Stephan Gasser and Margarethe Rammerstorfer and Karl Weinmayer

Markowitz Revisited: Social Portfolio Engineering

Article (Accepted for Publication)
(Refereed)

Original Citation:

Gasser, Stephan and Rammerstorfer, Margarethe and Weinmayer, Karl
(2017)
Markowitz Revisited: Social Portfolio Engineering.
European Journal of Operational Research, 258 (3).
pp. 1181-1190. ISSN 0377-2217

This version is available at: https://epub.wu.ac.at/4677/
Available in ePubWU: November 2015

epubWU, the institutional repository of the WU Vienna University of Economics and Business, is provided by the University Library and the IT-Services. The aim is to enable open access to the scholarly output of the WU.

This document is the version accepted for publication and — in case of peer review — incorporates referee comments.
Markowitz Revisited: 
Social Portfolio Engineering

Stephan M. Gasser∗ Thomas Kremser† 
Margarethe Rammerstorfer‡ Karl Weinmayer§

October 31, 2014

Abstract

In recent years socially responsible investing has become an increasingly more popular subject with both private and institutional investors. At the same time, a number of scientific papers have been published on socially responsible investments (SRIs), covering a broad range of topics, from what actually defines SRIs to the financial performance of SRI funds in contrast to non-SRI funds. In this paper, we revisit Markowitz’ Portfolio Selection Theory and propose a modification allowing to incorporate not only asset-specific return and risk but also a social responsibility measure into the investment decision making process. Together with a risk-free asset, this results in a three-dimensional capital allocation plane that allows investors to custom-tailor their asset allocations and incorporate all personal preferences regarding return, risk and social responsibility. We apply the model to a set of over 6,231 international stocks and find that investors opting to maximize the social impact of their investments do indeed face a statistically significant decrease in expected returns. However, the social responsibility/risk-optimal portfolio yields a statistically significant higher social responsibility rating than the return/risk-optimal portfolio.

Keywords: Socially Responsible Investments, Portfolio Optimization, International Financial Markets

JEL classification: G11, G15, A13

∗This paper was peer reviewed and accepted for presentation at the 27th Australasian Finance and Banking Conference. We thank Otto Randl for valuable feedback and suggestions. Any remaining errors are the responsibility of the authors.

∗WU (Vienna University of Economics and Business), Department of Finance, Accounting and Statistics, Wohlhandelsplatz 1, 1020 Vienna, Austria; email: stephan.gasser@wu.ac.at
†WU (Vienna University of Economics and Business), Department of Finance, Accounting and Statistics, Wohlhandelsplatz 1, 1020 Vienna, Austria; email: thomas.kremser@wu.ac.at
‡MODUL University, Department of International Management, Am Kahlenberg 1, 1190 Vienna, Austria; email: margarethe.rammerstorfer@modul.ac.at
§WU (Vienna University of Economics and Business), Department of Finance, Accounting and Statistics, Wohlhandelsplatz 1, 1020 Vienna, Austria; email: karl.weinmayer@wu.ac.at
1 Introduction

This paper provides new insights into the area of portfolio optimization by introducing a third criterion to the classical portfolio selection parameters of return and risk, in order to be able to build portfolios that are socially responsible (i.e. by using socially responsible investments or SRIs). Motivated by the increasing popularity of SRIs (see e.g. Sparkes and Cowton [2004]), we revisit Markowitz’ Portfolio Selection Theory (Markowitz [1952]) and propose a tri-criterion model that also incorporates a social responsibility measure into the investment decision making process. Together with a risk-free asset, this results in a non-dominated three-dimensional capital allocation plane (CAP) allowing investors to custom-tailor their asset-allocations and incorporate all personal preferences regarding risk, return as well as social responsibility. We apply this framework to an investment universe of 6,231 international publicly traded companies to obtain all feasible combinations of both a return/risk and a social responsibility/risk-optimal portfolio as well as a risk-free asset, thereby creating a CAP in a three-dimensional return/risk/social responsibility space. Hence, we provide a multiple criteria decision model to investors interested in generating a certain social impact with their investment decisions.

Utilizing a simulation approach to optimize both 20,000 return/risk and 20,000 social responsibility/risk-optimal portfolios, we find that investors opting to maximize the social responsibility of their investments do indeed face a statistically significant decrease in expected daily returns (0.1613% vs. 0.0357%). However, the social responsibility/risk-optimal portfolio also yields a significantly higher social responsibility rating (0.9435, with 1 being the maximum attainable level) than the return/risk-optimal portfolio (0.0855), while at the same time exhibiting a lower risk exposure (0.5381% vs. 0.7540%). In addition, the Sharpe Ratios are significantly lower for the social responsibility/risk-optimal
The CAP yields a graphical representation of all feasible portfolio combinations attainable for investors. We demonstrate the application of this approach for a number of different investor types and show, for example, how much return needs to be sacrificed and by how much the risk exposure decreases for increases of a portfolio’s social responsibility rating. The results are robust for multiple simulation runs based on our model framework.

On the basis of our computations and the resulting CAP, investors can easily decide on their optimal portfolio given their respective preference parameters for return, risk and social responsibility. The proposed model framework is thus a moderately complex modification of Markowitz’ Portfolio Selection Theory, allowing practitioners and academics to implement a social responsibility factor into their asset allocation strategies. Concerning the findings of our empirical analysis, it is interesting to note that investors caring about the social responsibility of their investments face a statistically significant decrease in both expected returns and risk exposure, with the Sharpe Ratios being also significantly lower than those of a return/risk-optimal portfolio. However, investors may attain optimal portfolios exhibiting a modest social responsibility rating, by accepting only a very limited decrease of the resulting portfolio’s Sharpe Ratio.

