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Does Elderly Employment have an Impact on Youth Employment? A General Equilibrium Approach

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Does Elderly Employment have an Impact on Youth Employment?

A General Equilibrium Approach

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Abstract

Does an increase of elderly employment cause a decline in youth employment? A simplified view of a demand driven economy would give a positive answer to this question. Econometric studies based on a single equation approach deliver little support for this belief. However, these studies typically suffer from identification problems to which no attention is paid in most cases. We therefore use a general equilibrium framework when trying to quantify these effects. Using yearly and quarterly Austrian labor and gdp data, we estimate two model variants by Bayesian methods: a) a standard equilibrium model where the degree of complementarity between old, young and primary labor is crucial for the sign and strength of the relevant effects and b) a simple, solely demand driven model which always leads to a crowding out of young through an increase in employment of the old. It turned out that the demand driven model is inferior in fitting the data compared to the standard model. Further, the degree of complementarity is estimated to be strong enough to lead to a small positive effect of elderly employment on youth employment.

JEL codes: A10, C11, E10, J01, J26

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1. Introduction

Is there a connection between employment of young, old and primaries? And if there is any connection – how does it look like? The answers to these questions are relevant for several reasons. For instance, most countries suffer from a more or less unsustainable pension system and, as a consequence, in many countries there are attempts to raise the average retirement age. Clearly, such reforms are disadvantageous for the people who are affected and therefore strong resistance is to be expected. However, in the resulting political discussion it is common not only to refer to arguments which are directly connected to those people but also to emphasize possible negative effects on employment rates of younger people. A notable example for this line of arguments is a statement of Werner Faymann, social democratic chancellor of Austria, published in the Kronen Zeitung on the 5th of May 2013:[1] *"…Furthermore, it is problematic to keep old people in employment if at the same time there are nearly six million unemployed young in the EU. … If elderly stay longer then there are fewer jobs for the young. This would be wrong and cynical."*

The general idea of such arguments is that employment is determined by aggregate demand and aggregate demand is somehow fixed with no feedback from labor markets. This line of thinking is often called the "lump of output" fallacy (see Layard, Nickell, Jackman, 1991, p. 502ff). This argument is quite compatible with simplified Keynesian views and those simplified views have played and still play an important role in political discussion.

There seems to be some empirical evidence, at least for Austria, favoring the notion of a fixed amount of labor which is distributed among different groups. The Austrian labor market is characterized by comparatively low elderly employment and extraordinary high youth employment at the same time. However, this is possibly a special feature of the Austrian labor market. It requires only a simple scatter-diagram of the EU-15-countries showing a strong positive correlation between the employment of young and old and Austria is just an outlier in such a plot (see Uhl, 2012, p. 16). Uhl argues that the extraordinary high youth employment rate in Austria might be due to the special dual educational and training system for young people.

In this paper we will take a closer look on possible connections between employment rates of different age groups from a general equilibrium perspective. In principle, we try answering these questions by employing standard economic theory using plausible parameter values. It seems clear that in such a framework the view of a fixed employment level of the economy is not supported. Employment is not only determined by aggregate demand but also dependent on labor market conditions, particularly on labor supply and labor productivity. As it will turn out, the degree of substitutability or complementarity among different labor groups will play a

[1] Original text in German: *"… Darüber hinaus sei es problematisch, wenn man Ältere länger in Arbeit halte und es zugleich fast sechs Millionen arbeitslose Jugendliche in der EU gibt. … Wenn Ältere länger bleiben gibt es für Junge weniger Jobs. Das wäre falsch und zynisch".*

major role for the impact of labor supply shocks of different working groups. Unfortunately, theory alone will not give us enough information on the sign and strength of these effects. So, basically this issue comes down to an empirical investigation.

We will start with a standard equilibrium model, which is simplified by ignoring intertemporal substitution effects through interest rates. This simplification strongly facilitates the theoretical as well as the empirical analysis and makes the basic operating effects more transparent. We will estimate the main parameters of this model by Bayesian methods. Starting with a concrete model and utilizing some prior information one can circumvent some troublesome identification problems which typically arise in these settings. In many econometric studies which investigate the relationship between elderly and young employment, these identification issues are not dealt with properly or are simply ignored (for instance Gruber, Wise, ed., 2010 and Uhl 2012).

Having estimated the model with Austrian data we can examine the effects of several shocks, like labor supply shocks of the elderly, on employment of different age groups with the help of theoretical impulse response functions. We will argue that the estimated degree of substitutability or complementarity is of utmost importance for the effects of interest. Additionally, we will estimate VARs as well as standard econometric time series regressions using simulated data of our model and compare these estimates with VARs and regressions resulting from actual data. Following the approach of Kehoe (2006) and Chari, Kehoe and McGrattan (2008), we thereby will check whether our theoretical model is capable of capturing the main features of actual data.

Additionally, we will investigate the properties of an extremely shortened Keynesian model with exogenous demand which is intended to represent the previously mentioned simple view that employment is somehow fixed and a larger labor supply of one group automatically leads to a crowding out of other groups from labor. As it will turn out, this model will not fit actual data as well as our standard model.

In section 2 we will give a critical review of the empirical literature and explain why severe identification problems render most of these results suspicious. In section 3 we will explain our methodology in more detail, followed by a presentation of the theoretical models in section 4. Section 5 discusses the data used and several problems connected to these, section 6 presents our results and section 7 concludes.

2. A critical review of the literature

Most of the existing literature rests on a single equation approach. Thereby, a variable representing employment of the youth, such as employment or unemployment rates, is regressed on a measure of employment of the elderly and on some control variables, such as gdp etc.

$$l_y\ (l_p)\ =\ \beta_1 l_o\ +\ \beta_2 y\ +\ldots+\ u \tag{1}$$

Here, l_y represents employment of the youth, l_p employment of the primaries, l_o employment of the old and y gdp (per capita). A notable example of this approach is the book of Gruber and Wise (2010) which contains thirteen papers examining this relationship for twelve different countries and additionally a panel study aggregating these particular countries.

For Canada, France, Italy, the Netherlands, UK and the Panel Study the results point to a clear positive correlation between employment rates of different age groups. So, according to these studies, there is no reason to believe that higher employment of the old, for instance due to pension reforms, crowds out employment of the young for those countries. For the other countries, Belgium, Denmark, Germany, Japan, Spain, Sweden and the United States the results are ambiguous as they found conflicting signs for β_1 in different specifications. Furthermore, in some cases, the results are simply not significant, in particular for the ones which seem to support the "lump of output" thinking.

In short, the empirical evidence contained in this book favors an even positive effect of the elderly employment on youth employment. In contrast, the "lump of output" view is not supported unambiguously for any of these countries.

