on a parametrisation which is comparable to the baseline model from Matsuyama et al. (2016). For a first analysis, we fix $\delta_h = 0.05$ and $\sigma_h = 0.5$ and check the dynamical system in the (μ, B) parameter space[5]. For the capital share we follow the standard macroeconomic literature and set $\alpha = 0.4$. Although we point out that this model is highly stylized therefore an exact calibration of the model seems not achievable. Thus, we set our parameters comparable close to Matsuyama et al. (2016). The two dimensional bifurcation diagram[6], displayed in Fig. 3, shows the effect of an increase in γ on the (μ, B) parameter space.

We observe a shrinkage of the area with high-order periodicity (white) but also an expansion of parameter combinations which eventually lead to period two and four cycles[7]. On the one hand we find a destabilisation as a stable fixed point is harder to achieve with intermediate values of γ but also irregular cycles (i.e. cycles with period higher than 11) are less likely to occur. Moreover, the parameter regions where a fixed point is observed also shrink, additionally pointing towards destabilising effects.

[5]This is consistent with the assumption that human capital depreciation is low, thus the accumulation highly persistent. Additionally, the strength of the external effect is chosen to be of intermediate magnitude.

[6]The textbook definition of a *bifurcation* refers to a qualitative change in the long-run dynamics as a model parameter changes.

[7]A note on definition: A period n-cycle is a cycle which has a duration length of n time steps.

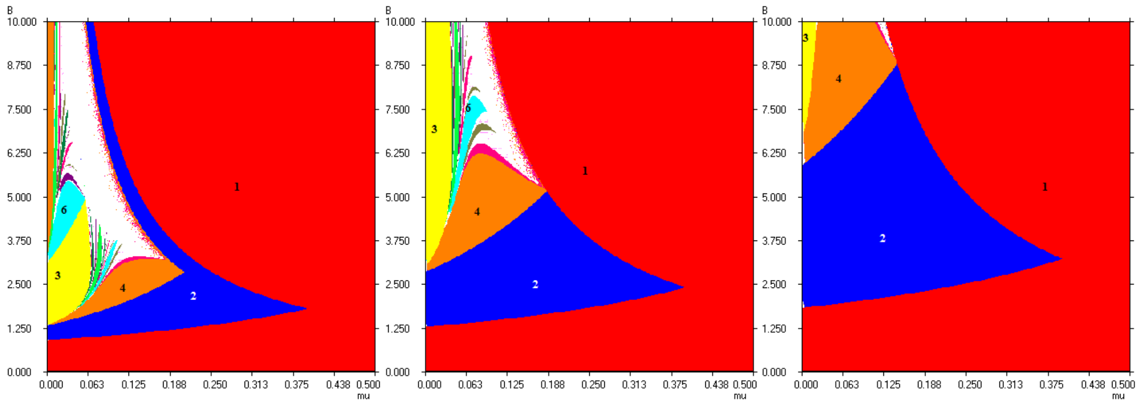


Fig. 3: (μ, B) plane with $\gamma = 0.05$ (left), $\gamma = 0.10$ (center) and $\gamma = 0.15$ (right). Numbers indicate the periodicity of stable cycles. The white regions indicate a periodicity higher than 12.

But, however, a much higher gross rate of return of Bad projects is necessary to enter the regions of high periodicity compared to the original model. Thus, we conclude that human capital serves as stabiliser in the following way: It pushes up the profitability of Good projects such that the net worth of young agents after their working period is higher. This enables young agents to invest in high profitable Bad projects as the net worth requirement (i.e. the borrowing constraint) is fully met. This leads under sufficiently low frictions (agents can pledge a higher amount of return, namely μB, see middle branch of Eq. (19), to the lender) to a credit shift towards Bad projects. Without human capital the result would be a deterioration of the next generation's net worth due to the lack of Good projects. However, human capital changes that situation as the net worth is still high enough which eventually avoids the propagated mechanism of boom and bust cycles. On the other hand, for intermediated high values of B we indeed observe rich and complex dynamic behaviour of k, the physical capital.

The crucial parameter for the importance of human capital is γ, technically indicating the output elasticity of human capital. Put differently, this also expresses the cost share of output whose stability impact we will discuss in the following section.

4.1.1 Corridor Stability

We stick now to the case, where we enter the region of high periodicity to check for intriguing dynamic phenomena. For instance, Matsuyama et al. (2016) reports the so-called *corridor stability* for the parameter μB which exhibits not only some interesting dynamic properties but also offers a strong economic rational. As we use an augmented production function we concentrate again on γ, the production share of human capital. We find the same phenomenon for certain γ-values depending on the magnitude of B and μ, where all other parameters left unchanged. The credit market friction parameter is set $\mu = 0.25$ such that a convergence towards a stable steady state could be achieved without human capital.

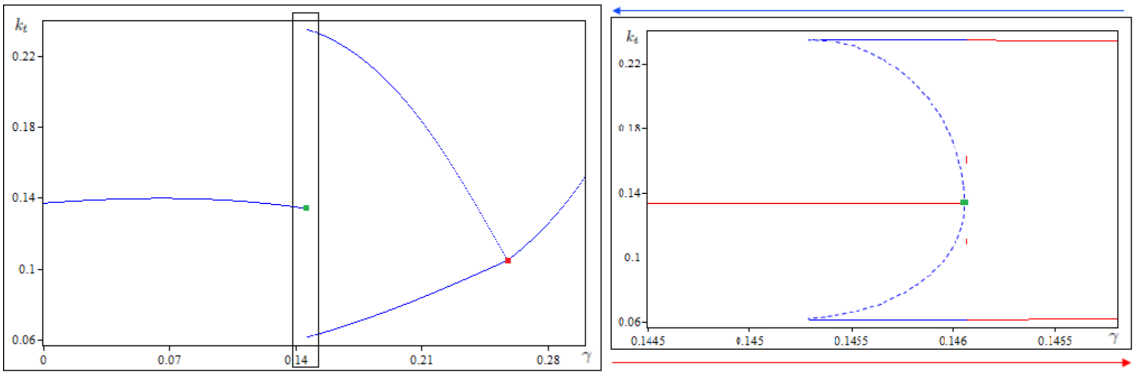


Fig. 4: Bifurcation scenario of γ (with $B = 5, \mu = 0.25$); arrows indicate the direction of computation