There are already a number of papers attempting to implement a variety of other criteria (be it financial or non-financial) into Markowitz’ Portfolio Selection Theory (see e.g. Ehrgott et al. 2004; Steuer et al. 2005; Hirschberger et al. 2013; Bilbao-Terol et al. 2012) for example, introduce a goal programming model for SRI portfolio selection that tries to enable investors to match their ethical and financial preferences. With a data set of UK mutual funds, the authors show that investors’ risk attitudes impact the loss of return triggered by choosing SRIs. Ballestero et al. (2012) also focus their study on socially responsible investments. They propose a “financial-ethical bi-criteria model” on the basis of two opportunity subsets consisting of 20 ethical (i.e. “green”) funds and 60 other assets,
respectively. Their results indicate that ethical investments are accompanied by risk exposure increases. In a 2013 study, Bilbao-Terol et al. implement a two-stage multi-objective framework for the selection of SRI portfolios by applying a “Hedonic Price Method”. On the basis of a data set of 160 SRI and conventional funds, their empirical results suggest that the financial penalties associated with SRIs are relatively minor for highly risk-averse investors. Finally, Utz et al. (2014) extend Markowitz’ Portfolio Selection Theory by introducing a social responsibility measure (i.e. ESG scores, see Section (2)). Their model is then applied to a data set of conventional and socially responsible mutual funds. They find no support for lower financial performance or risk exposure increases of SRI funds. Interestingly, they also conclude that SRI funds do not exhibit higher social responsibility scores compared to their conventional counterparts.

Besides these papers published in research areas close to our own, a fairly large set of studies focusing on the broad topic SRIs exists. The first strand of literature focuses on companies’ corporate social responsibility (CSR) schemes and tries to evaluate the impact of CSR activity on financial performance. Alexander and Buchholz (1978), Aupperle et al. (1985) and McWilliams and Siegel (2001) apply a variety of different approaches but fail to find any statistically significant relationship between a company’s level of CSR activity and its financial performance. Cochran and Wood (1984), however, manage to establish “weak support” for a non-negative connection between CSR and financial performance. Following Sen et al. (2006) and Du et al. (2011), the diverging results of these studies might be attributed to stakeholder awareness issues, since any positive effects of CSR efforts on financial performance critically depend on stakeholder awareness. Finally, Sparks and Cowton (2004) state that both SRIs and the practice of CSR can no
longer be considered niche products but are becoming “mainstream” with increasing SRI adoption by institutional investors.

Another strand of research compares the financial performance of SRIs and conventional investments (CIs). Hamilton et al. (1993) show that socially responsible mutual funds do not earn excess returns that are statistically significantly different from conventional mutual funds. This is supported by Sauer (1997), who states that socially responsible investments do not cause investors to forgo financial performance. Schröder (2007) agrees in his comparison of the performance characteristics of SRI equity indices and conventional benchmark indices; however, he also notes that many SRI indices experience higher levels of risk relative to their benchmarks. Renneboog et al. (2008) analyze the performance difference between SRI funds and conventional benchmark funds on a country-by-country basis and find no statistically significant results for the U.S. and the U.K., while e.g. in France, Japan and Sweden SRI investors experience sub-par financial performance. Gil-Bazo et al. (2010) find the performance of socially responsible funds - before fees and managed by SRI-specialized management companies - to outperform conventional benchmarks, while SRI funds run by generalist fund managers underperform conventional funds.

Besides performance comparison analyses, Benson and Humphrey (2008) conduct a study on the determinants of fund flows for SRI and CI funds. They find that SRI fund flows are less sensitive to historic fund performance than the fund flows of conventional funds and argue that the investment strategy of socially responsible investors is more persistent, i.e. that they are more likely to invest in a fund they already own. Finally, Kempf and Osthoff (2008) test the frequently made claim that the social and environmental standards of SRI funds are quite similar to those of conventional funds, and find SRI funds to have a significantly higher ethical ranking compared to conventional funds.
The contribution of this paper is threefold. First, in contrast to previous studies optimizing socially responsible portfolios, we focus our analysis on individual assets instead of funds, and employ an unbiased and independently provided measure to gauge their level of social responsibility (i.e. the ESG score). Second, we propose a modification of the Markowitz model allowing investors to incorporate not only asset-specific return and risk, but also ESG scores into the investment decision making process. Thus, instead of merely building a return/risk-optimal portfolio, a dual-step optimization is applied here and we compute a second ESG/risk-optimal portfolio. Together with a risk-free asset, this yields a three-dimensional CAP illustrating all feasible portfolio combinations. Investors are thus able to custom-tailor their asset-allocations and incorporate all personal preferences regarding risk, return and social responsibility. Third, using a simulation approach, we apply this model to a unique data set of 6,231 international stocks (including the complete universe of ESG-rated companies) in order to empirically examine the relationship between return, risk and social responsibility.

The structure of this paper is as follows. In Section (2) we present additional background information on Markowitz’ Portfolio Selection Theory before elaborating on our empirical data and the benefits of ESG scores. Section (3) introduces our Markowitz’ model modification, while Section (4) describes the empirical methodology applied. Section (5) contains the results of our analysis, while Section (6) concludes.

