



WU International Taxation Research Paper Series

No. 2015 - 19

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to international tax differentials**

Sebastian Beer

Editors:

Eva Eberhartinger, Michael Lang, Rupert Sausgruber and Martin Zagler (Vienna University of Economics and Business), and Erich Kirchler (University of Vienna)

Electronic copy available at: <http://ssrn.com/abstract=2610616>

Research Papers

Productive and unproductive reactions of MNEs to international tax differentials*

Sebastian Beer[†]

May 2015

Abstract

I provide initial evidence that international taxation impacts the magnitude of R&D investments in the UK. Relying on a simple theoretical model, I show that the observed response is consistent with the mispricing of intra-group transactions. My structural estimates suggest that transfer prices for internally provided innovation increase by around three percent in response to a one percentage point increase in foreign taxation. This response is more pronounced than previous estimates of aggregate profit shifting indicate, supporting concerns on the susceptibility of intangible assets to facilitate profit relocation.

*I would like to thank Jan Loeprick, Andreas Wagener, and Martin Zagler for their helpful comments. I also thank the DIBT program to provide a forum to discuss the findings of this paper. Financial support from the Austrian Science Fund (FWF: W 1235-G16) is gratefully acknowledged. All remaining errors and inaccuracies are, of course, my own.

[†]University of Economics and Business; email: sebastian.beer@wu.ac.at

1 Introduction

An increasing share of R&D activity in the UK is directed at enhancing sales in foreign markets and provided to affiliated companies (Griffith, Miller, and O’Connell, 2010). International taxation thus likely impacts two related decisions of innovation centers in the UK: how much research should be provided to affiliated companies and at what price. Studies of firm level micro data panels indicate that tax rates drive both the volume (Grubert, 2003) and the pricing (Clausing, 2003; Bernard, Jensen, and Schott, 2006) of multinational enterprises’ internal transactions. To date, the interaction of these decisions has, however, received little attention in empirical research.

In this paper, I show in a simple theoretical model that aggressive tax planning increases the optimal investment into R&D. Using a panel dataset of 1611 UK based firms, I investigate the sensitivity of research investments to changes in national and international taxation. Consistent with the theoretical prediction, I find a positive marginal impact of observable transfer mispricing on the magnitude of R&D investments. In other words, my empirical results suggest that the level of immobile research activity in the UK is higher than it would be if multinational enterprises (MNEs) could not leverage differences in international taxation. Research is an important driver of global productivity with significant spill-over effects both locally and internationally (Hall et al., 2009). An increased spending on R&D may thus at least partly offset the welfare losses associated with an eroding tax base.

The findings presented in this paper relate to earlier work by Becker and Riedel (2012), showing that corporate taxation at the parent level drives the investment decision of foreign subsidiaries. Specifically, the authors identify two transmission channels, the use of a common input and profit shifting, through which a tax externality can emerge.¹ Their empirical results indicate that profit shifting increases capital investment. Expanding on this initial observation, I show that a similar mechanism exists for investments into R&D. Given some flexibility in the setting of internal prices, R&D activity serves a double purpose: to enhance foreign sales and to lower the group’s effective tax rate. Due to the additional return, the optimal investment into R&D increases.

In order to quantify the implied price manipulations, I draw on prior work which assumes that the (unobservable) marginal costs of profit shifting are proportional to the

¹In an earlier version of the paper, Becker and Riedel show that credit constraints are another possible reason for cross-border tax effects.

ratio of shifted to true income (Hines and Rice, 1994; Huizinga and Laeven, 2008). This cost specification suggests that the marginal net return to profit shifting is a quadratic function of the tax difference to other affiliates. A correlation between research investments and a squared tax differential is thus indicative for the tax efficient alignment of internal prices. My results suggest that a one percentage point increase in foreign taxation inflates internally used prices by around 3%.

The observed response is stronger than previous estimates of aggregate profit shifting indicate. In a meta-study Heckemeyer and Overesch (2013) report an average semi-elasticity of taxable profits of around 1. Due to a lack of data, there only exists scarce direct evidence on the manipulation of internal prices. Using monthly observations on US trade from the International Price Program, Clausing (2003) finds that a one percentage point increase in foreign taxation decreases (increases) the internal price of exports (imports) by roughly two percent. Relying on the Linked/Longitudinal Firm Transactions Database, Bernard et al. (2006) find that the export price for intra-firm transactions is, on average, between 54% and 67% below its arm's length equivalent. Their results suggest that a one percentage point increase in foreign taxation reduces the gap between arm's-length and related-party prices by around 0.60 percent. In light of these findings, my estimates support concerns on the susceptibility of intangible assets to facilitate profit shifting.

My work thus contributes in two related ways to the existing literature. First, I show that international taxation and the ability to manipulate prices impacts real decisions. Prior research has predominantly focused on intangible concepts, including reported income (Hines and Rice, 1994; Huizinga and Laeven, 2008), the capital structure of the firm (Huizinga et al., 2008) or the location of intellectual property (Griffith et al., 2014). While of all these dynamics have important implications for policy makers, their economic impact is potentially confined to governmental budgets. Becker and Riedel (2012) clarify that the tax base eroding effect of profit shifting is partly compensated by the indirect effect of an increased investment level. My results provide more granularity by illustrating how transfer-mispricing increases the magnitude of R&D activity in the UK.

Second, I present a new strategy to quantify the magnitude of profit shifting of MNEs. Prior work (e.g. Hines and Rice, 1994; Huizinga and Laeven, 2008; Beer and Loeprick, 2015) typically explains reported profits with a range of firm-specific variables. A residual correlation with tax differentials is then seen as evidence for the tax-efficient manipulation of prices. These studies hinge on the assumption that all productive factors are observable. Otherwise, the residual correlation could be a consequence of credit constraints, the use of a

common good across MNE groups, or internal transactions at fair value. My identification strategy does not rely on this critical assumption. Importantly, my estimates do not suggest that prior findings are severely biased.