We report the bifurcation scenario of parameter γ by tracing Fig. 4 from left to right. The left panel of Fig. 4 shows the bifurcation structure for an intermediate range of γ-values. With sufficient low importance of human capital (i.e. a low γ-value) we observe a fixed point for k_t near 0.14. Reaching the green point, the computation of the system's eigenvalue yields minus unity, stating that the system lost its stability via a flip bifurcation. A period-two cycle is born. By enlarging the interval around $\gamma = 0.14$ (see boxed region) and varying the computing directions[8] we observe following phenomena: There is a coexisting stable period two cycle and a stable fixed point and, moreover, also a period two saddle point (indicated by the dotted line). To confirm those presumptions we perform some numerical simulations which lead to results reported in Tab. 3. There is a triple cycle coexistence, a fixed point, a stable and a saddle period two cycle.

Period	Classification	$L_x = \{(k^*, h^*)\}$	colour in Fig. 5
[1]	stable	$\{(0.137, 10.512)\}$	green
[2]	stable	$\{(0.062, 10.480), (0.236, 10.341)\}$	yellow
[2]	saddle	$\{(0.077, 10.509), (0.207, 10.404)\}$	white

Tab. 3: Numerical simulation results, $\gamma = 0.1458$, where L_x indicates the periodical fixed points.

[8] We initialise on the previous value of k_t.

This is exactly the phenomenon of *corridor stability* which Matsuyama et al. (2016) reported for the parameter product μB. The corridor is spanned by the unstable period two cycle (see dotted line). The parameter γ indicates the importance of human capital in the production process. If a shock hits this parameter the magnitude of the shock is crucial for the system. This corridor stabilty implies that the system is robust and self-correcting against small shocks but unstable against shocks with higher magnitude (see Leijonhufvud (1973) for a first comment and a qualitative treatment of this issue). In this situation even a small positive shock in γ becomes *catastrophic* and *irreversible* if the flip bifurcation point is crossed. The latter means by reverting to the original parameter value, we will not come back to the stable fixed point but remain at the (stable) period two cycle (transition from the red to the blue bifurcation path in Fig. 4). The former characteristic is crucial for our model: Human capital brings another possible component into the model as it produces instability for a wide parameter range of γ.

Moving on, the amplitude of the period-two cycles gradually decreases until the red point. The bifurcation at the red point is a *border collision bifurcation (BCB)* as the trajectory crosses the k_μ border, thus it moves from the second (Ψ_M from Eq. (19)) to the third regime. For $\gamma \gtrsim 0.257$ the trajectory stays at the third regime. Out of a period two cycle a fixed point is born and the stability of the system is ensured for an intermediate human capital share. At least at this parameter configuration human capital produces stability. Also in the augmented model the corridor stability remains present for the friction parameter μ. We refer for a detailed mathematical treatment to Sushko et al. (2014) and Matsuyama et al. (2016).

Most interestingly the basin of attraction[9], displayed in Fig. 5, shows some structures which we need to discuss in detail. The red area indicates a fixed point basin and the blue area a period two basin. The boundary between the period two and the fixed point basin is the k_μ threshold. That is where the borrowing constraint is not binding any more. The basin boundary is spanned by the unstable period two cycle, forming a *corridor*. The coloured dots refer to the period and its stability property (see Tab. 3 for numerical evidence).

From an economic perspective we want to stress the following: Human capital has an ambiguous effect on the system's stability. In an economy where human capital is either relatively negligible or sufficiently important, we do not observe any stability distortions. But, however, there exists a parameter region where human capital is a factor for instability due to the period two cycle. In this region we also observe the

[9] In terms of dynamical system theory, a basin of attraction of an attractor is the set of all initial conditions converging, after sufficient transient iterations, to that attractor.

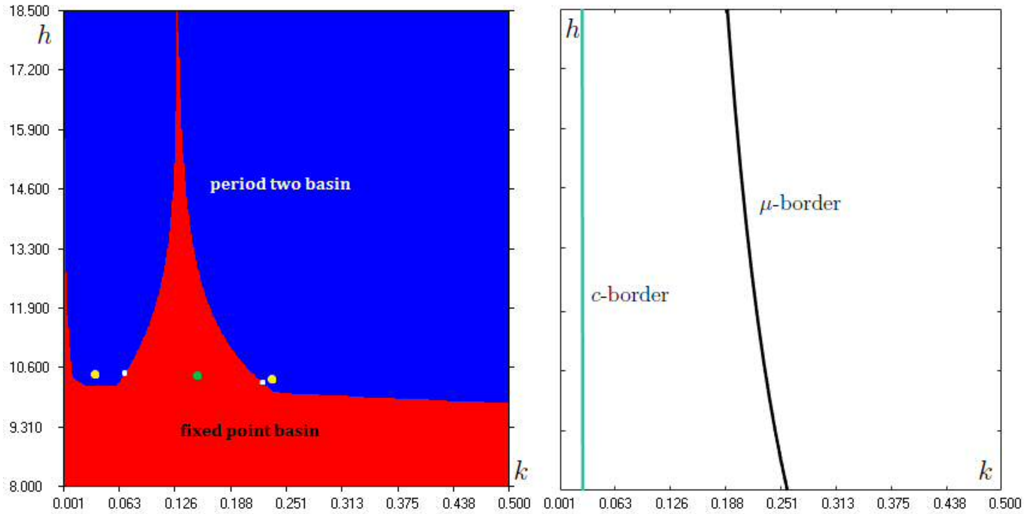


Fig. 5: Basin of attraction (left) and basin boundaries (right) with $\gamma = 0.1458$

phenomenon of corridor stability. So with a rising importance of human capital it is necessary to endure a transition period which might come with some instability. Put differently, the output shares or the cost share of human capital regarding the output reflects the compositions of sectors in an economy. Our result suggests that small sectoral shifts tend to be robust regarding the stability of output. In scenarios where large shifts occur (e.g. transition from agricultural based economies over industrial to service based economies) instability might be more persistent and comes with higher frequency of boom and bust cycles.