Similar results are obtained by Uhl (2012) by estimating the Gruber and Wise equations for Austria which are further enhanced by a richer specification with additional control variables, such as schooling rate and primary employment rate. The results obtained are in line with the Gruber and Wise study, pointing also to a positive effect of elderly employment on youth employment.

However, this cited literature generally lacks to clarify the reason for a positive correlation between young and old employment which seems to be somewhat counterintuitive. A byproduct or even a central point of the equilibrium approach used in this paper is an explanation of these possible positive effects. The basic argument is that young, primary and old labor are not perfect substitutes but are to some extent complements. So, a higher employment of the old could increase productivity of the others and thereby lead to a higher demand for them.

An alternative reason for this observed positive correlation between old and young employment could simply be a statistical one, i.e. the dependency of both on some other variables like total factor productivity, general preference shocks or general demand shocks for which these papers do not explicitly account for.

There is also a closely connected and deeper identification problem involved here. As demonstrated in Appendix A, using a single equation approach with a left hand variable youth employment and a right hand variable elderly employment automatically leads to an identification problem. The reason is that there always exists an equation with elderly employment on the left hand side and youth employment on the right hand side with the very same structure. This is also true for any linear combination of those two equations. The usual procedure dealing with this problem is an instrumental variable approach, i.e. to find some exogenous variables which enter in one of these equations and not in the other. So the tricky part is finding appropriate instruments. For the problem at hand we need an exogenous factor which affects employment of the old but does not directly influence employment of the young. Reforms in pension laws are clear candidates for this task. However, it is not sure that these pension reforms are indeed exogenous and are not a reaction to some economic circumstances. It is particularly problematic to use pension reforms as instruments for the case of Austria, since restrictive measures were often accompanied by other measures which actually facilitate early retirement, for instance an easier way to enter invalidity pension. Furthermore, one has to take into account the possibility that pension reforms are just one aspect of a bigger reform package concerning the whole labor market which makes it difficult to disentangle the specific effects. In particular, pension reforms are rather rare events in most countries. For example, there were five notably reforms in the last 40 years in Austria. So, it is doubtful, whether this variation is sufficient for a credible identification.

An interesting example for the instrumental variable approach is Vestad (2013) who examined the case of Norway. Vestad investigated whether the probability of young people becoming employed is connected to the number of early retirements within a regional panel data setting using a unique micro data set from 1994 to 2004. As early retirements could be influenced by general labor market conditions he instrumented this variable by the number of people who reach the minimum age for the Norwegian early retirement program. Since the minimum age has been lowered in Norway several times, there is enough variation in the data for the first stage regression.

Contrary to the work contained in Gruber and Wise (2010) and Uhl (2012) he finds a positive effect of early retirements on the probability of the young getting employed. In fact, the conclusion of this paper is that for every additional early retired one young is getting employed. This is somewhat hard to believe since one question arises immediately: Is there no substitution between elderly and primaries at all?

However, the study of Vestad is difficult to interpret and to compare to the standard approach, as he only deals with one side of the labor market, namely the entrants to labor and not the persons who leave labor. So, he estimates a sort of gross effect whereas the other papers investigate the net effect on youth employment and this is the measure we are actually interested in.

In our work, by estimating an explicit equilibrium model with Bayesian methods we are able to reduce identification problems as the interactions of the variables are fully taken into account and the use of prior information further facilitates the estimation procedure.

3. Methodology

Our starting point is a standard neoclassical labor market model without frictions, but we additionally distinguish three types of labor, i.e. the young, the primaries and the old. For estimation purposes we assign the group of an age of 20 to 24 to the young, 25 to 54 to the primaries and 55 to 64 is assigned to the old. These three types of labor are aggregated by a CES function to "combined" labor which is then used in a standard Cobb-Douglas production function. On the consumer side we also distinguish between those three types of consumers. Preferences of consumers are hit by specific and common shocks and the three distinct labor supply functions could differ in their degree of labor supply elasticity. Because of this simplicity the basic working effects are easily identified and further the analytic solution is traceable.

We proceed in the following way: After specifying the model we carry out estimation by Bayesian methods. However, great care has to be taken in specifying the appropriate priors. For instance, in this context Canova (2007) warns "*while it is hard to 'cheat' in a classical framework, it is not very difficult to give the impression that identification problems are absent in a Bayesian framework by choosing tight enough priors, presenting well-behaved posterior distributions and entirely side-stepping the comparison between priors and posteriors*", (Koop, Pesaran, Smith 2013, p. 305).

Having estimated the model, we study the consequences of innovations of serial correlated shocks in elderly labor supply on young employment as well as on the other variables of interest. We thereby follow the standard procedure by calculating impulse response functions, IRFs, together with their Bayesian confidence bands.

To check whether the model is capable of reproducing the main features of actual data great care is given to model evaluation. For this purpose we estimate a VAR with gdp and elderly, primary and young employment rates to capture the correlation structure of the data. To properly compare this VAR with our model we use the approach proposed and forcefully recommended by Kehoe (2006) and Chari, Kehoe, McGrattan (2008). The special aspect of this procedure is not to compare the model IRFs with the VAR-IRFs based on actual data because, as Kehoe (2006) has shown, lag truncation biases and small sample issues could be very substantial and might result in totally misleading conclusions. Instead, we use model generated data, estimate an equivalent VAR and compare those IRFs with the VAR-IRFs estimated with the actual data. A big advantage of this method is that we do not have to care about (doubtful) identification of the VAR, omitted variables or lag-length issues as we are only interested in the correlation structure of the data.

In addition, model generated data are also used to estimate single equation regression in the line of Gruber and Wise (2010) respectively Uhl (2012) and the results are compared to estimates of actual data. Last but not least, model generated data are used to compute various variances and covariances, which are then related to the moments of actual data, according to Real Business Cycle Theory tradition.

4. Two Models

As previously mentioned, we start with a simple general equilibrium model. Our basic model is essentially a simplified version of a full-fledged neoclassical macro model. We abstain from an intertemporal/dynamic analysis and consider a static case only as static and dynamic equilibrium models often produce very similar conclusions.[2] The model dynamics solely come from serial correlated shocks.

We additionally consider an ultra-keynesian view by introducing an alternative model with exogenous demand for goods which in term determines labor demand. This model is intended to reflect the idea of a fixed amount of labor which is divided among several types of labor groups. The rest of the model is specified in an identical manner to the basic model.