The remainder of this paper is structured as follows. In the first section, I develop the theoretical underpinnings to understand the effect of the international tax environment on R&D investment decisions in the UK. The model allows me to derive expressions for the domestic and foreign user costs of R&D investments. These expressions then guide my estimation strategy in section 3. I present my empirical results in section 4, assess their quantitative implication in section 5 and provide conclusions in section 6.

2 The optimal provision of research for foreign and domestic markets

In this section, I consider the optimal investment and pricing decision of an MNE's innovation center seeking to maximize global after-tax profits. The innovation center's output stimulates sales of domestic and foreign affiliates. In order to mitigate tax arbitrage opportunities, transfer pricing regulations stipulate that the internal price used for these transactions reflects the price which would prevail between independent parties.² In practice, however, markets for comparable transaction do often not exist and the true value of innovation provided within a group is, usually, difficult to establish.³ I thus assume that the innovation center has some leeway in the setting of internal prices.

Modeling the investment into and pricing of research as a simultaneous decision problem allows me to investigate the impact of price manipulations on real activity. The result of this model are simple expressions for the user cost of research which then guide my empirical approach in the next section. I start by describing the model's basic structure in section 2.1. Thereafter, I discuss the innovation center's decision problem and optimal magnitude of research provided (2.2). Section 2.3 presents the implied user cost of research.

2.1 Firm structure and technology

The MNE group consists of the innovation center and N affiliates, one of which is, like the innovation center, domiciled domestically. The other affiliates are located in $N - 1$ foreign

²The arm's length principle is defined in Article 9 of the OECD Model Tax Convention

³See BEPS Action 8 on hard to value intangibles.

jurisdictions. The innovation center employs a linear technology in the domestic economy (Country 1) to transform labor into research. A tax credit of size $\mu > 1$ subsidizes research expenses at a tax rate of τ_1 . The innovation center's after-tax profits thus read

$$\pi_{In}(p, R) = [(1 - \tau_1)p - (1 - \mu\tau_1)w]R, \quad (1)$$

where p is a transfer price, w is the wage rate and R is the total magnitude of research provided.

R&D activity fosters the group's global sales. The mechanism of how this takes place is not clear-cut and may vary across industries. While some innovations are easily transferable between jurisdictions (trademarks and brands), other technologies need to be adapted to market characteristics (the recipe of food products) or are not transferable at all. I account for this ambiguity by defining the following concave, country-specific sales functions

$$f_i(\alpha R_i + (1 - \alpha)R), \quad \text{for } i = 1, \dots, N \quad (2)$$

where $f_i(x) = \theta_i f(x)$ and $f(x)$ is a CES-production function, R_i denotes research directed at country i and $\alpha \in [0, 1]$ is a parameter. This sales function nests two extremes I will focus on in my empirical strategy: R&D activity may be deemed to produce a non-rival good, simultaneously contributing to the sales of an MNE's operations ($\alpha = 0$), or it may create a perfectly divisible and rival intermediate input ($\alpha = 1$). I will refer to these cases as the *common input* and *intermediate input* scenario, respectively.

The MNE's affiliates reimburse the innovation center for the service of stimulating sales. For accounting consistency, the total amount of research received and payed for needs to match the total amount of research produced. Responding to the variability of the sales function, affiliate i thus pays some price for the quantity $\alpha R_i + (1 - \alpha)\frac{R}{N}$. Summing this quantity over i gives R for all values of α .

In the absence of price manipulations, global after-tax profits read

$$\pi(p, \mathbf{R}) = \sum_{i=1}^N \left\{ f_i(\alpha R_i + (1 - \alpha)R) - p \left[\alpha R_i + (1 - \alpha)\frac{R}{N} \right] \right\} (1 - \tau^i) + \pi_{In}(p, R). \quad (3)$$

where τ_i denotes the corporate tax rate in country i and $\mathbf{R} = (R_1, \dots, R_N)$ is a vector of country-specific research investments. This expression shows that the optimal distribution and magnitude of research is clearly determined by tax rates and production possibilities.

What is more, global profits are a linear function of the internal price used. If there were no regulations determining this price, the MNE could, in principle, boundlessly leverage differences in taxation.

The arm's length principle thus stipulates that the price for internal transactions reflects a price which would arise between independent parties. Yet, due to a lack of comparable transactions, this principle is often difficult to enforce, giving the innovation center some leeway in the setting of internal prices. Specifically, the MNE may set a price p deviating from a price \hat{p} which is in line with transfer pricing regulations. I will refer to the latter price as the arm's length price. The manipulation of transfer prices comes at a cost, however. Prior work on profit shifting (see e.g. Hines and Rice, 1994; Huizinga and Laeven, 2008) typically assumes that related costs are proportional to the square of shifted income and inversely proportional to true income. In the current framework, shifted income is given by $(p - \hat{p})R$. If the innovation center's true profits are proportional to $\hat{p}R$, an assumption satisfied by the most common production functions when combined with optimizing behavior, this cost specification translates into

$$C(p, R) = \frac{1}{2\gamma} \frac{(p - \hat{p})^2}{\hat{p}} R. \quad (4)$$

The parameter $\gamma > 0$ determines the overall costs of profit shifting. As γ tends to zero, price manipulations become prohibitively expensive.