4.1.2 Interaction of frictions and human capital

This section analyses the interaction effects between the human capital parameter and the strength of the credit market friction. There exists a $[\mu, \gamma]$-parameter continuum where periods higher than order 12 occur. The left panel of Fig. 6 shows this situation and the right panel shows an enlargement of the boxed area on the left panel. The line traces the one dimensional bifurcation in Fig. 7.

The boundary between the fixed point and the period two cycle parameter range has a clear structure. The fixed point looses its stability through a flip bifurcation, i.e. the numerical evaluation of the eigenvalues of the Jacobian equals minus unity. Due to the complicated structure of the Jacobians (recall, that each of the three regimes has its own Jacobian) an analytical treatment is omitted. Nevertheless, numerical evaluations of the value of the Jacobian will be provided. A closer look shows the rich internal bifurcation structure of the parameter μ pointing towards fractal patterns.

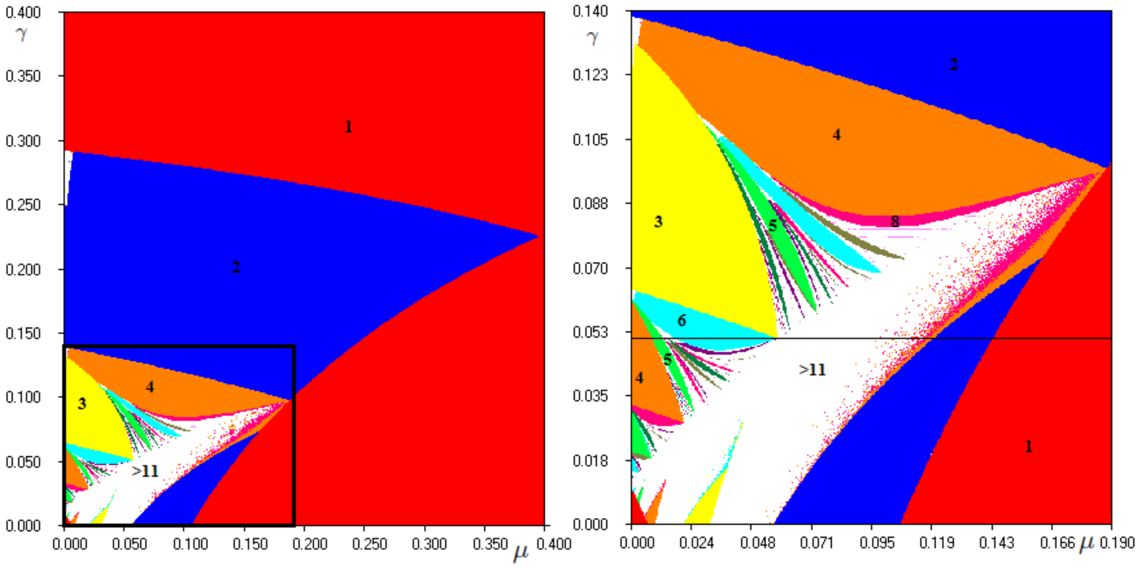


Fig. 6: (μ, γ) plane with $\alpha = 0.4$ (left) and enlarged box (right). Numbers indicate length of stable cycle.

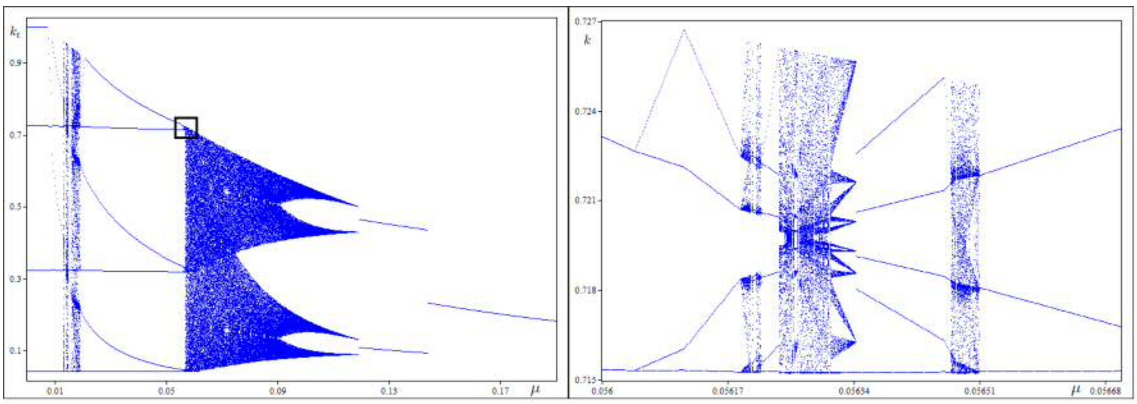


Fig. 7: Bifurcations scenario of μ (left) and enlarged box (right).