2 Background and Data

In 1952, Harry Markowitz introduced what has since become known as the Markowitz Portfolio Selection Theory. In this paper, Markowitz stipulates that - under certain conditions - any investor can build an optimal risky portfolio by considering asset-specific return ($\mu$) and risk ($\sigma$, i.e. standard deviation or volatility) as the two essential factors.
However, the resulting portfolio’s expected return and risk are not merely the sum of these variables, as the riskiness of the portfolio is not only dependent on the riskiness of the individual assets it is composed of, but also depends on the correlation of these assets. As a result, it is possible to combine assets in such a way that the resulting portfolio is characterized by a higher return to risk ratio than provided by every single asset by itself, an effect known as diversification. Numerous extensions and modifications to Markowitz’ Theory have been published, all building and contributing to today’s Modern Portfolio Theory, most notably Tobin (1958) and Sharpe (1966). Despite criticism mainly focusing on the model oversimplifying reality through some of its assumptions (e.g. normally distributed returns, efficient markets), the model is still being taught in business schools worldwide, is spawning new areas of research each year (e.g. the inclusion of additional criteria into the optimization selection process) and is widely being used as the tool of choice (albeit often featuring modifications) by practitioners. In this paper, we revisit Markowitz’ Portfolio Selection Theory and propose a modification allowing to incorporate not only asset-specific return and risk but also a social responsibility measure into the investment decision making process.

As already mentioned, most of the existing literature on socially responsible portfolio optimization focuses on funds instead of specific assets. This can be attributed to the limited availability of accessible and unbiased social responsibility ratings on individual companies. As a result, researchers often relied on pre-screened SRI funds in order to avoid having to ascertain the social responsibility level of specific assets by themselves. However, there are serious repercussions to this approach. The composition of investment funds underlies unpredictable changes due to either market developments or managerial requirements, and in addition other factors like differences between fund companies’ social responsibility screening processes or the fee structure of actively managed funds can also heavily influence the analysis of SRI fund performance. In this regard, Utz et al. (2014)
for example find that there is no statistically significant difference between the allocation of assets in conventional and socially responsible funds and that socially responsible funds do not exhibit a higher social responsibility rating compared to their conventional counterparts. Some researchers are trying to circumvent this issue by creating their own social responsibility measures, but the problems associated with their approaches are obvious. Such measures often only manage to partly capture real world effects (e.g. the case with dichotomous social responsibility variables), lack reproducibility by externals, and result in studies not comparable to each other.

To avoid these problems, we rely on an unbiased and independent external measure of the social responsibility of individual companies. Thomson Reuters ASSET4 provides access to so-called ESG (i.e. environmental, social, governance) data on more than 4,300 international companies, dating back to 2001. Their “overall ESG score” measures the social responsibility of companies on a scale between 0 and 100\(^2\), is comparable across all companies and markets, and allows for straightforward, reproducible quantitative analysis of SRI\(^3\).

A representative data set of global stocks is required for the empirical analysis in this paper. In order to build such a data set of both conventional and socially responsible investments, we use two data sources. First, the constituents list of the Thomson Reuters Equity Global Index, a broad and international index containing a total of 9,253 stocks, both of conventional as well as socially responsible companies. Second, the constituents list of the Thomson Reuters ASSET4 database mentioned above, in order to add all ESG-rated companies to the data set. Stocks included in both constituent lists are just added

\(^{2}\)The best-rated company in the ASSET4 company universe features an ESG score of 100, while the worst-rated company has an ESG score of 0 (Thomson Reuters, 2012). For the purpose of this paper we divide ESG scores by 100, resulting in ESG scores ranging between 0 and 1.

\(^{3}\)Besides of the “overall ESG scores” mentioned above, Thomson Reuters ASSET4 also provides ESG ratings on a large variety ESG sub criteria.
once to the final data set, all stocks not rated in the ASSET4 ESG database are assigned an ESG score of 0.

We retrieve daily stock prices and ESG score time series data for all stocks included in the data set via Thomson Reuters Datastream. The number of observations is thereby limited by the historic availability of ESG scores in the ASSET4 database, thus our data set ranges from 2001 to 2012, with a maximum of 3,111 observations of daily stock prices per stock. Following this, we compute daily stock returns ($\mu$) as a financial performance measure and the standard deviation ($\sigma$) of said returns as a measure of financial risk. ESG scores ($\theta$) are used as social responsibility indicators. We exclude all stocks in both data sets, which do not provide daily stock prices for the full observation period. The total number of stocks included in our sample is 6,231. 2,924 companies exhibit a positive ESG score (i.e. SRIs), while 3,307 firms do not provide ESG scores (i.e. CIs).

Table (1) classifies all stocks included in the data set by industry sector (industrial, utilities, transportation, banks and loan, insurance, other financials) and ESG score availability, and provides descriptive statistics (numbers of stocks, sample shares, mean returns, mean standard deviations, mean ESG scores).
More than 77% of all companies included in the sample belong to the industrial sector, with about 44% of these having a positive ESG score. Average ESG scores vary between 0.3450 (other financials) and 0.5910 (utilities). The mean daily returns of ESG-rated companies are lower than the mean daily returns of non-ESG companies, with the same being true for the standard deviation of daily stock returns (the exception being Insurance).

Table 2 classifies all stocks in the data set by continent and ESG score availability.
Approximately 51% of all companies in the sample are located in Asia, 23% in North America, and 18% in Europe. The remaining 8% are spread over South America, Australia and Africa (no country codes are provided for 0.34% of all companies). While only 25.67% of all Asian companies exhibit a positive ESG score, more than 90% of Australian companies also do so. For all remaining continents, the shares of positively ESG-rated companies are higher than 54%. The highest mean ESG score can be found in Europe (0.6130), with Asia once again bringing up the rear (0.3582). Mean daily returns of ESG-rated companies are once again lower than the mean daily returns of non-ESG companies (the exception being Africa and Australia), with the same being true for the standard deviation of daily stock returns (exception: Australia).