2.2 The optimal magnitude and price of research

The innovation center seeks to maximize global after-tax profits, by choosing research investments and transfer prices, while accounting for the costs of price manipulations. This optimization problem is summarized by:

$$\max_{p, \mathbf{R}} \pi^G = \pi(p, \mathbf{R}) - C(p, R) = \pi(\hat{p}, \mathbf{R}) + S(p, R). \quad (5)$$

The second equality rewrites the MNE's objective as the sum of two components including the after-tax global profits in the absence of price manipulations and the net return to

profit shifting:

$$S(p, \mathbf{R}) = (p - \hat{p}) \sum_{i=1}^N \left[\alpha R_i + (1 - \alpha) \frac{R_i}{N} \right] (\tau_i - \tau_1) - C(p, R). \quad (6)$$

This latter representation allows for a concise and intuitive description of the mechanics of the optimal solution. Global profits in the absence of profit shifting are clearly not affected by the choice of p . The profit-maximizing price is thus uniquely determined by the net return to profit shifting. Specifically, the first order condition, $\partial S(p, R)/\partial p = 0$, implies

$$\frac{p - \hat{p}}{\hat{p}} = \gamma [\alpha \tau_I + (1 - \alpha) \tau_C - \tau_1], \quad \text{where} \quad \tau_I = \frac{1}{R} \sum_{i=1}^N R_i \tau_i, \quad \text{and} \quad \tau_C = \frac{1}{N} \sum_{i=1}^N \tau_i \quad (7)$$

are a weighted and an unweighted average global tax rate, respectively. Accordingly, the optimal transfer price is inflated (i.e. above its arm's length equivalent) with the scale of manipulation depending on the cost parameter γ , if domestic taxation is below the foreign average.

Combining the optimal price with (4) and (6) implies two important conclusions. One is that the net return to profit shifting is strictly increasing in the absolute tax differential. Groups with larger differences in corporate taxation thus profit to a higher degree from the ability to manipulate prices. The other one is that the marginal net return to profit shifting, $\frac{\partial S(p, \mathbf{R})}{\partial R_i}$, is for a finite γ strictly positive at an optimal price. As a consequence, the set of first order conditions for an optimal level of research directed at market i , given by

$$\frac{\pi(p, \mathbf{R})}{\partial R_i} = - \frac{\partial S(p, \mathbf{R})}{\partial R_i} \quad \text{for all } i, \quad (8)$$

indicates that the magnitude of research directed at foreign markets is higher than it would be in the absence of profit shifting.⁴ The result follows the model's basic intuition. With some flexibility in the setting of internal prices, the provision of foreign research serves a double purpose: to stimulate sales in the foreign economy and to leverage differences in taxation.

⁴This follows from the concavity of the profit function.

2.3 The user cost of research

Building on the optimality conditions above, I derive explicit expressions for the user cost of research for the common input ($\alpha = 0$) and the intermediate input ($\alpha = 1$) scenario. These relations then guide my empirical strategy in the next section. In order to avoid multicollinearity and increase the accuracy of my estimates, I also derive an explicit expression for the arm's length price, the MNE takes as given in its decision problem.

The arm's length principle suggests that \hat{p} is the price balancing the demand (supply) for (of) research in a scenario where the affiliates are independent profit maximizers. For consistency with the above analysis, I assume that independent innovation centers employ the MNE's technology. Under perfect competition in factor and product markets, a linear production technology imposes a zero profit condition. Using equation (1), this suggests that the equilibrium price among independent producers is given by

$$\hat{p} = w \frac{1 - \mu\tau_1}{1 - \tau_1} \equiv \rho^D. \quad (9)$$

In practice, market prices do not necessarily reflect marginal production costs.⁵ This might particularly be true for intangible intensive industries. Building on the above expression, I thus consider a slightly more general definition of the arm's length price. Specifically, I assume that the arm's length price is of the form $\hat{p} = (1 + \varepsilon)\rho^D$, where ε denotes a mark up over marginal production costs.

By combining this price with the definition of global profits (3) and the first order conditions above yields the following explicit expressions for the user cost of research:⁶

$$h'(R) = \rho^D \left[\frac{1 - \tau_C}{1 - \tau_W} - \varepsilon \frac{\tau_C - \tau_1}{1 - \tau_W} - \gamma(1 + \varepsilon) \frac{1}{2} \frac{(\tau_C - \tau_1)^2}{(1 - \tau_W)} \right], \quad (10a)$$

$$f'_i(R_i) = \rho^D \left[1 - \varepsilon \frac{\tau_i - \tau_1}{1 - \tau_i} - \gamma(1 + \varepsilon) \frac{1}{2} \frac{(\tau_i - \tau_1)^2}{(1 - \tau_i)} \right], \quad \text{for all } i \neq 1. \quad (10b)$$

The first line is based on the common input scenario and relates the optimal investment into research to global sales, denoted by $h(R) = \sum_i \theta_i f(R)$. The variable $\tau_W = \sum_{i=1}^N \theta_i \tau^i / \sum_{j=1}^N \theta_j$ is a weighted average of global tax rates. The second line defines the optimal investment into an intermediate input directed at enhancing sales in country i .

The equations bear two important implications. First, a correlation between research

⁵See Martins et al. (1996) for a survey of profit margins in the OECD.

⁶See Annex A for the derivation.

expenses and foreign tax rates are not necessarily indicative of an illegal manipulation of prices. Even if the adjustment of transfer prices is prohibitively costly ($\gamma = 0$) and competition imposes a zero profit condition on MNE’s innovation centers ($\varepsilon = 0$), domestic research activity remains correlated with foreign tax rates unless global tax rates are equalized. Second, some flexibility in the setting of internal prices ($\gamma > 0$) unambiguously lowers the user cost of domestic research in both scenarios considered. The optimal investment into R&D is thus higher than it would be in the absence of profit shifting. Importantly, these equations show that a positive correlation between research expenses and a squared tax differential can be given a structural interpretation, indicating that the manipulation of prices is not prohibitively costly.

3 Empirical Approach

3.1 Sample selection and descriptive statistics

My firm-level data is extracted from the ORBIS database and covers the period from 2005 to 2012. I download information on balance sheet items, the profit and loss account as well as the group structure for all UK firms, reporting a positive value of research and development expenses. The vast majority of the sample, comprising approximately 2500 firms, is part of a larger group. To obtain estimates of the overall group size and on the extent of foreign operations, I identify registered group members (2175 subsidiaries and 2200 parents are associated with an ownership of more than 50%) and merge their firm-level data with the sample of R&D performing firms. I only account for foreign subsidiaries with unconsolidated accounts.

