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Investor Borrowing Heterogeneity in a Kiyotaki-Moore Style Macro Model

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Abstract

We allow for heterogeneity in investors' ability to borrow from collateral in a Kiyotaki-Moore style macro model. We calibrate the model to match the quintiles of the distribution of leverage ratios of US non-financial firms. We show that financial amplification of the model with heterogeneous investors can be orders of magnitude higher, because of more pronounced asset price reactions.

Keywords: Collateral Constraints, Leverage, Heterogeneity, Financial Amplification
JEL-Codes: E32, E44

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1 Introduction

We present an extension of a core macroeconomic model with financial frictions to account for heterogeneity in investors' ability to borrow from collateral. The literature leading this field, both the seminal contributions of Kiyotaki and Moore (1997) and Bernanke et al. (1999), but also the large literature thereafter[1], retains a high degree of aggregation: there typically exists a *representative* financially-constrained agent, and our existing models are typically calibrated to match an economy-wide average of leverage ratios. In the data, observed leverage ratios (assets to net worth) of US non-financial firms are, on average, at around 1.5 to 2 (see, e.g. CGFS (2009)), with wide cross-sectional variation. Leverage ratios of financial intermediators are substantially higher.[2]

We take the framework of Kiyotaki and Moore (1997) and, instead of a representative investor, introduce different investor types that each can manage different types of capital, which are collateralizable to different degrees. We calibrate the model to match the observed leverage ratios of the quintiles of the distribution of leverage ratios for US non-financial firms, using the dataset of Rauh and Sufi (2010). We find that the model with heterogeneous investors produces a much more pronounced financial amplification of shocks compared to a model version where collateral constraint parameters are calibrated to the economy-wide average (homogeneous investors). This is because investors with the highest leverage are the drivers of asset prices, not the economy wide average. Asset price drops in response to negative productivity shocks are therefore much stronger in the heterogeneous investors model, tightening the financial constraints of all investors, and leading to additional amplification.

2 The model

The model economy is populated by a representative saver (patient), investors (impatient), and a representative firm. Investors borrow from savers to invest in capital; they each manage a specific type of capital, which they rent to firms for use in production, and which is collateralizable to different degrees. For simplicity, we assume labor supply and the investors' types of capital are in fixed supply.

2.1 Investors

There is a continuum of investors, with measure 1. Investors come in I different types, each investor i's size is given by n_i, where $\sum_i n_i = 1$. Preferences of investor i, for $i = 1, .., I$, are:

$$E_t \sum_{s=t}^{\infty} \left(\beta^{I_i}\right)^s \frac{\left(C_t^{I_i}\right)^{1-\sigma}}{1-\sigma}, \tag{1}$$

[1]Some selective examples of recent influential contributions include Iacoviello (2005), Gertler and Kiyotaki (2010), Christiano et al. (2010), Karadi and Gertler (2011), Kiyotaki and Moore (2012), and Jermann and Quadrini (2012).

[2]Leverage ratios of US commercial banks at the center of the crisis were in the range of 15-20, those of US investment banks around 25-30 (CGFS (2009)).

where $C_t^{I_i}$ is consumption of a homogenous final good, σ the coefficient of relative risk aversion and β^{I_i} investor i's discount factor. The budget constraint reads:

$$C_t^{I_i} + q_{i,t} K_{i,t}^{I_i} = L^{I_i} W_t^{I_i} + \left(q_{i,t} + R_{Ki,t}\right) K_{i,t-1}^{I_i} + B_{i,t}^{I_i} - R_{i,t-1} B_{i,t-1}^{I_i}. \qquad (2)$$

$K_{i,t}^{I_i}$ is holdings of a type-i fixed asset, $q_{i,t}$ the asset price, $R_{Ki,t}$ the return the asset earns, $W_t^{I_i}$ is wage income, L^{I_i} her (inelastically supplied) labor input, $B_{i,t}^{I_i}$ is debt issued to savers, and $R_{i,t-1} B_{i,t-1}^{I_i}$ the payment on previously incurred debt. Investors also face a constraint on total leverage due to an inability to commit to repayment. Total debt of investor i is restricted to be no greater than κ_i times the market value of her assets, where $\kappa_i < 1$:

