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Dynamic Effects of Regulation and Deregulation in Goods and Labour Markets

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DYNAMIC EFFECTS OF REGULATION AND DEREGLATION IN GOODS AND LABOUR MARKETS

by

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Abstract
Modern macroeconomic models with a Keynesian flavour usually involve nominal rigidities in wages and goods prices. A typical model is static and combines wage bargaining in the labour markets and monopolistic competition in the goods markets. As central policy implication it follows that deregulating labour and/or goods markets increases equilibrium employment.

We reassess the consequences of deregulation in a dynamic model. It still increases employment at the fixed point, which corresponds to the static equilibrium solution. However, deregulation may also lead to stability loss and endogenous fluctuations.

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Keywords
Labour and goods markets deregulation, monopolistic competition, business cycles

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1. Introduction

Recently, Blanchard and Giavazzi (2003) analysed the macroeconomic effects of regulation and deregulation in goods and labour markets. Their framework combines monopolistic competition in the goods market with wage bargaining in the labour market. In the short run, deregulation primarily changes the distribution of rents; in the long run this induces via firms entry and exit changes in employment levels.

Their model belongs to the long tradition of macroeconomic models with a Keynesian flavour, which use Dixit-Stiglitz monopolistic competition as a microfoundation for nominal rigidities and price setting power in the goods market.\(^1\) Keynesian features studied

\(^1\) The widely used approach of specifying wage and price setting equations (see e.g. Nickel, 1990; Layard et al., 1991; or Blanchard, 2003; and for empirical applications various
in such models are the possibility of unemployment (e.g. Blanchard and Kiyotaki, 1987; d’Aspremont et al., 1990), wage price spirals (Blanchard, 1986) and multiple equilibria (e.g. d’Aspremont et al., 1995; Linnemann, 2001) and that fiscal policy has income multiplier effects (Heijdra and Litghart, 1997; Heijdra et al., 1998; see also Solow, 1998).

Typically, all these models focus on what Blanchard and Giavazzi (2003) call short run general equilibrium – i.e. a position with a fixed number of firms in which all firms choose the same price and quantity and in which goods markets clear – and on the long run general equilibrium – a position with zero profits due to firm entry and exit. How those equilibria are brought about is usually not modelled explicitly, with Maussner (1992, 1996) and, more recently, Jin (2001) and Currie and Kubin (2002) as the only exceptions known to us.

In particular, the transition to the short run equilibrium is not simple and obvious because even in equilibrium firms do not act upon a basis of knowing fully the model. Firms do not take into account that their decisions influence aggregate variables or that other firms may react. In this sense, their equilibrium pricing decisions are based on a price elasticity that is partial (i.e. it takes the aggregate income, the other prices and the price index as given) and that is higher than the one holding in the aggregate (which allows for all interdependencies). Given these limitations to knowledge in equilibrium, it would be inconsistent to assume full knowledge of the model out-of-equilibrium. This raises the question how the equilibrium is brought about, esp. how the equilibrium quantities are found.

analyses by the European Commission, e.g. McMorrow and Roeger, 2000; Bains et al., 2002) can also be seen in this tradition.
We suppose rational behaviour in the sense that agents hold outside equilibrium the same information as in a short run (or in a long run) equilibrium and act consistently. Following Maussner (1992, 1996) and Currie and Kubin (2002), we assume that firms know that the demand functions they face are iso-elastic and that they know the value of the partial price elasticity but that they do not know all the interdependencies of the aggregate model. In particular, they do not know the position of the demand functions which they anticipate on the basis of market positions realized in the past.  

We develop a model which closely follows Blanchard and Giavazzi’s (2003) set up but which is amended by a dynamic process focusing on the transition to the short run general equilibrium (and by a more explicit specification of the reservation wage). The short run general equilibrium solution is a fixed point of our dynamic model. However – and this is the main point – depending upon the parameters the fixed point may lose stability through a Flip-bifurcation giving rise to cyclical solutions and endogenous fluctuations. We show analytically that deregulation in goods and labour markets may lead to instability.

The paper is organized as follows: In the second section, we present our model: monopolistically competitive firms bargain with unions over employment and wage rates on the basis of firms’ anticipated demand functions and workers’ reservation wages. In the third section, we explicitly add two dynamic components. First, the adjustment of the

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2 Jin (2001) does not confine his analysis to the Dixit-Stiglitz variant of monopolistic competition. In his framework, the demand functions need not be iso-elastic and firms base their decisions on linear expected demand functions.

3 D’Aspremont et al. (1995) analyze endogenous fluctuations in what they call a Cournotian monopolistic competition model. In contrast to our approach, the dynamics in their model is equilibrium dynamics, which is driven by an overlapping generations structure and variable mark-ups.
anticipated demand function on the basis of realized market prices and quantities; second, the adjustment of the reservation wage on the basis of realized employment and wage rates. We show that the first process in isolation leads to diverging time paths: deregulation leads to instability. In the fourth and the fifth sections, we study two different specifications for the reservation wage adjustment, each of which mirrors a different institutional set up. We show that this second process may dampen the sharp instability result: deregulation may still destabilize the economy; however, time paths do not diverge but fluctuate following a period-two cycle or more complex trajectories. Some simulations complete the dynamic explorations. The last section is left for concluding remarks.

2. The economic framework

The analysis, being framed in a dynamic setting, requires dealing explicitly with time. We assume that during the time unit, which for expository purposes we call ‘Week $t$’, all events occur according to a well-defined sequence. On Monday of Week $t$, firm $i$ and the associated union bargain over employment and the nominal wage rate on the basis of firms’ anticipated demand and workers’ nominal reservation wage. Production occurs during the Week from Tuesday to Friday. On Friday, commodities are delivered to the market. Market equilibrium determines the realized price. Finally, on Saturday firms and unions update the information relevant for decisions in the following period concerning the employment, the nominal wage rate and the realized price.

2.1 Firms and Households

The economy comprises a fixed number $m$ of firms, each firm producing one differentiated good in a regime of monopolistic competition; and $L$ households, which own the existing
firms, each household supplying one unit of labour. Each firm \( i \) faces a union composed of
\[
M_i^i = \frac{L}{m} \quad \text{members, with } i = 1 \ldots m.
\]

All firms share the same production technology, which involves only one input, labour, used in a fixed proportion.\(^4\) Assuming that during Week \( t \) firm \( i \) employs \( N_i^t \) workers, the production of good \( i \) is:
\[
x_i^t = f(N_i^t) = N_i^t
\]
(1)

Household \( j \)'s preferences (\( j = 1 \ldots L \)) towards good \( i \) are represented by a CES utility function:
\[
U_{ji}^t = \left( m^{\frac{1}{\sigma}} \sum_{i=1}^{m} \left( c_{ji}^{t} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma}}
\]
(2)

where \( 1 < \sigma < \infty \) is the constant elasticity of substitution between goods and \( c_{ji}^{t} \) is the consumption level of good \( i \) for the period. Note that, because of the factor \( m^{\frac{1}{\sigma}} \) included in the utility function, this specification does not represent a preference for goods variety; \( \sigma \) is a parameter related only to the market power of a single good supplier (see Blanchard and Kiyotaki, 1987; and also the discussion in Benassy, 1996). In Blanchard and Giavazzi (2003), it is the central parameter for studying the effects of goods market deregulation.

