

***THE STABILITY OF FULL EMPLOYMENT.
A RECONSTRUCTION OF CHAPTER 19-KEYNESIANISM.***

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1. Introduction and Plan of the Paper

It has now become usual to interpret Keynesian theory as “sticky price-macroeconomics” (cf. e.g. McCallum 1986) and to consider a Keynesian-type stabilisation policy primarily as a substitute for wage flexibility. However, if the market failure of unemployment is just due to the “friction” of insufficiently flexible wages it is consequent not to opt for an active stabilisation policy but for wage flexibility. Indeed, this is the now prevalent prescription for macroeconomic policy. Therefore, from a Keynesian perspective the crucial question must be whether flexibility of nominal wages is in fact a proper remedy for eliminating effective demand failures and thereby unemployment. Some tentative answers to this question are to be found in chapter 19 of Keynes’s “General Theory”, “Changes in Money Wages”. From this point of departure a Keynesian tradition emanated – most prominently represented by Patinkin (1948) and Tobin (1975) – that based its arguments for policy activism on the potential instability of full employment equilibrium in a system of flexible wages and which might be called “Chapter 19-Keynesianism”.¹

As a contribution to this approach the following study tries to investigate the dynamic properties of the traditional Keynesian model. After some short notes on the development of the relevant literature the model is analysed in three steps: First the full-employment or steady-state equilibrium of the Keynesian model is established as a kind of benchmark. Second the instantaneous equilibrium is analysed where the nominal wage and expectations of inflation are given. Third the dynamics of the instantaneous equilibrium is formulated (in continuous time) making use of a Phillips curve (with inflationary expectations) as the wage-adjustment equation and of adaptive expectations (with rational expectations as a limiting case). Then the dynamic properties – stability vs. instability, monotonic vs. oscillatory adjustment – of the model are examined in the absence of active stabilisation policy, that is assuming, in particular, monetary policy to follow Friedman’s constant money growth-rule. Furthermore two regimes are distinguished: on the one hand the “flexible-interest-rate-regime” where the nominal interest rate is free to move and on the other hand the “zero-interest-rate-regime” (similar to the Keynesian “liquidity trap”) where the non-negativity restriction on the nominal interest rate becomes binding. At last, some conclusions are drawn from the analysis and the results are compared with those of other studies, in particular with Tobin (1975).

2. Some Notes on the Literature

As already pointed out, the *locus classicus* of the “instability-of-full-employment”-version of the Keynesian theory is chapter 19 of the “General Theory” (Keynes 1936). This chapter is based on what Hicks (1974, 59f.) called the “wage theorem”: If the rate of interest and thereby the quantity of money measured in wage units is kept constant, there will be no effect of changes in nominal wages on aggregate demand and employment: Any decrease in nominal wages would just generate a proportional decrease in the price level, thus leaving relative prices and output

¹ On the varieties of Keynesianism cf. for example Blaug (1997, 667f.).

constant. However, if the quantity of money is fixed in nominal terms then a fall in money wages will cause repercussions on aggregate demand giving rise to a non-proportional response of the price level and thereby to real effects.

In the case of a closed economy these possible effects on aggregate demand consist of the redistribution effect of unanticipated deflation (presumably negative), the price expectations effect (negative, if expectations are extrapolative), what was later to be called the Keynes-effect (positive) and the effect of the burden of private and public debt (presumably negative). Furthermore, if the economy experiences a liquidity trap (either from the outset or during the adjustment process) the Keynes effect will vanish so that the overall effect of wage reductions is more bound to amplify than to reduce unemployment. In sum Keynes's conclusion was that "there is, therefore, no ground for the belief that a flexible wage policy is capable of maintaining a state of continuous full employment ... The economic system cannot be made self-adjusting along these lines." (Keynes 1936, 267) However, as was soon pointed out (e.g. by Pigou 1943) the effect of (liquid outside) wealth on expenditure might guarantee the existence of full employment equilibrium and secure the stability of adjustment through wage reductions.

It should be noted, that most of the effects that Keynes discusses in chapter 19, were well-known in contemporary business cycle theory. Of course, the Keynes effect was at least implicitly hinted at in the writings of Wicksell. The effect of inflation and of inflationary expectations, respectively, on the real interest rate played a major role in neoclassical explanations of the cycle (cf. e.g. Fisher 1923 and Pigou 1933, 241ff.).² Furthermore, Fisher's debt-deflation theory of depressions (Fisher 1933) identified the interaction of over-indebtedness and deflation as an important cause of a contraction of demand. The redistribution effect of unanticipated inflation on aggregate demand was also present in Fisher's analysis and was therefore aptly termed "Fisher effect" by Tobin (1980, 10). Even the notion that during the adjustment process the existence of a minimum interest rate could inhibit the tendency towards full employment equilibrium could be found in contemporary business-cycle theory, e.g. in Pigou (1933, 212f.), in Röpke (1933, 436)³, and still earlier in Keynes's "Treatise".⁴

Among the various interpretations of Keynes it was Patinkin who most consistently emphasised the importance of chapter 19 (cf. Patinkin 1948). He ended up in considering Keynes's theory "a dynamic theory of unemployment *disequilibrium*" (Patinkin 1976, 113) – thus abandoning the concept of "unemployment equilibrium" – and in basing his arguments for Keynesian policy on the practical instability of the *laissez-faire* system. However, Patinkin's arguments for (practical) instability were only suggestive and not grounded in formal analysis. The discovery of the

² Strictly speaking, the effect of inflation on the real interest rate due to sluggish adjustment of expectations (as emphasised in Fisher 1923) should be distinguished from the insufficient adjustment of the nominal rate to inflationary expectations.

³ On Fisher cf. Dimand (1994, 1995) and Laidler (1991, especially 90ff.); on Pigou cf. Collard (1983, 1996) and Klausinger (1998, 60f.).

⁴ As Keynes (1930, 334f.) noted, if the market rate of interest falls far below the long-period norm, then open-market operations *à outrance* by the central bank may be inhibited by its fear of serious financial loss.

Phillips curve (Phillips 1958) can be seen in this regard as a formal representation of slow wage adjustment as suggested by Keynesian theory and, in fact, Phillips had also been the first to formally analyse the dynamics of the Keynesian system by calculating numerical examples (Phillips 1954). Moreover, a different aspect of stability had been tackled by Cagan (1956) in his analysis of the feedback from (expected) inflation on (the demand for) real balances and back to (actual) inflation: He showed that for hyperinflation not to be explosive without monetary accommodation this response must not be too strong. Yet, the first rigorous stability analysis of the Keynesian system was provided by Tobin (1975).⁵ The crucial distinction in this regard, that the price level effect is permanent and therefore cumulative whereas the price expectations effect is just a-once-for-all-shift, can already be found in Phillips (1954, 310f.).

As Keynes had recognised, the price expectations effect will be destabilising only when expectations are formed by extrapolating actual price changes. This was captured ever since Cagan (1956) by the model of adaptive expectations, which following Tobin (1975) is also used in this paper. The alternative combination of sluggish wage adjustment with perfect foresight of inflation leads to the well-known case of saddle-point (in-)stability so that a forward-looking solution would force the time path on the unique stable trajectory (cf. e.g. Wörgötter 1988). DeLong and Summers (1986) analysed the case with Taylor-like wage contracts, which are partly backward and partly forward looking, and found that for sensible parameter values increasing price flexibility might be destabilising.

Leijonhufvud (1973) introduced a novel feature into the debate by his notion of “corridor stability”. It pointed to the fact that the qualitative behaviour of a (non-linear) dynamic system could be different for different initial conditions, or put in other words for shocks of a different size. Examples of corridor stability were *inter alia* elaborated by Leijonhufvud (1973), Howitt (1978) and Löfgren (1979).⁶ In particular, the relevance of stocks for the smoothing of production and consumption, of long-term income expectations and of deflation-induced bankruptcy on aggregate demand were singled out as possible causes for corridor stability. Tobin (1993, 62) suggested that his dynamic model might also exhibit such a property due to the non-linear response of the rate of interest to changes in real balances. Finally, it should be noted that a notion similar to that of corridor stability is implied by the distinction made in neoclassical business cycle theory between primary and secondary depression, by such authors as e.g. Schumpeter, Röpke and Robertson.⁷

The role of a minimum interest rate, i.e. the vanishing of the Keynes effect due to the insensitivity of the rate of interest to real balances, in the adjustment process is referred to by e.g. Patinkin (1976, 106) and it is also implicit in Tobin’s analysis (cf. 1975, 200f.). Similarly,

⁵ Other examples of stability analyses are those of Sargent (1979, 113ff.) and Neldner (1982); cf. also Shone (1997, ch. 9).

⁶ Cf. also for a critique Grossman (1974).