Table 1

Sample Selection		
Step	Description	Firms
1.	Downloaded	2,163
2.	At least 2 years reported R&D expenditure	1,696
3.	Ebit Margin $\in (-1, 1)$, Research intensity < 1	1,611

I drop firms which report information on research expenditures for less than 2 years to increase the accuracy of my estimates. Furthermore, I remove observations with extreme profitability ratios (exceeding -1 or 1) and with research intensities (defined as the ratio of

current research expenses to total assets) exceeding one. This reduces my baseline sample to 1611 firms (see Table 1).

Table 2 presents information on the sample composition and the distribution of the main variables. In line with prior studies (Dischinger and Riedel, 2011) I add a small constant to the level of research expenditures in order to avoid losing information on subsidiaries that do not report research expenditures in a given year. On average, firms in my dataset spend around 3.6% of their domestic turnover on R&D. Around 38% of the observations are classified as SMEs (small or medium enterprises).⁷ For these firms, a higher tax credit applies. Around 36% of the observations in my baseline sample are classified as MNEs.

Table 2

Sample descriptives						
Variable	Observations	Min.	1st Qu.	Mean	3rd Qu.	Max.
R	5684	1	105	2890	1407	381000
y^D	5684	17	8118	78510	48310	5364000
y	4036	86	8212	91270	49560	3E+07
ρ	5684	0,61	0,67	0,81	0,91	0,92
SME	5684	0	0	0,38	1	1
Multi	5684	0	0	0,36	1	1
s	5684	0	0	0,67	0	729
τ^d	5425	-0,28	0	0,03	0	0,69
τ^{dd}	5425	0	0	0,01	0	0,21
τ^c	5434	-0,26	0	0,03	0	0,62
τ^{cc}	5434	0	0	0,01	0	0,26

The descriptive statistics on the first three variables, research expenses (R), domestic (y^D) and global (y) sales, is in thousands of pounds. ρ is my measure of the domestic user cost of research investments. SME , a dummy variable, is one for firms that are classified as a small or medium sized enterprise according to HMRC definition. $Multi$ takes the value of one if the observation is affiliated with foreign firms. s is the ratio of foreign to domestic sales. The tax variables are defined in the text.

⁷Until August 2008, the HMRC defined small and medium enterprises (SMEs) to be companies with fewer than 250 employees and either an annual turnover not exceeding 50 million or a balance sheet not exceeding 43 million. Thereafter all thresholds were doubled. Under the SME scheme the rate of relief for corporation tax purposes is 225% from April 2012 onwards. It was 150% until July 2008 and 175% until April 2011 where it increased to 200%. Until 2009 to be eligible the SME needs to own an intellectual property arising from R&D expenditure.

3.2 Estimation approach

Building on the theoretical model, my baseline regressions take two forms. For the common input scenario, equation (10a) suggests the following regression specification:⁸

$$\ln(R)_{it} = \alpha_1 y_{it} + \alpha_2 \ln(\rho_{it}) + \alpha_3 \tilde{\tau}_{it}^c + \alpha_4 \tilde{\tau}_{it}^{cc} + \gamma' \mathbf{z}_t + \nu_i + \epsilon_{it}, \quad (11)$$

where my dependent variable, R , is reported research and development expenses, y is the logarithm of global sales, the domestic user cost is given by $\rho = (1 - \mu\tau_1)/(1 - \tau_1)$ and the tax variables are calculated according to

$$\tilde{\tau}^c = \frac{\tau_C - \tau_1}{1 - \tau_W} \quad \text{and} \quad \tilde{\tau}^{cc} = \frac{(\tau_C - \tau_1)^2}{1 - \tau_W}, \quad (12)$$

where definitions of these variables are given above.

A straight-forward test of the intermediate input scenario requires information on the magnitude of R&D expenses incurred by MNEs in one country, to foster sales in other countries of operation. Unfortunately, this information is not publicly available. However, if research directed at foreign markets is small relative to research for the domestic economy, the available data on overall R&D expenses can be used to derive a testable regression specification (see Hines, 1993, for a similar approach). I present details on the derivation of my second estimating equation in Annex B:

$$\ln(R)_{it} = \beta_1 y_{it}^D + \beta_2 \ln(\rho_{it}) + \beta_3 s_{it} + \beta_4 s_{it} \tilde{\tau}_{it}^I + \beta_5 s_{it} \tilde{\tau}_{it}^{II} + \gamma' \mathbf{z}_t + \nu_i + \epsilon_{it}. \quad (13)$$

The variable y^D denotes the logarithm of domestic sales, s is the ratio of foreign to domestic sales and the tax variables are calculated according to

$$\tilde{\tau}^d = \frac{1}{N_i} \sum_{j \in A_i} \frac{\tau_j - \tau_1}{1 - \tau_j} \quad \text{and} \quad \tilde{\tau}^{dd} = \frac{1}{N_i} \sum_{j \in A_i} \frac{(\tau_j - \tau_1)^2}{1 - \tau_j}. \quad (14)$$

where the summation is over all foreign countries, observation i operates in which amounts to N_i countries.

The vector \mathbf{z} controls for time-specific shocks by including GDP per capita, the unemployment rate and the inflation rate. The error component allows for firm-specific fixed effects ν_i . Both specifications nest a standard cost of (research) capital specification for

⁸See Annex B for details on the derivation.

purely domestic firms.⁹ The coefficient α_2 (β_2) thus measures the sensitivity of research activity to changes in the domestic production structure.¹⁰ It is expected to take a negative sign and can be interpreted as the elasticity of substitution in global (domestic) production.

In the common input scenario, international taxation directly affects the optimal investment into R&D. The coefficient α_3 reflects the legal response to a changing tax environment while α_4 is associated with the potential of manipulating internal prices. Both tax variables are expected to decrease the user cost of research, implying positive coefficients. In the intermediate input scenario, the impact of international taxation depends on the share of research directed at foreign markets. The ratio of foreign to domestic sales, s , proxies for this share. The coefficient β_3 is expected to take a positive sign. Imperfect competition suggests that the coefficient β_4 is positive. Profit shifting opportunities are, again, captured by the second tax differential. If price manipulations are not prohibitively costly, β_5 is expected to take a positive sign.

