

**LONG-RUN MONETARY NON-NEUTRALITY**  
***in a Model of Endogenous Growth\****

MARTIN ZAGLER

*Working Paper No. 37*

*June 1996*

*\* The author wishes to thank Christian Keuschnigg, Christian Ragacs, Herbert Walther and the participants of the Spring '96 Department of Economics Research Seminar for discussion and helpful comments. All remaining errors are, of course, my own.*

*Address of the author:*

*Martin Zagler*

*Department of Economics*

*Vienna University of Economics*

*Augasse 2-6, A-1090 Wien*

*Elm: Zagler@isis.wu-wien.ac.at*

*Web: <http://www.wu-wien.ac.at/wwwu/institute/vw1/zagler.html>*

## **Abstract**

Empirical Analysis, indicating a negative tradeoff between long-run growth and economic stability appear sensitive with respect to policy intervention. I use a model of fully rational utility maximizing representative agents and profit maximizing firms acquiring rents by inventing a new product variety on which they have market power in a monopolistically competitive goods market.

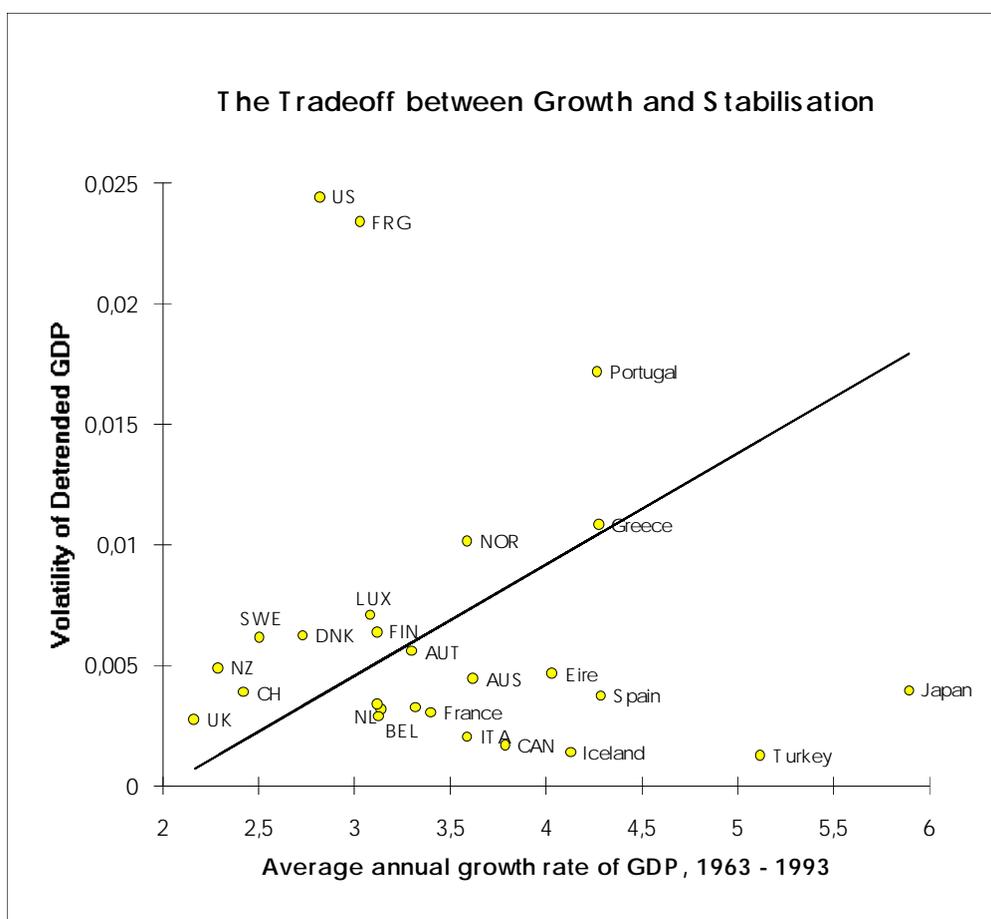
Monopolistic competition has been used in three contexts in modern economics: trade, growth and New Keynesianism. I shall use the latter two, together with a small menu cost argument enabling nominal price rigidity on the goods market, to show that monetary policy can stabilize the economy closer to potential output than laissez-faire in the short run, thereby inducing faster innovation driven endogenous growth in the long run. Whilst the effect of fiscal policy on growth and the effect of monetary policy on levels is not new to endogenous growth and New Keynesian models, respectively, the result of a growth effect of monetary policy, which the model describes, is genuine.

Once again, this model is policy-oriented, analyzing the traditional instrument of monetary policy. The result indicates that monetary policy can achieve more than just driving the price level, but that there exist both short-run static and long-run dynamic real effects of monetary policy. Though it is not intended to send a clear political signal to central bankers, the sound theoretical framework of the analysis does allow to conclude that these effects indeed exist, but empirical tests are certainly necessary to detect the correctness of the theoretical conclusion.

## A. Motivation

### A.1. Some empirical evidence

It is commonly believed that there exists a correlation between long-run growth rates of economy-wide GDP and the stabilization of aggregate supply and demand at the full employment level. Whilst Keynesians suggest that not using some of the capacity is a waste of resources and therefore growth is likely to decline, neoclassical and real business cycle economists support the thesis that the market outcome is efficient, under-utilization of only temporary nature, and therefore any stability-oriented policy is likely to reduce growth prospects. At first sight, the data seem to support the second group of authors. When looking at long-run growth rates of GDP of the OECD countries and the fluctuations around the trend, as done in figure 11 below, a negative correlation is suggested by the data.



**FIGURE 1:** The tradeoff between growth and volatility (*Data source: OECD*)

Using standard deviation as a measure of volatility, simple cross-country ordinary least squares regression of GDP growth on volatility yields the following results:

$$vol = -0.00929 + 0.00462 \text{ growth} + u_i$$

(-2.26)
(4.04)

Looking at T-statistics, but also at the uncentered  $R^2$  of 74.2 %, one is tempted to accept the hypothesis of negative correlation, at least for OECD countries. A sample greater than the 24 countries used, such as the Summers and Heston worldwide data might be interesting to examine. As the quality of these data is not the best, an application of that particular sample might harm the analysis more than it would benefit.