⁷ For a discussion of the relation between the concepts of secondary deflation and of corridor stability cf. Landmann (1981) and Klausinger (1999).

economic historians when employing the concept of a deflationary spiral as a possible explanation of the Great Depression have a situation in mind where the destabilising effect of deflation on the real rate of interest is due to a floor to the nominal rate (cf. Temin 1989, 56ff.)

During the monetarist controversy the interpretation of the Phillips curve as a stable inflation-unemployment trade off – as elaborated by Samuelson and Solow – centred the debate on the neutrality of equilibrium, whereas the instability critique was neglected. So when the existence of such a trade off was successfully challenged by taking account of inflationary expectations by Friedman and Phelps, the ensuing consensus on the natural-rate-hypothesis appeared as a deadly blow to Keynesian economics in a traditional sense. Of course, this became the more true after the revolution of rational expectations when finally price (or wage) flexibility was identified with market clearing, so that stability problems were somehow considered as the result of “frictions”.

However, as Tobin (1975) had demonstrated, the Phillips curve (and adaptive expectations of inflation) could have been integrated into the tradition of Chapter 19-Keynesianism.⁸ This might be considered the (truly Keynesian) way not taken by the mainstream of macroeconomics and to be reconstructed by the following model.

The reconstruction below and the model of Tobin (1975) differ in some important respects. On the one hand, in Tobin’s model the Phillips curve and the hypothesis of adaptive expectations are supplemented by a third dynamic relation involving the adjustment of output to expenditure, which is neglected here. On the other hand, Tobin’s assumption of rigid prices is replaced in the following by the assumption of rigid nominal wages (which adjust according to a wage Phillips curve). This has the advantage of highlighting the crucial question if and inasmuch real wages can be controlled by changing nominal wages. This is in some sense the core issue of Keynes’s chapter 19 (and also of chapter 3) which is however left out in Tobin’s analysis.⁹ Furthermore, the non-linear relation between the price level and the interest sensitivity of money demand is accounted by Tobin only in verbal terms and in his graphical illustrations but not in his algebra. This shortcoming is if only crudely remedied in this model by explicitly introducing a minimum rate of interest and by distinguishing between a regime of a flexible and of a fixed (zero) nominal interest rate.

3. The comparative statics of the Keynesian macro-model

As a starting point of the following dynamic analysis we first sketch the standard version of the steady-state equilibrium of the Keynesian macro-model (with full employment and steady inflation) and derive the comparative-static results. The same will then be done for the instantaneous equilibrium of the model. With regard to the instantaneous equilibrium two kinds of regime are distinguished, according to whether the nominal rate of interest is flexible or fixed by the zero floor.

⁸ Cf. e.g. for this type of critique Leijonhufvud (1998).

⁹ This advantage might however be bought at a price in that due to the explicit modelling of the production function the linearisation of the model is less easily defensible than in Tobin’s version.

The symbols used in the rest of the paper have the following meaning: y is real income (or output), z expenditure, r the real and i the nominal rate of interest, M the money supply, P the price level, m real balances, w the real wage, W the nominal wage, f the production function, l the demand for labour, n the supply of labour assumed to be fixed; $\mathbf{p} \equiv \dot{P}/P$ the rate of inflation (where $\dot{x} \equiv dx/dt$, t a time index), \mathbf{p}^e the expected rate of inflation, $\mathbf{m} \equiv \dot{M}/M$ and $\mathbf{w} \equiv \dot{W}/W$ the rate of growth of the money supply and of nominal wages, respectively.

3.1 Steady-state equilibrium

The structural form of the model¹⁰ consists of the following eight equations:

$$\begin{aligned}
 (1) \quad & y = z(y, r, m), \quad 1 > z_y > 0, z_r < 0, z_m \geq 0, \\
 (2) \quad & m \equiv M/P = m(y, i), \quad m_y > 0, m_i < 0, \\
 (3) \quad & i = r + \mathbf{p}^e, \\
 (4) \quad & y = f(l), \quad f' > 0, f'' < 0, \\
 (5) \quad & f'(l) = w \equiv W/P, \\
 (6) \quad & \mathbf{p} = \dot{P}/P, \\
 (7) \quad & l = n, \\
 (8) \quad & \mathbf{p} = \mathbf{p}^e.
 \end{aligned}$$

Equation (1) is the condition of income-expenditure equilibrium including the Pigou-effect of real balances on expenditure, (2) states the equilibrium of the money market, (3) implicitly defines the real rate of interest. Equation (4) is the production function with the usual properties, (5) is the profit maximising condition for employment and (6) defines the actual rate of inflation. (7) and (8) are the specific conditions for steady-state equilibrium: (7) is the condition for full-employment equilibrium (the existence and uniqueness of which is taken for granted) and (8) requires that expectations of inflation are fulfilled. It should be noted that there is no feedback from investment to productive capacity, thus we neglect the long-run effects of economic growth. The eight endogenous variables of the model are y , l , w , r , i , P , \mathbf{p}^e and \mathbf{p} , the exogenous variables are M , $\mathbf{m} \equiv \dot{M}/M$, and n . The assumption of an exogenous and constant \mathbf{m} implies, of course, a Friedman-type rule for the growth of the money supply without feedback.

Given the supply of labour, n , output, employment and the real wage are fixed by the equilibrium condition of the labour market. Thus we have only to look how the remaining variables depend on M and \mathbf{p} :¹¹

$$(9) \quad dr = -\frac{z_m m_i}{z_r + z_m m_i} d\mathbf{p},$$

¹⁰ For a textbook version of the steady-state model cf. McCallum (1989, ch. 6).

¹¹ Here and in the following when the solutions for an endogenous variable x are expressed as $x = x(M, \mathbf{m}, n)$, then the comparative-static multipliers are written as $x_M \equiv \mathcal{J}_x / \mathcal{J}_M$. Furthermore \tilde{x} denotes the value of x in steady-state equilibrium and in particular $\tilde{r}(0)$ is the value of r when $\mathbf{m} = 0$.

$$(10) \quad di = d\mathbf{p} + dr,$$

$$(11) \quad dP = \frac{1}{m} dM - \frac{P z_r m_i}{m(z_r + z_m m_i)} d\mathbf{p}.$$

These results are as expected, confirming the “classical dichotomy” with the sole exception of the effect of inflation on the real rate of interest.¹² This is due to the working of the Pigou effect; a higher rate of inflation induces lower real cash balances, which leads to lower consumption and therefore a lower real rate of interest is needed to secure full employment. If the Pigou effect is absent, the real rate of interest will be invariant to changes in inflation. Furthermore, we stipulate that the Pigou effect is not “too strong” so that the following condition holds:¹³

$$(12) \quad 1 - z_y - z_m m_y > 0.$$

Solving for the rate of inflation, by substitution from (11) and by noting that $\dot{M}/M = \mathbf{m}$, we arrive at $\mathbf{p} = \frac{1}{P} (\tilde{P}_M \dot{M} + \tilde{P}_p \dot{\mathbf{p}})$. In the steady state the rate of inflation must be constant for constant \mathbf{m} so that $\dot{\mathbf{p}} = 0$, therefore:

$$(13) \quad \tilde{\mathbf{p}} = \tilde{\mathbf{p}}^e = \mathbf{m}.$$

Because of (13) \mathbf{p} can be replaced by \mathbf{m} in the solutions (9) to (11). Similarly we can solve for the rate of growth of nominal wages:

$$(14) \quad \tilde{\mathbf{w}} = \tilde{\mathbf{p}} = \mathbf{m}.$$

Finally it must be noted that the (exogenous) rate of money growth \mathbf{m} is bounded below because of $\tilde{i} \geq 0$. Therefore after linearising around the steady state solution:

$$(15) \quad \mathbf{m} \geq -\tilde{r}(\mathbf{m}) = -\tilde{r}(0) - \tilde{r}_m \mathbf{m} \Rightarrow \mathbf{m} \geq \mathbf{m}_{\min} = -\frac{\tilde{r}(0)}{1 + \tilde{r}_m}.$$

Obviously, \mathbf{m}_{\min} is the rate of money growth required by the “optimal quantity of money”-rule of Friedman (1969) where the rate of deflation equals the real rate of interest.

3.2 Instantaneous equilibrium

The instantaneous equilibrium is characterised by the fact that the variables can be partitioned into slow and fast variables. The slow variables are changing through time but are given at any point of time; in the Keynesian macro model these slow variables are the nominal wage W and the expected rate of inflation \mathbf{p}^e , which are now treated as additional exogenous variables. Therefore in instantaneous equilibrium there is the possibility of unemployment (non-market clearing in the labour market) and of erroneous expectations of inflation. All the other endogenous variables are fast variables which take on their market-clearing levels immediately

¹² This is sometimes called the „Mundell effect“, cf. Mundell (1963).