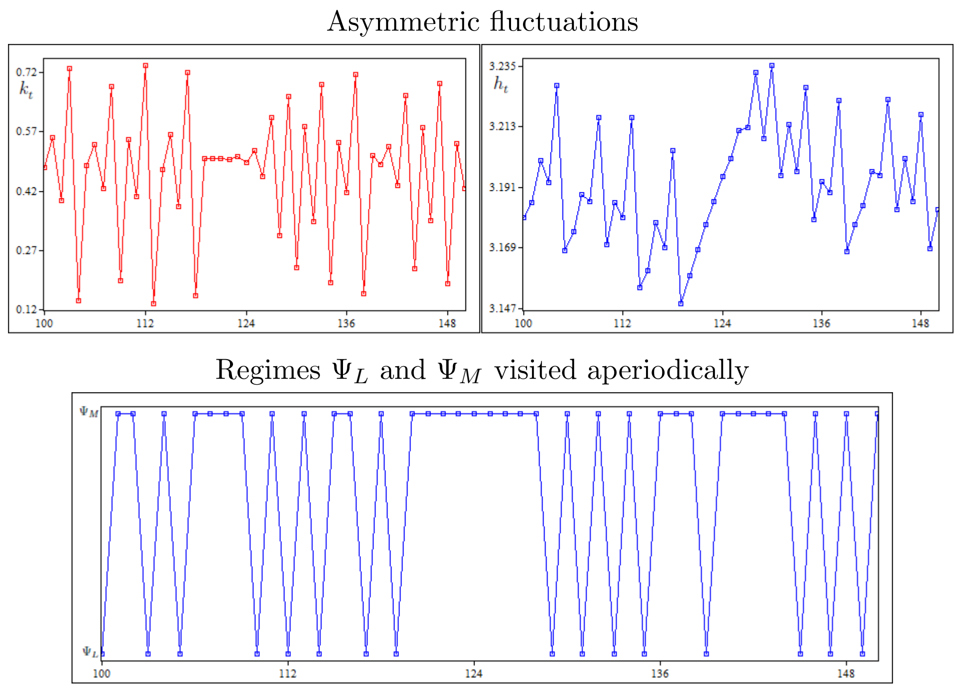
By fixing $\gamma = 0.088$ we trace the bifurcation of μ through the horizontal line and observe the structures which is shown in the left panel of Fig. 7. The boxed area is enlarged in the right panel. We observe a rich internal bifurcation structure, which can be seen on the right panel.

Not only the dynamics but also the economic implications are important. Especially in economies with a very low human capital share, each change (or shock) in the financial sphere due to a variation in μ, the credit friction parameter, might lead to instability. This is close to the general conclusions from empirical studies which also report that financial market development might come with some costs, see, e.g. Acemoglu & Zilibotti (1997), Greenwood & Smith (1997) and Martin & Rey (2004). This highly depends on the value of the human capital share. These findings serve as a clear indicator for the robustness providing character of human capital. With high human capital shares the economy becomes robust and more resilient for changes of financial frictions as the steady state (needless to say, not the value of the fixed point) becomes invariant under changes in μ.

4.2 Time series

As mentioned above those model specifications lead, under certain parameter continua, to aperiodic and 'stochastic looking' time series patterns even though the model is completely deterministic. Therefore we present some time series of the central dynamic variables to highlight this characteristic. As our model involves a three-regime law of motion with endogenous regime switching for physical capital k, we are able to trace the trajectory through the different regimes. In each figure discussed below the regimes corresponding to the upper trajectory is shown where each point of k_t is realised in one of the regime displayed in the lower figure.

In Fig. 8 we simulated a scenario where the human capital share is quite small. An asymmetric period 5-cycle is interrupted by periods of stability (around period 120) with a consecutive amplifying cyclicity. Human capital shows the same irregular pattern except of the following: At the beginning we also observe an asymmetric period 5-cycle but with a clear decreasing trend. During the stability period human capital experiences a steady increase again with a following up-and-down pattern towards a decreasing trend interrupted by a short period of growth. Looking at the regimes one can immediately see that during the periods of stability the trajectory remains in the middle branch of the map, even though this regime is not a guarantor for stability (see around period 140). By varying the initial conditions (not displayed here), k_0, h_0, we observe under this parameter set a completely different trajectory, pointing towards chaotic characteristics. By increasing the human capital share, γ, we observe (see Fig. 9) a symmetric period 3-cycle, following the up-up-down pattern. A bust is followed by two recovery periods with growth eventually busting again. The same can be found for human capital. The regime figure reports a periodic switching between all three regimes, following the pattern: Boom $(\Psi_M) \Rightarrow$ Bust $(\Psi_R) \Rightarrow$ Recovery $(\Psi_L) \Rightarrow$ Boom $(\Psi_M) \Rightarrow$ Bust $(\Psi_R) \Rightarrow$ ... Put differently, in terms of the the model's narrative: After the bust, the deterioration of the borrowers net worth causes that agents are not longer able to finance Bad projects (i.e. the borrowing constraint is severely binding) which results that the credit is shifted towards Good projects. This shift drives up the net worth which eventually eases the borrowing constraint and allows that credit starts again flowing into Bad projects, ultimately creating a boom phase. And the circle starts again. These findings are in line with the evolution of human capital which follows the same pattern, lagged for one period. Handing over to a scenario where $\gamma = 0.30$ (see Fig. 10), a symmetric period 2-cycle is observable. Increasing γ even more, the system's trajectories are attracted to its fixed point: a steady state in physical and human capital is reached after sufficient transient periods. For this scenario we see an alternating pattern between regime Ψ_L and Ψ_R, meaning that we are in the non-distortionary case (see Eq. (18)) where the borrowing constraint is never binding. Here, the intergenerational transfer of human capital can be clearly seen. A bust in physical capital leads to a one period later bust in human capital as the Bad projects fail to generate both, human capital and next generation labour demand. Notice, that in both latter scenarios the booms and busts are lagged for human capital. As an example consider Fig. 9. For period 100 physical capital experiences its boom whereas human capital is just in its recovery step, subsequently reaching its boom period. At this point the evolution of physical capital again busts. For a more clear comparison, see Fig. 10, where boom and bust characteristics are alternating for physical and human capital. Recalling the law of motion for human capital, $h_{t+1} = \frac{\sigma_h}{1-\alpha} k_t^{\alpha} h_t^{\gamma} + (1 - \delta_h) h_t$, this behaviour becomes clear. A boom in physical capital means that the majority of agents invested in Good projects in the previous period which produce human capital as external effect. This leads to a boom of human capital for the next generation as it is intergenerationally transferred. As the bust period for physical capital occurred the next generation spillover effects are declining which results in busting human capital. To certain extend human capital amplifies the recovery of the overall economy.