Moreover, when one would like to have a closer look at the data, it is the quality that matters. In particular, I would like to ask whether policy matters in that context. Evidently, some countries have been running programs of aggregate supply stabilization during the years in question, whilst other have been engaged in growth-oriented policies. When separating the OECD sample in 'institutional', or stability-oriented countries, and 'market economy', or growth-oriented countries<sup>1</sup>, the picture changes. As OLS is certainly not applicable for a sample smaller than around 20 observations, I shall not present a full regression here. But when looking at the previous graph, it becomes evident that the fugitive observations (in particular the U.S., FRG, and Japan) imply a positive tradeoff for conservative countries. It appears that countries which are less volatile, such as Japan, exhibit higher growth rates. Similarly, the stability-oriented countries (e.g. Sweden, Austria, France, Spain) form a positive tradeoff between stability and growth, but with much less volatility at low rates of growth, whilst the intercept with the horizontal axis lies at around 6 % growth for both conservative and progressive countries. The data seem to suggest a technical frontier to growth. The following analytical analysis tries to exploit possible theoretical reasons for this empirical observation. Before doing this, I will try to give an overview of the larger theoretical concept.

### ***A.2. A theoretical motivation***

New Classical Economics has targeted post-war Keynesians by repeatedly asking two questions. The first question addresses the problem of time inconsistency of the Keynesian IS-LM framework. It is the equivalent to the monetarist critique of the short-run Keynesian model, and it suggests that even if resources are allocated towards the present, they would lack in the future, and potentially more than have been used up today. The Keynesian models up to then were models of the very short run, with huge explanatory problems when it comes to discussing long-run implications.

The second question that started to puzzle me is the question of rationality. This question relates to the Lucas critique, which states that any model setup that is not based upon individual reasoning may be inconsistent on the aggregate level. On the individual level, economists are tempted to assume rationality on behalf of the agents, as one shall not make politics with the stupidity of the people. Besides an ethical problem, an economic problem might arise out of this effort as well. Therefore, one

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<sup>1</sup> I am using the principal ruling political party as a separating device, where conservatives are assumed to be growth oriented and progressive parties to be stability oriented. This device is certainly very problematic, therefore the results obtained should be treated with care.

shall always use the worst case for political intervention, namely rationality on behalf of the agents. Moreover, if one assumes that agents know the environment they are acting in much better than the non-involved observer, it is very likely that rational behavior describes economic behavior best.

For a long time, Keynesians have not found a reasonable response to this question, and government intervention has been widely refuted on theoretical grounds. It was not until the New Keynesian ideas started to come around, supported by the empirical threat of persistent unemployment, that government action became fashionable again within the profession. The breached literature uses the standard notion of rational, forward-looking agents in long-run models, introducing one or two assumptions, which seem very plausible, and obtains results that relate closely to a Keynesian analysis. The extensions consist of two factors: a monopolistic competition market structure, which ensures price-setting suppliers, and which does not seem implausible at all, and some kind of price rigidity, such as small menu costs, to obtain results that call for intervention. The New Keynesian models so far have two shortcomings. The first is that they do not yet apply welfare analysis, although the utilitarian setup would allow for that. Because of the fact that the welfare concept of the representative agent can be very misleading, and a mere focus on employment and growth could be just as good, or even a better measure of welfare than utility. The second is that the models do not explicitly model effects of government intervention, and because of a very different motivation of the results, it is not straightforward to assume that government policy in New Keynesian models is equivalent to government policy in the IS-LM framework. Therefore, unless this question is settled, one hesitates to apply simple IS-LM analysis to respond to current problems of the economy, as one does not yet know much about their effectiveness in rational long-run models. Therefore, the aim of this paper is to defend or to extend textbook models by arguing that despite a very different story that goes along, government policy has equivalent effects.

## **B. The Model**

### ***B.1. Households***

The economy consists of a potentially increasing variety of differentiated goods. As no household will produce all goods by itself, a market for exchange of these goods will be necessary. In order to facilitate trade on this market, and in order to separate transaction across place and time, the economy exhibits money. It is a standard representation to introduce money into the utility function. Households maximize lifetime utility subject to an intertemporal budget constraint. Assuming that some aggregate basket of consumer goods is homothetic in utility, the optimization problem can be separated into two stages. At the first, intertemporal stage, households choose between consuming, saving and money holdings. Assuming that lifetime utility for the representative, infinitely-lived, utilitarian household takes the form

$$U_0 = \int_0^{\infty} \frac{1}{1-\gamma} \left[ c_t^\alpha m_t^{1-\alpha} \right]^{1-\gamma} e^{-\theta t} dt, \quad (1)$$

where  $c_t$  is consumption of the composite good at time  $t$ ,  $m_t$  is real money demand at time  $t$ , the parameter  $\alpha$  represents the share of consumption in current expenditure,  $\gamma$  is the coefficient of relative risk aversion or the inverse of the intertemporal elasticity of substitution, and  $\theta$  is the individual rate of time preference<sup>2</sup>. The utility function is Cobb-Douglas in the tradeoff between consumption and money holdings, which implies that money demand will grow at the same rate as consumption. This specific functional form has been chosen as it describes very nicely the behavior of money in truly endogenous models of the monetary economy, as in Lucas (1980). Lucas describes a multi-good economy with stochastic consumption shares. He finds that in a model with money as an alternative to a credit economy where information is costly, money demand does not depend upon the number of goods (also more goods increase the social benefit of money holdings) but only upon the value of purchases. Deriving the 'quantity equation' for the economy, one finds that the velocity of money is constant due to the specific choice of utility, which is perfectly consistent with the Lucas model. The utility function is of the constant relative risk aversion type (CRRA) with respect to money and consumption, which implies that the intertemporal elasticity of substitution is independent of the level of consumption. The intertemporal household budget constraint takes the following nominal form,

$$\dot{A}_t + \dot{M}_t = i_t A_t + p_t w_t + p_t d_t + p_t \xi_t - p_t c_t.$$

This states that the change in nominal wealth  $A_t$  and additional nominal money demand  $M_t$  is equal to the nominal return on existing assets  $i_t a_t$ , nominal wages  $p_t w_t$ , nominal dividends  $p_t d_t$ , and nominal government transfers  $p_t \xi_t$  on the one hand, and nominal consumption  $p_t c_t$ . The style of the budget constraint does not change when dividing both sides by prices to obtain the budget constraint in real terms,

$$\dot{a}_t + \dot{m}_t = r_t a_t + w_t + d_t + \xi_t - c_t - \frac{\dot{p}_t}{p_t} m_t, \quad (2)$$

where  $a_t$  is real wealth,  $r_t$  is the real rate of interest, and  $\dot{p}_t/p_t$  is the inflation rate. Hamiltonian maximization of the objective function (1) with respect to consumption real money holdings and wealth subject to the intertemporal household budget constraint yields the following first-order conditions:

$$\alpha \left[ c_t^\alpha m_t^{1-\alpha} \right]^{1-\gamma} c_t^{-1} = \lambda_t, \quad (3)$$

$$(1-\alpha) \left[ c_t^\alpha m_t^{1-\alpha} \right]^{1-\gamma} m_t^{-1} - \frac{\dot{p}_t}{p_t} \lambda_t = \theta \lambda_t - \dot{\lambda}_t, \quad (4)$$

$$r_t \lambda_t = \theta \lambda_t - \dot{\lambda}_t. \quad (5)$$

$\lambda_t$  is the dynamic multiplier, expressing the value of an additional unit of household wealth. Substituting the left-hand side of equation (5) for the right-hand side of equation (4) and dividing this expression by equation (3), one obtains the money service yield (MSY), which expresses that the