¹³ Translated into the terms of the IS-LM-model this condition requires that an increase in real money balances will lower the rate of interest.

so that equilibrium in the goods and money market and the profit maximising choice of employment is guaranteed.¹⁴

The flexible-interest-rate regime

We first consider the case where the nominal rate of interest is strictly positive in instantaneous equilibrium. Then the structural form of the model is given by equations (1) to (6) from above, with the six endogenous variables y , l , r , i , P and \mathbf{p} , and the exogenous variables W , \mathbf{p}^e , M , and \mathbf{m} (For the sake of simplicity n is disregarded in the following.)

Solving for the comparative-static effects in instantaneous equilibrium we arrive at the following well-known results:

$$(16) \quad dy = \frac{1}{\Delta} \left[-z_r m_i d\mathbf{p}^e + \frac{1}{P} (z_m m_i + z_r) \left(dM - \frac{M}{W} dW \right) \right],$$

$$\Delta \equiv \Gamma - (z_m m_i + z_r) \frac{M}{W} \frac{f''}{f'} < \Gamma \equiv (1 - z_y) m_i + z_r m_y < 0.$$

$$(17) \quad dl = \frac{1}{f'} dy.$$

The signs of the multipliers are as expected. In particular, the negative sign of y_w is due to the Keynes as well as the Pigou effect. Furthermore, it can be easily seen that an equi-proportional change of M and W has no effect on y : $y_w W + y_M M = 0$. Defining $v \equiv W/M$, i.e. the nominal wage deflated by the money supply, it follows that $y_v = y_w M$ and similarly for the effects on employment l .¹⁵

For the real and the nominal rate of interest the results are:

$$(18) \quad dr = -\frac{1}{\Delta} \left[m_i \left(1 - z_y - z_m \frac{1}{v} \frac{f''}{f'} \right) d\mathbf{p}^e + (1 - z_y - z_m m_y) \frac{m}{v} dv \right],$$

$$(19) \quad di = d\mathbf{p}^e + dr.$$

Again an equi-proportional change of M and W has no effect on r and i , respectively; the effect of v is positive as expected because of the previous assumption (12).

The solution for the price level is:

$$(20) \quad dP = \frac{1}{\Delta} \left[\frac{f''}{f'} \frac{P}{w} z_r m_i d\mathbf{p}^e + \frac{1}{m} (\Delta - \Gamma) dM + \frac{1}{w} \Gamma dW \right].$$

¹⁴ As usual, in labour market disequilibrium actual employment is determined by labour demand; therefore positive as well as negative deviations from full employment are feasible.

¹⁵ Notice that $dM - \frac{1}{v} dW = -\frac{1}{v} (dW - v dM) = -\frac{M}{v} dv$.

In particular, the money and (nominal) wage elasticity of the price level can be defined as:

$$(21) \quad \mathbf{a} \equiv P_M \frac{M}{P} = \frac{\Delta - \Gamma}{\Delta} \geq 0 \quad \text{and} \quad 1 - \mathbf{a} \equiv P_W \frac{W}{P} = \frac{\Gamma}{\Delta} \geq 0,$$

so that the elasticity of the price level with respect to an equi-proportional change in M and W is, of course, unity. Note that \mathbf{a} can also be interpreted as the nominal wage elasticity of the real wage.

Finally, from (20) and (21) follows for the rate of inflation:

$$(22) \quad \mathbf{p} = \frac{1}{P} (P_W \dot{W} + P_M \dot{M} + P_p \dot{\mathbf{p}}^e) = \mathbf{a} \mathbf{m} + (1 - \mathbf{a}) \mathbf{w} + \frac{P_p}{P} \dot{\mathbf{p}}^e.$$

That is, in instantaneous equilibrium the rate of inflation is a weighted sum of the rate of growth of the money supply and of nominal wages, respectively, plus a term proportional to the change in the expected rate of inflation.

The zero-interest-rate regime

In Keynesian macroeconomics some significance has been attributed to the so-called “liquidity-trap” where the equilibrium solution is restricted by the condition that the (nominal) rate of interest must not fall below a (possibly positive) minimum value. In the following we investigate the consequence of this restriction for the system in instantaneous equilibrium (and later on for the dynamics of the system), where the floor to the nominal rate of interest rate is for simplicity set at zero. As usual we assume that at this zero rate of interest money demand becomes infinitely elastic.

In this case the equations (4) to (6) can be taken over from above whereas (1) to (3) are now replaced by the following four equations:

$$(23) \quad y = z(y, r, m), \quad 1 > z_y > 0, \quad z_r < 0, \quad z_m \geq 0;$$

$$(24) \quad m = m(y, 0), \quad m_y > 0, \quad m_i|_{i=0} \rightarrow -\infty;$$

$$(25) \quad i = 0;$$

$$(26) \quad r = -\mathbf{p}^e.$$

These equations are mostly self-explaining. Note, however, that because of the infinite elasticity of money demand the equilibrium value of P is no longer determined by equation (24), since it is fulfilled for any amount of real cash balances.

The comparative static effects of the model, when at the interest rate floor, can be derived by proper substitutions and are given below:¹⁶

$$(27) \quad dy = -\frac{1}{\Delta_0} \left(z_r d\mathbf{p}^e + z_m \frac{m}{v} dv \right), \quad \Delta_0 \equiv 1 - z_y - z_m \frac{1}{v} \frac{f''}{f'} > 0;$$

$$(28) \quad dl = \frac{1}{f'} dy;$$

¹⁶ The same results follow from (17) to (21) with m_i going to minus infinity.

$$(29) \quad di = 0;$$

$$(30) \quad dr = -d\mathbf{p}^e;$$

$$(31) \quad dP = \frac{1}{\Delta_0} \frac{f''}{f'} \frac{P}{w} z_r d\mathbf{p}^e + \mathbf{a}_0 \frac{1}{m} dM + (1 - \mathbf{a}_0) \frac{1}{w} dW, \quad 1 \geq \mathbf{a}_0 \equiv -\frac{1}{\Delta_0} z_m \frac{1}{v} \frac{f''}{f'} \geq 0;$$

$$(32) \quad \mathbf{p} = \mathbf{a}_0 \mathbf{m} + (1 - \mathbf{a}_0) \mathbf{w} + \frac{P}{P} \dot{\mathbf{p}}^e.$$

The zero-interest-rate regime: No Pigou effect

The case of the “liquidity trap proper”, well known from the literature, assumes the absence of the Pigou effect. Then the solutions for y , P and \mathbf{p} become very simple as the effects of the quantity of money virtually disappear (note that here $\mathbf{a}_0 = 0$):

$$(33) \quad dy = -\frac{1}{1 - z_y} z_r d\mathbf{p}^e;$$

$$(34) \quad dP = \frac{1}{1 - z_y} \frac{f''}{f'} \frac{P}{w} z_r d\mathbf{p}^e + \frac{1}{w} dW;$$

$$(35) \quad \mathbf{p} = \mathbf{w} + \frac{P}{P} \dot{\mathbf{p}}^e.$$

In this special case the rate of inflation equals the rate of growth of nominal wages plus, again, the term proportional to the change in expected inflation.

4. The dynamics and stability of the Keynesian macro-model

The dynamics of the system is determined by the sequences of instantaneous equilibrium as driven by the adjustment of the slow variables. The properties of this adjustment process, in particular the stability of the steady-state equilibrium, are analysed in the following. Just as before, we distinguish the dynamic behaviour of the system with a flexible (positive) nominal rate of interest from the behaviour when the nominal interest rate is at the zero floor. Finally, it is examined at which constellations the transition from the one regime to the other will take place and what is implied thereby for the behaviour of the system as a whole.

We start by specifying the dynamic relations governing the time paths of the slow variables, i.e. the nominal wage and the expected rate of inflation. These are given by:

$$(36) \quad \mathbf{w} = l(l - \bar{n}) + \mathbf{p}^e, \quad l > 0,$$

$$(37) \quad \dot{\mathbf{p}}^e = k(\mathbf{p} - \mathbf{p}^e), \quad k > 0.$$

Equation (36) is an expectations augmented Phillips curve and (37) states that expectations of inflation are formed adaptively.