Table 2 groups all companies included in the data set by ESG score levels (in steps of 20%) and highlights mean daily returns, mean standard deviations and mean ESG scores.
Here, companies with the highest mean ESG score (0.8830) exhibit the lowest mean return (0.0387%) and the lowest standard deviation of returns (2.1719%), while companies with the lowest positive mean ESG score (0.1085) exhibit mean returns similar to companies with no ESG Score (0.0720% vs 0.0716%), but lower standard deviation (2.80580% vs 2.9109%).

<table>
<thead>
<tr>
<th>θ</th>
<th>No. of Stocks</th>
<th>µ</th>
<th>σ</th>
<th>θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>81-100</td>
<td>492</td>
<td>0.0387%</td>
<td>2.1719%</td>
<td>0.8830</td>
</tr>
<tr>
<td>61-80</td>
<td>626</td>
<td>0.0474%</td>
<td>2.3577%</td>
<td>0.7027</td>
</tr>
<tr>
<td>41-60</td>
<td>575</td>
<td>0.0565%</td>
<td>2.5389%</td>
<td>0.4979</td>
</tr>
<tr>
<td>21-40</td>
<td>627</td>
<td>0.0610%</td>
<td>2.6851%</td>
<td>0.2976</td>
</tr>
<tr>
<td>1-20</td>
<td>604</td>
<td>0.0720%</td>
<td>2.8050%</td>
<td>0.1085</td>
</tr>
<tr>
<td>0</td>
<td>3,307</td>
<td>0.0716%</td>
<td>2.9109%</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 3: Data Set Descriptive Statistics - ESG Score Levels

In the following, we refer to this data set as Total Pool. For our empirical analysis, however, we also create a subset containing only stocks with positive ESG scores, i.e. a subset with socially responsible assets. We refer to this subset as SRI Pool.

Finally, we introduce a risk-free asset with an assumed daily return $\mu$ of 0 and an ESG score $\theta$ of 0.

3 Theoretical Model

In theory, investors interested in CIs and investors interested in SRIs are clearly separated and, thus, the performance of the different types of investments are analyzed utilizing different methods and theories. However, in reality, investors can usually not be clearly separated into "socially responsible investors" and "socially irresponsible investors". Hence, in the following, we shed light on the universe of feasible investment opportunities for risk-averse, $\mu/\sigma$-optimizing investors to limit themselves to utilizing only assets that are not deemed socially responsible.
investors deciding on the basis of their preferences for not only return and risk but also social responsibility.

Let \( n \) denote the number of risky securities \((i = 1, ..., n)\). In this group we further distinguish two subsets, SRI and CI denoted by \( n_{iSR} \) and \( n_{iCI} \), respectively.

In a traditional mean-variance portfolio selection, risk-averse investors maximize expected return \((\mu)\) and minimize return risk \((\sigma)\) as given by Equation 1:

\[
\max \alpha \mu - \beta \sigma, \tag{1}
\]

with \( \alpha \) representing the return preference parameter and \( \beta \) indicating the risk preference parameter measuring the level of risk aversion.

For our analysis we expand Equation 1 in order to allow investors to incorporate three preferences parameters:

\[
\max \alpha \mu + \gamma \theta - \beta \sigma, \tag{2}
\]

i.e. we enhance the well-known Markowitz (1952) approach by a third dimension, namely the social responsibility of risky assets. \( \theta \) denotes that social responsibility rating\(^5\) while \( \gamma \) indicates the social responsibility preference parameter of an investor. All preference parameters are expected to be \( \geq 0 \), since for a rational investor negative preference parameters would not make sense. The overall social responsibility of any portfolio (PF) of risky assets is thus given by \( \theta_{PF} = \sum_{i=1}^{n} \theta_i w_i \).

\[\text{The decision variables for investors are thus given by Equations 3 to 5.}\]

\[5\text{The social responsibility rating } \theta = 0 \text{ of all unrated CIs is } 0.\]
\[
\max \mu_{PF} = \sum_{i=1}^{n} \mu_i w_i 
\]
\[
\min \sigma_{PF} = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_i \sigma_j w_i w_j \rho_{ij} 
\]
\[
\max \theta_{PF} = \sum_{i=1}^{n} \theta_i w_i 
\]

Equations 3 and 4 are standard equations from Markowitz’ Portfolio Selection Model. Equation 5 assumes in line with Drut (2010) that social responsibility ratings are additive. This is also very often assumed by practitioners and rating agencies (see also Barracchini (2004) and Scholtens (2009) for an in-depth discussion). In addition, social responsibility ratings are expected to be time-independent from expected returns, i.e. a better social responsibility rating does not necessarily cause higher or lower returns. In line with several empirical studies (e.g. Basso and Funari (2014)), we assume the possibility that investors may be willing to give up a certain amount of return in order to reach the intended level of social responsibility (i.e. \(\alpha \to 0\)).

Following Dorfleitner and Utz (2012), we expect that investors do not care much about a change in the level of a company’s social responsibility rating after they have already decided to invest in that company. This means that we are able to neglect the risk of changes of social responsibility ratings.

The portfolio optimization as given by Equation 2 considering the constraint that the sum of portfolio weights equals one can easily be derived via maximizing the Lagrange function given by:

\[\text{This is also in line with Dorfleitner et al. (2012) and Basso and Funari (2014).}\]
\[
\max \Lambda : \alpha \mu + \gamma \theta - \beta \sigma + L(1 - \sum_{i=1}^{n} w_i)
\]  

(6)

We derive the necessary first order conditions for this optimization and simplify the separate terms as follows.