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<sup>2</sup> The standard restrictions on the parameters are  $0 < \alpha < 1$ ,  $0 < \theta < 1$ ,  $\gamma > 0$  and  $\gamma \neq 1$ . Empirically  $\gamma$  is close to but smaller than unity. Note that for  $\gamma = 1$  one has to use log-utility.

private marginal utility of holding money instead of consuming must equal the price of holding money, inflation and lost real interest payments,

$$\frac{(1-\alpha)/m_t}{\alpha/c_t} = r_t + \frac{\dot{p}_t}{p_t}. \quad (\text{MSY})$$

For a given nominal interest rate, the preceding equation also describes how much real money balances are demanded for a given level of consumption. Rearranging the Money Service Yield to get  $m_t$  explicitly  $m_t = [(1-\alpha)/\alpha] c_t/i_t$ , describes a money demand function, resembling the LM-curve, as in an economy without capital goods, national income is equal to  $c_t$ . It is important to note that in the model described here, real money balances are determined endogenously, whereas nominal money is exogenous. This is a principal difference to post-Keynesian models, where nominal money is endogenously determined by liquidity preferences, whilst real money is an exogenous policy variable that drives output.

Taking logs and derivatives of equation (3), using equation (5) to eliminate  $\lambda_t$ 's, yields the Keynes-Ramsey rule (KRR). This particular form of the Euler equation expresses the optimal evolution of consumption across time,

$$\frac{\dot{c}_t}{c_t} = \frac{1}{1-\alpha+\alpha\gamma} [r_t - \theta] + \frac{(1-\alpha)(1-\gamma)}{1-\alpha+\alpha\gamma} \left[ \frac{\dot{m}_t}{m_t} \right]. \quad (\text{KRR})$$

If the real interest rate is larger than the individual rate of time preference, it is preferable to postpone consumption until marginal utility gains outweigh interest gains. Part of the interest gain is allocated to money holdings, of course. This is best seen when setting the growth rate of money equal to the growth rate of consumption. Rearranging terms, one obtains the Keynes-Ramsey rule in a non-monetary economy. This can be checked by setting the share of money in expenditure  $(1-\alpha)$  to zero.

The static optimization problem which the household is facing is that it must choose between the variety of consumer goods. The respective utility function takes the form

$$c_t = \left[ \int_0^{n_t} x_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}. \quad (6)$$

There is a changing variety of goods allocated on the continuous interval from zero to  $n_t$ , where  $n_t$  represents the index of the most advanced variety. The subutility function is of the constant elasticity of substitution type (CES), with the elasticity of substitution between two goods  $x_{i,t}$  and  $x_{j,t}$  is equal to  $\varepsilon^3$ . This specific functional form has become known as the Dixit-Stiglitz demand function. The static household budget constraint takes the form

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<sup>3</sup> In order to have substitutes,  $\varepsilon$  is required to be greater than unity.

$$\int_0^{n_t} p_{i,t} x_{i,t} di \leq p_t c_t, \quad (7)$$

where  $p_{i,t}$  is the price of good  $x_{i,t}$ . The first-order conditions with respect to any  $x_{i,t}$  all take the form

$$c_t = \left[ \int_0^{n_t} x_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{1}{\varepsilon-1}} x_{j,t}^{-\frac{1}{\varepsilon}} = \mu_t p_{j,t}, \quad (8)$$

where  $\mu_t$  is the Lagrange multiplier. Multiplying both sides with  $x_{i,t}$ , and integrating over all goods, we find the price index  $p_t$  is equal to the inverse of the shadow value  $\mu_t$ . Eliminating the Lagrange multiplier in the first order condition (8), and applying the definition of the composite good (6) for the term in square brackets, one obtains an indirect demand function of the representative agent for a particular good  $x_{i,t}$ , conditioned upon aggregate demand  $c_t$ , the price index  $p_t$  and the price of the particular commodity  $p_{i,t}$ . Normalizing population to unity, this is also aggregate demand for good  $x_{i,t}$  within the economy,

$$x_{i,t} = \left( \frac{p_{i,t}}{p_t} \right)^{-\varepsilon} c_t. \quad (9)$$

An increase in aggregate demand also increases demand for the particular good  $x_{i,t}$ . Whilst an increase in the price index increases demand for  $x_{i,t}$ , an increase in  $p_{i,t}$  reduces demand. These observations are consistent with the law of demand, hence the demand function is well-behaved. Integration of the demand function (9) over all  $i$  gives the definition of the price index for the model economy,

$$p_t = \left[ \int_0^{n_t} p_{i,t}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}.$$

This closes discussion of the demand side of the economy. Note, however, that in accordance with Ethier (1982) the demand function (6) can be interpreted as a production function for intermediate goods used in the production process of the unique consumption good  $c_t$ . The intermediate inputs would thus represent the capital goods of the economy, a way which has been explicitly used in Romer (1991). As the large number of products within the economy gives additional microfoundation for introducing money into the economy, the interpretation of a variety of consumption goods is preferable in this setting. Under the alternative interpretation, if the quantity of a particular commodity  $x_{i,t}$  is independent of  $i$ , and constant for all  $x$ , a result which is supported by the symmetry of the model, the production function (6) can be reduced to

$$c_t = x_t n_t^{\frac{\varepsilon}{\varepsilon-1}}.$$

As the aggregate number of intermediaries is then equal to  $n_t x_t$ , total factor productivity, i.e. the number of final products per unit of intermediary input, thus equals  $n_t \exp\{1/\varepsilon\}$ . Thus, as the number of products increases, ever more of the composite good can be produced with a given number of inputs. Long-run growth is therefore possible even with a given factor endowment. The engine of growth is

therefore not factor accumulation, as in Romer (1986), but product diversification. But this leads the analysis towards treatment of the supply side of the economy.