Both equations need some reformulation before they are suitable for analysing the dynamics in the neighbourhood of steady-state equilibrium. To begin with, we note that in the steady-state $\tilde{l} = n$, $\tilde{\mathbf{p}} = \tilde{\mathbf{p}}^e = \tilde{\mathbf{w}} = \mathbf{m}$. For simplicity equation (36) is reformulated so that the time derivative of the relevant variable vanishes in steady-state equilibrium. This can be accomplished by

looking not at the nominal wage W but instead at $v \equiv W/M$. Accordingly, equation (36) is replaced by

$$(38) \quad \dot{v} = v(\mathbf{w} - \mathbf{m}) = v[\mathbf{I}(l - n) + \mathbf{p}^e - \mathbf{m}].$$

For equation (37) some manipulation is required, too. Substituting in (37) for the rate of inflation \mathbf{p} from (22) and rearranging yields

$$(39) \quad \dot{\mathbf{p}}^e = \mathbf{x}[\mathbf{a}\mathbf{m} + (1 - \mathbf{a})\mathbf{w} - \mathbf{p}^e], \quad \text{where } \mathbf{x} \equiv \frac{\mathbf{k}}{1 - (\mathbf{k}P_p/P)}.$$

Taking these equations as points of departure, in the following the dynamics of the system is analysed by looking at the linear approximations to the non-linear “true” system.

4.1 The flexible-interest-rate regime

In this regime the relevant effects of v and \mathbf{p}^e are determined by the multipliers as given above, (16) to (22). Qualitative information on the dynamics of the system can then be deduced from the properties of the Jacobian matrix. So at first by differentiating (38) and (39) we arrive at

$$(40) \quad d\dot{v} = v d\mathbf{w} + (\tilde{\mathbf{w}} - \mathbf{m}) dv = v[\mathbf{I}l_v dv + (\mathbf{I}l_p + 1) d\mathbf{p}^e], \quad \text{and}$$

$$(41) \quad d\dot{\mathbf{p}}^e = \mathbf{x}[(1 - \mathbf{a}) d\mathbf{w} - d\mathbf{p}^e] = \mathbf{x}(1 - \mathbf{a})\mathbf{I}l_v dv + \mathbf{x}[(1 - \mathbf{a})(\mathbf{I}l_p + 1) - 1] d\mathbf{p}^e,$$

where we have made use of the fact that $\tilde{\mathbf{w}} - \mathbf{m} = 0$ in steady-state equilibrium and $l_v = l_w M$.

Asymptotic stability now requires that the trace of the Jacobian be negative and the determinant positive. Looking first at the latter condition, we find:

$$(42) \quad \det(J) = v\mathbf{I}l_v \mathbf{x}[(1 - \mathbf{a})(\mathbf{I}l_p + 1) - 1] - v(\mathbf{I}l_p + 1)\mathbf{x}(1 - \mathbf{a})\mathbf{I}l_v = -\mathbf{x}v\mathbf{I}l_v > 0.$$

Therefore stability requires

$$(43) \quad \mathbf{x} > 0 \Leftrightarrow 0 < \mathbf{k} < \mathbf{k}^* \equiv P/P_p,$$

that is the speed of adjustment of expectations, \mathbf{k} , must not be too great. This condition is familiar from the Cagan model of hyperinflation (Cagan 1956) and it rules out the possibility of explosive inflation with a constant quantity of money.

It should be noted that for the limiting case of $\mathbf{k} \text{ @ } \mathbb{Y}$ adaptive expectations converge to perfect foresight. Then $\det(J)$ obviously becomes negative and the system exhibits saddle-point (in-) stability: If perfect foresight is interpreted as forward-looking then conditional stability can be imposed on the solution, whereas if it is backward looking then the adjustment path will be unstable.¹⁷ However, in the following we will in general assume condition (43) to be fulfilled.

¹⁷ For a more extensive analysis of the Keynesian model with perfect foresight cf. Wörgötter (1988, 23ff. and 51ff.).

The second stability condition refers to the trace of the Jacobian:

$$(44) \quad \text{tr}(J) = \underbrace{\nu l_\nu}_{\mathbf{h}} + \mathbf{x} \underbrace{[(1-\mathbf{a})l_p - \mathbf{a}]}_{\mathbf{\Omega}} < 0.$$

Obviously the first term, \mathbf{h} , is negative; then with condition (43) fulfilled stability depends on the value of the term $\mathbf{\Omega}$. The crucial value of $\mathbf{\Omega}$ is given by

$$(45) \quad \text{tr} < 0 \Leftrightarrow \mathbf{\Omega} < \mathbf{\Omega}^* \equiv \mathbf{h}/\mathbf{x}, \quad \mathbf{\Omega}^* > 0, ;$$

hence instability cannot be ruled out as a value of $\mathbf{\Omega} > \mathbf{\Omega}^*$ is not excluded by parameter restrictions.

Some insights into the meaning of condition (45) can be gained by examining whether nominal wage adjustment is destabilising (deviation-amplifying) or not in the first instant, i.e. what Howitt (1976, 268f.) called the “direct stability” of the system. For example, if there has been a shock, say leading to unemployment, then as our system tells us nominal wages and in turn inflationary expectations will fall with opposite effects (the Keynes and Pigou effect versus the price expectation effect) on aggregate demand. Now, which of these effects will dominate in the first instant when the adjustment starts? To answer this question we have to look at the effect on employment l of the combined adjustment of ν and \mathbf{p}^e . Linearising around the steady-state equilibrium (with $\tilde{\mathbf{p}}^e = \mathbf{m}$) and inserting from (38) and (39) we obtain:

$$(46) \quad d\dot{l} = l_\nu d\nu + l_p \frac{\mathbb{J}\tilde{\mathbf{p}}^e}{\mathbb{J}\dot{\nu}} d\dot{\nu} = \frac{1}{\nu} [\nu l_\nu + l_p \mathbf{x}(1-\mathbf{a})] d\dot{\nu}.$$

Therefore the condition for direct stability turns out as

$$(47) \quad \nu l_\nu + \mathbf{x}(1-\mathbf{a})l_p \equiv \Phi < 0.$$

Or, put in another way, if $\Phi > 0$, then the price expectations effect, $\mathbf{x}(1-\mathbf{a})l_p$, will dominate the combined Keynes and Pigou effect, νl_ν , and in the first instant the effect of wage adjustment will aggravate the deviation from full employment. However, $\Phi > 0$ is a weaker condition than $\mathbf{\Omega} > 0$ and therefore direct instability is only necessary, but not sufficient for asymptotic instability. The obvious reason is that wage reductions lead to permanent decreases of the price level and therefore these price level effects on aggregate demand accumulate over time. Conversely, price expectations produce only transitory effects on aggregate demand. Thus to generate asymptotic instability the expectations effect must outweigh the price level effects not just at an instant of time but the integral of these effects over time.¹⁸ To accomplish this, a decrease in inflationary expectations must cause a still greater decrease in the rate of inflation which then again feeds back into expectations. Inflationary expectations affect inflation through two channels: First, expectations enter directly into the wage adjustment equation and, second, they influence wages

¹⁸ This can also easily be formulated within the traditional AS-AD-framework: Wage reductions on the one hand lead to downward shifts of the AS-curve which increase equilibrium output, for a given downward sloping AD-curve. On the other hand, the price expectations effect causes a downward shift of the AD-curve. However, whereas successive wage reductions cause successive downward shifts of AS, the effect of unchanging expectations of deflation would just be a once-and-for-all shift of AD.

indirectly via aggregate demand and employment. From these wage changes a proportion of $1-\mathbf{a}$ is transmitted to the inflation rate. Thus the effect on inflation is given by $(1-\mathbf{a})(\mathbf{I}l_p + 1)$, and for instability it must be (sufficiently) greater than the change in expectations itself, that is greater than one, which implies $\Omega > 0$. Therefore the condition for asymptotic instability is much more restrictive than that for direct instability.

In this regard it might also be interesting to analyse the relevance of the structural form parameters for the asymptotic stability of the system. As the det-condition is fulfilled for all positive values of \mathbf{I} and \mathbf{x} , the question of stability turns on the sign of the trace and its dependence on the structural parameters.

At first we look at the parameter \mathbf{I} that governs the response of the nominal wage, that is we try to answer the question: “Is wage flexibility stabilising?” After slightly rearranging terms we see that

$$(48) \quad \text{tr}(J) = \mathbf{I} \underbrace{[vl_v + \mathbf{x}(1-\mathbf{a})l_p]}_{\Phi = ?} - \mathbf{x}\mathbf{a}.$$

The effect of increased wage flexibility hinges therefore on the sign of the very term Φ that determines the direct stability of the system. From (48) it is evident that for \mathbf{I} sufficiently small the trace is unequivocally negative and asymptotic stability is thereby guaranteed.¹⁹ This has a rather surprising implication: Either $\Phi < 0$, that is wage flexibility is stabilising (implying also direct stability), then the parameter restrictions rule out the possibility of asymptotically unstable behaviour for any value of \mathbf{I} ; or $\Phi > 0$, then asymptotic instability is possible for some values of \mathbf{I} , and it will result only when \mathbf{I} is sufficiently great.²⁰ This is so because a high value of \mathbf{I} means rapid adjustment of wages to unemployment which is a precondition for allowing the expectations effect staying ahead of the price level effects by increasing rates of deflation.