4 Results

4.1 Baseline results

Table 4 presents my estimation results. I investigate the absolute price sensitivity of research investments in the UK with my first specification. The estimated coefficient suggests that a one percent increase in the user cost decreases R&D activity by 0.73 percent. In the second specification, I estimate the output-constant price effect. Since output responds to changes in the cost structure, my finding of a partial elasticity of 0.51 for the price variable understates the total impact of changes in user cost. Both effects are statistically significant at the 1% and 10% level and consistent with prior results (see Hall and Van Reenen, 2000, for a summary of the literature).

In the third specification, I add employment rates, income levels, and inflation rates as control variables at the country level to account for cost factors that are not related to tax. The coefficients of these controls are significant and follow standard assumptions. As employment rises, wages increase and add to the cost of research activity. GDP per capita affects research investments positively and inflation negatively.

⁹In the common input scenario, $\tau^{Fc} = \tau_1$ for purely domestic firms. The tax variables are thus zero. In the intermediate input scenario, the sales ratio is zero for domestic firms.

¹⁰I calculate the domestic user cost according to: $\rho_{it} = (1 - \mu_{it}\tau_{it}^D)/(1 - \tau_{it}^D)$, where τ_{it} denotes the firm-specific tax rate in year t and μ_{it} denotes a tax credit.

In columns (4) to (7), I investigate the impact of international taxation on research investments in the UK. The first two specifications are based on the assumption that R&D activity produces a common input, simultaneously stimulating the sales of an MNE's operations. In this scenario, the optimal magnitude of research activity is a function of global sales, domestic and international tax incentives. The coefficients on the domestic user cost and on global sales vary slightly, when compared to the previous specification. Their sum, however, remains remarkably constant. The partial effect of the international tax variables is, as expected, positive. The estimated coefficients are, however, not statistically significant in column (4). My theoretical model suggests that the first tax variable coefficient captures the mark up of independent producers. Under competitive pricing, this coefficient is zero. When incorporating this restriction in specification (5), the coefficient on the squared tax differential increases with significance at the 10% level. In line with my predictions developed in section 2, this partial correlation provides indirect evidence on profit shifting behavior.

Specifications (6) and (7) are based on the intuition of research expenditures financing a perfectly divisible and rival intermediate input, most of which is provided to domestic affiliates. In this scenario, the optimal magnitude of R&D activity is a function of domestic sales and tax incentive, as well as the ratio of foreign to domestic sales, interacted with foreign tax variables. The estimated coefficient on the sales ratio is positive. Holding domestic sales constant, this suggests that more sales abroad increase research activity in the UK, confirming the premise that a share of domestic R&D activity is directed at enhancing sales in foreign markets. The estimated effect is, however, only barely significant when controlling for macro economic shocks (specification 7).

Table 3

Fixed effects OLS estimation, Years: 2005-2012							
Dependent: logarithm of R&D expenses							
Specification Assumptions				Common Input		Intermediate Input	
Explanatory variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\ln(\rho)$	-0.73*** (0.27)	-0.51* (0.26)	-0.62** (0.29)	-0.66** (0.31)	-0.61** (0.31)	-0.52** (0.26)	-0.62** (0.29)
y		0.55*** (0.06)	0.52*** (0.06)	0.47*** (0.07)	0.47*** (0.07)	0.52*** (0.05)	0.50*** (0.06)
s						0.01 (0.00)	0.01* (0.00)
$\tilde{\tau}$				0.62 (0.96)			
$\tilde{\tau}^2$				1.91 (2.59)	2.97* (1.80)		
$s:\tilde{\tau}$						0.02** (0.01)	0.02* (0.01)
$s:\tilde{\tau}^2$						0.12*** (0.04)	0.12*** (0.04)
$\ln(\text{GdpPc})$			0.93** (0.43)	1.47*** (0.48)	1.52*** (0.49)		1.08** (0.43)
Inflation			-0.04** (0.02)	-0.06*** (0.02)	-0.06*** (0.02)		-0.04** (0.02)
$\ln(\text{Emp})$			-5.76** (2.69)	-9.36*** (3.46)	-9.05*** (3.43)		-6.47** (2.75)
Observations	5684	5684	5585	3962	3962	5425	5335
Adj. R^2	0.00	0.03	0.04	0.04	0.04	0.03	0.04

Notes: *, **, and *** indicate significance at the 10%, 5% and 1% level. Robust standard errors in parentheses.

The coefficients on the tax variable interactions represent the legal and illegal response to a changing tax environment. The coefficient on the first tax variable is positive and significant in both specifications. As opposed to a common input scenario, I thus find indirect evidence of a positive mark up among independent producers. Tax differences to foreign affiliates may thus help explain variations in the magnitude of research investments in the UK. Importantly, my model suggests that this correlation is not related to profit shifting activity. The second coefficient on the squared tax variable, however, captures the effect of price manipulations. It is positive and significant in both specifications; indicating

that groups with larger tax differences spend more on R&D. Notably, the effect is not driven by size, which is held constant by including domestic sales.

4.2 Robustness checks

I perform a series of specification and robustness checks in Table 4 below. In the first three columns I follow a general-to-specific approach to investigate the relative performance of my theoretical model. The first column includes all regressors of my baseline regression. The interaction of the sales ratio with the squared tax differential is positive and significant at the 10% level. The other tax variables and the logarithm of global sales are not significant. In the second and third specification, I drop variables with the lowest p-value. While the coefficient on the squared tax differential remains positive and is measured with increasing accuracy, it does not achieve significance. Overall, this suggests that the intermediate input scenario is a more plausible model in the context of my analysis.

In columns (4) to (7), I investigate the robustness of my results by dropping firms which do not provide at least 4 observations on R&D expenditures. This reduces my sample by around 50%. The result is an increase in the estimated price sensitivity to around 1. Concurrently, the estimated partial effect of turnover decreases. The overall response to a changing production structure, i.e. the sum of these two coefficients, is largely unaffected.