Define \(\alpha \mu\) as \(\vec{\alpha}\):

\[
\vec{\alpha} = \begin{pmatrix}
\alpha_{\mu_1} \\
\vdots \\
\alpha_{\mu_n} \\
1
\end{pmatrix}
\]  

(7)

Define \(\gamma \theta\) as \(\vec{\gamma}\):

\[
\vec{\gamma} = \begin{pmatrix}
\gamma_{\theta_1} \\
\vdots \\
\gamma_{\theta_n} \\
0
\end{pmatrix}
\]  

(8)

Define \(w\) as \(\vec{w}\):

\[
\vec{w} = \begin{pmatrix}
w_1 \\
\vdots \\
w_n \\
L
\end{pmatrix}
\]  

(9)
The variances and covariances are given as a doubled covariance matrix $C$:

$$
C = \begin{pmatrix}
\beta^2 C_{11} & \beta^2 C_{12} & \ldots & \beta^2 C_{1n} & 1 \\
\beta^2 C_{21} & \beta^2 C_{22} & \ldots & \beta^2 C_{2n} & 1 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\beta^2 C_{n1} & \ldots & \ldots & \beta^2 C_{nn} & 1 \\
1 & 1 & 1 & 1 & 0
\end{pmatrix}
$$

(10)

Finally, this leads to:

$$C^{-1}\vec{\alpha} + C^{-1}\vec{\gamma} = \vec{w}
$$

(11)

Based on the model derived above, we can now distinguish between different types of efficiency as summarized by Table 4. A fictitious risky asset $A$ is strictly preferred to asset $B$, if one of the rules depicted in Table 4 is true. The rules in the first column are the well-known basis of most $\mu/\sigma$-portfolio optimization approaches. A similar line of reasoning can be applied to $\theta/\sigma$-portfolio optimizations (second column), where social responsibility ratings take the place of asset returns. The third column indicates efficiency rules for investors optimizing their portfolios on the basis of their preferences of return, risk and social responsibility.

<table>
<thead>
<tr>
<th>$\mu/\sigma$ Efficiency</th>
<th>$\theta/\sigma$ Efficiency</th>
<th>$\mu/\theta/\sigma$ Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A &gt; B$</td>
<td>$\mu_A = \mu_B &amp; \sigma_A &lt; \sigma_B$</td>
<td>$\theta_A = \theta_B &amp; \sigma_A &lt; \sigma_B$</td>
</tr>
<tr>
<td>$\mu_A &gt; \mu_B &amp; \sigma_A = \sigma_B$</td>
<td>$\theta_A &gt; \theta_B &amp; \sigma_A = \sigma_B$</td>
<td>$\mu_A = \mu_B &amp; \theta_A &gt; \theta_B &amp; \sigma_A = \sigma_B$</td>
</tr>
<tr>
<td>$\mu_A &gt; \mu_B &amp; \sigma_A &lt; \sigma_B$</td>
<td>$\theta_A &gt; \theta_B &amp; \sigma_A &lt; \sigma_B$</td>
<td>$\mu_A = \mu_B &amp; \theta_A = \theta_B &amp; \sigma_A &lt; \sigma_B$</td>
</tr>
</tbody>
</table>

Table 4: Portfolio Efficiencies
4 Methodology

As outlined in Section (3), our model requires us to set up a covariance matrix (see Equations (10) and (11)) to compute the weights of optimized portfolios given specific sets of preference parameters. However, not all assets in our data sets were available over the entire observation period, which raises an issue since the shortest data availability of a single asset determines the length of the observation period for all assets included in the covariance matrix. Since we want to ensure the empirical validity of our results (i.e. included all assets with the maximal number of observations), we therefore choose to deviate from this model approach. For the empirical analysis in Section (5), we thus implement the theoretical model introduced in Section (3) via a dual-step optimization process detailed in the following.

A simulation approach is applied to be able to handle the large data set (6,231 stock, 12 years of daily returns data) outlined in Section (2). Firstly, a set of 50 stocks is picked out of the Total Pool superset using a random-draw procedure\(^7\). The probability of a stock being drawn is thereby uniformly distributed. Once this set has been chosen, the portfolio weights of the global minimum-variance-portfolio (MVP) consisting of these 50 stocks are calculated as given by Equation (12):

$$\min \sigma_{PF}, \text{s.t. } \sum_{i=1}^{n} w_i = 1,$$

which implies $\alpha = 0$, $\beta = 1$ and $\gamma = 0$.

Secondly, we run the dual-step optimization referred to above, with the MVP being the starting point for the construction of two different efficient frontiers, one return/risk-efficient ($\mu/\sigma$) frontier and one social responsibility/risk-efficient ($\theta/\sigma$) frontier. In the first

\(^7\)Following Statman (1987), we choose a set size of 50 stocks as basis for our optimization.
step, individual average stock returns and the covariance matrix of the set of 50 stocks are used as input variables for the optimization process. Using the MVP as the lower boundary and the maximum return portfolio $\text{MRP}$ as the upper boundary, we build 100 additional portfolios in between, in order to create the efficient frontier. This efficient frontier is thus composed of $\mu/\sigma$-optimal portfolios $PF_{\mu\sigma}$ with stepwise increasing returns. After the efficient frontier has been generated, a risk-free asset $rf$ ($\mu = 0, \sigma = 0, \theta = 0$) is introduced to the set of feasible assets and Sharpe Ratios $S$ (see Equation 13, following (Sharpe, 1966)) are calculated for all portfolios on the efficient frontier, to find the single-best risky portfolio $PF_{\text{max}}$ exhibiting the maximum Sharpe Ratio.