### **B.2. Manufacturing firms**

For matters of convenience, I shall separate the manufacturing and the research and development sector of the model economy. Whilst the manufacturing industry uses a given blueprint of a new product and manufactures with a given technology, a competitive research sector aims to develop new products, whenever the marginal product of innovation, i.e. the expected returns, exceed the marginal cost. Manufacturing firms use labor as the only input. This simplification is purely a matter of convenience, it would not change the characteristics of the solution, yet introducing capital makes the analysis more complicated. Moreover, as already mentioned, intermediate goods can be viewed as capital goods subject to full depreciation. Assuming that the production function is linear homogenous, a proper choice of units allows us to define the production function of the representative manufacturing firm as,

$$x_{i,t} = f(l_{i,t}) = l_{i,t}, \quad (10)$$

where  $l_{i,t}$  is the quantity of labor applied in the process of producing one unit of good  $i$ . Assuming that there is only one supplier for each product, an observation which is supported by Bertrand competition and fixed costs of product imitation, which would turn profits for both the existing firm and the market entrant negative, the representative firm maximizes real profits subject to the production technology (10) and the demand function (9). The firm takes the action of firms operating in other markets as given, although the price they are setting will influence the price index, and the quantity they are producing will influence aggregate consumption. Yet the influence is considered to be very small, hence it is near-rational to assume the price index and aggregate demand as given,

$$\pi_{i,t} = (p_{i,t}/p_t)x_{i,t} - w_t l_{i,t}. \quad (11)$$

The first-order condition for optimality with respect to labor demand can be reduced to give the well-known relation that in a market of monopolistic competition the price equals the wage times the mark-up, which is defined by the elasticity of demand,

$$\frac{p_{i,t}}{p_t} = \frac{\varepsilon}{\varepsilon-1} w_t. \quad (12)$$

Note that in a competitive equilibrium, where all products are perfect substitutes, and the elasticity of substitution approaches infinity, the mark-up will vanish, and the price would equal marginal costs. The price is independent of the particular product, hence we can use the formula for the price index to identify the wage in terms of the product index,

$$w_t = \frac{\varepsilon-1}{\varepsilon} n_t^{\frac{1}{\varepsilon-1}}. \quad (13)$$

With the definitions of the individual prices and the price index, the demand for a particular product can now be derived out of the demand function (9),

$$x_{i,t} = n_t^{\frac{\varepsilon}{1-\varepsilon}} c_t. \quad (9')$$

This formulation is consistent with the observations at the end of the section on the demand side. Finally, substituting these results into the profit function (11), real profits can be expressed as a function of aggregate consumption and the number of products in the economy,

$$\pi_{i,t} = \frac{c_t}{\varepsilon n_t}. \quad (11')$$

As this is true for all  $n_t$  producers, aggregate profits of the economy equal  $c_t/\varepsilon$ . The value of the individual firm is defined as the discounted sum of all future profits, of course. The value function is therefore defined as,

$$V_{i,t} = \int_t^{\infty} \pi_{i,t} e^{-\int_t^s r_s ds} d\tau.$$

Taking derivatives with respect to time, the following no-arbitrage condition emerges. It expresses that investing into a firm, which distributes all of its profits in the form of dividends and which is subject to potential capital gains, must be as profitable as investing all that money in interest yielding bonds. If this were not the case, arbitrage between the bond market and the stock market would take place until the price of the stock adjusts so that the no-arbitrage condition holds with equality. Note however, that it is the infinite horizon of the population which ensures via the transversality condition that no stock market bubble prevails this fundamental solution. The no-arbitrage condition on the firm level equals,

$$\pi_{i,t} + \dot{V}_{i,t} = r_t V_{i,t}.$$

Under the assumption of a static population, aggregation of the no-arbitrage condition across products and substituting aggregate profits from (11') yields the economy-wide no-arbitrage condition,

$$\frac{c_t}{\varepsilon V_t} + \frac{\dot{V}_t}{V_t} - \frac{\dot{n}_t}{n_t} = r_t. \quad (\text{NAC})$$

An increase in products on the market reduces market power of the individual firm, hence the no-arbitrage condition is enhanced by the rate of innovation.

### B.3. R & D firms

Having explained the optimization behavior of firms which hold an existing blueprint, the next chapter shall describe which economic forces drive new innovations. I shall distinguish between manufacturing firms and R & D firms in a way that R & D firms aim to develop a new product and sell it to the highest bidding manufacturing firm. Competition amongst manufacturers bids the price of the blueprint up to the point where it equals the discounted future stream of profits. The value of the firm at the initial stage will therefore equal the cost of the product blueprint. A firm or laboratory in the R & D sector is exposed to the following technology,

$$\Delta n_t = \phi \bar{l}_t n_t \Delta t.$$

It can add incrementally to the existing number of products by devoting  $\bar{l}_t$  units of labor for a time interval  $\Delta t$ . Labor productivity in R & D is equal to  $\phi$ . Moreover, the preceding technology assumes

that the number of existing products has a positive effect on future innovations.  $n_t$  is used as a measure of knowledge in the economy. It is assumed to enter linearly into the production knowledge for convenience. Helpman/Grossman (1991) show that more general specifications do not change general results. The continuous case of the above 'production' function is obtained as  $t$  gets infinitely small, and takes the form,

$$\dot{n}_t = \phi \bar{I}_t n_t. \quad (14)$$

With Chamberlain free-entry to R & D, the preceding technology specifications make labor demand go to infinity whenever the total revenue of undertaking research exceeds total costs. This is incompatible with the notion of general equilibrium, so the free-entry condition reads,

$$\frac{W_t}{\phi n_t} \geq v_{i,t}. \quad (15)$$

This expression must hold with equality whenever the number of products is increasing. The latter case is the equilibrium with R & D, and as it is the more interesting case, the following analysis will apply this case. Eliminating wages via condition (13), and substituting the aggregate stock value of the economy for  $v_{i,t}$ ,  $v \bar{I}_t = n \bar{I}_t v_{i,t}$ , gives the Helpman-Schumpeter equilibrium innovation locus (HSL)<sup>4</sup>,

$$\frac{\varepsilon \phi}{\varepsilon - 1} v_t = n_t^{\frac{1}{\varepsilon - 1}}. \quad (\text{HSL})$$

This condition defines the locus along which the economy is on a growth path. The locus lacks dynamics, as the model immediately jumps to equilibrium. In order to close the model, a labor market clearing condition will ensure that wages are adjusted in order to ensure full employment. Labor demand by manufacturing firms is given from the production function (10). Labor demand from the R & D sector is taken from the R & D production function (14). As it is insignificant whether one lab produces all the blueprints for a particular time, or whether a number of labs share the innovation market segment, no aggregation is required for R & D labor demand. As labor supply is normalized to unity, the labor market clearing condition takes the form,

$$\frac{\dot{n}_t}{\phi n_t} + \int_0^{n_t} I_{i,t} di = 1.$$

Using aggregate demand (9') to eliminate the integral from the above relation, the labor market clearing condition can be used to yield

$$\frac{\dot{n}_t}{\phi n_t} + n_t^{\frac{1}{1-\varepsilon}} c_t = 1. \quad (\text{LMC})$$