4.1 The Basic Model

On the production side we assume a representative firm with a standard Cobb-Douglas production function where capital stock is omitted (assumed to be constant) for the sake of simplicity:[3]

$$Y = A\overline{L}^{(1-\alpha)}. \tag{2}$$

As usual, the term $(1 - \alpha)$ represents the elasticity of output with respect to labor and is also assumed to correspond to the share of labor income on total income. Total labor $\overline{L}$, which exhibits diminishing returns, is a combination of our three types of labor, i.e. L_o the old, L_p the primaries and L_y the young, combined by a CES-aggregator:

$$\overline{L} = (b_o L_o^{\,r} + b_p L_p^{\,r} + b_y L_y^{\,r})^{\frac{1}{r}}. \tag{3}$$

The parameter r determines the elasticity of substitution s between the different types of labor by $s = 1/(1 - r)$. Thereby $r = 1$ represents the case of perfect substitutes, $r = 0$ the Cobb-Douglas case of a substitution elasticity of one and negative values of r represent the cases of elasticities of substitution smaller than one. The b_is should represent share parameters which sum up to one. However, as it is well-known, these share parameters are not independent from the exponent r which makes calibration and interpretation of the model problematic.[4] We

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[2] For instance, Williamson (2014) analysis a static and a dynamic version of his basic model and most key results hold for both versions.

[3] For formal derivations of all the model equations see Appendix B.

[4] See for instance Klump, Saam (2006), Cantore, Levine (2012) or Temple (2012).

therefore follow an approach originally suggested by Senhadji (1997) and use b_i^{1-r} instead of b_i. This variant somewhat surprisingly leads to the very simple log-linearization:

$$\bar{l} = b_o l_o + b_p l_p + b_y l_y . \tag{4}$$

Lower cases for the variables now generally represent log deviations from their steady-state values and the b_is are defined as *volume* shares of labor type i to total labor.

The specification of combined labor by a CES-aggregator is clearly restrictive because the substitution elasticity is assumed to be the same among all three types of labor. Finding counter-examples is not a difficult task; for instance, it seems plausible that a retiring experienced worker is substituted by another experienced person, who comes most likely from the primary segment, rather than by an unexperienced young. On the other hand, especially in some parts of the public sector the reverse case is also reasonable. Imagine, a retiring teacher who is almost always replaced by a new entrant. So, we simply have to assume that the CES-specification does not harm our results in a significant way. Actually, the different shares b_i and, more importantly, different labor supply elasticities permit asymmetric effects between different labor groups, even if the substitution elasticities are all equal.

Profit maximization of the firm is carried out in two steps. In the first step the amount of combined labor is determined according to the following standard first order condition; real wage w equals marginal product of labor, *mpl* (log-linearized):

$$\bar{w} = mpl = a - \alpha \bar{l} . \tag{5}$$

Thereby, a equals log deviations of total factor productivity A and $\bar{w}$ represents the wage for combined labor, combined by the CES-aggregator.

$$\bar{w} = b_o w_o + b_p w_p + b_y w_y . \tag{6}$$

This log-linearization is valid when the wage rates for the different types of labor w_i are all equal in the steady-state which is actually implied by the used variant of the CES-aggregator.

In the second step, given the value of combined labor the firm allocates the different types of labor optimally. This cost minimization problem leads to the usual conditions; marginal rate of substitution equals relative wages (log-linearized).

$$\begin{aligned} w_o - w_y &= (r-1)(l_o - l_y), \\ w_o - w_p &= (r-1)(l_o - l_p), \\ w_y - w_p &= (r-1)(l_y - l_p). \end{aligned} \tag{7}$$

Note that given $\bar{l}$ only two of the three conditions are independent. As (7) confirms, the elasticity of substitution between different labor actually equals $1/(1-r)$.

Turning to the consumer's problem, we consider three types of consumer/laborer with the following CRRA utility functions depending on consumption C and labor L:

$$U_i(C,L) \;=\; \frac{C_i^{1-\sigma}}{1-\sigma} \;-\; X_i \frac{L_i^{1+\varphi_i}}{1+\varphi_i}\,. \tag{8}$$

Note that we allow that the parameter φ differs across the three types of consumers, which implies different values of the Frisch labor supply elasticity. The parameter σ is assumed to be constant across the three types. Further the parameter X_i represents a preference shock which we interpret as labor supply shock. Maximization of utility subject to the static budget constraint, which requires consumption to be equal to wage income $W{\cdot}L$ plus the share b_i of total profits Π minus lump sum taxes T

$$C_i \;=\; W_i L_i \;+\; b_i\,(\Pi \;-\; T), \tag{9}$$

leads to the following first order condition which are shown in a log-linearized form:

$$w_i \;=\; \sigma c_i \;+\; \varphi_i l_i \;+\; \chi_i\;. \tag{10}$$

Equation (10) could be interpreted as labor supply functions.

For given aggregate consumption, given labor supply and given wage income share $(1 - \alpha)$, relative consumption follows from the budget constraints (9), once again as log-linearization:

$$c_i - c_j \;=\; (1-\alpha)(w_i + l_i - w_j - l_j)\,. \tag{11}$$

The model is closed by definition of aggregate demand assuming a share d of some exogenous demand components D on total demand Y:

$$y \;=\; c \;+\; d\,, \tag{12}$$

where the demand component d actually represents a shock in the share of D.

In sum, our model consists of the following thirteen log-linear equations for the thirteen endogenous variables:

$$y \;=\; a + (1-\alpha)\overline{l} \qquad \text{production function}$$

$$\overline{w} \;=\; mpl \;=\; a \;-\; \alpha\,\overline{l} \qquad \text{profit maximization}$$

$$\begin{aligned} w_o - w_y \;&=\; (r-1)\left(l_o \;-\; l_y\right) \\ w_y - w_p \;&=\; (r-1)\left(l_y \;-\; l_p\right) \end{aligned} \qquad \text{cost minimization}$$

$$l_o = \frac{1}{\varphi_o} w_o - \frac{\sigma}{\varphi_o} c_o + z + g$$

$$l_p = \frac{1}{\varphi_p} w_p - \frac{\sigma}{\varphi_p} c_p + u + g \qquad \text{labor supply}$$

$$l_y = \frac{1}{\varphi_y} w_y - \frac{\sigma}{\varphi_y} c_y + v + g$$

$$c_o - c_y = (1-\alpha)(w_o + l_o - w_y - l_y)$$

$$c_o - c_p = (1-\alpha)(w_o + l_o - w_p - l_p) \qquad \text{relative consumption}$$

$$\bar{l} = b_o l_o + (1 - b_o - b_y) l_p + b_y l_y \qquad \text{combined labor}$$

$$\bar{c} = b_o c_o + (1 - b_o - b_y) c_p + b_y c_y \qquad \text{aggregate consumption}$$

$$\bar{w} = b_o w_o + (1 - b_o - b_y) w_p + b_y w_y \qquad \text{aggregate real wage}$$

$$y = c + d \qquad \text{aggregate demand}$$

Note that we have split the labor supply shock χ into a specific and a general component. The specific shocks are *z*, *u* and *v* for elderly, primary and young labor and *g* represents a general labor supply shock. They are also normalized so that a unit shock leads to a one percent rise in labor supply. In sum we consider six types of shocks in this model: besides the four labor supply shocks we have a productivity shock *a* and an aggregate demand shock *d*. Each shock is modeled as an AR(1)-process:

$$\begin{aligned} z_t &= \rho_z z_{t-1} + e_{z,t}, \quad & u_t &= \rho_u u_{t-1} + e_{u,t}, \quad & v_t &= \rho_v v_{t-1} + e_{v,t}, \\ g_t &= \rho_g g_{t-1} + e_{g,t}, \quad & a_t &= \rho_a a_{t-1} + e_{a,t}, \quad & d_t &= \rho_d d_{t-1} + e_{d,t}. \end{aligned} \tag{13}$$

We further assume that all innovations $e_{i,t}$ are uncorrelated with each other.