$$\max S = \frac{\mu_{PF} - \mu_{rf}}{\sigma_{PF}}$$ (13)

In the second step, individual average ESG scores and the same covariance matrix as in the first optimization are used as input variables for the optimization process (see Section (3)). Using the MVP again as the lower boundary and the maximum ESG portfolio $\text{MEP}$ as the upper boundary, we now build 100 additional portfolios to create the $\theta/\sigma$-efficient frontier. The portfolios $PF_{\theta\sigma}$ on this efficient frontier thus feature stepwise increasing ESG scores, while the risk of every portfolio is again minimized. Following this, we calculate the Delta Ratio $\delta$ (an efficiency ratio that relates social responsibility and risk in a way similar to how the Sharpe Ratio relates return and risk; see Equation 14)) for each portfolio on the efficient frontier, to find the single risky portfolio $PF_{\delta_{\text{max}}}$ exhibiting the maximum Delta ratio.

$$\max \delta = \frac{\theta_{PF}}{\sigma_{PF}}$$ (14)

In this paper, we limit the MRP by the highest average return of a single stock in the set of 50 stocks. While still allowing for some short-selling, this setup prevents the weights of individual stocks from reaching extreme values.

Analogous to the MRP, we limit the MEP by the highest average ESG score of a single stock in the set of 50 stocks.
Thirdly, we simulate this dual-step optimization 20,000 times with varying 50-stock-sets, in order to be able to obtain results that are representative of an optimization involving all stocks included in the chosen data set. This yields 20,000 Sharp Ratio-maximized portfolios \( PF_{S_{\text{max}}} \) and 20,000 Delta Ratio-maximized portfolios \( PF_{\delta_{\text{max}}} \) that can be plotted as portfolio clouds in a three-dimensional return/risk/social responsibility space.

Fourthly, we repeat all this with the SRI Pool subset.

Finally, we define two representative portfolios \( \hat{PF}_{S_{\text{max}}} \) and \( \hat{PF}_{\delta_{\text{max}}} \). These portfolios are characterized by expected daily returns, standard deviations and ESG scores equal to the means of the respective 20,000 Sharpe Ratio and the 20,000 Delta Ratio-optimized portfolios. Following \[\text{Tobin (1958)}\], we introduce the risk-free asset and proceed to set up a three-dimensional capital allocation plane (CAP) indicating all feasible combinations of the risk-free asset \( rf \) and the risky portfolios \( \hat{PF}_{S_{\text{max}}} \) and \( \hat{PF}_{\delta_{\text{max}}} \).

5 Results

In this Section, we first examine the results of the two 20,000 portfolio optimization simulations. As already mentioned, we run the simulations twice, once for all stocks in the Total Pool superset, and once for the subset of socially responsible investments (SRI Pool). Second, we build the capital allocation plane (CAP), spanning the representative \( \mu/\sigma \) and \( \delta/\sigma \)-optimized portfolios \( \hat{PF}_{S_{\text{max}}} \) and \( \hat{PF}_{\delta_{\text{max}}} \) as well as the risk-free asset \( rf \). Third, we shed light on the different possibilities investors have on choosing an optimal portfolio on the basis of the CAP.

Table (5) illustrates the mean results of the 20,000 Total Pool and the 20,000 SRI Pool simulations for the respective Sharpe Ratio-maximized portfolios \( PF_{S_{\text{max}}} \), and the Delta Ratio-maximized portfolios \( PF_{\delta_{\text{max}}} \).\[10]\]

\[10\] Histograms depicting Total Pool and SRI Pool results in more detail are presented in the appendix, see Figures 4(a) to 8(b) and 9(a) to 13(b) respectively.
Table 5: Simulation Results: Means of Risky Portfolios $PF_{\text{max}}$ and $PF_{\delta\text{max}}$

<table>
<thead>
<tr>
<th></th>
<th>Total Pool</th>
<th>SRI Pool</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Expected Daily Return $\bar{\mu}$</td>
<td>0.1613%</td>
<td>0.0346%</td>
</tr>
<tr>
<td>Mean Expected SD of Returns $\bar{\sigma}$</td>
<td>0.7540%</td>
<td>0.6445%</td>
</tr>
<tr>
<td>Mean Expected ESG Score $\bar{\theta}$</td>
<td>0.0855</td>
<td>0.9230</td>
</tr>
<tr>
<td>Mean Expected Sharpe Ratio $\bar{\gamma}$</td>
<td>0.2139</td>
<td>0.0537</td>
</tr>
<tr>
<td>Mean Expected Delta Ratio $\bar{\delta}$</td>
<td>0.1134</td>
<td>1.4321</td>
</tr>
</tbody>
</table>

In both the Total Pool as well as the SRI Pool, the mean daily returns of the Sharpe Ratio-optimized portfolios $PF_{\text{max}}$ (0.1613% and 0.1755%) are statistically significantly higher than those of the Delta Ratio-optimized portfolios $PF_{\delta\text{max}}$ (0.0346% and 0.0357%). Interestingly, at the same time, the returns of the $PF_{\text{max}}$ are not significantly different between pools, i.e. the stock screening process and limitation to SRIs (SRI Pool) does not seem to negatively affect expected returns for $\mu/\sigma$-optimized portfolios. This finding is in line with Renneboog et al. (2008), who show that funds investing exclusively in SRIs (but still determining fund composition via a $\mu/\sigma$-optimization) do not exhibit returns significantly different from conventional funds.