$$B_{i,t}^{I_i} \leq \kappa_i q_{i,t} K_{it}^{I_i}. \qquad (3)$$

Borrowing constraint parameters κ_i differ across investors, because investors hold different types of capital, which are collateralizable to different degrees. In addition, lenders' liquidation technologies w.r.t. different investors may differ, because informational asymmetries may be differently strongly pronounced. The first-order conditions of investor i's optimal choice of asset holdings, $K_{i,t}^{I_i}$, and borrowing, $B_{i,t}^{I_i}$, are

$$\left(C_t^{I_i}\right)^{-\sigma} = \beta^{I_i} E_t \left[\left(C_{t+1}^{I_i}\right)^{-\sigma}\right] R_{i,t} + \mu_t^{I_i}, \qquad (4)$$

$$q_{i,t}\left(C_t^{I_i}\right)^{-\sigma} = \beta^{I_i} E_t \left[\left(C_{t+1}^{I_i}\right)^{-\sigma} \left(q_{i,t+1} + R_{Ki,t+1}\right)\right] + \mu_t^{I_i} q_{i,t}. \qquad (5)$$

Variable $\mu_t^{I_i}$ represents the shadow value of relaxing the leverage constraint by one unit. When $\mu_t^{I_i} > 0$, the expected return on the asset exceeds the cost of borrowing, and the collateral constraint (3) holds with equality (resulting from the Kuhn-Tucker condition).[3]

2.2 Savers

There is a representative saver, of measure 1, with preferences:

$$E_t \sum_{s=t}^{\infty} \left(\beta^S\right)^s \frac{\left(C_t^S\right)^{1-\sigma}}{1-\sigma}, \qquad (6)$$

where C_t^S and β^S are the saver's consumption and discount factor, respectively. Savers are more patient than investors, so that $\beta^S > \beta^{I_i}$, $\forall\ i$. The saver maximizes (6) subject to:

$$C_s^S + \sum_i \left(B_{is}^S - R_{is-1} B_{is-1}^S\right) + \sum_i q_{is} K_{is}^S = L^S W_s^S + \sum_i q_{is} K_{is-1}^S + G\left(K_{s-1}^S\right). \qquad (7)$$

[3] The assumption that investors are more impatient guarantees that investors' leverage constraints hold with equality at the deterministic steady-state. In a stochastic world, this is not necessarily the case, for example, when investors have strong precautionary motives. We follow a large literature, in assuming that the constraint always binds – which, with small shock volatility and a sufficiently low investors' discount factor, is likely.

We realize that our proposed model is too stylized in some dimensions. E.g., it is unrealistic to assume that all US non-financial firms are constrained. It would be possible to introduce a fraction of firms that are unconstrained. Similarly, we assume only short-term (one-period) debt, whereas in reality firms may become constrained, because they took on long-term debt under more optimistic lending conditions. All these are possible interesting avenues for future research.

K^S_{is} denotes the saver's holdings of type-i fixed assets, and $-B^S_{is}$ denotes lending to the i'th investor. The saver obtains wage W^S_t on her (inelastically supplied) labor. The saver uses all i-types of fixed assets, K^S_{is-1}, as inputs into a backyard production function, given by $Y^S_t = G\left(K^S_{s-1}\right) = Z\left(K^S_{t-1}\right)^{\omega}$, with $G'\left(K^S_{s-1}\right) > 0$ and $G''\left(K^S_{s-1}\right) < 0$. The saver's aggregate fixed asset holdings are modeled as a CES-composite of the individual i-types of fixed assets, with substitution elastiticity θ:

$$K^S_t = \left[\sum_i n_i^{\frac{1}{\theta}}\left(K^S_{i,t}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}. \tag{8}$$

The saver's first-order conditions for optimal choices of K^S_{is} and B^S_{is}, $\forall\ i$, are given by:

$$\begin{aligned}
\left(C^S_t\right)^{-\sigma} &= \beta^S E_t\left[\left(C^S_{t+1}\right)^{-\sigma}\right]R_{i,t}, && (9)\\
q_{i,t}\left(C^S_t\right)^{-\sigma} &= \beta^S E_t\left[\left(C^S_{t+1}\right)^{-\sigma}\left(q_{i,t+1} + \omega Z\left(K^S_{t-1}\right)^{(\omega-1)}\frac{n_i^{\frac{1}{\theta}}\left(K^S_{i,t-1}\right)^{\frac{-1}{\theta}}}{\left(K^S_{t-1}\right)^{-\frac{1}{\theta}}}\right)\right]. && (10)
\end{aligned}$$