Notes: Parameter specification:$\alpha = 0.4, m = 1.05, B = 5, \mu = 0.05, \sigma_h = 0.5, \delta_h = 0.05$

Fig. 8: $\gamma = 0.05$; Upper panel: Trajectory for physical (left) and human (right) capital after 100 transient periods. Lower panel: Visited regime (Ψ_L, Ψ_M, Ψ_R) of k_t.

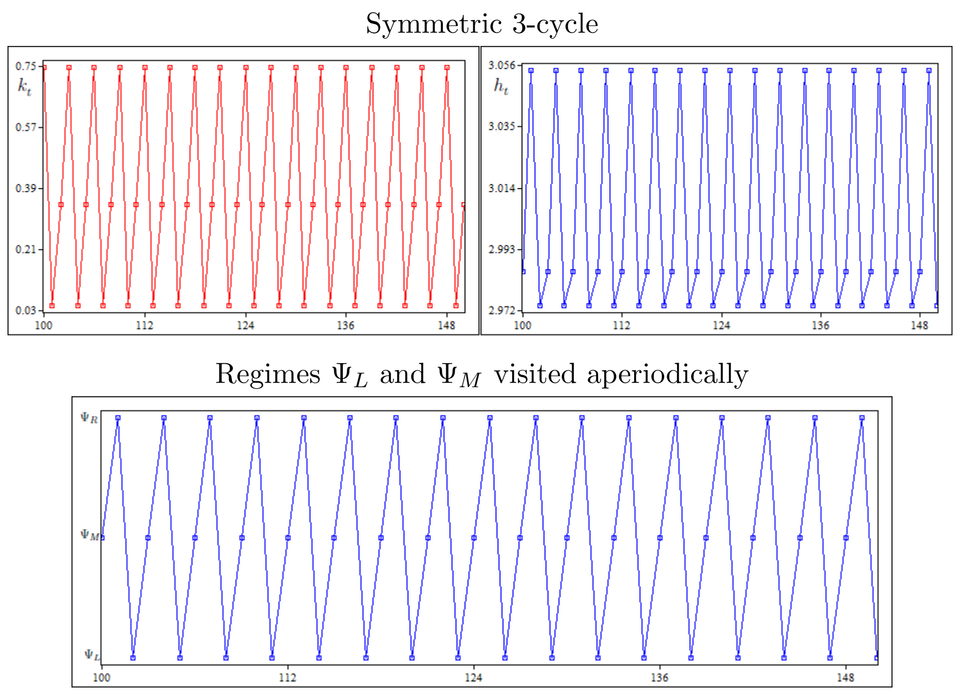
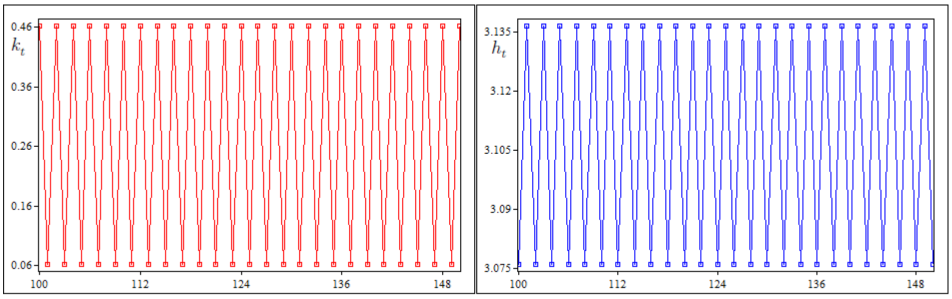


Fig. 9: $\gamma = 0.15$

Symmetric 2-cycle



Periodic alternation between regimes Ψ_L and Ψ_R

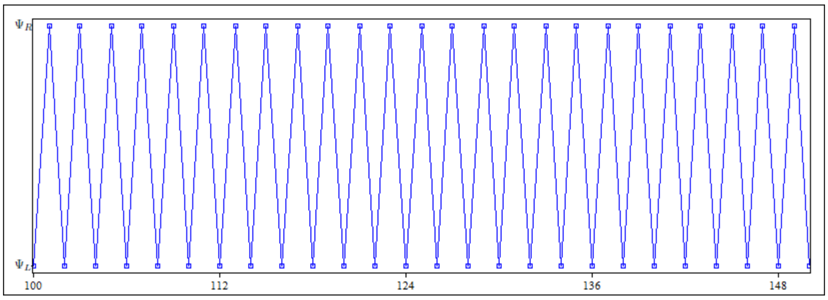


Fig. 10: $\gamma = 0.30$

5 Income assessment

Since the law of motion is derived and a first dynamical analysis of the system is conducted, we move on to a closer look to the factor income distribution. The model set-up allows us to distinguish between labour and capital income. Whereas the former only is earned by young agents the latter is distributed in various ways between the old agents.

5.1 Definition and Computation

First of all, we give some clarification about the definition of *income* in this augmented model. We need to take the intergenerational links and structures as well the different forms of income into account. So the overall income, defined as labour and investment income from a functional income perspective, in point $t+2$ is contributed by two generations, the young (belonging to generation L_{t+1}) and the old (generation L_t) agents. As the young simply receive labour income w_{t+1} from working during period $t+1$ to $t+2$ this generation's contribution is easy to grasp. Whereas old agents get return from net worth allocation (i.e. investment) at point $t+1$ depending on investment decision. To briefly recap possible choices: (a) starting a Good project, (b) starting a Bad project or (c) lend to other agents. As the availability of finance for Bad projects is determined by the available net worth, we have carefully check which regime of the dynamics governing map, Eq. (19), applies. We now start to describe the income I_{t+2} composition step-by-step where L_t denotes the old and L_{t+1} the young generation, respectively[10]. In equilibrium, $r_{t+1} = \rho_{t+1} = \frac{\alpha}{1-\alpha} k_t^{\alpha-1} h_t^{\gamma}$ and $w_t = k_t^{\alpha} h_t^{\gamma}$ hold. Fig. 11 sketches the generational income structure.