Finally, we can use all the above relations, in particular the no-arbitrage-condition (NAC), to solve the intertemporal household budget constraint, and show that Walras' Law implies the following relation for monetary policy,

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<sup>4</sup> This locus expresses the innovative Schumpeterian entrepreneurs optimal reaction. This particular interpretation was first proclaimed by Elhanan Helpman, who calls this locus the Schumpeter-line. For me, it seems natural to re-label the locus in the above manner.

$$\xi_t = \dot{m}_t + \frac{p_t}{p_t} m_t. \quad (\text{MPR})$$

The Keynes-Ramsey rule (KRR), the Money Service Yield (MSY), the no-arbitrage-condition (NAC), the Helpman-Schumpeter locus (HSL), the labor market clearing condition (LMC) and the monetary policy rule (MPR) define a dynamic system of six equations in seven parameters,  $c_t$ ,  $n_t$ ,  $m_t$ ,  $p_t$ ,  $v_t$ ,  $r_t$  and  $\xi_t$ . A seventh equation would define a process for  $\xi_t$  chosen by monetary authorities, or a distribution over the  $\xi_t$ 's as expected by individual agents, whenever monetary policy is discrete. Yet the model is already solvable for all variables except for prices in five equations only. Unless policy determines the price level, the model exhibits nominal indeterminacy. Rearranging the five fundamental equations and taking logs and derivatives along the balanced growth path, that is the path along which the growth rate of a variable  $x$  is constant,  $g_x = \dot{x}_t/x_t$ , it turns out that  $g_r = 0$ , and  $g_n = (\varepsilon - 1)g_c = (\varepsilon - 1)g_v = (\varepsilon - 1)g_m$ . Using these definitions, eliminating the number of products from the labor market clearing condition (LMC) via the Helpman-Schumpeter locus (HSL), then eliminating consumption per unit of wealth,  $c_t/v_t$ , with the help of the no-arbitrage-condition (NAC), and finally eliminating interest rates with the Keynes-Ramsey rule, an equilibrium growth rate for consumption in the economy shows up, namely

$$\frac{\dot{c}_t}{c_t} = \frac{\frac{\phi}{\varepsilon-1} - \theta}{\gamma + \varepsilon - 1}. \quad (16)$$

The economy grows whenever  $\phi/(\varepsilon - 1) > \theta$ . This condition is satisfied whenever research productivity does not lag too far behind the productivity in the manufacturing industry, which is by normalization the relevant correspondence. It is also satisfied when the elasticity of substitution is low, which makes just about any research profitable, as market power for the newly invented product will be large. An increase in the rate of time preference unambiguously reduces growth. Whenever agents value the present more than the future, consumption today will increase whilst investment for tomorrow, more innovation, will decline. Impatient societies will consume more in the beginning, and less in the future. An increase in research productivity increases the growth rate of the economy as well. Whenever less workers are needed for a given number of new inventions, the remaining labor force can devote efforts to more consumption today and to additional innovation, which boosts growth. When the condition for a positive growth rate is fulfilled, the growth rate declines, when the intertemporal elasticity of substitution decreases, or the coefficient of relative risk aversion increases. This seems to be obvious, as a higher potential to postpone consumption enforces this behavior. The effect of an increase in the elasticity of substitution between goods is ambiguous. The derivative takes the form,

$$\frac{\partial g_c}{\partial \varepsilon} = -\frac{\phi(\varepsilon - 1)^{-2}}{\gamma + \varepsilon - 1} - \frac{\phi(\varepsilon - 1)^{-1} - \theta}{(\gamma + \varepsilon - 1)^2}.$$

Whilst the first term is clearly negative, the second term can take either sign. Under a growth equilibrium, the second term is negative as well. Therefore, in a growing economy, an increase in the elasticity of substitution reduces growth. The reason is that a larger elasticity reduces monopoly

power, therefore profits, and therefore the return on innovation. As innovation declines, overall growth will decline as well.

#### ***B.4. Externalities and social optimum***

The model exhibits three kinds of externalities. The first externality is money, which exhibits private costs, although the social cost of producing money is virtually zero. In order to eliminate the pareto-inefficiency of money in the model, it is necessary to set the marginal utility of money equal to zero, or to set the growth rate of money supply, which drives inflation, equal to the negative real interest rate, a result which becomes evident when looking at the money service yield (MSY). This rule for monetary policy is the well-known Friedman rule.

The second type of externality is the monopolistic competition market structure. Evidently, the inclusion of demand in the profit calculus of the firm reduces potential output. On the other hand, perfect competition would fully eliminate innovation, as rents of the manufacturing firms, with which research is paid, would equal zero. Whilst money reduces the level of consumption, it does not reduce the growth rate of consumption. One can easily verify that by noting that the share of real money balances,  $\alpha$ , does not affect the equilibrium growth rate.

Finally, there is public knowledge capital in the research and development sector, hence product innovation does not pay its social marginal product, but its private marginal product. R & D tends to be too low in equilibrium.

Until now, the model is completely classical in a sense that prices and wages adjust immediately to ensure market clearing. Technically speaking, the model is homogenous of degree zero in wages, prices and real money balances. In particular, monetary policy is Sidrauski-neutral in the sense that consumption determines money demand, and not vice-versa. The next chapter will be devoted to introducing a market inefficiency which will lead the model to a state where money is effective and where it is useful to enact monetary policy.

### **C. Small Menu Costs**

The model has assumed complete flexibility of prices. There has always been an economic challenge to this neoclassical approach. Recently, New Keynesians have contributed a plausible microfoundation for sticky prices. They argue that menu costs, the cost of announcing prices so that all economic agents obtain this information, might play a central role in the explanation of the business cycle. As firms set prices initially optimal, the gain of a small price change after new information changed expectations is small as well. Therefore small menu costs, larger than the additional profit from changing price, can prevent price adjustment. It is important that out of a Pareto equilibrium,

prices are not set to optimize welfare, hence small price changes can have large, and positive, effects on welfare.

Within the intertemporal framework that I shall present, these menu costs do exhibit additional intertemporal effects. First of all, the choice of changing a price does not only have current, but also future effects. In particular, when it is likely that the reason for changing prices is temporary in nature, firms might not change the price, whilst consistent changes in important variables will make it even more likely that prices adjust quickly.

Second, there is a question of how to treat newly entrant firms, which for instant, offer a product that has been newly invented. When prices of firms that are already present in the market differ from their individual optimum, a new entrant can obtain a higher market share by choosing the optimal price. Moreover, as new stochastic information arrives all the time, the model will sooner or later arrive at a distribution of firms over the price interval. Solutions will turn complicate and solvable only with computable general equilibrium models. Here, it is assumed for the sake of simplicity that newly entering firms can choose everybody else's price at zero cost, whilst promoting a different price is costly. This assumption seems very sensitive to empirical findings.