Determinants of the effect of elderly labor supply shocks

What determines the effect of elderly employment on the employment of other groups in this basic model? Here, essentially four effects are at work. Firstly, a positive shock in elderly labor supply reduces elderly wages, which by means of cost-minimization leads to less labor demand for the other groups. Secondly, nevertheless, in total more people are employed because of a lower aggregate wage rate. This gives rise to the third effect. Higher aggregate employment reduces marginal productivity of combined labor and also of each labor group because of less capital per employee. The fourth effect has to compensate the first and third; higher elderly employment increases marginal productivity of the young and primaries because of complementarities between the labor groups. Such complementarities could arise for several reasons. Typically young employees are more creative and physically capable. On the other hand, old employees exhibit more human capital, especially firm specific human capital, and are generally more experienced. Generally, firms demand a broad variety of different skills. But apparently, these different skills are unevenly distributed among distinct age groups and so positive externalities are possible and probable. It now depends on the degree of these complementarities whether we observe a positive or a negative effect of elderly on young employment.

The analytic solution of the model is quite complicated and the terms involved are hard to analyze[5]. For space-saving reasons we therefore do not present it here. But somewhat surprisingly, one gets a very simple term when calculating which degree of complementarity, in terms of r or s, is required to get a positive effect of elderly employment on youth employment. Such a positive effect occurs if

$$r \equiv 1-\frac{1}{s} < \frac{(1-\alpha)\cdot(1-\sigma)}{\sigma\cdot(\alpha-1)+1}, \quad \text{or} \tag{14}$$

$$s < \frac{\sigma\cdot(\alpha-1)+1}{\alpha}. \tag{15}$$

For the case of $\sigma = 1$ (the substitution and wealth effect in labor supply cancel each other), one needs $r < 0$ (and $s < 1$) for a positive effect. In this case output elasticity of labor ($1-\alpha$) is irrelevant. In general, differentiating (14) with respect to σ leads to

$$\frac{(\alpha-1)\alpha}{\left((\alpha-1)\sigma+1\right)^2}. \tag{16}$$

As this term is negative, we need a lower r and lower s (a higher degree of complementarity) for larger values of σ in order to get a positive effect of elderly employment on youth employment.

To examine the effect of α we differentiate (14) with respect to α and get

$$\frac{(\sigma-1)}{\left((\alpha-1)\sigma+1\right)^2}. \tag{17}$$

The necessary degree of complementarity in order to receive a positive effect on youth employment from a shock in elderly employment now depends on whether σ is larger or smaller than one. If $\sigma > 1$ (the wealth effect in labor supply outweighs the substitution effect), less complementarity is required (a larger r and s) for the case of an increasing α and vice versa.

[5] The full solution as Maple sheet is clearly available upon request.

4.2 The Exogenous Demand Model

The exogenous demand model differs from the basic model only by two equations. Equation (2) and (5) are replaced by a specification of aggregate demand which is now an exogenous AR(1)-process

$$y_t = \rho_y y_{t-1} + e_{y,t}, \tag{18}$$

and, accordingly, labor demand results from the inverse of the production function

$$\bar{l} = -\frac{a}{1-\alpha} + \frac{y}{1-\alpha}. \tag{19}$$

All other equations remain the same. Like the basic model this model is also shocked by productivity shocks a, specific labor supply shocks z, v and u as well as general labor supply shocks g. Contrary to the basic model, we omit the particular demand shock d because this shock was only extremely weak identified in the exogenous demand model for obvious reasons. However, no result is affected in any way by this omission.

For the whole relevant parameter space, one can show by the analytical solution that a positive shock in elderly labor supply always leads to a lower level of youth employment in this model.

5. Data

To bring the model to the data, we use the following data base, which is typically used in the previously mentioned studies using a single equation approach: Real GDP per capita (Source: OECD); employment rates of young (20-24), primaries (25-54) and elderly (55-64), all in logs.

We investigate several variants, differing by frequency, employment measure and detrending methods:

- Yearly data from 1974 – 2012 (Source: Statistik Austria) versus quarterly data from 1998Q1 – 2013Q3 (Source: Eurostat), seasonally adjusted by a standard X11-procedure.
- Total employment rates versus employment rates of male only. The specification of male employment only is often used [6] because female employment rates depict several effects, such as a strong positive trend in female participation rate, a very high degree of part-time employment, especially in the primary segment with up to 50% part-time, a very low full-time employment rate of elderly women, hardly above 10% which are further disproportionately found in the public sector or an unusually high allocation of

[6] For instance, in several papers of Gruber and Wise, ed. (2010) or in Uhl (2012).

employment in the service sector with up to 80%.[7] Since these particular features are not captured by our model we will refer to the male employment only case as our basic specification and use full employment data as an interesting extension.

- Linear detrending versus other detrending methods. As our model presupposes data which tend to a steady state we follow the procedure commonly used in the literature, for instance in Smets, Wouters (2003), and use linear detrended data as the standard specification. However, several unit root tests of these detrended data do not rule out unit roots. Although there are a lot of reasons why one should not take unit root tests too seriously[8] we additionally use up to 3rd order polynomials time trends, HP-filter detrended and first order differenced data in order to check for detrending robustness.

In sum, we are using four detrended time series (y, l_p, l_o, l_y) for each specification described above and interpret them as log-deviations from steady-state.

Regarding the Austrian employment rates we additionally have to deal with two breaks in the calculation procedure of the Austrian statistical agency. In 1994 they switched from the previously used 'Subsistence-Concept', which is based on the idea that people are only counted as employed if their earnings are sufficient for subsistence, to the ILO-concept, where people are counted as employed if they are working at least two hours per week. Fortunately, for the year 1994 data for both concepts are available and so we were able to adjust the time series after 1993 accordingly by using the differences of these two measures.

For the second break in 2004, which is clearly noticeable even by visual inspection, there are no data available of both statistical computation procedures for this particular year. Data from 2004 onwards are therefore shifted by a factor which is calculated by the difference of 2003 and 2004 which is further adjusted by the difference of 2004 and 2005.