The second question pertains to the relevance of the parameter \mathbf{k} , that governs the speed of adjustment of inflationary expectations. Here we look only at values that do not violate the det-condition (43), that is $0 < \mathbf{k} < \mathbf{k}^*$. Examination of $\text{tr}(J)$ then leads to

$$(49) \quad \frac{\partial \text{tr}}{\partial \mathbf{k}} = \Omega \frac{\partial \mathbf{x}}{\partial \mathbf{k}} > 0 \text{ if } \Omega > 0.$$

Thus in the relevant domain, i.e. where instability is possible at all, increasing \mathbf{k} makes the fulfilment of the stability condition more difficult.

The relation between stability and the parameters of the static model can be analysed by similar methods. Again restricting the domain of analysis to $\Omega > 0$, it turns out that the results for the

¹⁹ However, a low value of \mathbf{I} while guaranteeing stability may imply that it takes much time for the system to reach a neighbourhood of the steady-state.

²⁰ Of course, the formulation that wage flexibility “is destabilising” or makes instability “more probable” is just a short-cut for saying that the set of the constellations of the remaining parameters that imply asymptotic stability decreases with decreasing \mathbf{I} etc.

strength of the Pigou effect, z_m , and for the interest sensitivity of money demand, m_i , are unequivocal: Increasing the strength of the Pigou effect and decreasing (in absolute value) the interest sensitivity of money demand makes the system definitely more stable. Obviously, unstable behaviour is most likely when the Pigou effect vanishes or when the interest sensitivity approaches (negative) infinity, respectively.

Finally we consider the role of the interest sensitivity of aggregate demand. As might have been expected, the effect is ambiguous. On the one hand a greater (absolute) value of z_r reinforces the Keynes effect of nominal wages on employment, but on the other hand it aggravates the price expectations effect, so that in sum the outcome is indeterminate. However, we may derive for z_r a result like that with respect to I . If aggregate demand is totally interest inelastic, that is for $z_r = 0$, the price expectations effect will vanish completely whereas due to the Pigou effect the stabilising influence of nominal wages on employment will persist, so that the system will be stable with $\text{tr}(J) < 0$. Therefore, if instability is possible for some value of z_r then the effect of an increasing interest sensitivity on aggregate demand must be destabilising. Or conversely: If this effect is stabilising, then the system will be stable for all possible values of z_r .

Another issue besides that of stability is whether the adjustment process is monotonic or oscillatory. This depends, of course, on the sign of the discriminant of the characteristic equation, $D = \text{tr}(J)^2 - 4 \det(J)$, which can be written as a function of Ω :

$$(50) \quad D(\Omega) = (\mathbf{h} + \mathbf{x}\Omega)^2 + 4\mathbf{h}\mathbf{x} = \mathbf{x}^2\Omega^2 + 2\mathbf{h}\mathbf{x}\Omega + \mathbf{h}(\mathbf{h} + 4\mathbf{x}).$$

Obviously, $D(\Omega^*) < 0$, so that at the crucial value of Ω where the system turns from a stable into an unstable one, the process is oscillatory in any case. The region of oscillatory behaviour is then determined by solving for $D(\Omega) = 0$ which leads to:

$$(51) \quad \Omega_{12} = \Omega^* \pm \frac{2\sqrt{-\mathbf{h}\mathbf{x}}}{\mathbf{x}}.$$

Because of $-\mathbf{h}\mathbf{x} > 0$ these roots are real, and they are both positive if $\mathbf{h} + 4\mathbf{x} < 0$, otherwise there is a positive and a negative root.

Examining the dynamics of the system as it depends on Ω , for great (in absolute value) negative values of Ω the time paths are stable and monotonic (therefore the steady-state equilibrium is a stable node). With Ω increasing (algebraically) to the value of Ω_2 , negative or positive, the system starts to exhibit oscillatory movements while remaining stable (the steady-state equilibrium becomes a stable focus). When Ω^* , a definitely positive value, is reached, the system enters the region of unstable (and still oscillatory) behaviour, and then increasing beyond Ω_1 , the behaviour remains unstable and again becomes monotonic.

The dynamic behaviour of the system can be visualised by means of phase diagrams.²¹ At first, we must therefore derive the equations of the curves for $d\dot{v} = 0$ and $d\dot{p}^e = 0$, the so-called

²¹ On the use of phase diagrams cf. Gandolfo (1996, 341ff.) and Shone (1997, ch. 4).

isokines (Gandolfo 1996, 350), by differentiating (38) and (39). This leads to:

$$(52) \quad \left. \frac{d\mathbf{p}^e}{dv} \right|_{\dot{v}=0} = -\frac{Il_v}{Il_p + 1} > 0,$$

where we have taken account of that $\mathbf{w} - \mathbf{m} = 0$ along $\dot{v} = 0$, and

$$(53) \quad \left. \frac{d\mathbf{p}^e}{dv} \right|_{\dot{\mathbf{p}}^e=0} = -\frac{(1-\mathbf{a})Il_v}{(1-\mathbf{a})(Il_p + 1) - 1} = -\frac{(1-\mathbf{a})Il_v}{\Omega} = ?$$

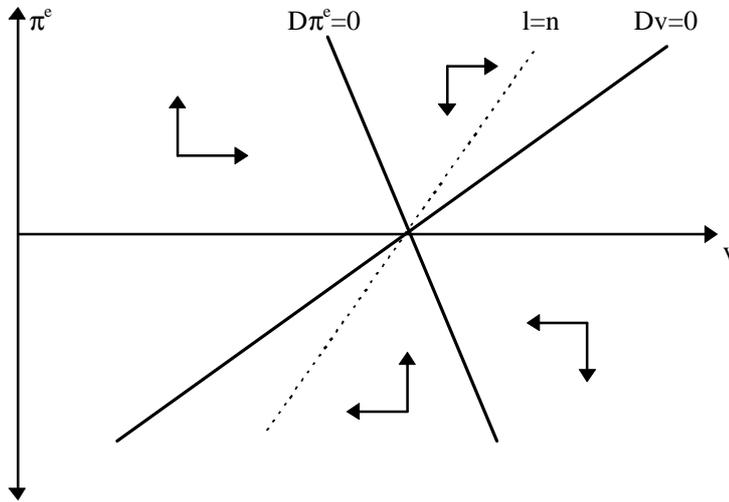
Whereas the slope of (52) is definitely positive, that of (53) is indeterminate and depends on the value of Ω . The meaning of these two isokines is, of course, that for $\dot{v} = 0$ the rate of change of nominal wages is in accordance with that of the money supply and that for $\dot{\mathbf{p}}^e = 0$ expected equals actual inflation.

Furthermore, we determine those values of v and \mathbf{p}^e that generate full employment. The corresponding curve can be derived by simply differentiating the condition $l = n$, which gives:

$$(54) \quad \left. \frac{d\mathbf{p}^e}{dv} \right|_{l=n} = -\frac{l_v}{l_p} > 0.$$

For the relevant phase diagrams two cases must be distinguished according to the respective sign of Ω . If $\Omega < 0$, then the isokine for \mathbf{p}^e has a negative slope and the phase diagram looks like that of figure 1.²² We know from the calculations above that the dynamics is stable in this case but may admit oscillatory movements.

Fig. 1: $i > 0$, $\mathbf{x} > 0$, $\Omega < 0$, $\mathbf{m} = 0$.



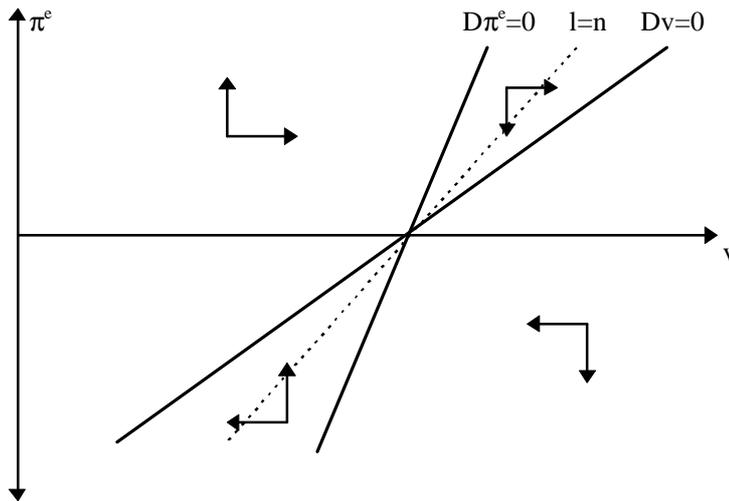
If $\Omega > 0$, then the isokine for \mathbf{p}^e has a positive slope and, furthermore, it can be shown that

$$(55) \quad \left. \frac{d\mathbf{p}^e}{dv} \right|_{\dot{\mathbf{p}}^e=0} > \left. \frac{d\mathbf{p}^e}{dv} \right|_{l=n} > \left. \frac{d\mathbf{p}^e}{dv} \right|_{\dot{v}=0}.$$

²² In the following figures the symbol D is used for $Dx \equiv \dot{x}$.