This chapter is concerned the active role of monetary policy in the framework of a New Keynesian model, and in particular, the question whether monetary policy plays an important for the growth rate of an economy. The following three propositions prove that there is indeed an effect of monetary policy on growth. As the model only intends to give an indication on the direction of the effect, the following passus will focus on a marginal analysis only.

**PROPOSITION 1:** The increase in real money balances has a positive effect on welfare. Moreover, it increases profit expectations and therefore the equilibrium growth rate of the economy.

**PROOF:** By rearranging the money service yield, it is easily shown that an increase in real money balances increase the level of aggregate demand. Taking derivatives,

$$\left. \frac{\partial c_t}{\partial m_t} \right|_{MSY} = \frac{\alpha i_t}{1-\alpha} + \frac{\alpha m_t}{1-\alpha} \frac{\partial i_t}{\partial m_t}.$$

The first term of this expression is positive, as the model requires positive rates of interest in order to motivate positive money holdings. The second term is zero, as the effect of changes in real money balances on nominal interest rates is zero. This is due to the neoclassical result of chapter B, which states that the model is free of money illusion. As aggregate profits are defined as  $c_t/\varepsilon$ , the effect of monetary policy on profits is positive as well,

$$\left. \frac{\partial \pi_t}{\partial c_t} \right|_{(1r)} = \frac{1}{\varepsilon}.$$

As expected profits increase, the activities of research laboratories will turn more interesting, and the speed of innovation will increase. As innovation drives growth within the model framework, the overall growth rate of the economy will increase as well.

Finally, as overall utility depends upon current levels of real money balances and final consumption, and the overall growth rate of the economy only,

$$U_t = \frac{1}{1-\gamma} c_t^{1-\gamma} m_t^{1-\gamma} \int_t^{\infty} e^{(g_c - \theta)(\tau-t)} d\tau,$$

the effect of an increase in real money balances exhibits a first-order increase in utility. Applying the utility function as a natural indicator for welfare, there is a positive effect on welfare as well. Hence, it has been shown that an increase in real money balances exhibits a positive effect on utility and growth. Q.E.D.

Yet monetary authorities are not capable of influencing real money balances, as already mentioned in the preceding chapter. Monetary authorities can only influence nominal balances, whilst real balances are determined on the money demand side only, just like in any other monetarist model. Given menu costs, there is of course a real effect of nominal variables on real variables. This will be shown in two steps. First, proposition two will describe the effect of nominal money changes on nominal variables, in particular prices, whilst proposition three treats the effect of nominal changes on real variables and welfare.

**PROPOSITION 2:** Whenever manufacturing firms produce at decreasing private returns to scale, a marginal increase in nominal money balances exhibits only a second order incentive to change prices.

**PROOF:** When setting the initial price, firms have to choose a price that maximizes profits. Therefore, we can restrict the analysis to the effect of nominal changes around the equilibrium without loss of generality, as the model has been restricted to analyze the effects around equilibrium anyhow. A more profound analysis would necessitate average nominal shocks as well, as for instance in Blanchard/Kiyotaki. Using asterisks to denote optimal variables, profits are defined as

$$\pi_{i,t}^* = (p_{i,t}^*/p_t)^{1-\varepsilon} c_t - \frac{\varepsilon-1}{\varepsilon} (p_{i,t}^*/p_t)^{1-\varepsilon} c_t.$$

The effect of a nominal money change on profits is then defined as,

$$\frac{\partial \pi_{i,t}^*}{\partial M_t} = \frac{\partial \pi_{i,t}^*}{\partial p_{i,t}^*} \frac{\partial p_{i,t}^*}{\partial M_t} + \frac{\partial \pi_{i,t}^*}{\partial c_t} \frac{\partial c_t}{\partial M_t}.$$

The first term is zero by definition, as firms have chosen prices to optimize profits, therefore the tangent on the profit curve is horizontal, and the derivative is zero. The first term is also considered to be the direct channel of nominal changes, therefore the effect of nominal money changes on profits is zero to the first order. The second-order effect, the impact of aggregate demand changes real profits is zero as well. Yet, this depends crucially on the assumption of constant private returns to scale in the

production function of the manufacturing industry. Evidently, for a firm that produces at constant cost for any quantity demanded, the change of demand does not imply a change in prices. In the appendix I develop a fancier model where the private returns to scale are decreasing. Then, the effect is unambiguously positive. Separating the very last term into two parts,

$$\partial c_t / \partial M_t = (\partial c_t / \partial m_t) (\partial m_t / \partial M_t) = (\partial c_t / \partial m_t) (\partial m_t / \partial p_t m_t),$$

it is easily verified that the second-order impact of changing prices back to the optimum is positive, as the effect of an increase of real money balances on consumption is positive<sup>5</sup>, and the effect of nominal money balances on real money balances is just equal to the inverse of the price index, which is defined to strictly positive as well. Q.E.D.

The last two propositions now enable us to obtain the principal result of the chapter on the effectiveness of monetary policy and its precise influence within the model framework.

**PROPOSITION 3:** If menu costs of changing prices are large enough to prevent price adjustment, then monetary policy is non-neutral. A monetary expansion will unambiguously foster utility and growth.

**PROOF:** Menu costs enter the model exogenously, therefore we can certainly find a cost at which changing prices is too costly for individual firms. Yet the menu costs need not be too large, as there is only a second-order effect of cost of changing prices, whilst the first-order or direct effect is zero in optimum. This has already been shown in proposition one. Menu costs can therefore be reasonably small to satisfy empirical evidence. As small menu costs prevent prices from adjusting to satisfy individual optimization, an expansion of nominal money balances induces an expansion of real money balances. All that needs to be shown now is that a change in real money balances has positive effects on output and welfare, or, that monetary authorities, which are assumed to be able to control nominal money, can influence real variables. But proposition one has already proven that an increase in real money balances increases utility and fosters growth. Q.E.D.

The preceding theoretical steps, have confirmed theoretically what the empirical evidence, as described in the introduction, has suggested. There is no trade-off between growth and stabilization policy. Quite contrary, there exists a political degree of freedom, and countries that have seeded stabilization rules have harvested higher rates of growth as well.

## D. Conclusions and Extensions

This paper finds evidence that stabilization policy fosters growth. The definition of stabilization is not used in the sense of real business cycle economics, but in the Keynesian sense of full-employment

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<sup>5</sup> This has already been shown in the Proof of Proposition 1.

policy. Whenever the market is not capable of ensuring full utilization of capacity, government intervention, as shown for the example of monetary policy, can contribute to drive the economy towards the pareto-optimum.