In following the common practice (Gruber, Wise (2010), Uhl (2012)) we will use employment rates rather than employment in levels in our econometric analysis. Besides of some econometric advantages of rates – such as reduced collinearity and reduced trends – employment in levels further depicts severe demographic shifts. Such shifts would change the steady-state in a manner which is not modeled within our theoretical framework.

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[7] See Statistik Austria (2007).

[8] For instance, because trend and difference stationary processes are nearly observationally equivalent in finite samples. See Campbell, Perron (1991), p. 157 ff. See also Cochrane (1991), p. 202 ff. for very critical remarks on using unit root tests. Among others he argues that uncertainty about the true nature of the possible nonlinear deterministic trend renders unit root tests more or less useless. We nevertheless also use first differenced data. Our main result of nearly neglectable effects of elderly labor supply on youth employment still holds with this detrending method. However, identification of the parameters deteriorates and mainly relies on the assumed priors. This is a hint that by first differencing one is simply throwing away too much important data information. For a very enlightening comment on this 'over-differencing' problem see the note of Cochrane (2012). See also Gorodnichenko, Ng (2009) who show that improper detrending can sometimes lead to biased estimates in DSGE models.

6. Results

6.1 Basic model and basic specification

In this section, we present the case for males, yearly linear detrended data to which we will further refer to as the basic specification. The other variants (alternative specifiactions) are used as robustness checks and are reported in section 6.2.
Our model contains 20 parameters which either have to be calibrated or estimated. From these only the two share parameters for the young, b_y, and elderly, b_o (see equation (4)) are calibrated by their averages over the whole sample. Typically, these values are about 0.09 for b_o and 0.11 for b_y. The share for the primaries, b_p, is simply $1 - b_o - b_y$.
All other parameters are therefore estimated. As outlined in section 3, we have been very careful in specifying prior distributions and generally assume rather loose priors.

Prior Specifications

In section 4 it was argued that the two parameters r (which determines the substitution elasticity between the factors) and σ (which defines smoothing behavior in consumption and in conjunction with φ determines the strength of the wealth effect in labor supply) are the most important parameters for assessing the effects of old employment on young employment. The parameter α also exhibits some influence but numerically its effect is of rather minor importance.
In particular, we use the following priors for those key parameters, shown in Table 1.

Parameter	Description	Prior type	Prior mean	Prior STD	Min.	Max.
r	1 – (1/substitution elasticity)	beta	-1	0.86	-3	1
σ	Consumption smoothing parameter	normal	1	0.3	-	-
α	Returns of scale parameter	beta	0.33	0.15	0.05	0.85

Table 1: Prior distributions of r, σ and α

For the parameter r a very loose prior is chosen. Clearly, r could not be larger than one (the case of perfect substitutes) and values below minus three are rather unlikely as this corresponds to a very low elasticity of substitution of only 0.25. The standard deviation of the used beta distribution is chosen so that values between –3 and 1 are nearly equally probable, almost approximating a uniform distribution for that range. However, we have abstained from using a uniform prior because of its discontinuities at its endpoints.

Regarding σ the standard prior calibration by choosing a normal distribution with mean 1 is followed.[9] As a positive side-effect we do not presuppose whether σ is larger or smaller than

[9] See for instance Gali, J. (2008) pg. 52.

one which is crucial for the effect of α and further for the effects of productivity shocks on employment. The standard deviation is chosen rather loose also in this case.

The prior of α is assumed to be beta-distributed with mean 0.33, a standard deviation of 0.15 and an allowed range of 0.05 to 0.85. Clearly, 0.33 is the standard value for this parameter and is often calibrated (and not estimated) in this way. However, as is well-known, permitting variable capital utilization would lead to a more linear behavior of the production function with respect to labor ($\alpha < 0.33$) as this case approximates constant returns to scale. On the other hand, a broader definition of capital including human capital (which is commonly used in growth theory) would lead to a higher value of α. For these stated reasons we decided to estimate α instead of calibrating it.

The remaining parameters are the Frisch labor supply elasticities for the age groups ($1/\varphi_i$) and the standard deviations and autocorrelation coefficients of the various shocks. In particular, we assume priors for the respective parameters as presented in Table 2.

Calibrating labor supply elasticities is a little bit problematic as several micro-studies point to low labor supply elasticities (intensive margin) whereas macro-studies typically reach to much higher values (extensive margin).[10] So we decided to choose an expected value of 1 which is a rough average of the values presented in the literature. Standard deviations are also loosely specified.

Turning to the standard errors of the innovations of the exogenous shocks we simply do not know which of these four labor supply shocks, productivity shocks or general demand shocks are predominant in explaining the data. Therefore, we specified the standard deviations so that in principle every shock is capable of generating the observed variation in the data by itself. However, as the data for the young and especially old employment rates exhibit a considerably larger variation than the other two variables, we allow a somewhat larger standard error of the labor supply innovations for elderly and young. This is ‘compensated’ by a larger value for the prior standard deviation.

Parameter	Description	Prior type	Prior mean	Prior STD
epsp	Labor supply elasticity of primaries ($1/\varphi_p$)	normal	1	0.3
epso	Labor supply elasticity of old ($1/\varphi_o$)	normal	1	0.3
epsy	Labor supply elasticity of young ($1/\varphi_y$)	normal	1	0.3
σ_eu	STD of primary labor supply shocks	gamma	0.02	0.01
σ_ez	STD of elderly labor supply shocks	gamma	0.03	0.02
σ_ev	STD of young labor supply shocks	gamma	0.03	0.02
σ_eg	STD of general labor supply shocks	gamma	0.02	0.01
σ_ed	STD of demand shocks	gamma	0.02	0.01
σ_ea	STD of productivity shocks	gamma	0.02	0.01

[10] See for instance Keane, M.P., Rogerson, R. (2011).

ρu	Rho of primary labor supply shocks	normal	0.8	0.3
ρz	Rho of elderly labor supply shocks	normal	0.9	0.2
ρv	Rho of young labor supply shocks	normal	0.8	0.3
ρg	Rho of general labor supply shocks	normal	0.8	0.3
ρd	Rho of demand shocks	normal	0.8	0.3
ρa	Rho of productivity shocks	normal	0.8	0.3

Table 2: Prior distributions of the other parameters

Finally, we have to specify priors for the autocorrelation coefficients for the various shocks. Here we generally assume a mean value of 0.8 and a very loose standard deviation of 0.3. We assume a higher degree of persistence only for the labor supply shocks of elderly because these shocks are in part due to pension reforms or other legal measures which actually should have longer lasting effects, even in detrended data.