Household \( j \)'s demand for good \( i \) is given by
\[
d_{ji}^t = \frac{Y_{jt}}{mP_i^t} \left( \frac{p_{ji}^t}{P_i^t} \right)^{-\sigma}
\]

\(^4\) Note that Maussner (1992, 1996) uses a more general production function with a variable marginal product of labour.
where \( Y_j \) is the household \( j \)'s nominal income (profit income, work and non-work income)\(^5\) and where

\[
P_t = \left( \frac{1}{m} \sum_{i=1}^{m} (P_t^i)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}
\]  

(3)

is the price index. Summing through \( j \), we obtain the demand for goods \( i \) during Week \( t \)

\[
d_i = \frac{Y}{mP_t} \left( \frac{p_t^i}{P_t} \right)^{-\sigma}
\]

(4)

where \( Y \) is the overall nominal income of the economy. Taking the income and the price index as given,\(^6\) the partial elasticity of demand with respect to the own price therefore reduces to \(-\sigma\).

If the price of each good is the same, \( p_t^i = p_t \), the price index becomes \( P_t = p_t \) and the demand for an individual good simplifies to

\[
d_t = \frac{Y}{mp_t}
\]

(5)

in which the elasticity of demand with respect to the price is equal to \(-1\).

\(^5\) Note that the demand decision can also be interpreted as a decision under a cash-in-advance constraint, see below section 2.3.

\(^6\) We thus neglect the Price Index Effect and the Ford Effect (see Yang and Heijdra, 1993; Dixit and Stiglitz, 1993; and d’Aspremont et al., 1990, 1996). Linnemeier (2001) showed that both effects are at the root of a possible multiplicity of equilibria. We prefer to demonstrate our results in the simplest possible framework with a unique equilibrium.
Moving on to the description of the events occurring during ‘Week \( t \)’, most of our discussion will be devoted to the crucial ones, that is, the bargaining process and the determination of the market equilibrium.

### 2.2 Bargaining

As Blanchard and Giavazzi (2003), we employ the efficient bargaining model (McDonald and Solow, 1983; Layard et al., 1990): During the Monday of Week \( t \) the parties (firm \( i \) and the associated union) bargain over employment \( N_i^t \) and the nominal wage rate \( W_i^t \) for the current period as to maximize

\[
\left( \frac{W_i^t - WR_i}{\bar{P}_i} N_i^t \right)^{\beta} \left( \frac{\bar{P}_i - W_i^t}{\bar{P}_i} N_i^t \right)^{1-\beta}
\]

where \( 0 < \beta < 1 \) represents the relative bargaining power of the unions. We assume a utilitarian trade union; and we observe that the CES indirect utility is given by the nominal income deflated by an expected price index \( \bar{P}_i \), that all profits are distributed to the owners’ households, and that workers not finding employment in the firm under consideration are expected to receive an alternative income or reservation wage in nominal terms, \( WR_i \).

Finally, \( \bar{P}_i \) denotes the anticipated price for good \( i \); i.e. the price at which the quantity produced is expected to be sold in the market on Friday.

The determination of the anticipated price is crucial to our model specification. Central to the Dixit-Stiglitz model of monopolistic competition is that – even in equilibrium – firms do not allow for all the interdependencies. Particularly, in their price setting decision, they

\[\text{[7] Blanchard and Giavazzi (2003) specify the alternative income in real terms. Further down, in section 4, we provide a full discussion of this issue.}\]
do not take into account that their decisions influence aggregate variables or that other firms could react. They set equilibrium prices on the basis of a constant price elasticity equal to \(-\sigma\). Given the fact that even in equilibrium firms do not act on the basis of knowing fully the model, it would be inconsistent to assume this knowledge outside equilibrium. Therefore, following Maussner (1992, 1996) and Currie and Kubin (2002), we assume that out-of-equilibrium firms have the same knowledge as in equilibrium; i.e. they know that their demand function is iso-elastic and consider the elasticity to be equal to \(-\sigma\). However, they do not take into account all interdependencies. Consequently, firms have to rely on past observations of market prices and quantities to anticipate the position of their demand function. Each firm knows that it sold in the previous period a quantity \(\hat{d}_{t-1}\) at a price \(\hat{p}_{t-1}\). If the firm assumes that it can sell the same quantity at the same price as in the previous period and that sales will react to a price change according to the elasticity of \(-\sigma\), it will anticipate its demand function for the current period to be given by:

\[
\hat{d}_t^e = \frac{\hat{d}_{t-1}}{(\hat{p}_t)^\sigma} \left(\frac{\hat{K}_t}{\hat{p}_t}\right)^\sigma
\]

where \(\hat{K}_t\) is the position of the anticipated demand function.\(^8\)

Using equations (1) and (7), we obtain the anticipated price \(\hat{p}_t^e\) as

\[
\hat{p}_t^e = \left(\frac{\hat{K}_t}{\hat{N}_t}\right)^{\frac{1}{\sigma}}
\]

\(^8\) In the main body of the paper we present the model with naïve expectations, i.e. only information of the last period is used to estimate the position of the demand function; in the Appendix we study also a version with adaptive expectations which is more general and more complex but does not change the basic dynamic mechanism at work.
Returning to the bargaining problem of equation (6), we assume that the anticipated price as given in equation (8) is common knowledge to both bargaining parties. Observing that – consonant with the monopolistic competition set-up – the parties assume the expected price index $\bar{p}_i$ to be independent of their decisions, the bargaining results in:

$$ W'_i = \left( \frac{\sigma - 1 + \beta}{\sigma - 1} \right) WR_i = \left( 1 + \beta \frac{1}{\sigma - 1} \right) WR_i \quad (9) $$

$$ N'_i = K_i \left[ \left( \frac{\sigma}{\sigma - 1} \right) WR_i \right]^{-\sigma} \quad (10) $$

$$ \bar{MR} = \frac{\sigma - 1}{\sigma} \beta p_i = WR_i \quad (11) $$

Note that equations (10) and (11) are not independent results, equation (11) can be obtained from equation (10) by taking constraint (8) into account.

Note in addition the familiar result from the efficient bargaining problem, namely that a position is chosen off the profit maximum. In equation (9), the marginal revenue is equated to the nominal reservation wage and not to the marginal cost (corresponding to the nominal wage rate). It follows that the price is set as a mark-up over the reservation wage,

$$ p_i = \left( 1 + \frac{1}{\sigma - 1} \right) WR_i. $$

Following Blanchard and Giavazzi (2003), we interpret a decrease in the mark-up (i.e. an increase in $\sigma$) as goods market deregulation. It reduces the size of firm $i$’s anticipated rent per unit of output, $(\bar{p}_i - WR_i) = \left( \frac{1}{\sigma - 1} \right) WR_i$. Labour market deregulation is reflected in Blanchard and Giavazzi (2003) by a decrease in the relative bargaining power of the union, $\beta$, which reduces the workers’ share in the rent, $\beta \left( \frac{1}{\sigma - 1} \right) WR_i$ (see equation (9)). In
addition, our model, as developed in the following sections, allows for labour market deregulation in the form of changing the unemployment insurance scheme. This form of labour market deregulation may also impinge on the bargaining outcome (and on the size of the rent) through its effects on the nominal reservation wage.

During Week \( t \), at the end of contracting, firm \( i \) employs \( N_i^t \) workers, pays the nominal wage rate \( W_i^t \) and anticipates the price \( p_i^t \). Figure 1 depicts the bargaining equilibrium. Given the reservation wage \( WR_i \) and the position of the anticipated demand function \( K_i \), the bargain determines the employment level \( N_i^t \) and the wage rate \( W_i^t \) in firm \( i \). Each firm expects to sell its output at an anticipated price \( p_i^t \).