This case is depicted in figure 2. We know again from the calculations above that this constellation is compatible with any kind of dynamic behaviour, stable or unstable, monotonic or oscillatory. Instability would then mean either cyclical movements in employment and inflation of ever increasing amplitude or a kind of deflationary (or inflationary) spiral with e.g. ever increasing deflation running ahead of expectations.

Fig. 2: $i > 0$, $\mathbf{x} > 0$, $\Omega > 0$, $\mathbf{m} = 0$.



Finally we may look at the effect of including inflationary expectations in the wage adjustment equation. This means comparing the expectations-augmented Phillips curve (36) with the naive Phillips curve, i.e. with

$$(36') \quad \mathbf{w} = I(l - n), \quad I > 0.$$

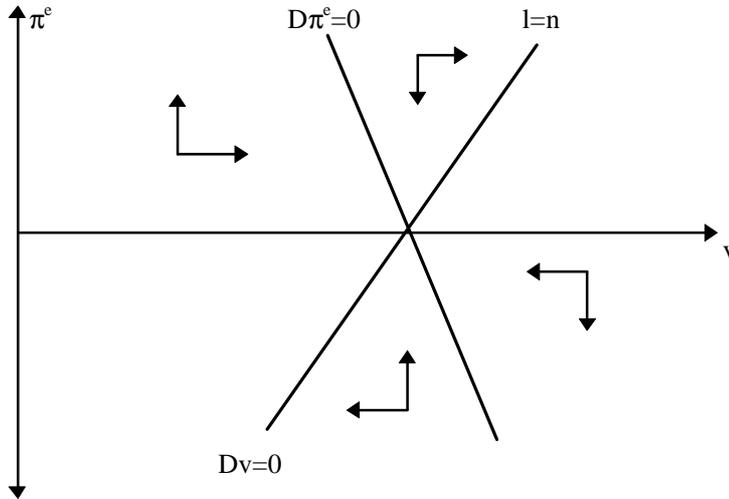
Looking at the dynamics of the system, we see that whereas the det-condition is left unaffected in the tr-condition the indeterminate term Ω is now replaced by

$$(44') \quad \Omega^+ \equiv (1 - \mathbf{a})I l_p - 1 = \Omega - (1 - \mathbf{a}) = ?$$

Obviously, Ω^+ is more likely to be negative and therefore to fulfil the stability condition.

Therefore taking inflationary expectations in the Phillips curve into account makes the system more unstable.

This fact can also be depicted in a phase diagram. In the case of the naive Phillips curve the isokine for v coincides with the full employment curve, that is $\dot{v} = 0$ implies $l = n$. Thus the phase diagram (for example with $\Omega^+ < 0$) looks like figure 3.

Fig. 3: $i > 0$, $\mathbf{x} > 0$, $\Omega^+ < 0$.

4.2 The zero-interest-rate-regime

Next the dynamics of the model is analysed for the case when the zero floor for the nominal rate of interest becomes binding. As we stick to the assumption that the nominal rate of interest is positive in the steady-state, we can no longer restrict the linearisation to the neighbourhood of (steady-state) equilibrium, where $\mathbf{w} = \mathbf{m}$. Therefore by differentiating (38) and (39) we now obtain:

$$(56) \quad d\dot{v} = [vIl_v + (\mathbf{w} - \mathbf{m})]dv + v(Il_p + 1)d\mathbf{p}^e, \text{ and}$$

$$(57) \quad d\dot{\mathbf{p}}^e = \mathbf{x}(1 - \mathbf{a}_0)Il_v dv + \underbrace{\mathbf{x}[(1 - \mathbf{a}_0)Il_p - \mathbf{a}_0]}_{\Omega_0} d\mathbf{p}^e.$$

Note that the partial derivatives referred to in (56) and (57) are those from (27) to (32).

When examining the qualitative properties of this dynamic system we have to acknowledge that the results do no longer refer to the stability of the system as a whole but only to its behaviour within the zero-interest-rate-regime. Calculating the trace and the determinant of the Jacobian matrix that correspond to (56) and (57) we obtain:

$$(58) \quad \text{tr}(J) = vIl_v + (\mathbf{w} - \mathbf{m}) + \mathbf{x}\Omega_0 = ?$$

$$(59) \quad \det(J) = \mathbf{x}[(\mathbf{w} - \mathbf{m})\Omega_0 - vIl_v] = ?$$

Two features of this result should be noticed: On the one hand, the sign of Ω_0 depends on \mathbf{a}_0 : $\Omega_0 > 0 \Leftrightarrow 0 \leq \mathbf{a}_0 < Il_p / (1 + Il_p) < 1$. In the zero-interest-case the positive value of \mathbf{a}_0 is solely due to the Pigou effect and is proportional to its strength; therefore it may be reasonably supposed to be small so that $\Omega_0 > 0$ is a plausible outcome. Moreover, the qualitative properties hinge also on the sign of $(\mathbf{w} - \mathbf{m})$, which means that the behaviour might be different according to

the points on which side of the isokine for v we consider.²³ For example, with $(\mathbf{w} - \mathbf{m}) < 0$ and $\Omega_0 > 0$ a negative value of $\text{tr}(J)$ as well as of $\det(J)$ becomes more probable.

The peculiarities of the dynamics in the zero-interest-rate-regime are brought out more clearly when considering the special case with $z_m = 0$. In this case the results are clear-cut. As is evident from (33) to (35), $l_v = 0$ and therefore $\mathbf{a}_0 = 0$ which implies $\Omega_0 > 0$. Thus the relevant terms are now given by:

$$(60) \quad \text{tr}(J) = \mathbf{w} - \mathbf{m} + \mathbf{x}\Omega_0 = ?$$

$$(61) \quad \det(J) = \mathbf{x}(\mathbf{w} - \mathbf{m})\Omega_0 = ?$$

Then for $\mathbf{w} - \mathbf{m} < 0$, that is for points below the isokine for v , $\det(J) < 0$ so that the system behaves like that of a saddle point, whereas for $\mathbf{w} - \mathbf{m} > 0$, that is for points above the isokine for v , $\text{tr}(J) > 0$ so that the system behaves like that of an unstable node or focus. (A more thorough investigation of the type of adjustment thereby generated is postponed after discussing the corresponding phase diagrams below.) However, this result for the special case sheds some light on the behaviour of the system at the zero interest rate floor in general: As $\text{tr}(J)$ and $\det(J)$ are continuous functions of z_m the results for a zero Pigou effect must also be valid for a sufficiently small Pigou effect.

4.3 The boundary of the zero-interest-rate regime

The next step before we are able to analyse the dynamic system as a whole is to determine the boundary (in v - \mathbf{p}^e -space) between the two regimes, i.e. where the nominal interest rate becomes zero in instantaneous equilibrium.

In order to derive this boundary we first recapitulate the linear approximation for the nominal interest rate in instantaneous equilibrium from (19):

$$(62) \quad di = i_p d\mathbf{p}^e - i_M \frac{m}{v} dv.$$

Inserting for di and $d\mathbf{p}^e$ from the steady-state solution:

$$(63) \quad \begin{aligned} di &\equiv i - \tilde{i} = i - \mathbf{m} - \tilde{r} = i - \mathbf{m} - \tilde{r}(0) - \tilde{r}_m \mathbf{m}, \\ d\mathbf{p}^e &\equiv \mathbf{p}^e - \tilde{\mathbf{p}}^e = \mathbf{p}^e - \mathbf{m}, \end{aligned}$$

and setting $i = 0$ results in

$$(64) \quad i = 0 = i_p (\mathbf{p}^e - \mathbf{m}) - i_M \frac{m}{v} (v - \tilde{v}) + \mathbf{m} + \tilde{r}(0) + \tilde{r}_m \mathbf{m}$$

²³ Obviously, $\mathbf{w} - \mathbf{m} = 0$ implies $\dot{v} = 0$.