Posterior Estimates

The estimation results for our benchmark case – yearly data, males only, linear detrended – are presented in Table 3.[11]

Parameters	Post. Mode	STD of Mode	Post. Mean	Confidence Interval	
r	-1.5065	0.6041	-1.4891	-2.7375	-0.4683
σ	0.7750	0.1053	0.7872	0.5869	0.9879
α	0.3822	0.1981	0.3956	0.1363	0.6458
epsp	1.1035	0.2891	1.0919	0.6262	1.5680
epso	0.6203	0.3016	0.6996	0.2099	1.1482
epsy	0.9655	0.2850	1.0026	0.5486	1.4853
σ_eu	0.0060	0.0025	0.0064	0.0023	0.0102
σ_ez	0.0847	0.0209	0.0906	0.0553	0.1250
σ_ev	0.0388	0.0087	0.0406	0.0251	0.0551
σ_eg	0.0065	0.0027	0.0070	0.0029	0.0111
σ_ed	0.0074	0.0032	0.0081	0.0024	0.0126
σ_ea	0.0146	0.0017	0.0153	0.0125	0.0183
ρu	0.6767	0.1883	0.6436	0.3673	0.9343
ρz	0.9730	0.0286	0.9592	0.9214	0.9990
ρv	0.6597	0.1182	0.6566	0.4633	0.8475
ρg	0.6904	0.1972	0.6724	0.4166	0.9651
ρd	0.6846	0.1963	0.6634	0.3892	0.9502
ρa	0.8911	0.0820	0.8623	0.7515	0.9828

Table 3: Estimation results for model parameters; yearly data, males only, linear detrended. Modes are calculated by maximization of the log posterior kernel. The standard deviation of the mode is calculated in Dynare under the assumption of a normal distribution. Posterior mean and confidence intervals are obtained by applying the Metropolis-Hastings sampling algorithm using 50,000 draws and an acceptation rate of 0.33.

[11] Estimation is carried out with Dynare, version 4.3.4.

The posterior mode and mean of our key parameter r of approximately -1.5 corresponds to a substitution elasticity of 0.4. So, both estimates imply a quite high degree of complementarity between the different age groups. Standard deviation of the mode and estimated confidence intervals show that the case of perfect substitutes ($r = 0$) is extremely unlikely. Comparing the posterior distribution of r with the assumed prior in Figure 1 further shows that there is enough information in the data to assign most of the probability mass to be below zero. As argued in section 4, we would expect a positive effect of elderly labor supply shocks on youth employment rates because of the relative high degree of complementarity between the age groups in production process.

Our second key parameter is σ which determines consumption smoothing and, probably more importantly, the degree of wealth effects in labor supply. The estimated mean is below one which, according to the discussion in section 4, even enhances the complementarity effect a little bit. The relatively small confidence interval and a comparison of prior and posterior distribution of σ in Figure 1 reveal that this parameter is quite well identified by the data.

Turning to α we get estimates of 0.38 and 0.39, respectively. Comparing prior and posterior of α in Figure 1 could lead to the somewhat misleading conclusion that the parameter is only weak identified by the data. However, alternative prior specifications reveal that the estimate of α is quite robust and in the range of 0.3 to 0.45.

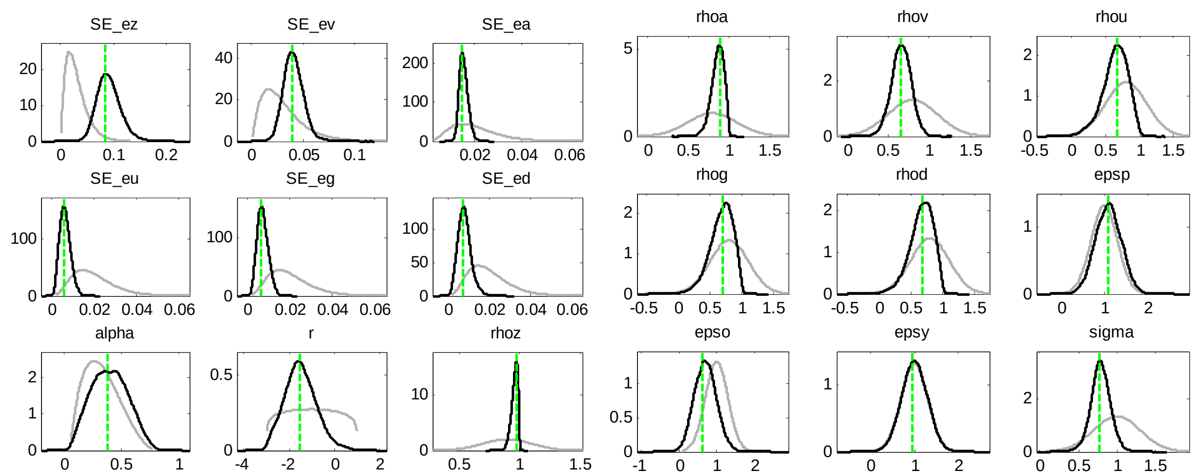

Figure 1: Priors and estimated posterior distributions, basic model

Since prior and posterior distributions of the labor supply elasticities are very similar in Figure 1 one can reach the conclusion that sample information is simply not sufficient. Consequently, these parameters are only weakly identified. However, none of our main results are affected by plausible alternative prior specifications in any significant way.[12]

As previously mentioned, we have been very cautious in specifying the priors for the standard deviations of our six innovations as we simply did not know a priori which factors are the

[12] This fact is also confirmed by the analytical solution of the model, see equation (14).

most dominant in explaining the data. As it now turned out, the point estimates for the specific innovations in elderly and youth employment exhibits the largest variance, followed by general productivity innovations. The comparison of priors and posteriors in Figure 1 shows that this finding is strongly supported by the data.

Shocks of primary labor supply as well as general labor supply shocks only play a minor role. It is further interesting to note that the estimated autocorrelation coefficients for elderly labor supply and productivity shocks show that these two factors are the ones with the highest degree of persistence. This makes perfect sense as in the case of elderly labor supply there have been several pension reforms and other legal measures during the sample period which naturally implies a higher persistence. Regarding productivity shocks, a high degree of persistence is what one would expect on a priori grounds. Although we have permitted values for the autocorrelation coefficient to be larger than one (non-stationarity) by our prior specification, Figure 1 shows that such values are regarded as extremely unlikely by the estimation procedure.

Model Impulse Responses, Basic Model, Basic Specification

As our main interest lies in the effects of shocks to the elderly labor supply, we will show only these effects in Figure 2 in more detail. All other impulse responses can be found in Appendix C, Figure 11.