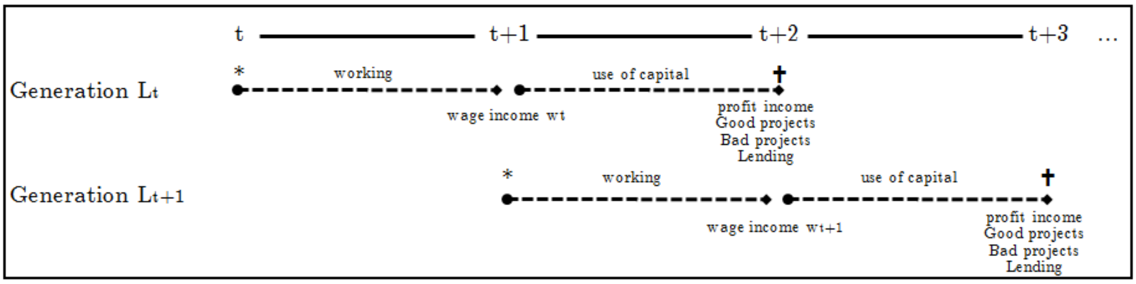


Fig. 11: Generational income structure

First, assume net worth is too low or the financial friction is too severe such that Bad projects cannot be started (we are located at regime Ψ_L of map Eq. (19)). Therefore the income is determined by

$$I_{t+2}|\Psi_L = \underbrace{(k_{t+1}^{\alpha} h_{t+1}^{\gamma})}_{\text{wage young}} \cdot L_{t+1} + \underbrace{(\frac{\alpha}{1-\alpha} k_{t+1}^{\alpha-1} h_{t+1}^{\gamma} w_t)}_{\text{return old (Good)}} \cdot L_t \tag{20}$$

Now, assume that Bad projects can and will be started (we are located either at the Ψ_M or Ψ_R branch of Eq. (19)). Therefore the income is determined by young agents' labour remuneration and investment return from either Good or Bad projects or lending. This brings up the question how big the generational share of lenders, Good and Bad investors actually is. In period $t+2$ this question only concerns the old generation L_t. From the condition that aggregate credit supply must equal aggregate credit demand, we derive that $w_t = k_{t+1} + mX_t$ where X_t denotes the measure of Bad projects, hence the share of agents investing in Bad. This yields the following identities where the lenders are simply the residual agents who neither run a Good nor a Bad project. The profits from Bad projects are determined by return minus credit repayments, $mB-(m-w_t)r_{t+1}$ and the return for Good projects analogously to the first case. Lenders simply receive the interest payments from Bad

[10]Recall that in each generation a unit mass of agents is born, thus $L_t = L_{t+1} = 1$. By adding those generational identifiers, the reader can easily trace through the intergenerational traps.

investors.

$$L_t^G = \frac{k_{t+1}}{w_t} \quad \text{with return} \quad R_{t+1}^G = \frac{\alpha}{1-\alpha} k_{t+1}^{\alpha-1} h_{t+1}^{\gamma} w_t \tag{21}$$

$$L_t^B = \frac{w_t - k_{t+1}}{m} \quad \text{with return} \quad R_{t+1}^B = mB - (m - w_t) r_{t+1} \tag{22}$$

$$L_t^L = L_t - L_t^G - L_t^B \quad \text{with return} \quad R_{t+1}^L = \frac{\alpha}{1-\alpha} k_{t+1}^{\alpha-1} h_{t+1}^{\gamma} (m - w_t) \tag{23}$$

Knowing that, we construct the income equations for the subsequent regimes as follows, which completes the income assessment.

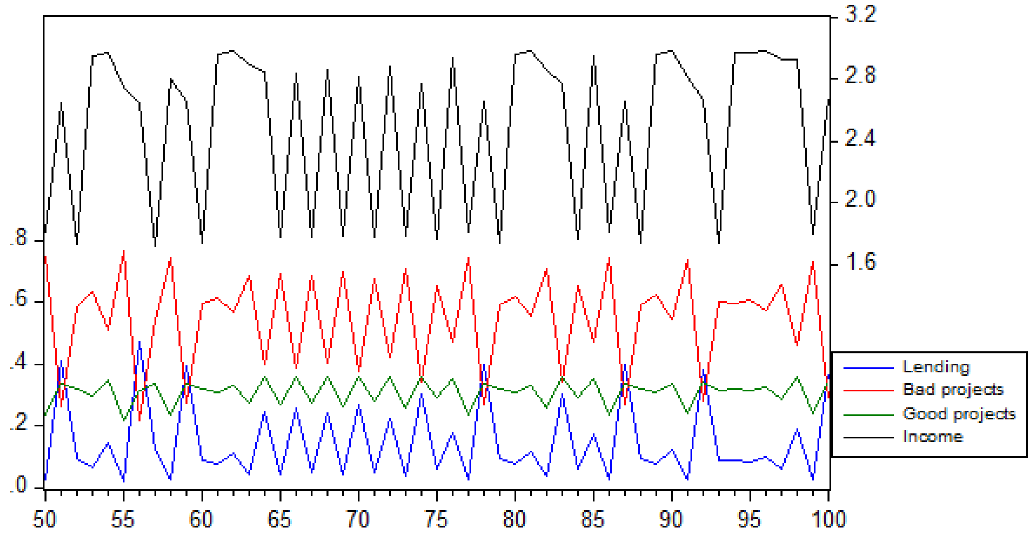
$$I_{t+2} | \Psi_M, \Psi_R = \underbrace{(k_{t+1}^{\alpha} h_{t+1}^{\gamma})}_{\text{wage young}} \cdot L_{t+1} + \underbrace{R_{t+1}^G \cdot L_t^G + R_{t+1}^B \cdot L_t^B + R_{t+1}^L \cdot L_t^G}_{\text{income old}} \tag{24}$$