This gives us finally the equation for the boundary between the two regimes as:

$$(65) \quad \mathbf{p}^e|_{i=0} = \frac{1}{i_p} \left[i_M \frac{m}{v} (v - \tilde{v}) - \tilde{r}(0) - (1 + \tilde{r}_m - i_p) \mathbf{m} \right],$$

$$\text{with } \frac{\mathcal{J}\mathbf{p}^e}{\mathcal{J}\mathbf{m}} = -\frac{1 + \tilde{r}_m - i_p}{i_p} < 0; \quad \frac{\mathcal{J}\mathbf{p}^e}{\mathcal{J}v} = \frac{i_M}{i_p} \frac{m}{v} < 0.$$

Therefore the boundary has a negative slope in v - \mathbf{p}^e -space and its position at $v = \tilde{v}$ is such that the expected deflation, $-\mathbf{p}^e$, is at least as great as the real rate of interest corresponding to the given rate of money growth, i.e.

$$(66) \quad \mathbf{m} \geq \mathbf{m}_{\min} \Rightarrow \mathbf{p}^e|_{i=0, v=\tilde{v}} \leq -\tilde{r}(\mathbf{m}).$$

4.4 The dynamics of the system as a whole

To describe the dynamics of the system as a whole we must now combine the results for the two regimes which are separated in v - \mathbf{p}^e -space by the boundary as calculated above.²⁴ Thus we shall examine the dynamics of the system as a whole by making heuristic use of the respective phase diagrams (see figures 4 and 5). In particular we start by investigating the special case with $z_m = 0$ (no Pigou effect) and furthermore we impose the restriction on the isokine for \mathbf{p}^e that whenever its slope is negative it must be steeper than that of the boundary. (This restriction is more fully discussed below.)

Then it is evident that without the Pigou effect the two isokines and the full-employment-curve become horizontal when entering the region of the zero-interest-rate regime because of $l_v = 0$ and $\mathbf{a}_0 = 0$. v will decrease below the isokine for v and increase above it, similarly \mathbf{p}^e will decrease below the isokine for \mathbf{p}^e and increase above it. It follows from this (and from the calculations above) that the dynamics of the system becomes monotonic and unstable when a process enters the zero-interest-rate-regime in the region below $\dot{\mathbf{p}}^e = 0$. However, when a process enters it in the region lying above $\dot{\mathbf{p}}^e = 0$ it will eventually return to the flexible-interest-rate regime and its stability then depends upon the dynamic properties of that regime.

This is illustrated by the following phase diagrams. Figure 4 describes the case where $\Omega > 0$ so that the isokine for \mathbf{p}^e has a positive slope and figure 5 the case where $\Omega < 0$ but with the (negative) slope of the isokine for \mathbf{p}^e being steeper than that of the boundary.

Both diagrams reveal the fact that there exists a critical rate of inflation (in fact: a rate of deflation), marked in these diagrams by the point of intersection of the boundary and the isokine for \mathbf{p}^e : If the (expected) rate of inflation ever (in any regime) falls below this critical rate, the process will necessarily become unstable and a return to the steady-state equilibrium will be impossible. The mechanism working below the critical rate is obvious: There is unemployment that causes wage reductions which in turn (because of the absence of the Pigou effect) lead to a proportional lowering of the price level, so that the real wage is left unaffected. However, the

²⁴ Again it should be noticed that the system is described by linear approximations.

deflation generates expectations of deflation which decrease aggregate demand and thereby lower the price level still further and thus raise the real wage. Therefore unemployment increases giving rise to more wage reductions and so on – the typical case of a deflationary spiral.

Fig. 4: $x > 0, \Omega > 0, m = 0$

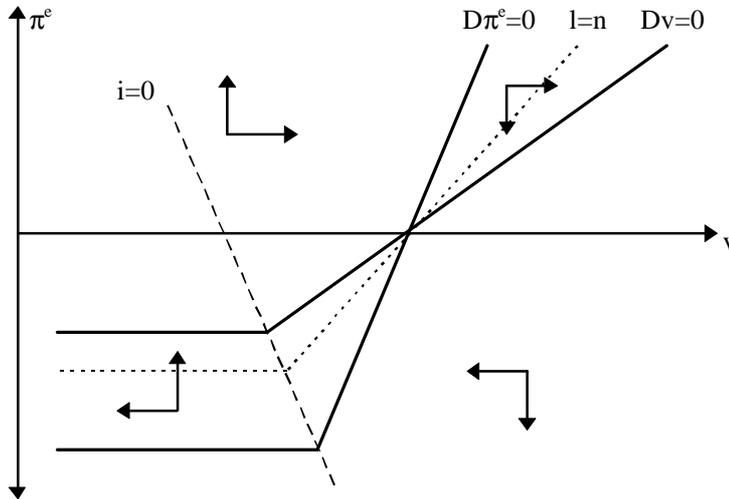
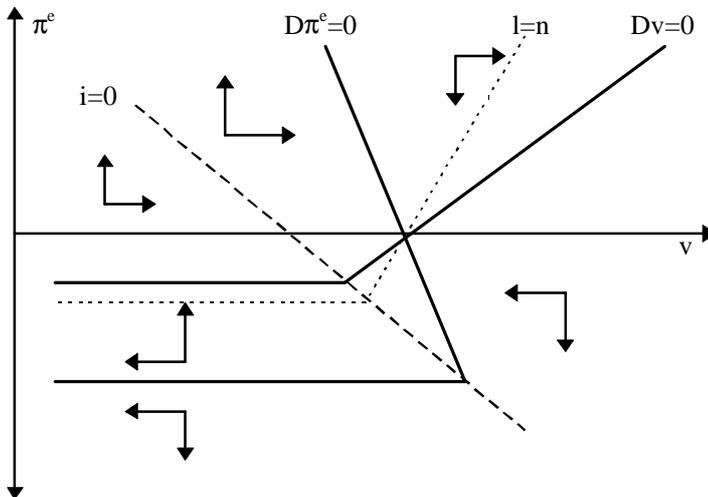


Fig. 5: $x > 0, \Omega < 0, m = 0$



The critical rate of inflation, \hat{p}^e , can be computed, by using linear approximations, from (65) and from the equation of the p^e -isokine, i.e.:

$$(67) \quad p^e = -\frac{(1-a)ll_v}{\Omega}(v - \tilde{v}) + m,$$

rendering:

$$(68) \quad \hat{p}^e = \frac{\Lambda + i_p - 1}{\Lambda + i_p} m - \frac{1}{\Lambda + i_p} \tilde{r}(0), \quad \Lambda \equiv i_M \frac{m}{v} \frac{\Omega}{(1-a)ll_v} = ?$$

Checking for the effect on i of changes of \mathbf{p}^e along the \mathbf{p}^e -isokine gives:

$$(69) \quad di = i_p d\mathbf{p}^e + i_M \frac{m}{v} dv = (\Lambda + i_p) d\mathbf{p}^e;$$

which confirms that the nominal rate of interest is positive within the flexible-interest-rate-region.

In the equations above the term $\Lambda + i_p$ is indeterminate. However, as the existence of a sensible critical rate of inflation depends upon this term being positive we provisionally stipulate the condition:

$$(70) \quad \Lambda + i_p > 0.$$

As can be ascertained this condition is identical to the restriction imposed above on the slope of the isokine for \mathbf{p}^e . Moreover, $\text{sgn}(\Lambda) = \text{sgn}(\Omega)$ so that condition (70) is compatible with all types of adjustment processes in the flexible-interest-rate-regime.

So what would a violation of (70) amount to? In any case such a violation cannot be ruled out a priori as it is possible to designate arbitrarily great negative values to Λ by making I sufficiently small. However, that would mean that the relative strength of the impact of \mathbf{p}^e (as compared to v) is greater on the nominal rate of interest than on inflationary expectations. As a consequence the boundary of the two regimes and the isokine for \mathbf{p}^e would intersect to the left and above the steady-state equilibrium. Thereby the only possibility of the nominal rate of interest to become zero at fully anticipated inflation would be at a positive rate of inflation combined with over-full employment. This does not appear to be a plausible situation and it is what assumption (70) does exclude. Anyway, in the following we shall take assumption (70) for granted.

Then the most interesting feature of this result is the response of the critical rate of inflation to the steady-state rate of money growth:

$$(71) \quad \frac{\mathcal{J}\hat{\mathbf{p}}^e}{\mathcal{J}\mathbf{m}} = \frac{\Lambda + i_p - 1}{\Lambda + i_p} \quad \text{and} \quad \frac{\mathcal{J}(\mathbf{m} - \hat{\mathbf{p}}^e)}{\mathcal{J}\mathbf{m}} = \frac{1}{\Lambda + i_p} > 0, \quad \text{if } \Lambda + i_p > 0.$$

That is, the gap between the steady-state rate and the critical rate of inflation widens with higher rates of inflation. As a corollary it can be deduced that with the optimal-quantity-of-money-rule of Friedman (1969) the steady-state rate of inflation coincides with the critical rate:

$$(72) \quad \mathbf{m} = \mathbf{m}_{\min} = -\tilde{r}(\mathbf{m}) = \tilde{\mathbf{p}} = \hat{\mathbf{p}}^e.$$

Two conclusions emerge from the above analysis: First, we may compare these results with those valid for the flexible-interest-rate regime. Taking account of the zero interest rate floor obviously widens the range of parameter constellations where instability is a possible outcome. Even when the process would be stable within the flexible-interest-rate region, at least for oscillating adjustment paths and with a sufficiently great disturbance the possibility of undershooting the critical rate of inflation cannot be excluded. However, if such undershooting takes place the process is irrevocably unstable. Thus, instability in the Keynesian macro model

may arise even if the rather mild condition for stability within the flexible-interest-rate-region is fulfilled.