2.3 Firms

Final good firms produce using labor and the fixed asset with standard production function, $Y^I_t = A_t L^{\varepsilon}\left(K^I_{t-1}\right)^{\varepsilon}$. A_t is productivity. Aggregate employment L is constant (because of our assumption of inelastic labor supply) and normalized to 1. K^I_{t-1} is a CES-composite of the individual i-types of assets of investors i:

$$K^I_t = \left[\sum_i n_i^{\frac{1}{\theta}}\left(K^{I_i}_{it}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{11}$$

First-order conditions give:

$$\begin{aligned}
W_t &= (1-\varepsilon)A_t\left(K^I_t\right)^{\varepsilon-1}, && (12)\\
R_{Ki,t} &= \varepsilon A_t\left(K^I_t\right)^{(\varepsilon-1)}\frac{n_i^{\frac{1}{\theta}}\left(K^{I_i}_t\right)^{\frac{-1}{\theta}}}{\left(K^I_t\right)^{-\frac{1}{\theta}}}. && (13)
\end{aligned}$$

2.4 Equilibrium

Equilibrium in the markets for labor, fixed assets, and debt implies:

$$\begin{aligned}
\sum_i L^{I_i} + L^S &= L \equiv 1, && (14)\\
K^{I_i}_{i,t} + K^S_{i,t} &= n_i, i = 1, ...I, && (15)\\
B^{I_i}_{i,t} + B^S_{i,t} &= 0, i = 1, ...I, && (16)
\end{aligned}$$

The resource constraint is:

$$\sum_i C^{I_i}_t + C^S_t = A_t L^{\varepsilon}\left(K^I_{t-1}\right)^{\varepsilon} + G\left(K^S_{t-1}\right). \tag{17}$$

3 Parameterization

Table 1 summarizes parameter values. To save space, the table provides references used for the specification of conventional parameters. For less conventional parameters, e.g. the substitution elasticities of the i-types of capital, θ, we will provide sensitivity analysis. Z, the productivity in the backyard production sector, is set such that investors hold 80% of fixed assets at steady-state. The main novelty of our paper is to take seriously the large heterogeneity in investors' leverage ratios. We consider $I = 5$ and calibrate parameters κ_i, for $i = 1, ..5$, to match the 90, 70, 50, 30, and 10-th percentiles of the distribution of debt ratios (measured as total debt to total assets at book value[4]) of US non-financial firms, using the dataset of Rauh and Sufi (2010). This dataset contains 2453 public US non-financial firms, for the period of 1996-2006. Each investor's size is then given by $n_i = 1/5$. The leverage ratios (assets-to-net worth) corresponding to κ_i, for $i = 1, ..5$, are given by $\frac{1}{1-\kappa_i}$, which are 6.45, 2.54, 1.92, 1.51, and 1.11.

Parameter		Value	
Discount factor, savers	β^S	0.99	
Discount factor, investors	β^{I_i}	0.97	
Risk aversion	σ	2.00	
Productivity, autocor.	ρ_A	0.9	Perri and Quadrini (1991)
Productivity, shock vol.	σ_A	0.005	Perri and Quadrini (1991)
Formal production	ϵ	0.39	Young et al. (2010)
Informal production	ω	0.08	Benhabib et al. (1991)
Labor share, savers	$l^S = \frac{L^S}{\sum_i L^{I_i}+L^S}$	0.8	
Substitution elasticity, CES	θ	5	
Size of i-type capital, CES	n_i	1/I=1/5	
Constr., heterogen. invest.	κ_1,κ_2,κ_3,κ_4,κ_5	0.84,0.61,0.48,0.34,0.10	
Constr., homogen. invest.	κ_i, $i = 1, ...5$	0.48	

Table 1: Parameters

4 Results

We present impulse responses for two model versions: in the first, all investors I face the same leverage constraint, i.e. $\kappa_i = , \forall\ i$. This is identical to a model version of a representative, or average investor, as commonly used in the literature. We refer to this as the 'homogeneous investors' case. Our alternative model version, called 'heterogeneous investors' case, allows our investors $i = 1, ...I$ to differ in their ability to borrow from collateral.

Figure 1 presents impulse responses to a 1% productivity decrease for the case of 'homogeneous investors'. The reduced productivity in formal production reduces wages for both investors and savers, and, because the fall is persistent, reduces the return on investment in the fixed assets for investors. These effects imply a fall in the price of the fixed asset (panel D), which causes a tightening of leverage constraints and leads investors to decrease their borrowing (panel E). Panel A of figure 1 shows that total output, Y_t, the sum of formal and

[4]Alternatively, one could use debt to the market value of assets. The values for the quintiles of this measure are somewhat lower: $\kappa_1 = 0.54$, $\kappa_2 = 0.34$, $\kappa_3 = 0.24$, $\kappa_4 = 0.14$, $\kappa_5 = 0.02$.