2.3 Temporary goods markets equilibrium

The characteristics of the goods market equilibrium follow from the symmetry assumption, that is, all firms behave identically. Each firm pays the same wage \( W_i^t = W_i \), hires the same number of workers, \( N_i^t = N_i \), and produces and supplies the same quantity \( x_i^t = x_i \). They envisage to sell these quantities at an identical anticipated price \( p_i^t = p_i \). At the end of the production period, on Friday, each firm supplies \( x_i \) to the (identical) true demand function \( d_i \) (see equation (5)). In contrast to the anticipated demand function, the true demand function takes into account the reactions of all other firms; and, therefore, it has an elasticity of \(-1\)

\[9\] With efficient bargaining the employment level and thus the output is determined in the bargaining process. In contrast, Maussner (1992, 1996) assumes that output levels are adjusted to the demand coming forward at the set prices. This is compatible with constant wages and with monopoly trade unions (the cases he analyses) but not with efficient bargaining. See also Jin (2001).
(instead of $-\sigma$). Producers cannot realize their price and quantity anticipations. In the following, we assume that firms sell the entire quantity produced at the market-clearing price $\hat{p}_t$ (different from the anticipated price).\textsuperscript{10} Using (5), the realized price is determined as:

$$\hat{d}_i = x_i = N_i, \quad \hat{p}_i = \frac{Y}{m\pi_i} = \frac{Y}{mN_i}$$

(12)

Figure 2 depicts the temporary goods market equilibrium.

At this point in the paper, it might be worthwhile to trace explicitly the money circuit corresponding to the market transactions described so far. Money is only used as a means of transaction and its quantity, denoted by $Q$, is assumed to be given. On Monday morning, the households in their quality of firms’ owners hold the entire quantity of money. At the conclusion of the bargaining process, firms borrow part of it to pay the workers’ wages. Taking into account the unemployment benefits, part of the quantity of money is redistributed to the unemployed. On Friday morning, the entire quantity of money is in the hand of the households in the form of wages, unemployment benefits and money holdings. On Friday, the households’ consumption demand is determined under a \textit{cash in advance} constraint. Market prices are therefore determined as $\hat{p}_i = \frac{Q}{m\pi_i} = \frac{Q}{mN_i}$; i.e. by the pure

\textsuperscript{10} In principle, producers as monopolists can also decide to sell a smaller quantity than the produced one when entering the market. However, they do not have an incentive to do so: On the basis of an elasticity of demand greater (anticipated demand) or equal to 1 (true demand) in absolute terms, revenues do not increase with a quantity restriction. In addition, at that moment, production costs are already sunk and maximizing revenues also maximizes profits. Therefore, quantity restrictions cannot increase profits. Currie and Kubin (2002) also explore an alternative quantity-rationing scheme.
quantity theory of money. Firms receive as revenue the entire quantity of money, which they redistribute totally to households as realized profits, given by the difference between revenues and wage payments, and as debt repayments corresponding to the wages paid in advance.\(^\text{11}\) Therefore, total realized nominal income \(Y\) as the sum of nominal wage payments and realized nominal profits is always equal to the quantity of money \(Q\); the *cash in advance* constraint coincides with an income constraint in the consumer maximization problem. From now on, we take the quantity of money and thus the nominal income as numéraire.

Finally, on Saturday firms and unions update their information. The position of firm \(i\)'s anticipated demand function changes on account of the realized price \(\hat{p}_t\) and the realized demand \(\hat{d}_t\). Inserting this information into equation (7) determines the position of the anticipated demand for the period \(t+1\):

\[
K_{i,t+1} = \hat{d}_t \hat{p}_t^\sigma
\]

Similarly, workers' reservation wage adjusts in the light of the realized nominal wage \(W_t\) and of the realized employment \(N_t\).

3. The dynamic system

3.1. Outline

The dynamic behaviour of the model involves two processes. First, as suggested above, the position of the anticipated demand function (and thus of the anticipated marginal revenue)

\(^{11}\) In order to keep our analysis simple, we assume no interest payments.
shifts over time. For a given nominal reservation wage and marginal revenue function, the efficient bargaining outcome, $MR = WR$, determines employment. Since all firms are identical, equation (10) can be written as:

$$ e_t = \frac{m}{L} K \left\{ \left( \frac{\sigma}{\sigma-1} \right)^{WR} \right\}^{-\sigma} \quad (14) $$

where $e_t = \frac{mN_t}{L}$ denotes the employment rate. Taking into account equations (12) and (13), it can also be written as

$$ e_t = \left( \frac{Y}{L} \right)^{\sigma} e_{t-1}^{1-\sigma} \left( \frac{\sigma}{\sigma-1} \right)^{-\sigma} (WR)^{-\sigma} \quad (15) $$

Second, the nominal reservation wage, $WR$, is also adjusted over time.

For expository purposes, in what follows we begin our study with the dynamics of the first process in isolation by assuming a nominal reservation wage invariant over time. After that we specify the adjustment of the reservation wage explicitly and explore the dynamic properties of the full system.

3.2. Goods market dynamics in isolation

If we assume a reservation wage fixed at an arbitrarily chosen level, $WR = \bar{WR}$, the implied dynamic process is

$$ e_t = \left( \frac{Y}{L} \right)^{\sigma} e_{t-1}^{1-\sigma} \left( \frac{\sigma}{\sigma-1} \right)^{-\sigma} (\bar{WR})^{-\sigma} \quad (16) $$

Equation (16) is a one-dimensional first-return map with the following fixed point and first derivative
\[ \bar{c} = \left( \frac{Y}{L} \right) \left( \frac{\sigma}{\sigma-1} \right)^{-1} \left( \overline{WR} \right)^{-1} \]  

(17)

\[ \frac{\partial e_t}{\partial e_{t-1}} = \left( \frac{Y}{L} \right)^{\sigma} \left( \frac{\sigma}{\sigma-1} \right)^{-\sigma} (1-\sigma) e_t^{\sigma} e_{t-1}^{1-\sigma} = 1-\sigma \]  

(18)

As long as \( \sigma > 1 \), the fixed point increases with \( \sigma \) but loses stability at \( \sigma = 2 \). Equation (18) shows that the derivative of the first return map is negative for all values of the employment rate; therefore, the time path diverges for \( \sigma > 2 \). Deregulating the goods market increases stationary employment but may eventually lead to instability.

Figure 3 illustrates the period 2 cycle occurring precisely at \( \sigma = 2 \). The employment rate alternates between \( e_1 \) and \( e_2 \); the anticipated demand and anticipated marginal revenue functions shift accordingly. Starting with the anticipated demand function 1 and the corresponding anticipated marginal revenue function 1 the bargaining process results in the higher employment rate \( e_1 \). The realized market price is below the anticipated one. The anticipated demand and marginal revenue functions shift downward to the position 2. The next bargaining results in the lower employment rate \( e_2 \). The market price is now above the anticipated one inducing an upward shift of the anticipated demand and marginal revenue back to their respective position 1.