The mean standard deviation is the lowest in the two $PF_{\delta\text{max}}$, with 0.6445% and 0.5381%, as compared to the $PF_{\text{max}}$ (0.7540% and 0.8311%). ESG scores also vary greatly, from a mean of only 0.0855 (Total Pool: $PF_{\text{max}}$) to a mean of 0.9435 (SRI Pool: $PF_{\delta\text{max}}$). Similar to before, the ESG Scores of the $PF_{\delta\text{max}}$ are not significantly different from each other. Despite the $PF_{\text{max}}$ of the SRI Pool illustrating slightly higher returns, the higher mean Sharpe Ratio of the Total Pool: $PF_{\text{max}}$ (0.2139 vs. 0.2111), which is statistically significant, is caused by the decrease in risk as compared to the SRI Pool. Finally, Delta Ratios are also significantly different between pools, with the Total Pool $PF_{\delta\text{max}}$ exhibiting a ratio of 1.4321 and the SRI Pool $PF_{\delta\text{max}}$ featuring a ratio of 1.7534.

\footnote{Utz et al. (2014) emphasize that while SRI fund managers pre-screen assets for their social responsibility before adding them to asset pool of an SRI fund, they focus on financial performance (i.e. the are $\mu/\sigma$-optimizers) for determining the SRI fund’s final composition.}
On the basis of these results, it becomes clear that the choice between the Total Pool and the SRI Pool makes a more subtle difference, but the choice between the Sharpe Ratio-maximization and the Delta Ratio-maximization approach heavily impacts the expected return, risk and social responsibility of the respective optimal portfolio. One the one hand, with a view to the mean Sharpe Ratios, it is feasible to assume that investors aiming to maximize return would choose the Total Pool Sharpe Ratio-maximized Portfolio $PF_{S_{\text{max}}}$.

On the other hand, on the basis of the mean Delta Ratios, investors aiming to maximize social responsibility would choose the SRI Pool Delta Ratio-maximized portfolio $PF_{\delta_{\text{max}}}$.

For the setting up of the capital allocation plane, we thus define the first representative portfolio $PF_{S_{\text{max}}}$ as the mean of the Total Pool Sharpe Ratio-maximized portfolios $PF_{S_{\text{max}}}$ (see the black portfolio cloud in Figure 1) and the second representative portfolio $PF_{\delta_{\text{max}}}$ as the mean of the SRI Pool Delta Ratio-maximized portfolios $PF_{\delta_{\text{max}}}$ (see the grey portfolio cloud in Figure 1). Table 6 summarizes the properties of the two representative portfolios.

<table>
<thead>
<tr>
<th></th>
<th>Total Pool $PF_{S_{\text{max}}}$</th>
<th>SRI Pool $PF_{\delta_{\text{max}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Daily Return $\mu$</td>
<td>0.1613%</td>
<td>0.0357%</td>
</tr>
<tr>
<td>Expected Daily SD of Returns $\sigma$</td>
<td>0.7540%</td>
<td>0.5381%</td>
</tr>
<tr>
<td>Expected ESG Score $\theta$</td>
<td>0.0855</td>
<td>0.9435</td>
</tr>
<tr>
<td>Expected Sharpe Ratio $S$</td>
<td>0.2139</td>
<td>0.0663</td>
</tr>
<tr>
<td>Expected Delta Ratio $\delta$</td>
<td>0.1134</td>
<td>1.7534</td>
</tr>
</tbody>
</table>

**Table 6:** Simulation Results: Properties of Representative Risky Portfolios $PF_{S_{\text{max}}}$ and $PF_{\delta_{\text{max}}}$

Based on these two representative portfolios and a risk-free asset $(\mu = 0, \sigma = 0, \theta = 0)$, we are now able to span the capital allocation plane (see Figure 2).

---

12 We assume the correlation coefficient of returns between $PF_{S_{\text{max}}}$ and $PF_{\delta_{\text{max}}}$ to be 0.3. A slightly positive correlation coefficient seems the most likely choice, with the SRI Pool being a subset of the Total Pool superset. A lower correlation coefficient would decrease the set of feasible portfolio combinations, while a higher correlation coefficient would increase the set of feasible portfolio combinations.
Figure 1: The black portfolio cloud represents the 20,000 Sharp Ratio-maximized portfolios $PF_{S_{\text{max}}}$, calculated on the basis of the Total Pool. The gray portfolio cloud represents the 20,000 Delta Ratio-maximized portfolios $PF_{\delta_{\text{max}}}$, computed on the basis of the SRI Pool. Figure 1(a) depicts a 3D view of the results, while Figures 1(b) and 1(c) focus on 2D views of the return/risk and return/ESG axis respectively.
Figure 2: Point A designates the representative Sharpe Ratio-maximized portfolio $\hat{P}F_{S_{\text{max}}}$, while Point B designates the representative Delta Ratio-maximized portfolio $\hat{P}F_{\delta_{\text{max}}}$. Point C designates a risk-free asset ($\mu = 0$, $\sigma = 0$, $\theta = 0$). The gray capital allocation plane spanning points A, B and C represents the resulting set of feasible portfolio choices. Figure 2(a) depicts a 3D view of the results, while Figures 2(b) and 2(c) show 2D views of the return/risk and return/ESG axis respectively. The plane is constructed under the constraint of no short-selling.
Point A designates the representative Sharpe Ratio-maximized portfolio $\hat{PF}_{S_{\text{max}}}$, while Point B designates the representative Delta Ratio-maximized portfolio $\hat{PF}_{\delta_{\text{max}}}$. Point C designates a risk-free asset ($\mu = 0$, $\sigma = 0$, $\theta = 0$). The gray capital allocation plane spanning points A, B and C represents the resulting set of feasible portfolio choices for investors. Figure 2(a) depicts a 3D view of the results, while Figures 2(b) and 2(c) show 2D views of the return/risk and return/ESG axis respectively. The plane is constructed under the constraint of no short-selling.