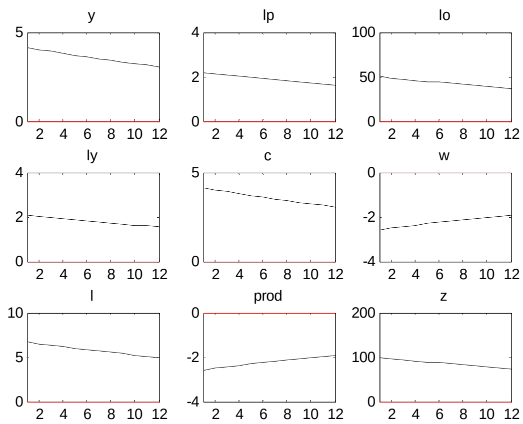

Figure 2: Model impulse responses due to a shock in elderly labor supply, z, basic model

In particular, Figure 2 shows the consequences of a unit shock in elderly labor supply z in terms of percent. So, this shock leads to an effect of only 50% on elderly employment l_o, or to an approximately 4.5% effect on output y, both measured in terms of the size of the original shock. The reason for the only 50% employment effect is a decline in elderly wages (not displayed here) which also leads to a decline in aggregate wage, w. However, the most interesting part here is that a labor supply shock of elderly leads to a *positive* effect on primary and young employment, although the effects are rather small, i.e. about 2% in terms of the original shock. The main reason for this effect is complementarity in production of the different age groups (recall that our estimate of the substitution elasticity is only about 0.4) as well as a decline in aggregate wage which leads to slightly more aggregate employment and output of about 6.5 respectively 4.5% in terms of the original shock. As aggregate employment increases, the higher labor supply of elderly is not automatically compensated by lower employment of the other age groups. General productivity is declining slightly as employment is increasing a little bit more than output. Further, the high degree of persistence of the shock in elderly employment and of its consequences is also worth noting.

To sum up, the estimation results for our reference model point towards a, although very small, crowding-in of other labor groups after a labor supply shock of a specific group. In contrast, assuming a fixed amount of labor and output would predict a severe crowding-out of the other groups, i.e. a decline of employment in the other groups.

In the following we will discuss model impulse responses along with their 90% Bayesian confidence bands. These impulse responses and confidence bands are based on the posterior distributions and posterior means of the model parameters in contrast to the impulse responses shown in Figure 2 which are based on the posterior modes. Furthermore, we consider a one standard deviation shock and not a unit shock. Generally, these figures are in accordance with the impulse responses discussed above.

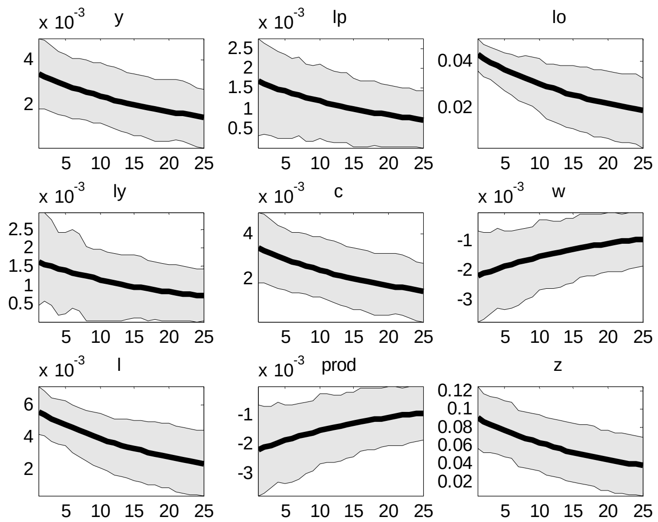

Figure 3: Impulse responses and 90% Bayesian confidence intervals for a one standard deviation shock in labor supply of elderly on the other observed variables, basic model

In Figure 3 a one standard deviation shock in elderly labor supply also has positive effects on young and primary employment. The interesting point is that according to the estimated confidence bands a negative effect can be ruled out with high probability or, in the language of classical econometrics, we have a significant positive effect. But note that, once again, the estimated positive effects are very small; a one standard deviation shock of about 8% in elderly labor supply leads to an impact effect on elderly employment of about 4% and further leads to an effect in young and primary employment of only 1.5 per mill. So this effect is rather negligible by all means.

In Appendix C, Figure 12 we additionally report the corresponding impulse responses of shocks in primary and youth labor supply, respectively.

6.2 Basic model, Alternative Specifications

In the following, we discuss alternative versions of the basic model. As a first variation of the baseline model we consider the effect of different detrending methods as well as different data frequencies. In particular, hp-filter, first difference and second order polynomial detrending is applied to both yearly and quarterly data.[13]

Model Impulse Responses of the Basic Model for alternative specifications

Figure 4 presents the effects of a one standard deviation shock in elderly labor supply on the other observable variables. For comparative reasons, we once more present the effects of our standard specification (i.e. Figure 3) in Panel a. The results for yearly data with different detrending methods are reported in Panel b, c and d and are in line with our basic model: A one standard deviation increase in elderly employment results in an even smaller but still positive effect on youth employment. Yet, according to the confidence bands, a very tiny negative effect cannot be ruled out anymore, i.e. the positive effect is not statistically significant, in particular for the first difference case.

The results for quarterly data are presented in Panel e, f, g and h. Note that for these specifications the estimation period is now from 1998Q1 to 2013Q3 due to unavailability of quarterly data prior to 1998. As one can see, the basic results remain the same, which indicates that our estimates are robust to a reduction in the estimation period. Once again, we can expect very small positive effects (about 2% of the original change in elderly employment) but very tiny negative effects cannot be ruled out, in particular for the first yearly difference case.[14]

[13] Additionally, we applied third order polynomial detrending but since the results of third order polynomial and hp-filter detrending hardly differ, only the hp-filter case is presented here. The other results are available on request.

[14] However, it should be noted that the specification of the shocks as AR(1)-processes is not very well-suited for quarterly data, since the typical hump-shaped impulse responses of quarterly VARs cannot be reproduced with this specification. One possibility would be to specify the shocks as higher order AR-processed but in this case the advantage of the larger number of observations in the quarterly data set would rapidly diminish.