3.3. The nominal reservation wage as a function of the employment rate

Next, we introduce a nominal reservation wage that depends positively on the employment rate, \( WR_t = WR(e_{t-1}) \): A higher employment rate is considered to increase the expected employment probabilities outside the firm under consideration and thus to increase the reservation wage. As a consequence, the adjustment of the nominal reservation wage affects the goods market dynamics. The resulting dynamic system is still one-dimensional:
The fixed point is

$$
\bar{e} = \left( \frac{Y}{L} \right) \left( \frac{\sigma}{\sigma - 1} \right)^{-1} (WR(\bar{e}))^{-1}
$$

Note that rising $\sigma$ still increases the equilibrium employment rate

$$
\frac{\partial \bar{e}}{\partial \sigma} = \frac{\bar{e}}{\sigma(\sigma - 1)} \left( 1 + \frac{\bar{e}}{WR} \frac{\partial WR}{\partial \bar{e}} \right) > 0
$$

The first derivative of (19) is

$$
\frac{\partial e_t}{\partial e_{t-1}} = \left( \frac{Y}{L} \right)^{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} e_t^{1-\sigma} \left( 1 - \sigma - \sigma \frac{e_{t-1}}{WR} \frac{\partial WR}{\partial e_{t-1}} \right) = 1 - \sigma - \sigma \frac{\bar{e}}{WR} \frac{\partial WR}{\partial e_{t-1}}
$$

The fixed point now loses stability at some $\sigma < 2$; beyond that value the time path diverges. Therefore, a positive influence of the employment rate on the nominal reservation wage destabilizes the economy.

Figure 3 also illustrates this destabilizing effect. Start from the market result 1 and consider the determination of position 2. The anticipated demand and marginal revenue functions shift again towards their position 2; but now, in addition, the nominal reservation wage is increased because of the high realized employment rate $e_1$. Therefore, the new employment rate 2 will be below $e_2$ in Figure 3. The increase in the nominal reservation wage thus increases the fluctuations. The process is destabilized.

### 3.4. The nominal reservation wage as a function of the employment rate and of its lagged value

Finally we allow for a positive influence of the lagged reservation wage upon its current value:
For the moment, we assume the function \(WR(g)\) to be monotonically increasing in both arguments. The rationale for the dependence of the nominal reservation wage on the employment rate is as sketched above; for the second one it runs along the following lines: a high reservation wage in \(t-1\) will result in a high bargained nominal wage rate in \(t-1\), which in turn is expected to raise the nominal reservation wage in \(t\).

The dynamic system is now two-dimensional and given by equations (15) and (22).

Figure 3 can be used to illustrate that the additional effect introduced by equation (22) is potentially stabilizing. Start again with the market position realized in period 1 and consider the determination of the position 2. The anticipated demand and marginal revenue functions shift to their respective position 2. Two factors change the nominal reservation wage. As in the previous case, the high employment rate \(e_1\) tends to increase it. At the same time, the high employment rate \(e_1\) implies that the bargained nominal wage in period 1 was comparatively low. This would reduce the nominal reservation wage, thus introducing a stabilizing element.

Without specifying explicitly the dynamic adjustment process for the nominal reservation wage, it is difficult to assess the stability properties. We provide such a specification in the following section and explore the dynamics of the full model.

4. Fully specified dynamics and numerical simulations

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\(WR_t = WR(e_{t-1}, WR_{t-1})\) (22)

12 But see below the discussion in section 5.
We model the reservation wage as the income expected by the trade union for members who do not find employment in the firm under consideration (see Layard et al., 1991). It therefore depends on the expected probability of finding employment in other firms, on the wage rate expected to be paid by other firms and on the unemployment benefit $B_t$.\textsuperscript{13}

Consonant with the monopolistic competition set up, we assume that the trade unions do not take into consideration reactions of other firms and the impact of their own decisions on the aggregate variables. The expected probability of finding employment in other firms and the expected wage rate outside the firm under consideration is therefore given by the respective values realized in the previous period.\textsuperscript{14}

\[
WR_t = e_{t-1} W_{t-1} + (1 - e_{t-1}) B_t
\]  

\textsuperscript{13} We did not incorporate explicitly the financing of the unemployment benefit. However, we studied the case of financing it out of a general labour income tax that applies both to workers and unemployed (similar to Calmfors and Johansson, 2001): Equation (6) would be modified to

\[
\left[1 - \rho \right] \left( \frac{W^i_t - WR_t}{R_t^0 - N_t^0} \right)^{\beta} \left( \frac{p_t^0 - W^i_t}{p_t^0 - N_t^0} \right)^{1-\beta}
\]

where $\rho$ denotes the anticipated tax rate, with $\rho = \tau_{t-1}$. On Tuesday, the tax rate is adjusted to match the benefit payments required by the realized unemployment:

$\tau_t \sum_i W^i_t N^i_t = \tau_t W_t e_t L = (1 - \tau_t) B_t (1 - e_t) L$ and money holdings are redistributed accordingly. This extension does not change the bargaining results and the following analysis.

\textsuperscript{14} Equation (18) involves trade unions anticipating a nominal reservation wage on the basis of naïve expectations. In the Appendix, we explore also the case of adaptive expectations.
The unemployment benefit is determined by applying a fixed replacement ratio $\phi$ to the nominal wage rate earned in the lost job, $B_t = \phi W_{t-1}$.

$$WR_t = e_{t-1}W_{t-1} + (1-e_{t-1})\phi W_{t-1}$$  \hfill (24)

This formulation is more explicit than the one provided by Blanchard and Giavazzi (2003); in contrast to their specification in real terms, it comes more natural in nominal terms.\textsuperscript{15}

Using equation (9) the system (15) and (24) can be written as

$$e_t = \left(\frac{Y_t}{L_t}\right)\sigma^\frac{1-\sigma}{\sigma-1} \left\{ \left(\frac{\sigma}{\sigma-1}\right)WR_t \right\}^{-\sigma}$$  \hfill (25)

$$WR_t = e_{t-1}\left(\frac{\sigma-1+\beta}{\sigma-1}\right)WR_{t-1} + (1-e_{t-1})\phi \left(\frac{\sigma-1+\beta}{\sigma-1}\right)WR_{t-1}$$  \hfill (26)

Appendix 1 investigates analytically the properties of the dynamic system (25)-(26).

The fixed point solutions are given by

\textsuperscript{15} Note that our instability result does not depend upon this difference in specification. With a real reservation rate $w_r$, the bargaining problem in equation (6) is modified to

$$\left(\frac{W'_t}{p'_t} - w_r\right)N'_t \left(\frac{\beta p'_t-W'_t}{\beta p'_t} N'_t\right)^{1-\beta}$$

and the results in equation (11) to $\frac{\sigma-1}{\sigma} \frac{\beta p'_t}{p'_t} = w_r$. Blanchard and Giavazzi (2003) only allow for a dependence of the reservation wage upon the (un)-employment rate. The central dynamic equation therefore is

$$e_t = e_{t-1}\left[\frac{\beta p'_t}{p'_t}\right]^\sigma = e_{t-1}\left[\frac{\sigma}{\sigma-1} wr(e_{t-1})\right]^\sigma$$

and the derivative at the fixed point is given by $\frac{\partial e_t}{\partial e_{t-1}} = 1 - \sigma \frac{\partial wr}{wr \partial e_{t-1}}$. Thus, also in this case the fixed point loses stability for sufficiently high values of $\sigma$. 