5.2 Analysis

We will take now a deeper look how the income distribution between the young and the old generation as well as the group size of the old generation (regarding investment) will change during times of cyclicity. As a consequence of the model framework the income distribution underlies the same cyclical behaviour as the other variables. Remember from the previous sections that human capital in general drives up the net worth of the young agents as they earn both, the marginal product of it and the labour remuneration. This, in general, eases the borrowing constraint enabling the credit to flow in the most profitable projects. The transition from 'worker' earning labour income to 'capitalist' earning investment income results from the overlapping generations set-up. Fig. 12 displays a situation with high frictions and low human capital importance.

Apparently, Bad projects are subject to a higher change in the amplitude throughout the whole simulation whereas Good projects suffer from less volatile movements. We see that a high share of Good projects does not necessarily causes a stable income path. Moreover, it seems that especially in times where the net worth is high (indicated by the high share of Bad projects and low share of lenders) a more or less persistent high income can be observed. As the volatility in Good and Bad projects starts again an immediate translation into a high income fluctuation can be observed.

To check whether these patterns are sensitive to a change in the human capital importance we show some histograms, in Fig. 13, of income with different values of γ. We can conclude that in general human capital stabilises the income fluctuations



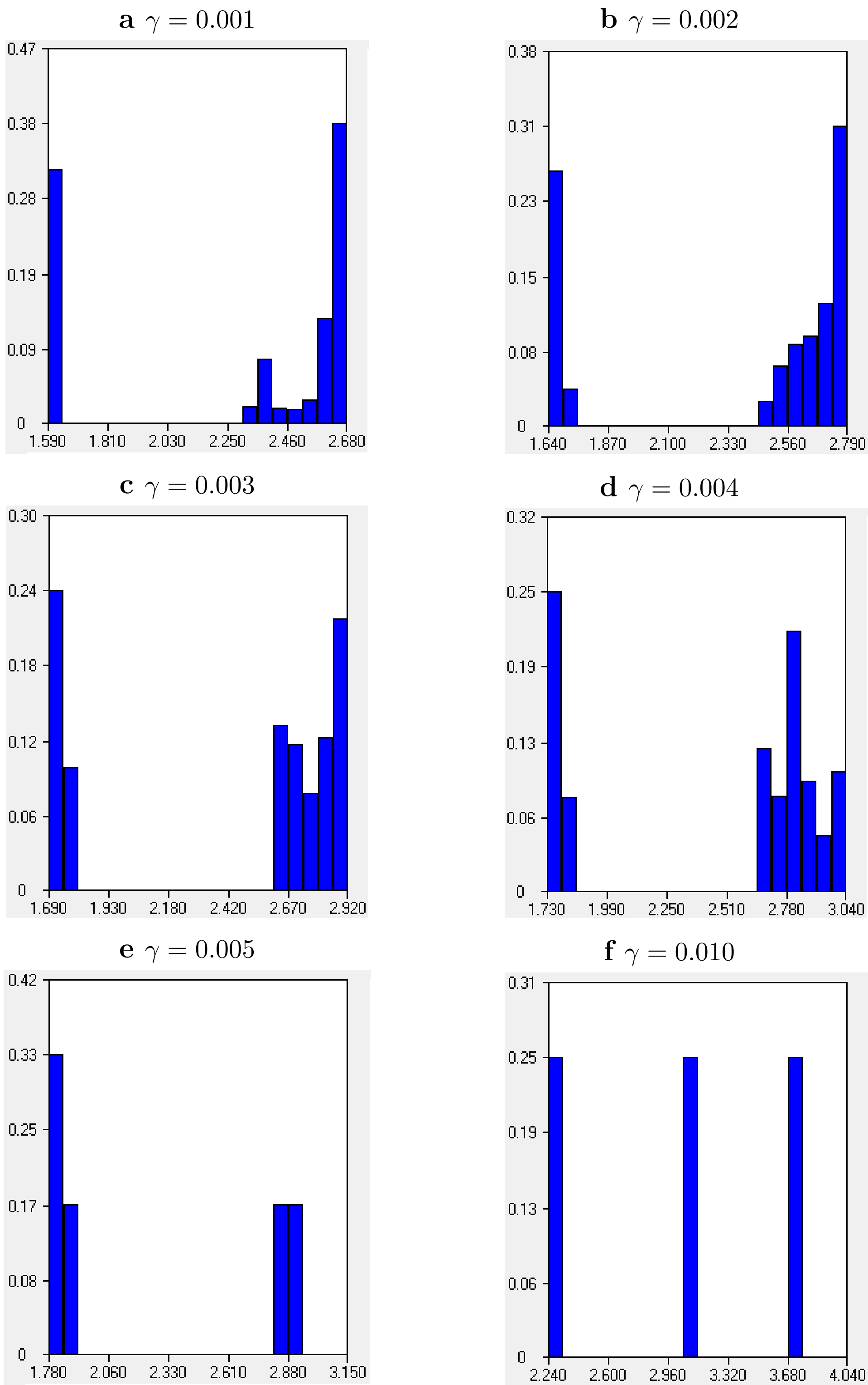
Notes: Parameter specification:$\alpha = 0.4, \gamma = 0.035, m = 1.05, B = 5, \mu = 0.048, \sigma_h = 0.5, \delta_h = 0.05$

Fig. 12: Shares of projects (left scale) and income (right scale)

when we are located in a region where asymmetric high order periodicity is prevalent. But we also observe an expansion of low income regimes when we rise γ even though the level of income experienced a rise.