Secondly, the above conclusions are strictly true only for the special case when the Pigou effect vanishes. However, it follows from a kind of continuity argument that similar results will also be valid for a sufficiently small Pigou effect. Put in terms of the above phase diagrams, it remains true that when entering the zero-interest-rate-region both isokines will become flatter (and the more approaching a horizontal line the smaller the Pigou effect). Although then instability will not be the inevitable consequence of an undershooting of the critical rate of inflation, the eventual return to the flexible-interest-rate-region, necessary for stability, is still no longer guaranteed. Therefore even in the more general case of a positive Pigou effect the stability properties of the system as a whole will deteriorate as compared to those of the flexible-interest-rate-regime.

5. Interpreting the Results: Some Keynesian Conjectures

By drawing on the foregoing analysis we now attempt to formulate some implications for Keynesian theory both from a history-of-thought and from a macroeconomic perspective.

The stability of the Keynesian system in general

The first implication refers to the stability of full employment, when no account is taken of the zero-interest-rate floor. In general the result of the foregoing analysis is that all kinds of dynamic behaviour are consistent with the model. Put in another way, there is no sensible parameter restriction that can rule out a priori that the full employment equilibrium is unstable. Of course, stability is facilitated by the strength of the Keynes and the Pigou effect, and it is impeded by the effect of expected deflation on effective demand. However, the dominance of the price expectations effect in the first instant is not sufficient for generating instability, as the weight of the stabilising price level effects is cumulative over time. Therefore, a strongly positive feedback from expected on actual inflation is needed to generate instability, a condition which might be considered rather stringent.

The relation between the structural parameters and stability is by and large as expected. The strength of the Pigou effect enhances stability whereas it is rendered more difficult by a high interest elasticity of money demand and by (too) rapid adjustment of inflationary expectations. With regard to wage flexibility and the interest elasticity of aggregate demand the results are ambiguous: While the sign of the relation is uncertain in general, if instability is possible at all, then only at high levels of wage flexibility and of the interest elasticity of aggregate demand. Finally, it has been shown that the inclusion of inflationary expectations into the wage Phillips curve is not favourable to the stability of the system.

The relevance of the zero-interest-rate floor

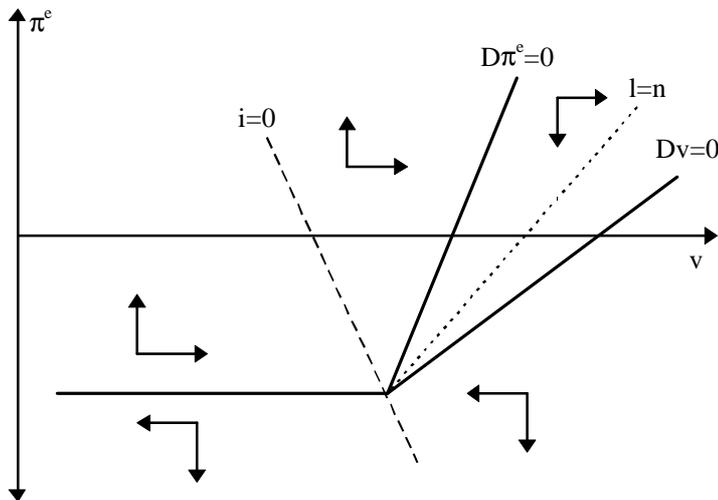
A specific feature of this analysis is the distinction between the dynamic behaviour when the nominal rate of interest is flexible and that when it is fixed by the zero floor. In the latter case the comparative static results of the instantaneous equilibrium are those well known from the Keynesian liquidity trap case. However, there are two distinct aspects. First, the floor to the nominal interest rate is a property of the system experienced during the adjustment process and not one pertaining to the steady-state equilibrium. The existence of that floor might therefore be relevant even if the interest rate compatible with full employment is well beyond the floor. And second, it is irrelevant for the qualitative results whether the floor to the nominal interest rate is zero or a positive value associated with the existence of a liquidity premium.

The central thesis is, anyway, that the existence of such a floor considerably aggravates the stability problem. For the trajectories that enter the region where the floor is a binding constraint there are more parameter constellations that give rise to instability than if they would have stayed in the flexible-interest-rate region all the time. In this sense the system becomes “more unstable”. Moreover, at the zero floor the stability of the system depends in a different way on the parameters of the structural form than before, and it is here that the existence of the Pigou effect is most crucial.

The existence of a critical rate of inflation

It has been shown that the stronger the Pigou effect the more likely is the system to be stable with a flexible as well as with a zero nominal interest rate. However, in the zero-interest-rate regime the Pigou effect is crucial insofar as without it the dynamic system is definitely unstable. Referring to the system as a whole any path that during the adjustment process falls below a critical rate of inflation (determined by the steady-state rate of inflation and the corresponding real rate of interest) will never return to the steady-state equilibrium. This property of the dynamic system (with no Pigou effect) gives rise to three observations: First, the system exhibits the characteristics of “corridor-stability”, that is it is stable for small shocks but unstable for great shocks. In the case of small shocks the system may return to equilibrium while all the time remaining in the flexible-interest-rate region (or if entering the zero-interest-rate region above the critical rate, it will eventually leave it again). To the contrary, for sufficiently great shocks the adjustment process will undershoot the critical rate of inflation and become unstable after entering the zero-interest-rate region. Second, for a shock of given size the risk that the adjustment process will fall below the critical rate of inflation is the smaller the higher the steady-state rate of inflation. In this sense lower steady-state inflation means a higher risk of instability. Third, as a special case, the optimal-quantity-of-money rule as proposed by Friedman is under these circumstances a knife-edge-solution. For here the steady-state and the critical rate of inflation are identical so that any deviation downwards would generate instability. This property can be easily visualised by means of the phase diagram of figure 6.

Fig. 6: $x > 0$, $\Omega > 0$, $m = m_{\min}$, $z_m = 0$.



These three particular features have been (heuristically) derived for the special case when there is no Pigou effect. However, as argued earlier, the qualitative properties will be retained at least for sufficiently small values of the Pigou effect, too. So even with a small positive Pigou effect these considerations might still be valid as a description of the system behaviour and therefore as a guidance for a Keynesian policy.

6. Conclusion

The Keynesian model analysed in this paper bears some resemblance to the seminal one of Tobin (1975). However, the modifications of that model, in particular taking explicit account of the wage-price relationship and of the existence of a zero floor to the nominal rate of interest, are reflected in the results, too. First, on the one hand stability is more easily achieved when the analysis is restricted to the flexible-interest-rate regime, i.e. when disregarding the existence of a minimum rate of interest. Yet, on the other hand when looking at the system as a whole the region of instability may indeed have widened, as (under plausible parameter restrictions) any time path that eventually generates deflation that falls below a critical level may become unstable. Second, this model verifies some of the conjectures made by Tobin. For example there is the possibility of corridor stability (cf. Tobin 1975, 201n.4), and there are destabilising effects of increasing wage (or price) flexibility or the adjustment speed of expectations (cf. Tobin 1993, 63).

Finally, some results of this model are apparently new: One is the relation between direct stability, the stabilising effect of wage flexibility and asymptotic stability. Another is the (positive) relation between steady-state inflation and the stability of full-employment due to the working of the zero-interest-floor which, although rather trivial, is not explicitly mentioned in the Keynesian literature. The same is true for the application of this result to Friedman's optimal money rule. However, it was well-known that equilibrium with the nominal interest rate on cash balances set equal to the rate of time preference would place the economy on a knife-edge

(Niehans 1978, 96).²⁵ Recently Krugman (1998) argued in a modern re-interpretation that the liquidity trap might be due to the failure of the central bank to commit itself credibly to a policy of inflation, which with the nominal rate at the zero floor could produce negative real rates if correctly expected. Although derived from a different model the implication seems similar to the above one that the probability of experiencing such a trap situation is enhanced by a policy of low inflation rates.

²⁵ Niehans (1976, 96n.) explicitly refers to such a situation as a kind of „liquidity trap“.

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