They exhibit the usual comparative static properties: goods and labour market deregulation – as reflected in a higher value of $\sigma$ and in lower values of $\beta$ and $\phi$, respectively – engender a higher employment rate. The analysis further shows that the fixed point is stable only for low values of $\sigma$ and for high values of $\beta$ and $\phi$. However, in contrast to the one-dimensional models studied previously, the stability loss now occurs through a Flip bifurcation giving rise to attracting period-two cycles. Therefore, goods and labour markets deregulation – though increasing the stationary employment rate – may destabilize the economy, but it does not lead to diverging time paths. Appendix 1 also shows that adaptive expectations stabilize the system but the main conclusions still carry over: stability is lost through a Flip bifurcation; it occurs at higher degrees of deregulation and the engendered fluctuations are less pronounced.

So far, we have explored analytically the dynamic properties of the system (25)-(26). We now turn to a small calibration exercise. Note that the fixed point solutions only depend upon three parameters: the parameter reflecting the degree of monopoly power in the goods market, $\sigma$, and the two parameters related to the extent of labour market regulation, namely the bargaining power of trade unions $\beta$ and the replacement ratio $\phi$. These parameters are subject to the following boundaries: $1 < \sigma$, $0 < \beta < 1$ and $0 < \phi < \frac{\sigma - 1}{\sigma - 1 - \beta}$ - the latter ensuring a positive employment rate at the fixed point. Choosing an upper limit for $\sigma$ therefore allows studying the entire parameter space numerically. In our exercise, we assumed $\sigma \leq 30$ (implying a mark-up larger than 0.034).

We searched for numerical values of $\beta$, $\phi$ and $\sigma$ such that the fixed point approximates two macroeconomic stylized facts; namely that in 1990s Continental Europe the employment
rate typically assumes values above 0.8 and that the wage share typically assumes values slightly below 0.6. In addition, we ask which dynamic properties the time path exhibits close to the fixed point. Our numerical exploration shows that parameter constellations exist which result in plausible fixed points, which are stable for highly regulated markets and which lose stability with goods or labour market deregulation. With naïve expectations, fluctuations are typically strong and the employment rate hits quickly its upper boundary of one; with adaptive expectations, cyclical time paths exist with comparatively small amplitudes, hitting no boundary condition.

5. An alternative institutional set-up

In this section, we consider a different specification for the unemployment benefit mirroring an institutional set-up close to a social assistance scheme (see e.g. Layard et al., 1991; and Pissarides, 1998). According to such specification, the compensation for the unemployed is fixed in real terms (corresponding to a certain CES utility level \( \omega \)) and the nominal payment is adjusted each period to the price index, \( B_t = \omega P_{t-1} \). The unemployment benefit systems in the OECD countries are found in between this alternative specification and the one considered in the previous section where a fraction of the past period earned wage was assigned to the unemployed (see Goerke, 2000).

The dynamic system now is

\[ B_t = \omega P_{t-1} \]

\(^{16}\) Taking the average over a significant group of European Countries, Giammarioli et al. (2002) estimate that in the 1990s the labour share was 0.586 in the Business sector, 0.58 in Industry and 0.536 in the Tradable Services.
A crucial feature of this system is that the realized market position impacts on the dynamics both through shifts of the demand function, implicit in equation (28), and through the determination of the nominal reservation wage, as shown in equation (29). The analytical structure of this specification is more complex than the one presented in the previous section, in which the realized market position was affecting the dynamics only through shifts of the demand function. Note that – in contrast to the assumption in the previous sections – the partial derivative need not be positive since with the assumed specification a high employment rate in period \( t-1 \) not only means a high expected re-employment probability in period \( t \) but also – via a low realized price in \( t-1 \) – a low nominal unemployment benefit in \( t \). The latter indirect effect of the employment rate on the nominal reservation wage was not present in the previous sections.

The fixed point corresponding to the system (28) and (29) is given by

\[
\bar{e} = \left( \frac{Y}{L} \right)^{\sigma} \left( e_{t-1} \right)^{1-\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{WR_{t}} \]

\[
WR_{t} = e_{t-1} \left( \frac{\sigma - 1 + \beta}{\sigma - 1} \right) WR_{t-1} + (1 - e_{t-1}) \frac{Y}{Le_{t-1}} - \omega
\]

The major conclusions from the previous case carry over: As is shown in Appendix 2, deregulation in the goods market – as reflected in a higher value of \( \sigma \) – or in the labour market – as reflected in lower values of \( \beta \) and \( \omega \) – increases the stationary employment rate, but may eventually lead to a Flip bifurcation of the fixed point giving rise to attracting period two cycles. Therefore, the basic trade off remains present: Deregulation increases employment rates but may destabilize the economy. Appendix 2 also shows that adaptive
expectations reduce local instability as the Flip bifurcation occurs at higher degrees of deregulation. However, this case is more complex than the previous one: Too strong regulation may lead to instability via a Hopf bifurcation of the fixed point which gives rise to an attracting circle. Figure 4a illustrates this point by means of a bifurcation diagram, which plots the changes in the attractor (i.e., the resting position) of the system as the parameter \( \sigma \) is varied, keeping the other parameters constant: we assume \( 3.63 \leq \sigma \leq 6.43 \), \( \beta = 0.35 \), \( \omega = 0.67 \); in addition, we assume naïve expectations for trade unions and adaptive expectations for firms, with \( \alpha = 0.35 \) representing the speeds at which firms revise their anticipations on the position of the demand function. Below \( \sigma_{\text{Hopf}} \), deregulating the goods market stabilises the system. For \( \sigma_{\text{Hopf}} < \sigma < \sigma_{\text{Flip}} \) the system converges to the fixed point. However, additional deregulation may destabilise the fixed point giving rise to a stable period-two cycle, as \( \sigma \) passes through \( \sigma_{\text{Flip}} \), or to a more complex dynamics, as \( \sigma \) is further increased above \( \sigma_{\text{Flip}} \), where the system undergoes a Hopf bifurcation of the second iterate: the period two cycle loses stability and another stable attractor is created that consists of two circles. Figures 4b and 4c consider specifically the latter case for \( \sigma = 6.35 > \sigma_{\text{Flip}} \): Figure 4b shows the two-part attractor and figure 4c plots a time path for the employment rate. Note that it exhibits a pattern much closer to the typical features of a business cycle than the period-two cycle after the Flip bifurcation.
6. Conclusion

In this paper, we have analyzed the dynamics\textsuperscript{17} of a model that follows closely the prototype specification put forward by Blanchard and Giavazzi (2003) combining monopolistic competition in the goods market and efficient bargaining in the labour market. The dynamics results from two sources: First, inherent to monopolistic competition is the assumption that even in equilibrium firms do not act upon knowing the full model. In particular, each supplier sets prices according to a partial demand price elasticity (which is higher than the value that takes into account all interdependencies). Assuming the same limitations to knowledge outside equilibrium, each firm will adjust the position of the anticipated demand function according to previous market realizations. Second, bargaining in the labour market is based on a reservation wage reflecting the expected income outside the firm under consideration. This anticipation is also adjusted in the light of realized market results.

While the bargaining process may be specified in various forms, the goods market dynamics is directly implied by the mechanism at the core of the monopolistic competition model. It is difficult to imagine fundamentally different specifications without leaving the assumed market structure. We showed in the paper, that the first process destabilizes the economy: Deregulation does increase the stationary employment rate, but engenders diverging time paths. Introducing various plausible specifications for the adjustment process of the reservation wage dampens this sharp result: The stability loss occurs through a Flip bifurcation giving rise not to diverging time paths but to attracting period-two cycles

\textsuperscript{17} We deliberately focused on the transition to the short run general equilibrium. As Currie and Kubin (2002) showed the transition to a long run general equilibrium does not mitigate the short run instability properties.
and eventually to complex time paths. However, the basic trade-off remains: Deregulation, while increasing the stationary employment, may lead to instability.