I test the robustness of the common input scenario in columns (4) and (5). The estimated coefficients are qualitatively similar to the baseline regression. In the first specification, both coefficients are positive and measured imprecisely. Dropping the first tax differential reduces the standard error of the squared tax differential. In column (5), the squared tax differential is positive and statistically significant.

Finally, with the sixth and seventh specification, I test the robustness of the intermediate input hypothesis. The coefficient on the first tax variable interaction is not significant in column (6). This observation is consistent with the premise that the arm's length price reflects marginal production costs. In the following specification (7), I incorporate this restriction by dropping the first tax variable. This reduces the standard error on the squared tax differential considerably. The interaction with the squared tax differential is positive in both specification and significant at the 5% and 1% level respectively. This provides indirect evidence for the tax efficient alignment of internal prices.

Table 4

Fixed effects OLS estimation, Years: 2005-2012							
Dependent: logarithm of R&D expenses							
Test	Specification			Robustness			
				Common input		Intermediate input	
Specification assumptions	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Explanatory variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\ln(\rho)$	-0.65** (0.31)	-0.68** (0.30)	-0.66** (0.29)	-1.05** (0.45)	-0.90** (0.45)	-0.98** (0.45)	-1.00** (0.45)
y^D	0.51*** (0.12)	0.50*** (0.06)	0.50*** (0.06)	0.30*** (0.08)	0.31*** (0.08)	0.37*** (0.09)	0.37*** (0.09)
s	0.02* (0.01)	0.01* (0.00)	0.01* (0.00)			0.03* (0.02)	0.01*** (0.00)
$\tilde{\tau}$	0.45 (0.99)	0.17 (0.72)		2.58 (1.74)			
$\tilde{\tau}^2$	1.79 (3.08)	1.29 (1.88)	1.50 (1.65)	0.58 (3.81)	4.62* (2.37)		
s: $\tilde{\tau}$	-0.09 (0.08)	0.02 (0.01)	0.02 (0.01)			-0.19 (0.13)	
s: $\tilde{\tau}^2$	0.42* (0.21)	0.11** (0.05)	0.11** (0.05)			1.08** (0.51)	0.27*** (0.08)
y	0.00 (0.10)						
$\ln(\text{GdpPc})$	1.41*** (0.48)	0.99** (0.45)	1.02** (0.44)	1.15* (0.64)	1.83*** (0.68)	1.18* (0.64)	1.62** (0.68)
Inflation	-0.06*** (0.02)	-0.04** (0.02)	-0.04** (0.02)	0.00 (0.03)	-0.02 (0.04)	0.00 (0.03)	-0.03 (0.04)
$\ln(\text{Emp})$	-9.24*** (3.48)	-6.81** (2.87)	-6.66** (2.76)	-3.73 (4.73)	-7.31 (5.47)	-3.65 (4.73)	-8.12 (5.44)
Observations	3957	5335	5335	1430	1430	1926	1926
Adj. R^2	0.04	0.04	0.04	0.06	0.06	0.05	0.05

Notes: *, **, and *** indicate significance at the 10%, 5% and 1% level. Robust standard errors in parentheses.

5 Interpretation

The findings of my empirical analysis suggest that there is a robust, positive correlation between research investments in the UK and the squared tax difference to intra-group affiliates. One possible mechanism explaining this relationship is the tax efficient alignment of transfer prices. Specifically, the assumptions outlined in section 2 imply that the net

return to price manipulations is increasing in a squared tax differential. The estimated partial effect thus allows for a structural interpretation. In this section, I combine the empirical results with my theoretical model to obtain estimates for the gap between arm’s length and internal prices as well as their sensitivity to foreign taxation.

The common input scenario provides a straight forward way to retrieve the structural cost parameter. Specifically, when testing the model’s basic structure in specification (4) of Table 4, the first tax variable’s coefficient is not statistically different from zero. This suggests that arm’s length prices reflect marginal production costs. The distribution of the second tax coefficient estimate in the following specification (5), where this restriction is incorporated, thus comprises all information necessary to retrieve an estimate for the structural cost parameter:¹¹

$$\widehat{\gamma} = 2\widehat{\alpha}_4. \tag{15}$$

Combined with optimal decision making (equation (7)), this estimate suggests that transfer prices between related parties are, on average, around 11 percent above their arm’s length equivalent. Increasing the average foreign tax rate by one percentage point further increases this gap by around 3 percent.¹²

This semi elasticity is a rough estimate for the innovation center’s semi elasticity of taxable profits.¹³ The estimated response is much larger than prior studies on aggregate profit shifting suggest (Clausing, 2003; Bernard et al., 2006; Heckemeyer and Overesch, 2013). However, the results correspond to the identification of intangible assets being an important driver in the magnitude of observable profit shifting (Dischinger and Riedel, 2011; Beer and Loeprick, 2015) and provide further support to concerns about the susceptibility of intangible assets to facilitate income relocation.

In the intermediate input scenario, the coefficient estimate on the first tax differential is positive (Table 4, Column (7)), suggesting that independent producers charge a mark

¹¹I assume that $\sigma = 1$ which is not rejected by my regression results

¹²Substituting the coefficient estimate of specification (5) in Table 4 gives $\widehat{\gamma} = 5.94$. The associated standard error is 3.6. The average difference $(\tau_C - \tau_1)$ is given by 0.018, suggesting an average δ of 10.69%. The semi-elasticity of the wedge between arm’s length and internal prices is given by $\Delta\delta/(1 + \delta)$. The definition of the optimal price implies $\Delta\delta = \gamma \frac{N-1}{N} \Delta\tau_i$. With an average number of foreign affiliates, $(N-1)$, amounting to 1.22 for the subsample of MNEs in my dataset, and $\Delta\tau_i = 0.01$, I obtain $\Delta\delta = 0.033$. Dividing this by 1.1069 gives a semi-elasticity of 2.98.