As shown in chapter B, a standard neo-classical model does not exhibit a connection between monetary policy and growth. Money may affect levels of GDP in the short run, in the long run money is considered to be Sidrauski-neutral. New Keynesian models up to now have not yet challenged this position, due to a lack of intertemporal models with endogenous rates of growth. Whilst endogenous growth models have always supported fiscal policy as a means of fostering economic growth, they have denied that monetary policy is effective. This paper refutes the result and shows that, at least at the margin, money expansion exhibits an unambiguously positive impact on economic growth.

This effect is achieved through two distinct transmission mechanisms, which are both necessary to ensure the existence. The first is the small menu cost argument, which is necessary to have real effects nominal money changes. Whenever monetary authorities raise nominal money balances, the cost of increasing money prices prevents agents from reducing real money balances. As money holdings are too low in equilibrium<sup>6</sup>, agents are better off and therefore increase current aggregate demand. This is where the second effect, the particular engine of growth, comes in. As profit opportunities increase when aggregate demand increases, a monetary expansion induces a higher R & D labor demand, higher wages, a higher rate of innovation and therefore a higher and welfare-increasing rate of economic growth.

Work on this topic is, of course, not completed yet. The paper demands extensions to show whether some typical fallacies of macroeconomic models will appear in this context as well. The first extension is certainly the modeling of an open economy, where diffusion effects of monetary policy — that is foreign crowding out — need to be investigated.

The second extension concerns the public sector. So far, the government in our model issues fiat money, and finances transfers to private agents. Obviously, there is public consumption and public investment to be considered as well. Moreover, government is not financed by monetary expansion alone. Therefore a taxation scheme would be an interesting extension to see particular effects of certain policies. Finally, as there is a government trade-off between debt- and tax-finance, whenever agents have finite horizons. An overlapping generations setup allowing for an intertemporal tradeoff of government policy can be of interest. Yet, as it is likely that the results will not differ much from the standard findings, I have spared the analysis for later investigations.

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<sup>6</sup> As mentioned in chapter B, it is optimal to have a price of money equal to the marginal cost of producing money. As the public cost is virtually zero and therefore much lower than the private opportunity cost of holding money, real money balances are higher in optimum.

## Appendix

The production technology (10) in the main text is rather simple. On the one hand, this is beneficial for finding nice, interpretable solutions, but on the other hand, it might be too much of a simplification to be realistic. In particular the fact that firms optimally do not adjust their price when aggregate demand changes seems counterintuitive. There are several alternatives for how to model the production process. One is to introduce physical capital. This seems realistic, yet unimportant for the model solution. Moreover, capital is already captured within the variety of consumption goods. Another is to introduce human capital. One can either assume that human capital is a private production factor, but then it would not be any different from physical capital. Yet another possibility is to assume that human capital is external to the firm, and, for instance, a byproduct of research and development. For the model, assume that the production function is Cobb-Douglas in labor and human capital, where firms can only influence their labor demand,

$$x_{i,t} = l_{i,t}^\beta n_t^{1-\beta}. \quad (\text{A10})$$

As the number of blueprints developed in the R & D process is certainly a good proxy for human capital, we shall assume that the number of blueprints drives the human capital. For the sake of simplicity, it is assumed that this impact is linear. Therefore, the new production function reads,

$$x_{i,t} = l_{i,t}^\beta n_t^{1-\beta}. \quad (\text{A10}')$$

Substituting this expression into the profit function (11) and maximizing with respect to profits, the price mark-up equation (12) now takes the form,

$$\frac{p_{i,t}}{p_t} = \left[ \frac{\varepsilon w_t}{\beta(\varepsilon - 1)} \right]^{\frac{\beta}{\varepsilon - \varepsilon\beta + \beta}} \left[ \frac{c_t}{n_t} \right]^{\frac{1-\beta}{\varepsilon - \varepsilon\beta + \beta}}. \quad (\text{A12})$$

Substituting this into the definition of the price index, wages are defined as

$$w_t = \frac{\beta(\varepsilon - 1)}{\varepsilon} \left( \frac{c_t}{n_t} \right)^{\frac{\beta-1}{\beta}} n_t^{\frac{\varepsilon - \varepsilon\beta + \beta}{\beta(\varepsilon - 1)}}. \quad (\text{A13})$$

Therefore, demand for a particular product can be derived to yield,

$$x_{i,t} = \left[ \frac{\varepsilon w_t}{\beta(\varepsilon - 1)} \right]^{\frac{-\varepsilon\beta}{\varepsilon - \varepsilon\beta + \beta}} \left[ \frac{c_t}{n_t} \right]^{\frac{-\varepsilon(1-\beta)}{\varepsilon - \varepsilon\beta + \beta}} c_t. \quad (\text{A9}')$$

In optimum, individual profits are then defined to be,

$$\pi_{i,t} = \frac{\varepsilon - \varepsilon\beta + \beta}{\varepsilon} \frac{c_t}{n_t}. \quad (\text{A11}')$$

The no-arbitrage condition now reads,

$$\frac{\varepsilon - \varepsilon\beta + \beta}{\varepsilon} \frac{c_t}{v_t} + \frac{\dot{v}_t}{v_t} - \frac{\dot{n}_t}{n_t} = r_t. \quad (\text{A.NAC})$$

As nothing changes on the demand side, the Money Service Yield (MSY) and the Keynes-Ramsey rule (KRR) do not change, the new Helpman-Schumpeter locus (HSL) and the new labor market

clearing condition (LMC) to derive the new equilibrium growth locus. Inverting the production function and eliminating individual product demand via (A.9') and wages via equation (A.13), the new labor market clearing condition now takes the form,

$$\frac{\dot{n}_t}{\phi n_t} + c_t^{\frac{1}{\beta}} n_t^{\frac{2\varepsilon\beta+2\beta-2\varepsilon+1}{\beta(\varepsilon-1)}} = 1. \quad (\text{A.LMC})$$

Finally, substituting wages into equation (15), we obtain the Helpman-Schumpeter locus,

$$\frac{\beta(\varepsilon-1)}{\varepsilon\phi} \left( \frac{c_t}{n_t} \right)^{\frac{\beta-1}{\beta}} n_t^{\frac{\varepsilon-\varepsilon\beta+\beta}{\beta(\varepsilon-1)}} = v_t. \quad (\text{A.HSL})$$

Taking logs and derivatives, we find that along the balanced growth path,  $g_r = 0$  and  $g_c = g_v = g_m = g_n = (2\varepsilon - 2\varepsilon\beta + 2\beta - 1) / (\varepsilon - 1)$ . Substituting these result back into reduced form equations, we find the following solution for the equilibrium growth rate of consumption<sup>7</sup>,

$$\frac{\dot{c}_t}{c_t} = \left[ \frac{\varepsilon - \beta(\varepsilon - 1)}{\beta(\varepsilon - 1)} \phi - \theta \right] / \left[ \frac{\varepsilon / \beta + 2\beta - 2\varepsilon\beta + 1}{2\varepsilon + 2\beta - 2\varepsilon\beta - 1} + \gamma \right]. \quad (\text{A16})$$