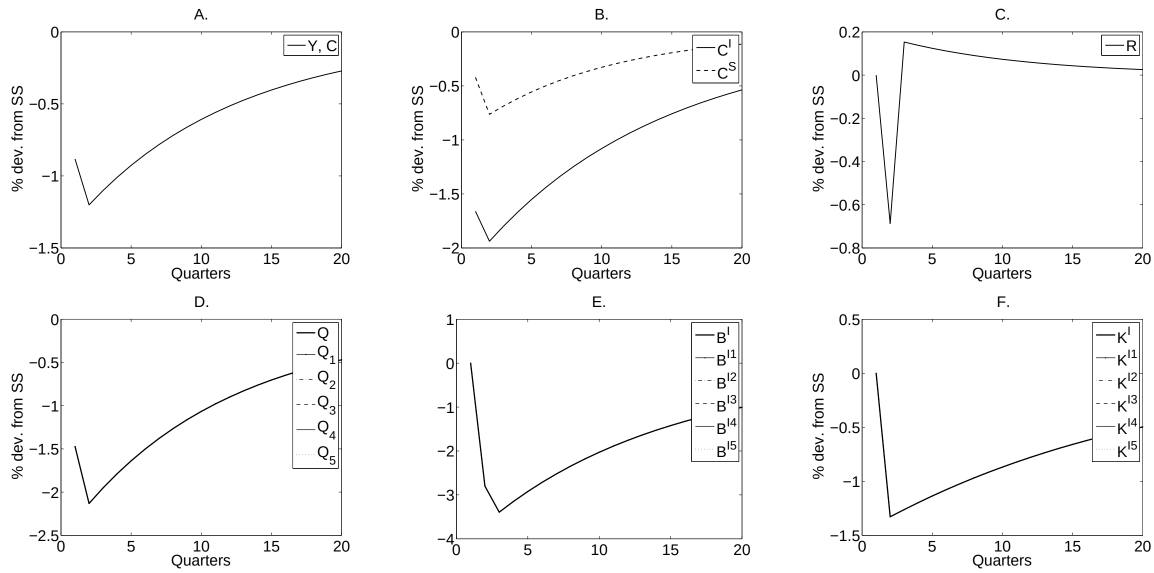

Figure 1: Impulse responses to a 1% productivity decrease, model with homogeneous investors.

backyard production, falls. Output in the formal sector, Y_t^I, declines both because of the direct effect of lower productivity, but also because in response to lower borrowing investors can now finance less of their holdings of fixed assets. Investors reduce their holdings of the fixed asset (panel F), which was used in final good sector. Since more of the fixed asset is allocated to backyard production, Y_t^S, increases, but this increase is not enough to compensate the fall in Y_t^I. The binding leverage constraint thus leads to an additional dip in output in period 2. Because both savers and investors have temporarily lower consumption (panel B), they reduce their savings. However, investors reduce their demand for investment funds even more strongly, since the tightening of leverage constraints forces them to reduce their total borrowing to finance investment. With binding constraints the drop in demand thus exceeds the drop in the supply of funds, and the real interest rate must fall (panel C).

Figure 2 presents impulse responses for the case of heterogeneous investors. Qualitatively, the behavior of economic variables is similar as before. But quantitatively the responses differ markedly. Investor 1, the most levered investor, has to sharply decrease her borrowing and, as a result, her holdings of the fixed asset, by 23.44% and 19.68% respectively. The asset price corresponding to investor 1's fixed assets drops by almost 5.49% at peak – because of the high leverage ratio this constitutes a much more pronounced asset price drop than in the 'homogeneous investors' version. Because asset prices of different types of capital are tightly linked to each other via savers' intertemporal optimality conditions, they experience similarly pronounced declines. The (CES-based) aggregate asset price index now falls by 3.35% at peak, compared to the more moderate drop of 2.13% in the model version with homogeneous investors. The stronger asset prices declines, in turn, lead to a more substantial tightening of financial constraints also for the other, less levered, investors.[5] This translates

[5] Even though in a very different framework, the idea that asset prices are driven not by the average investor (but the marginal investor) features prominently in the work of John Geanakoplos (see, e.g. Geanakoplos (2009)). In our setup asset prices are driven primarily by the agents with the highest leverage.