¹³While the regression results show that research investments are increasing in foreign taxation, the effect is small. The relative change in the innovation center’s taxable profits are thus approximated by $\Delta p/p$, which corresponds to the percentage change in the gap between arm’s length and internal prices.

up over marginal production costs. I obtain the following conditional estimate:

$$\widehat{\gamma} = \frac{\widehat{\beta}_5}{\widehat{\beta}_4} \frac{2\widehat{\varepsilon}}{1 + \widehat{\varepsilon}}. \quad (16)$$

Identification of the structural cost parameter thus requires knowledge of ε . The regression results suggest that independent producers charge a mark up of 3.2.¹⁴ Combined with the first order conditions of the theoretical model, this indicates that internal prices are around 40% above their arm’s length equivalent. Increasing foreign taxation by one percentage point further increases the internal price by around 6%.¹⁵

The point estimates in the intermediate input scenario imply an even more pronounced response to international tax incentives. However, they are based on a mark up which seems excessive and which is measured only imprecisely. The standard error associated with this estimate does indeed not reject the competitive pricing hypothesis $\varepsilon = 0$. As a result, the estimated price manipulation is also not significantly different from zero.

Table 5

Relative manipulation of prices			
Estimates conditional on ε			
Mark up (ε)	0.06	0.09	0.12
Average percentage manipulation	2.93 (1.15)	4.28 (1.68)	5.55 (2.18)
Semi elasticity	0.63 (0.24)	0.91 (0.34)	1.17 (0.44)

First line (Average percentage manipulation) gives percentage deviation of internal prices from a hypothetical equilibrium price: $(p - \hat{p})/\hat{p} = \gamma(\tau_i - \tau_1)$. The second line indicates the semi elasticity of the wedge between arm’s length and internal prices: $(1 + \delta)^{-1}\partial(1 + \delta)/\partial\tau_i$. The cost parameter γ is retrieved conditional on ε . The mean tax differential ($\tau_i - \tau_1$) is 0.045. All values are significantly different from zero. Standard errors are calculated with the delta method and given in parentheses.

To further investigate the intermediate input scenario, I present estimates for the percentage manipulation of prices conditional on plausible mark ups in Table 5.¹⁶ The first

¹⁴The model implies $\widehat{\varepsilon} = \widehat{\beta}_4(\widehat{\beta}_3\widehat{\beta}_2)^{-1} = 3.2$

¹⁵The average difference ($\tau^I - \tau_1$) is 4.5% and the cost parameter estimate is $\widehat{\gamma}_I = 9.14$, suggesting that the average wedge between arm’s length and internal prices is $\delta = 0.4114$. The semi-elasticity of internal prices is given by $\Delta\delta/(1 + \delta) = 0.0647$.

¹⁶In a separate regression (not reported in this paper), I investigate the pricing strategy of independent

line reports the average percentage deviation of arm's length and internal prices. Given a mark up of 9 percent, I estimate that internal prices are, on average, around 4.3 percent above their arm's length equivalent. The second line indicates that this gap increases by slightly less than 1 percent, in response to a 1 percentage point increase in the average foreign tax rate. These estimates are consistent with prior indirect evidence on the scale of profit shifting.

6 Conclusion

Using a panel data set of 1611 multinational and domestic firms, I investigate the sensitivity of research investments in the UK to changes in national and international taxation. I find a robust positive correlation between R&D expenditures and the squared tax difference to intra-group affiliates. This findings provides indirect evidence on the tax efficient manipulation of transfer prices. Blending the empirical results with a simple theoretical model suggests that internal prices increase by three percent in response to a one-percentage point increase in foreign taxation. Notably, my results indicate that the magnitude of immobile R&D activity in the UK is higher than it would be if price manipulations were prohibitively costly. While theoretical work has provided examples of why aggressive tax minimization may be desirable from a social perspective (e.g. Hong and Smart, 2010), applied work has largely focused on its detrimental effects.

My analysis provides an additional avenue to applied research in exploring profit shifting of MNEs. Prior work has, for the most part, investigated the sensitivity of taxable profits, rather than the sensitivity of prices. While this approach provides more direct evidence on the scale of observable profit shifting, it rests on the important assumptions that all productive factors are observable. Failure to account for intermediate inputs could result in biased estimates. Importantly, my identification strategy does not presume the observability of all input factors. The point estimates, though associated with considerable error margins, suggest that elasticities identified in prior work are not biased downwards.

firms in my sample. By drawing on non-parametric estimation methodologies proposed by Hall et al. (1986); Hall (1988) and extended by Roeger (1995) I find that the average mark up in my sample of independent firms is around 9%. This finding is consistent with other estimates of the wedge between marginal revenues and costs for UK firms (Martins et al., 1996).

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A Derivation of the user cost

Combining the first order conditions for an optimal magnitude of research directed at market i with the definition of global profits (3) and the net return to profit shifting (4),

(6) gives

$$\begin{aligned} & \left[f'_i - \hat{p} \left[\alpha + (1 - \alpha) \frac{1}{N} \right] \right] (1 - \tau_1) + (1 - \alpha) \sum_{j \neq i} \left[f'_j - \hat{p} \frac{1}{N} \right] (1 - \tau_j) \\ & + (1 - \tau_1) \hat{p} - (1 - \mu \tau_1) w = -(p - \hat{p}) \left[\alpha + \sum_{i=1}^N \frac{1 - \alpha}{N} \right] (\tau_i - \tau_1) + \frac{1}{2\gamma} \frac{(p - \hat{p})^2}{\hat{p}} \end{aligned} \quad (17)$$

Rearranging this expression, setting $\alpha = 0$ and using $f_i = \theta_i f$ gives

$$f' \sum_{i=1}^N \theta_i (1 - \tau_i) = (1 - \mu \tau_1) w - p(\tau_C - \tau_1) - \frac{1}{2\gamma} \frac{(p - \hat{p})^2}{\hat{p}}. \quad (18)$$

Substituting the optimal price (7) and using $h(R) = \sum_i \theta_i f$ yields

$$h' \frac{\sum_i \theta_i (1 - \tau_i)}{\sum_i \theta_i} = (1 - \mu \tau_1) w - \hat{p}(\tau_C - \tau_1) - \hat{p} \frac{1}{2} \gamma (\tau_C - \tau_1)^2. \quad (19)$$

Finally, using the arm's length price $\hat{p} = (1 + \varepsilon) \rho^D$, where $\rho^D = w \frac{1 - \mu \tau_1}{1 - \tau_1}$, defining $\frac{\sum_i \theta_i (1 - \tau_i)}{\sum_i \theta_i} = 1 - \tau_W$ and rearranging gives the user cost of research for the common input scenario (10a).