References:


Appendix 1

In this appendix we examine some of the properties of the dynamic system related to the case in which the employment benefit is a proportion of the nominal wage rate earned in the lost job, $B_t = \phi W_{t-1}$. Assuming adaptive expectations, the dynamic system is

$$K_t = (1 - \alpha) K_{t-1} + \alpha \frac{L}{m} \left( \frac{Y}{L} \right)^{\sigma} (e_{t-1})^{1-\sigma} \quad (A1.1)$$

$$W_{R_t} = (1 - \gamma) W_{R_{t-1}} + \gamma \left[ e_{t-1} \left( \frac{\sigma - 1 + \beta}{\sigma - 1} \right) W_{R_{t-1}} + (1 - e_{t-1}) \phi \left( \frac{\sigma - 1 + \beta}{\sigma - 1} \right) W_{R_{t-1}} \right] \quad (A1.2)$$

According to equation (A1.1) firms update their information over the position of the demand function on the basis of the realised market position at the speed measured by the parameter $\alpha$, where $0 \leq \alpha \leq 1$. Similarly, equation (A1.2) clarifies that trade unions update their expectations on the reservation wage on the basis of the bargaining outcome at the speed measured by the parameter $\gamma$.

Using $e_{t-1} = \frac{m}{L} K_{t-1} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} (W_{R_{t-1}})^{-\sigma}$ the system is two dimensional in $K_t$ and $W_{R_t}$.

For naïve expectations, i.e. for $\alpha = \gamma = 1$, the system reduces to the system as given in (25) and (26):

$$K_t = \frac{L}{m} \left( \frac{Y}{L} \right)^{\sigma} (e_{t-1})^{1-\sigma} \quad \text{or} \quad e_t = \left( \frac{Y}{L} \right)^{\sigma} (e_{t-1})^{1-\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} \quad (A1.3)$$

$$W_{R_t} = e_{t-1} \left( \frac{\sigma - 1 + \beta}{\sigma - 1} \right) W_{R_{t-1}} + (1 - e_{t-1}) \phi \left( \frac{\sigma - 1 + \beta}{\sigma - 1} \right) W_{R_{t-1}} \quad (A1.4)$$

**A1.1 Fixed point and comparative statics**

From (A1.1) and (A1.2), we solve for the stationary values of the employment rate, the position of the demand function and the reservation wage
The stationary price is $\bar{p} = \frac{Y}{\bar{v}L}$ and the stationary nominal wage is $\bar{W} = \frac{\sigma - 1 + \beta}{\sigma - 1} \bar{W}_R$. It follows that the stationary real wage and the stationary real reservation wage are given by:

$$\bar{w} = \frac{\sigma - 1 + \beta}{\sigma} \quad \bar{w}_R = \frac{\sigma - 1}{\sigma}$$ \hspace{1cm} (A1.9)

From (A1.5) to (A1.9), the stationary state solutions depend on $\phi$, $\beta$ and $\sigma$. The comparative statics with respect to $\sigma$ involves:

$$\frac{\partial \bar{v}}{\partial \sigma} = \frac{\beta}{(\sigma - 1 + \beta)^2 (1 - \phi)} > 0$$ \hspace{1cm} (A1.10)

$$\frac{\partial \bar{W}_R}{\partial \sigma} = \frac{1}{(\sigma \bar{v})^2} \frac{Y}{L} \left( \bar{v} - \sigma (\sigma - 1) \frac{\partial \bar{v}}{\partial \sigma} \right) \quad \frac{\partial \bar{w}}{\partial \sigma} = \frac{1 - \beta}{\sigma^2} > 0 \quad \frac{\partial \bar{w}_R}{\partial \sigma} = \frac{1}{\sigma^2} > 0$$ \hspace{1cm} (A1.11)

with $\frac{\partial \bar{W}_R}{\partial \sigma} \geq (\leq) 0$ for $\frac{\partial \bar{v}}{\partial \sigma} \leq (> \frac{\bar{v}}{\sigma (\sigma - 1)}$

$\beta$ impacts on $\bar{v}$, $\bar{W}_R$ and $\bar{w}$ in the following way:

$$\frac{\partial \bar{v}}{\partial \beta} = -\frac{\sigma - 1}{(\sigma - 1 + \beta)^2 (1 - \phi)} < 0$$ \hspace{1cm} (A1.12)

$$\frac{\partial \bar{W}_R}{\partial \beta} = -\left( \frac{\sigma - 1}{\sigma} \right) \frac{Y}{L \bar{v}^2} \frac{\partial \bar{v}}{\partial \beta} > 0 \quad \frac{\partial \bar{w}}{\partial \beta} = \frac{1}{\sigma} > 0$$ \hspace{1cm} (A1.13)
\( \phi \) impacts on \( \overline{\varepsilon} \) and \( \overline{WR} \) in the following way

\[
\frac{\partial \overline{\varepsilon}}{\partial \phi} = -\frac{\beta}{(\sigma-1+\beta)(1-\phi)} < 0 \tag{A1.14}
\]

\[
\frac{\partial \overline{WR}}{\partial \phi} = \left(\frac{\sigma-1}{\sigma}\right) Y \frac{\partial \overline{\varepsilon}}{\partial \phi} > 0 \tag{A1.15}
\]

### A1.2 Bifurcation analysis

The Jacobian evaluated at the fixed point is

\[
J_E = \begin{bmatrix}
1-\alpha \sigma & \alpha \sigma^2 \frac{\overline{\varepsilon} L}{m} \left(\frac{Y}{\overline{\varepsilon}}\right)^{\sigma-1} \\
\gamma \frac{(\sigma-1)(1-\phi)-\phi \beta}{\sigma} \frac{m}{\overline{\varepsilon}L} \left(\frac{Y}{\overline{\varepsilon}}\right)^{-\sigma} & 1-\sigma \frac{(\sigma-1)(1-\phi)-\phi \beta}{\sigma-1}
\end{bmatrix} \tag{A1.16}
\]

Its Determinant and Trace are:

\[
det J_E = 1-\alpha \sigma - \gamma \sigma (1-\alpha) \frac{(\sigma-1)(1-\phi)-\phi \beta}{\sigma-1} \tag{A1.17}
\]

\[
tr J_E = 2-\sigma \sigma - \gamma \sigma (1-\phi)-\phi \beta \frac{(\sigma-1)(1-\phi)-\phi \beta}{\sigma-1} \tag{A1.18}
\]

with \( -\infty < det J_E < 0 \).

The violation of one of the following conditions would lead to instability for the system (in brackets the type of bifurcation involved):

(i) \( 1-\det J_E = \alpha \sigma + \gamma \sigma (1-\alpha) \frac{(\sigma-1)(1-\phi)-\phi \beta}{\sigma-1} > 0 \) (Hopf bifurcation);

(ii) \( 1-\tr J_E + \det J_E = \alpha \sigma \frac{\sigma}{\sigma-1} \left[ (\sigma-1)(1-\phi)-\phi \beta \right] > 0 \) (Saddle node bifurcation);
(iii) \[ 1 + \text{tr} \, J_e + \det J_e = 2(2 - a \sigma) - (2 - \alpha) \gamma \frac{\sigma}{\sigma - 1} [(\sigma - 1)(1 - \phi) - \phi \beta] > 0 \]

(Flip bifurcation).