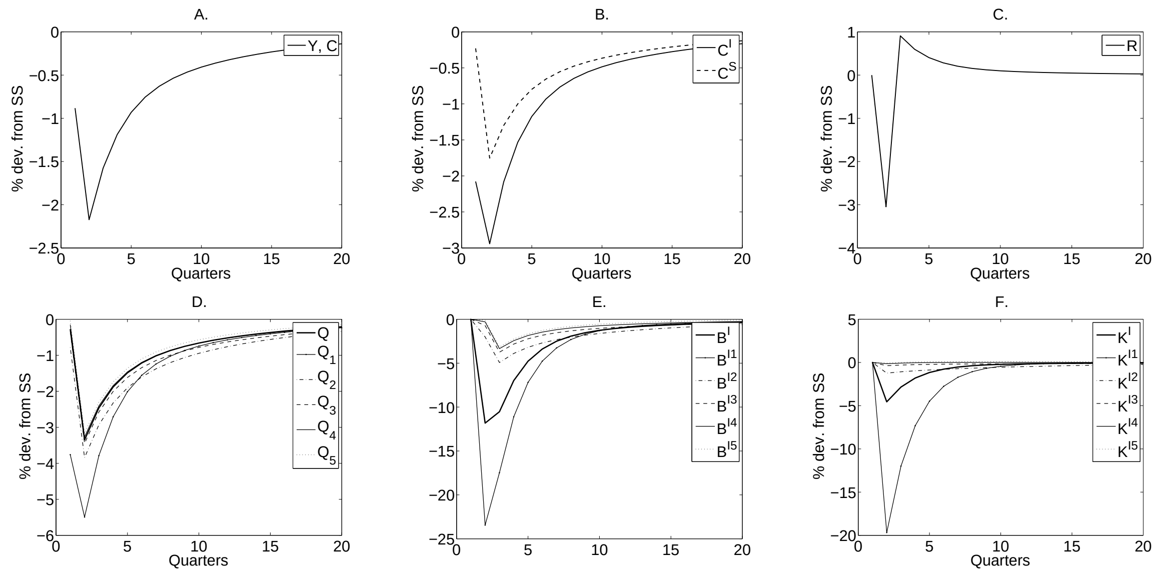

Figure 2: Impulse responses to a 1% productivity decrease, model with heterogeneous investors.

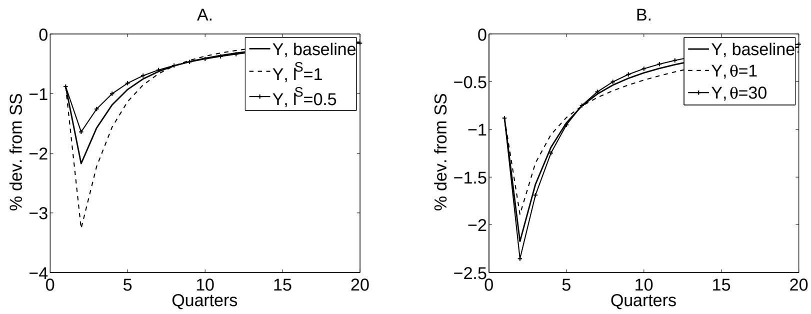

Figure 3: Impulse responses to a 1% productivity decrease, sensitivity analysis.

into an additional financial amplification compared to the model version where investors are identical, even though *on average* leverage constraints are not more severe: output in the economy drops, at peak, by 2.17% in our model version with investors who are heterogeneous in their ability to borrow, compared to just 1.20% in the model version with homogeneous investors.

Figure 3 presents sensitivity analysis of our findings, focusing on the impulse response of total output. Panel A demonstrates that a higher share of labor supply coming from savers, l^S, means that investors have less resources from which to finance their acquisition of capital – thus they are less constrained, and financial amplification is smaller. Panel B varies the substitution elasticity with which different types of fixed assets are aggregated in the CES-composite. A higher elasticity between varieties of capital implies higher amplification.

5 Conclusion

We have presented a simple modification to a stylized Kiyotaki-Moore style model with collateral constraints: allowing for multiple types of investors, each of which has a different ability to borrow from collateral, calibrated to match the means of the quintiles of the distribution of leverage ratios of US non-financial firms. We find that in such extension, the financial amplification and acceleration mechanism may be orders of magnitude stronger. Such modifications can be relevant also when using models of financial frictions for the study of economic policy.

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