## References

- Agell, Jonas, and Lommerud, Kjell Erik,** *Egalitarianism and Growth*, Scandinavian Journal of Economics, Vol. 95, 1993, pp. 559-579
- Alesina, Alberto, and Rodrik, Dani,** *Distributive Politics and Economic Growth*, National Bureau of Economic Research Working Paper No. 3668, 1991
- Alogoskoufis, George, and Ploeg, Frederick van de,** *On Budgetary Policies and Economic Growth*, CEPR Discussion Paper No. 496, 1990
- Alogoskoufis, George, and Ploeg, Frederick van de,** *Money and Endogenous Growth*, CentER for Economic Research Discussion Paper No. 9109, Tilburg, 1991
- Alogoskoufis, George, and Ploeg, Frederick van de,** *Endogenous Growth, Convergence and Fiscal Policies in an Interdependent World*, In: Helmut Frisch, Andreas Wörgötter, Open Economy Macroeconomics, IEA Conference Proceedings Macmillan Press Ltd., London, 1993
- Azariades, Costas,** *Intertemporal Macroeconomics*, Blackwell Publishers, Oxford, 1993

<sup>7</sup> Note that with  $\beta = 1$ , the case of labor as the only production function, the solution evidently reduces to the solution in the main text, equation (16).

**Bean, Charles, and Pissarides, Christopher,** *Unemployment, Consumption and Growth*, European Economic Review, Vol. 37, 1993, pp. 837-859

**Blanchard, Olivier J., and Fischer, Stanley,** *Lectures in Macroeconomics*, MIT Press, Cambridge, London, 1989

**Blanchard, Olivier J., and Kiyotaki, Nobuhiro,** *Monopolistic Competition and the Effects of Aggregate Demand*, American Economic Review, 1987, pp. 647-666

**Ethier, Wilfred,** *National and International Returns to Scale in the Modern Theory of International Trade*, American Economic Review, Vol. 72, 1982, pp. 389-405

**Grossman, Gene M., and Helpman, Elhanan,** *Product Development and International Trade*, Journal of Political Economy, Vol. 97, 1989, pp. 1261-1283

**Grossman, Gene M., and Helpman, Elhanan,** *Innovation and Growth in the Global Economy*, The MIT Press, Cambridge and London, 1991

**Grossman, Gene M., and Helpman, Elhanan,** *Quality Ladders in the Theory of Growth*, Review of Economic Studies, Vol. 58, 1991, pp. 43-61

**Grossman, Gene M., and Helpman, Elhanan,** *Quality Ladders and Product Cycles*, The Quarterly Journal of Economics, 1993, pp. 557-586

**Grossman, Gene M., and Helpman, Elhanan,** *Endogenous Innovation in the Theory of Growth*, The Journal of Economic Perspectives, 1994

**King, Mervyn A.,** *Growth and Distribution*, European Economic Review, Vol. 36, 1992, pp. 585-592

**King, Robert G., and Rebelo, Sergio T.,** *Business Cycles with Endogenous Growth*, mimeo, University of Rochester, 1988

**King, Robert G., and Rebelo, Sergio T.,** *Public Policy and Economic Growth: Developing Neoclassical Implications*, Journal of Political Economy, Vol. 98, 1990, pp. S126-S150

**Koopmans, Tjalling C.,** *Economic Growth at a Maximal Rate*, Quarterly Journal of Economics, Vol. 78, 1964, pp. 355-394

**Lucas, Robert E.,** *Equilibrium in a Pure Currency Economy*, In: Kareken, Neil, and Wallace, John H., *Models of Monetary Economies*, Federal Reserve Bank of Minneapolis, 1980

**Lucas, Robert E.,** *On the Mechanics of Economic Development*, Journal of Monetary Economy, 1988, pp. 3-42

**Lucas, Robert E.,** *Making a Miracle*, Econometrica, Vol. 61, 1993, pp. 251-272

**Mankiw, N. Gregory, Romer, David, and Weil, David N.,** *New Keynesian Economics*, Vol. 1, MIT Press, Cambridge, 1991

**Mankiw, N. Gregory, Romer, David, and Weil, David N.,** *New Keynesian Economics*, Vol. 2, MIT Press, Cambridge, 1991

**Matsuyama, Kiminori,** *Increasing Returns, Industrialization, and Indeterminacy of Equilibrium*, The Quarterly Journal of Economics, 1993, pp. 617-650

**Matsuyama, Kiminori,** *Modeling Complementarity in Monopolistic Competition*, Stockholm University Seminar Paper, No. 534, 1993

**Mulligan, Casey B., and Sala-i-Martin, Xavier,** *Transitional Dynamics in Two-sector Models of Endogenous Growth*, National Bureau of Economic Research Working Paper No. 3986, 1992

**OECD,** *National Accounts, Main Aggregates, 1960-1990*, OECD Publications, Paris, 1993

**Pelloni, Alessandra,** *Nominal Rigidities and Increasing Returns*, European University Institute, mimeo, 1994

**Ramsey, Frank P.,** *A Mathematical Theory of Saving*, Economic Journal, Vol. 38, 1928, pp. 543-559

**Romer, Paul M.,** *Dynamic Competitive Equilibria with Externalities, Increasing Returns and Unbounded Growth*, Ph. D. Dissertation,

University of Chicago, Department of Economics, 1983

**Romer, Paul M.,** *Growth Based on Increasing Returns Due to Specialization*, American Economic Review, Papers and Proceedings, Vol. 77, 1987, pp. 56-62

**Romer, Paul M.,** *Endogenous Technological Change*, Journal of Political Economy, Vol. 98, 1990, pp. S71-S102

**Sala-i-Martin, Xavier,** *Lecture Notes on Economic Growth (I): Introduction to the Literature and Neoclassical Models*, National Bureau of Economic Research, Cambridge, Working Paper No. 3563, 1990

**Sala-i-Martin, Xavier,** *Lecture Notes on Economic Growth (II): Five Prototype Models of Endogenous Growth*, National Bureau of Economic Research, Cambridge, Working Paper No. 3564, 1990

**Schumpeter, Joseph A.,** *The Explanation of the Business Cycle*, Econometrica, Vol. 7, 1927, pp. 286-311

**Solow, Robert M.,** *A Contribution to the Theory of Economic Growth*, Quarterly Journal of Economics, 1956, pp. 65-94

**Solow, Robert M.,** *Growth Theory, An Exposition*, Clarendon Press, Oxford, 1970

**Solow, Robert M.,** *Growth Theory and After*, American Economic Review, 1988, pp. 307-317