Conditions (i) and (ii) always hold as long as condition (A1.8) holds. The Flip bifurcation is the only possible.

We first study the case of naive expectations in which \( \alpha = \gamma = 1 \). Condition (iii) then reduces to

\[ 2(2 - \sigma) - \frac{\sigma}{\sigma - 1} [(\sigma - 1)(1 - \phi) - \phi \beta] > 0 \quad \text{(A1.19)} \]

Note that condition (A1.8) and (A1.19) can only hold simultaneously for \( 1 < \sigma < 2 \), which therefore is a necessary condition for a positive and stable fixed point employment rate. If condition (A1.19) is not satisfied, the system loses stability through a Flip bifurcation.

Figure 5, upper panel, visualizes condition (A1.8) and (A1.19), i.e. the parameter set for which the fixed point employment is positive and stable, for \( \beta = 0.35 \).

Analytically, the parameter set has to satisfy the following conditions (\( \alpha^{\text{Pos}} \) denotes the parameter value for which condition (A1.8) holds with an equality sign, \( \alpha^{\text{Flip}} \) denotes the parameter value for which condition (A1.19) holds with an equality sign):

\[ \beta^{\text{Flip}} = (\sigma - 1) \frac{1 - \phi}{\phi} - 2 \frac{(2 - \sigma)(\sigma - 1)}{\phi \sigma} < \beta < (\sigma - 1) \frac{1 - \phi}{\phi} = \beta^{\text{Pos}} \quad \text{(A1.20)} \]

\[ \phi^{\text{Flip}} = \frac{\sigma - 1}{\sigma - 1 + \beta} - 2 \frac{(2 - \sigma)(\sigma - 1)}{\sigma(\sigma - 1 + \beta)} < \phi < \frac{\sigma - 1}{\sigma - 1 + \beta} = \phi^{\text{Pos}} < 1 \quad \text{(A1.21)} \]

\[ \sigma^{\text{Flip}} = 1 + \frac{1 + \phi + \phi \beta}{2(3 - \phi)} + \sqrt{\left[ 1 + \frac{1 + \phi + \phi \beta}{2(3 - \phi)} \right]^{2} + \frac{\phi \beta}{3 - \phi}} > 1 + \frac{\phi \beta}{1 - \phi} = \sigma^{\text{Pos}} \quad \text{(A1.22)} \]
Therefore, deregulating labour and goods markets (i.e reducing $\beta$ or $\phi$ or increasing $\sigma$) eventually leads to a Flip bifurcation.

For the general case of adaptive expectations, i.e. $0 < \alpha < 1$ and $0 < \gamma < 1$, it can be shown that

\[
\det J_{J} (0 < \alpha < 1; 0 < \gamma < 1) < \det J_{J} (\alpha = \gamma = 1) \tag{A1.23}
\]

\[
\frac{\partial \det J_{J} (0 < \alpha < 1; 0 < \gamma < 1)}{\partial \alpha} < 0 \quad \frac{\partial \det J_{J} (0 < \alpha < 1; 0 < \gamma < 1)}{\partial \gamma} < 0 \tag{A1.24}
\]

\[
\text{tr} J_{J} (0 < \alpha < 1; 0 < \gamma < 1) < \text{tr} J_{J} (\alpha = \gamma = 1) \tag{A1.25}
\]

\[
\frac{\partial \text{tr} J_{J} (0 < \alpha < 1; 0 < \gamma < 1)}{\partial \alpha} < 0 \quad \frac{\partial \text{tr} J_{J} (0 < \alpha < 1; 0 < \gamma < 1)}{\partial \gamma} < 0 \tag{A1.26}
\]

Therefore, adaptive expectations stabilize the system.
Appendix 2

In this appendix we study the dynamic system related to the case in which the employment benefit is fixed in real terms and its nominal counterpart is adjusted to the realized price, \( B_t = \omega P_{t-1} \). The dynamic system is

\[
K_t = (1-\alpha)K_{t-1} + \alpha \frac{L}{Y} \left( \frac{Y}{m \cdot L} \right) (e_{t-1})^{1-\sigma} \quad (A2.1)
\]

\[
WR_t = (1-\gamma)WR_{t-1} + \gamma \left( \frac{\sigma - 1 + \beta}{\sigma - 1} e_{t-1}WR_{t-1} + \frac{1-e_{t-1}}{e_{t-1}} \frac{Y}{L} \right) \quad (A2.2)
\]

Using \( e_{t-1} = \frac{m}{L} K_{t-1} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} (WR_{t-1})^{-\sigma} \) the system is two dimensional in \( K_t \) and \( WR_t \). For naïve expectations, i.e. for \( \alpha = \gamma = 1 \), the system reduces to the system as given in (28) and (29)

\[
K_t = \frac{L}{m} \left( \frac{Y}{L} \right)^{\sigma} (e_{t-1})^{1-\sigma} \quad \text{or} \quad e_t = \left( \frac{Y}{L} \right)^{\sigma} (e_{t-1})^{1-\sigma} \left( \frac{\sigma}{\sigma - 1} \frac{WR_{t-1}}{WR_{t-1}} \right)^{-\sigma}
\]

\[
WR_t = \frac{\sigma - 1 + \beta}{\sigma - 1} e_{t-1}WR_{t-1} + \frac{1-e_{t-1}}{e_{t-1}} \frac{Y}{L} \omega \quad (A2.3)
\]

A2.1 Fixed point and comparative statics

Apart from the employment rate, the stationary values for all other variables correspond to those of the previous model; also the partial derivatives with respect to \( \sigma \) and \( \beta \) are as before. The stationary employment rate is given by

\[
\bar{e} = \frac{\sigma - 1 - \omega \sigma}{\sigma - 1 + \beta - \omega \sigma} \quad (A2.4)
\]

For further convenience note
\[
\overline{\varepsilon} = \frac{\omega - \omega_0}{\overline{w} - \omega_0}
\] (A2.5)

If \( \omega < \omega_0 \) and \( \beta > 0 \), then \( \omega < \overline{w} \) and \( 0 < \overline{\varepsilon} < 1 \).

We have

\[
\frac{\partial \overline{\varepsilon}}{\partial \sigma} = \frac{\beta \left[ \omega (1 - \beta) + \sigma (\overline{w} - \omega) \right]}{\overline{w} \left[ \sigma (\overline{w} - \omega) \right]^2} > 0
\] (A2.6)

and

\[
\frac{\partial \overline{\varepsilon}}{\partial \beta} = -\frac{\omega - \omega_0}{\sigma (\overline{w} - \omega)^2} < 0
\] (A2.7)

The impact of changes in \( \omega \) is given by

\[
\frac{\partial \overline{\varepsilon}}{\partial \omega} = -\frac{\beta}{\sigma (\overline{w} - \omega)^2} < 0 \quad \frac{\partial \overline{WR}}{\partial \omega} = -\left( \frac{\sigma - 1}{\sigma} \right) \frac{Y}{\overline{L}e^2} \frac{\partial \overline{\varepsilon}}{\partial \omega} > 0
\] (A2.8)