On the basis of the CAP, investors can easily decide on their optimal portfolio given their respective preference parameters $\alpha$, $\beta$, and $\gamma$. $\mu/\sigma$-optimizing investors (the “Conventional Investors”), not caring about the social responsibility of their investments, will therefore choose a portfolio on the line between points C (the risk-free asset $rf$) and A (the representative Sharpe Ratio-maximized portfolio), their choice solely depending on how much of their budget they want to invest in the risk-free asset and in the risky portfolio, respectively. $\theta/\sigma$-optimizing investors (the “Socially responsible Investors”), disregarding the return of their risky portfolio, will choose a portfolio on the line between the risk-free asset $rf$ (point C) and the representative Delta Ratio-maximized portfolio (point B), their choice depending on how much of their budget they want to invest in the risk-free asset and in the risky portfolio, respectively. Apart from these two types of investors, the capital allocation plane provides the means for all types of investors to choose their optimal combination of both risky portfolios and the risk-free asset. Thus, by using the CAP, it becomes easy for investors to determine for example how much expected return has to be “sacrificed” in order to achieve a certain ESG score or how high the ESG score would be for a specific expected return and a given level of risk.\textsuperscript{13}

\textsuperscript{13}Please note that even though the CAP depicted in Figure 2 is explicitly constructed under a no-short selling constraint, it is of course possible for all types of investors to short-sell either one of the risky portfolios $\hat{PF}_{S_{\text{max}}}$ or $\hat{PF}_{\delta_{\text{max}}}$ or the risk-free asset in order to reach even more beneficial portfolio combinations (again, given the investors’ individual preference parameters) on an extended CAP.
There are two ways that investors might use to select their optimal portfolio on the basis of the CAP.

After setting up the CAP, investors could simply determine the level of one of the three output parameters (i.e. return, risk, social responsibility). This results in a limited set of feasible portfolios that are all exhibiting the same level of that one fixed parameter, but that are at the same time different from each other in terms of the two non-fixed parameters. Figure 3 demonstrates this graphically for three exemplary cases. Line I indicates all feasible portfolios with a fixed expected return of 0.1%. Line II indicates all feasible portfolios with a fixed expected standard deviation of 0.25 and line III indicates all feasible portfolios with a fixed expected ESG score of 0.3.

Furthermore, given that investors have knowledge about their individual preference parameters, it would of course also be possible that investors insert their specific preference parameters into Equation (6). This would also yield an optimal portfolio situated on the CAP.

\footnote{The fixed values used for the output parameters in this example were picked at random.}
Figure 3: Point A designates the representative Sharpe Ratio-maximized portfolio $\hat{PF}_{S_{\text{max}}}$, while Point B designates the representative Delta Ratio-maximized portfolio $\hat{PF}_{\delta_{\text{max}}}$. Point C designates a risk-free asset ($\mu = 0$, $\sigma = 0$, $\theta = 0$). The gray capital allocation plane spanning points A, B and C represents the resulting set of feasible portfolio choices. I indicates all feasible portfolios exhibiting a fixed expected return of 0.1%. II indicates all feasible portfolios exhibiting a fixed expected standard deviation of 0.25. III indicates all feasible portfolios exhibiting a fixed expected ESG score of 0.3.
6 Conclusion

In this paper, we revisit Markowitz’ Portfolio Selection Theory and propose a modification allowing to incorporate not only asset-specific return and risk expectations but also a social responsibility measure (i.e. Datastream’s ESG scores) into the investment decision making process.

We apply a two-step simulation approach to calculate a representative Sharpe Ratio-optimal portfolio and a representative Delta Ratio-optimal (i.e. ESG/risk-optimal) portfolio. Together with a risk-free asset, we are thus able to create a capital allocation plane in a three-dimensional return/risk/social responsibility space. This capital allocation plane allows investors to custom-tailor their asset-allocations and incorporate all personal preferences regarding risk, return and social responsibility into their portfolio choice.

In our empirical analysis on the basis of a data set with 6,231 publicly traded companies, we find that investors caring about the social responsibility of their investments do face a statistically significant decrease in both expected returns and risk exposure, with the Sharpe Ratios being also significantly lower than those of a return/risk-optimal portfolio. However, it is interesting to note that investors may attain optimal portfolios exhibiting a modest social responsibility rating by accepting only a very limited decrease of the resulting portfolio’s Sharpe Ratio.
References


7 Appendix

Figure 4: Distribution of Return of Total Pool Simulation
Figure 5: Distribution of Standard Deviation of Total Pool Simulation
Figure 6: Distribution of ESG Score of Total Pool Simulation
Figure 7: Distribution of Sharpe Ratio of Total Pool Simulation
Figure 8: Distribution of Delta Ratio of Total Pool Simulation
Figure 9: Distribution of Return of ESG Pool Simulation
Figure 10: Distribution of Standard Deviation of ESG Pool Simulation
Figure 11: Distribution of ESG Score of ESG Pool Simulation
Figure 12: Distribution of Sharpe Ratio of ESG Pool Simulation
Figure 13: Distribution of Delta Ratio of ESG Pool Simulation