For the intermediate input scenario, I set $\alpha = 1$ in (17) and rearrange to get

$$f'_i (1 - \tau_i) = (1 - \mu \tau_1) w - p(\tau_i - \tau_1) + \frac{1}{2\gamma} \frac{(p - \hat{p})^2}{\hat{p}}. \quad (20)$$

Substituting the optimal price gives

$$f'_i (1 - \tau_i) = (1 - \mu \tau_1) w - \hat{p}(\tau_i - \tau_1) - \gamma \hat{p} (\tau_I - \tau_1) (\tau_i - \tau_1) + \frac{1}{2} \gamma \hat{p} (\tau_I - \tau_1)^2. \quad (21)$$

Finally, using the arm's length price $\hat{p} = (1 + \varepsilon) \rho^D$, approximating $(\tau_I - \tau_1)(\tau_i - \tau_1)$ by $(\tau_I - \tau_1)^2$ and rearranging gives (10b).

B Derivation of the regression specification

Country-specific sales $Y_i = f_i(x) = \theta_i f(x)$ are of a CES-type and thus satisfy

$$f'_i(x) = \theta_i \delta \left(\frac{f(x)}{x} \right)^{\frac{1}{\sigma}}, \quad \text{for all } i \quad (22)$$

where the elasticity of substitution between research and other factors of production is denoted by $\sigma \in [0, \infty]$. For the common input scenario, this implies that marginal global sales are given by

$$h' = \delta \left(\frac{f(R)}{R} \right)^{1/\theta} \sum_i \theta_i. \quad (23)$$

Taking logarithms, rearranging and substituting the user cost of research (10a) gives

$$r_i = \mu_i - \sigma \ln(\rho) + \sigma \left\{ \left[\varepsilon \frac{\tau_C - \tau_1}{1 - \tau_W} + \gamma(1 + \varepsilon) \frac{1}{2} \frac{(\tau_C - \tau_1)^2}{(1 - \tau_W)} \right] \right\} + y_i \quad (24)$$

where μ_i captures constants, lower case latin letters denote the logarithm of upper case letters, $\rho = \frac{1 - \mu\tau_1}{1 - \tau_1}$ and I used $1 - \tau_C/(1 - \tau_W) \approx 1$ and $\ln(1 + x) \approx x$. This relation is the basis for my first estimating equation. When interpreting the empirical results I assume a Cobb-Douglas specification, implying that $\sigma = 1$.

For the intermediate input scenario, note that the the first order condition for an optimal level of domestic research implies $f'_1 = \rho^D$. Combining this with (22) yields

$$\frac{R_i}{R_1} = \frac{Y_i}{Y_1} \left(\frac{\theta_i}{\theta_1} \right)^\sigma \left(\frac{\rho^D}{\rho^i} \right)^\sigma \approx \frac{Y_i}{Y_1} \left(\frac{\theta_i}{\theta_1} \right)^\sigma \left(1 + \sigma \frac{\rho^D - \rho^i}{\rho^i} \right), \quad (25)$$

where ρ^i denotes the user cost of research in the intermediate input scenario (the right hand side of (10b)) and the approximation is around $\rho^D = \rho^i$. The relative difference between the domestic and foreign user cost of research is given by

$$\frac{\rho^D - \rho^i}{\rho^i} \approx \varepsilon \frac{\tau_i - \tau_1}{1 - \tau_i} + \gamma(1 + \varepsilon) \frac{1}{2} \frac{(\tau_I - \tau_1)^2}{1 - \tau_i}, \quad (26)$$

where I used the approximation $\ln(1 + x) \approx x$. Next, I assume that the magnitude of research directed at foreign jurisdictions is approximately equal, i.e. $R_i = R_f$ for all $i \neq 1$. This implies

$$\sum_{i \neq 1} \frac{R_i}{R_1} = \left(\frac{\theta_f}{\theta_1} \right)^\sigma \frac{(N - 1)Y_f}{Y_1} \left[1 + \sigma \left(\varepsilon \frac{1}{N - 1} \sum_{i \neq 1} \frac{\tau_i - \tau_1}{1 - \tau^i} + \gamma(1 + \varepsilon) \frac{1}{2} \frac{1}{N - 1} \sum_{i \neq 1} \frac{(\tau_I - \tau_1)^2}{1 - \tau^i} \right) \right] \quad (27)$$

Finally, combining the above expressions with $\ln(R) \approx \ln(R_1) + \sum_{i \neq 1} \frac{R_i}{R_1}$ which holds for $\sum_{i \neq 1} R_i/R_1$ small shows that the logarithm of total research and development expenses in the intermediate input scenario may be written as

$$r \approx \mu_d - \sigma \ln(\rho) + y_1 + \frac{(N-1)Y_f}{Y_1} \left(\frac{\theta_f}{\theta_1} \right)^\sigma \left[1 + \sigma \left(\varepsilon \frac{1}{N-1} \sum_{i \neq 1} \frac{\tau_i - \tau_1}{1 - \tau^i} + \gamma(1 + \varepsilon) \frac{1}{2} \frac{1}{N-1} \sum_{i=2}^N \frac{(\tau_i - \tau_1)^2}{1 - \tau^i} \right) \right] \quad (28)$$

implying the estimating equation (13).