**A1.2 Bifurcation analysis**

The Jacobian evaluated at the fixed point is

\[
J_{E} = \begin{bmatrix}
1 - \alpha \sigma & \alpha \sigma^2 \left( \frac{Y}{\overline{L}e} \right)^{\sigma - 1} \left( 1 - \gamma \gamma \left( \sigma + \beta - 1 \right) \overline{\varepsilon} - \frac{\sigma^2 \omega}{\sigma - 1} \right)
\end{bmatrix}
\] (A2.9)

Its Determinant and Trace are:

\[
\det J_{E} = 1 - \alpha \sigma - \gamma - \gamma \left( \sigma - 1 + \beta \right) \overline{\varepsilon} - \frac{\sigma^2 \omega}{\sigma - 1} + \gamma \alpha \sigma \left( 1 - \frac{\sigma \omega}{\sigma - 1} \right)
\] (A2.10)

\[
\text{tr} J_{E} = 2 - \alpha \sigma - \gamma \gamma \left( \sigma - 1 + \beta \right) \overline{\varepsilon} - \frac{\sigma^2 \omega}{\sigma - 1}
\] (A2.11)
The violation of one of the following conditions would involve instability for the system (in brackets the type of bifurcation involved):

(i) \[ 1 - \text{det} J_{E} = \alpha (1 - \gamma) \sigma + \gamma (\sigma - 1 + \beta) \bar{\epsilon} - \gamma (1 - \alpha) \frac{\sigma \omega}{\sigma - 1} > 0 \] (Hopf bifurcation);

(ii) \[ 1 - \text{tr} J_{E} + \text{det} J_{E} = \gamma \alpha \sigma \left( 1 - \frac{\sigma \omega}{\sigma - 1} \right) = \gamma \alpha \sigma \left( 1 - \frac{\omega}{w r} \right) > 0 \] (Saddle node bifurcation);

(iii) \[ 1 + \text{tr} J_{E} + \text{det} J_{E} = 4 - 2 \alpha \sigma - 2 \gamma \left( \sigma - 1 + \beta \right) \bar{\epsilon} - \gamma (1 - \alpha) \frac{\sigma \omega}{\sigma - 1} > 0 \] (Flip bifurcation).

Conditions (ii) always holds as long as and \( w r > \omega \).

In the case of naïve expectations \( \alpha = \gamma = 1 \) (i) and (iii) reduce to

(i) \[ 1 - \text{det} J_{E} = 1 + (\sigma - 1 + \beta) \bar{\epsilon} > 0 \] (Hopf bifurcation);

(iii) \[ 1 + \text{tr} J_{E} + \text{det} J_{E} = 2 - 2 \left[ (\sigma - 1 + \beta) \bar{\epsilon} \right] - \sigma \left( 1 - \frac{\omega}{w r} \right) > 0 \] (Flip bifurcation)

In the case of naïve expectations, condition (i) always holds; the Flip bifurcation is the only possible. Condition (iii) corresponds to

\[
\frac{\left[ (3 \sigma - 2) \omega + (4 - 3 \sigma) w r \right] \beta \bar{w r} - (\sigma - 1) (\bar{w r} - \omega) \left[ (3 \sigma - 4) \bar{w r} - \sigma \omega \right]}{\left( \sigma - 1 \right) (\bar{w r} - \omega) + \beta \bar{w r}} > 0 \quad \text{(A2.12)}
\]

In this condition for stability, the denominator is always positive. Therefore, the system is stable for

\[
\left[ (3 \sigma - 2) \omega + (4 - 3 \sigma) w r \right] \beta \bar{w r} > (\sigma - 1) (\bar{w r} - \omega) \left[ (3 \sigma - 4) \bar{w r} - \sigma \omega \right]
\]

We may distinguish three cases depending on \( \sigma \).
Case 1: $\sigma < \frac{4}{3}$. The stability condition is satisfied for all other parameter values:

Case 2: $\frac{4}{3} < \sigma < 2$. The stability of the system depends upon the parameter values. There are three sub-cases that we have to take into account depending on $\omega$

Case 2a: $\omega < \frac{3\sigma - 4}{3\sigma - 2} \frac{\omega r}{\sigma}$. It follows $(3\sigma - 4)\frac{\omega r}{\sigma} > (3\sigma - 2)\omega > 2\sigma \omega$. In this case, the stability condition is never satisfied.

Case 2b: $\frac{3\sigma - 4}{3\sigma - 2} \frac{\omega r}{\sigma} < \omega < \frac{3\sigma - 4}{\sigma} \frac{\omega r}{\sigma}$. The system is stable for

$$\beta > \beta^{flip} = \frac{(\sigma - 1)\left(\frac{\omega r}{\sigma} - \omega\right)\left[(3\sigma - 4)\frac{\omega r}{\sigma} - \sigma \omega\right]}{(3\sigma - 2)\omega + (4 - 3\sigma)\frac{\omega r}{\sigma}} > 0$$ (A2.13)

If $\beta$ falls below this value, the system loses stability through a Flip bifurcation.

Case 2c: $\frac{3\sigma - 4}{3\sigma - 2} \frac{\omega r}{\sigma} < \omega < \frac{3\sigma - 4}{\sigma} \frac{\omega r}{\sigma}$. The system is stable for

$$\beta > 0 \geq \beta^{flip} = \frac{(\sigma - 1)\left(\frac{\omega r}{\sigma} - \omega\right)\left[(3\sigma - 4)\frac{\omega r}{\sigma} - \sigma \omega\right]}{(3\sigma - 2)\omega + (4 - 3\sigma)\frac{\omega r}{\sigma}}$$

That is, the system is always stable.

Case 3: $2 < \sigma$. As for Case 2, the stability of the system depends upon the parameter values. Depending on $\omega$ we may identify two sub-cases:

Case 3a: $\omega < \frac{3\sigma - 4}{3\sigma - 2} \frac{\omega r}{\sigma}$. As in Case 2a, the stability condition is never satisfied.

Case 3b: $\frac{3\sigma - 4}{3\sigma - 2} \frac{\omega r}{\sigma} < \omega < \frac{3\sigma - 4}{\sigma} \frac{\omega r}{\sigma}$. As in Case 2b, the system is stable for
\[
\beta > \beta^{\text{flip}} = \frac{(\sigma - 1)(\text{wr} - \omega)[(3\sigma - 4\text{wr} - \sigma \omega)]}{(3\sigma - 2\omega + (4 - 3\sigma)\text{wr})}\text{wr} > 0
\] (A2.14)

If \( \beta \) falls below this value, the system loses stability through a Flip bifurcation. Figure 5, lower panel, visualizes the stability conditions for \( \beta = 0.35 \).

Not much can be said for the general case \( 0 < \alpha < 1 \) and \( 0 < \gamma < 1 \). The only clear-cut results are

\[
\frac{\partial \det J_{\mu}}{\partial \alpha} \left( 0 < \alpha < 1; 0 < \gamma < 1 \right) < 0 \quad \text{and} \quad \frac{\partial \text{tr} J_{\mu}}{\partial \alpha} \left( 0 < \alpha < 1; 0 < \gamma < 1 \right) < 0
\] (A2.15)

Therefore, reducing \( \alpha \) reduces the likelihood of a Flip bifurcation; in this sense it stabilises the system. However, in the general case, a Hopf bifurcation, which adds a destabilising moment to the model, is not excluded.

Numerical investigations confirm these results.
Figure 1: Bargaining equilibrium Week $t$
Figure 2: Short run commodity market equilibrium Week $t$
Figure 3: Period-two Cycle
Figure 5: Stability Regions