In Fig. 13e-Fig. 13f the trajectory is trapped in a symmetric period six and period four cycle[11], explaining the regular shaped pattern.

[11]This is not immediately obvious when checking the histograms as the difference of the realised values is not sufficiently large to be captured in a different bin.



Notes: Parameter specification: $\alpha = 0.4, \sigma_h = 0.5, \delta_h = 0.05, B = 5, \mu = 0.048, m = 1.05$, 10000 transients. Cycles observed: (a)-(d) asymmetric high periodicity, (e) symmetric period-six cycle, (f) symmetric period-four cycle.

Fig. 13: Histograms of income under different γ-specifications

For lower frictions we already found that physical and human capital reach a fixed point. Therefore this holds also true for the income. However, it is of major interest to check how the investment proportions of the old generation change under different friction and human capital specifications. Following Tab. 4 and Tab. 5 report numerical simulation results.

HC share	Good projects	Bad projects	Lending	Income
$\gamma = 0.035$	0.23	0.31	0.46	2.60
$\gamma = 0.100$	0.20	0.37	0.43	2.87
$\gamma = 0.200$	0.15	0.51	0.34	3.40
$\gamma = 0.250$	0.13	0.62	0.25	3.92

Tab. 4: Investment shares with $\mu = 0.35$, Numerical simulation results after 5000 transients.

Friction	Good projects	Bad projects	Lending	Income
$\mu = 0.15$	0.38	0.35	0.27	2.79
$\mu = 0.20$	0.33	0.35	0.32	2.69
$\mu = 0.25$	0.29	0.33	0.38	2.63
$\mu = 0.30$	0.25	0.32	0.43	2.60

Tab. 5: Investment shares with $\gamma = 0.035$, Numerical simulation results after 5000 transients

We immediately observe by inspecting Tab. 4 that higher human capital importance shifts investments towards Bad projects. This is mainly caused by the higher net worth of the young generation which enables them, as the borrowing constraint is eased, to invest in higher profitable investments. Even though the expected return of Good projects is increased by human capital, Bad projects are still more profitable. Due to the higher net worth we also observe the deterioration of lending as simply less credit is required. Even though the share of Good projects declines the raising profitability of Good projects compensate that shift and the aggregated income goes up. For a (fixed) low importance of human capital (see Tab. 5) we confirm the results also reported in Matsuyama et al. (2016): The system gains stability but the overall income decreases as a result of a deterioration of the net worth. Agents need to borrow more to start Bad projects even though it is easier resulting from an eased borrowing constraint.

6 Concluding remarks

In this paper we examined the impact of human capital on an economy with irregular output fluctuations which occurred due to credit flows into different investment projects. Human capital serves as technological component by reducing the effect of diminishing returns on physical capital. A learning-by-producing approach is applied to model human capital and it is assumed to be transferred intergenerational. We observed new dynamical features which have both, an immediate impact on the system stability and an economic rational. It appears that the human capital share of production has a significant though ambiguous impact on the stability. In general, a higher human capital share tends to stabilise output measured in per capita physical capital. But especially when the credit market friction is sufficiently high (i.e. μ is sufficiently low) low human capital shares introduce some instability by amplifying the cyclicity. The baseline model which we augment, developed by Matsuyama et al. (2016), reports the feature of *corridor stability* for the credit market frictions parameter. We identified the same feature for the human capital share. As human capital serves as component of technological progress we highlight the importance of technological shocks. A vast branch of literature in macroeconomic business cycle modelling deals with exogenous shocks (for example, shocks in total factor productivity, demand or supply) and their impact on stability characteristics. The shocks are usually assumed to be stochastic and following a mean reverting process. In the aftermath of the shock the system eventually returns to the equilibrium (the duration depends on the shock's persistence). Applied to our situation (i.e. the parameter change affects the corridor stability region), the system is robust to small shocks but suffers strongly from intermediate high shocks as it permanently looses its stability. Tracing the human capital share through various scenarios of credit market frictions, we observe that in situations where the human capital sector has a small proportion, credit market frictions have a strong impact on the economy's stability. On the other hand, a high human capital share tends to make the system resistant and resilient to shocks from the credit market as our simulation exercise clearly showed. With a high human capital share irregular fluctuations eventually vanish. This leads us to a first message regarding technological shocks. Transition periods where production shares of the final good production change the economy might slip into an unstable development path. We showed that this transition leads, under the presence of corridor stability, to an irreversible and catastrophic change of the output evolution. Such features unfortunately get lost as linear approximations are applied which is commonly done for standard models. In a worst case this might lead to simply wrong policy recommendations. Thus, we want to stress the fact that drawing policy recommendation out of a linearised model might induce contrary effects like destabilising an economy and should therefore carefully be considered.

Moreover, an income distribution assessment shows that human capital has an additional effect: in regions where asymmetric cycles occur a rise of human capital importance leads to an expansion of low income regimes but also to an enhancement of high income regimes. Additionally, an increase in income levels can be observed. For parameter regions where a fixed point can be achieved (i.e. a region with sufficiently low credit market frictions) human capital enhances the net worth of young agents enabling credit to flow in the most profitable projects thus driving up the general aggregated income. Thus selective policy actions towards human capital might help to boost the overall income situation and the general resilience of the economic system even under the presence of credit market frictions. A look at the investment shares (i.e. Good and Bad projects and lending) shows the following: High human capital shares lead to a shift in Bad projects and a decrease in Good projects. Due to the rise of net worth we observe an increase of the income but not the usual bust pattern. We conclude that, under the presence of human capital spillovers, major destabilising features are weakened ultimately leading to a stable output evolution.

Once again we would like to stress that human capital is an essential factor in economic processes and therefore needs to be considered when dealing with business cycle models. Moreover, the importance of regime-switching models in economics in both, theoretical and empirical related work is also highlighted as important dynamic phenomena can be captured with such an approach.

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