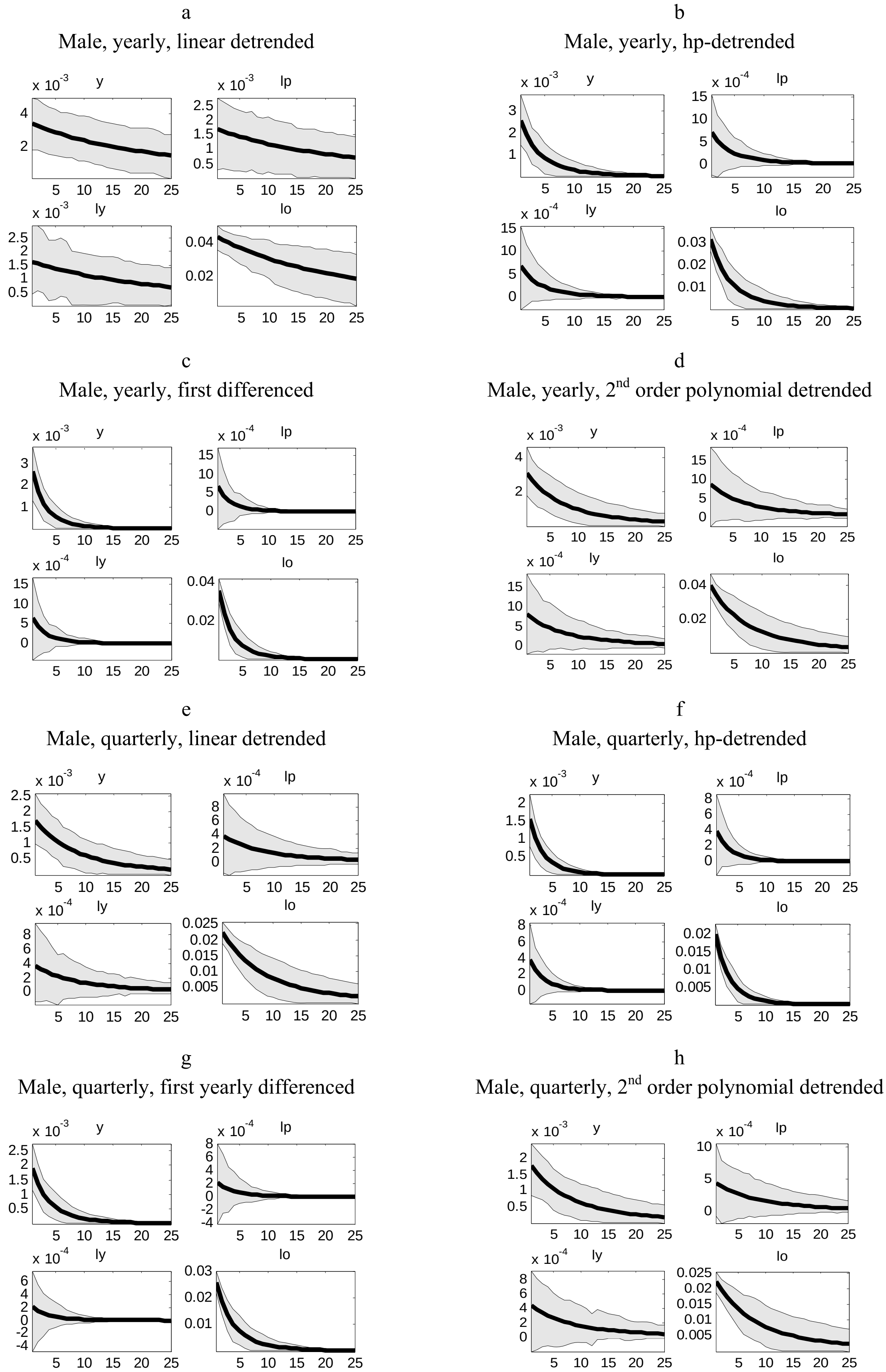

Figure 4: Effect of different specifications on impulse responses and 90% Bayesian confidence intervals for a one standard deviation shock in labor supply of elderly on the other observed variables, basic model, alternative specifications, male employment

Figure 5 essentially shows the same specifications as in Figure 4 but instead of male employment data total employment data are used now. As already mentioned in section 5, female employment rates depict secular trends which are not modelled within our framework and are possibly not well treated by our detrending methods. Additionally, extreme part-time shares in some age cohorts and a very low elderly participation rate where public employees are further overrepresented might lead to difficulties in interpretation or even to biased results in empirical analysis.

However, it is quite possible that females are concentrated in sectors in which complementarities play only a minor role. For example, retiring teachers are nearly exclusively replaced by young entrants. Clearly, in this case, and maybe in other parts of the public sector, we almost have a perfect substitution relationship between elderly and young. Now, 69% of the teachers in Austria are females who typically retire later than females in the private sector.[15] Obviously, this overrepresentation could lead to a higher overall degree of substitutability if female employment is taken into account.

Nevertheless, the estimated effects of shocks in elderly labor supply generally remain very small and slightly positive and are therefore in accordance with the results for men only. Only in the specification of yearly 2nd order polynomial detrended data we now observe a small negative effect. Overall, besides the case of quarterly hp-filtered data, negative effects cannot be ruled out anymore. Calculating the largest plausible *negative* effect, based on our confidence bands, a one standard deviation shock in elderly employment leads to an increase in elderly employment of about 4%, which in turn reduces youth employment at most by 0.05%, which implies an impulse cause ratio of 70:1. Clearly, small positive effects of the same order are practically equally likely as this negative effect.

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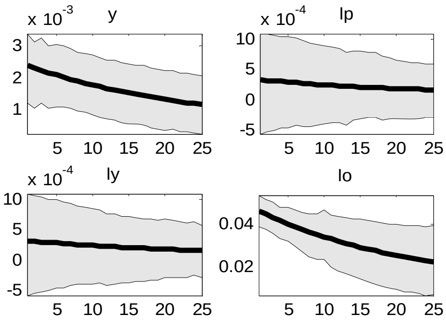
[15] Statistik Austria (2007), p.14.

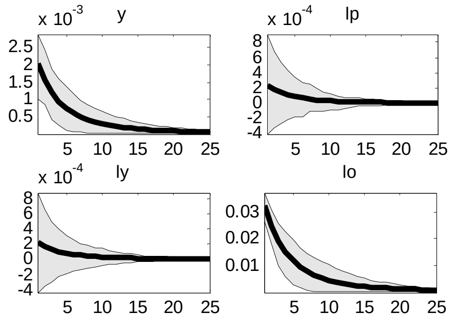
a

Total, yearly, linear detrended



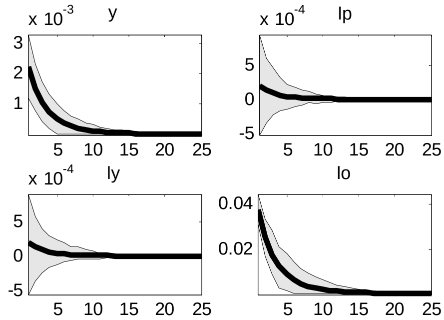
B

Total, yearly, hp-detrended



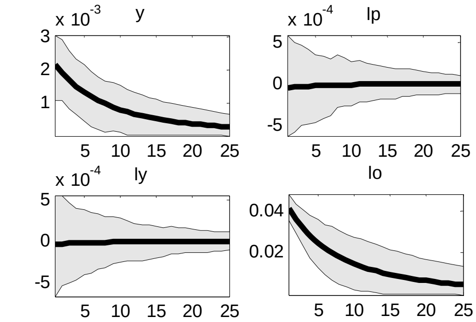
c

Total, yearly, first differenced



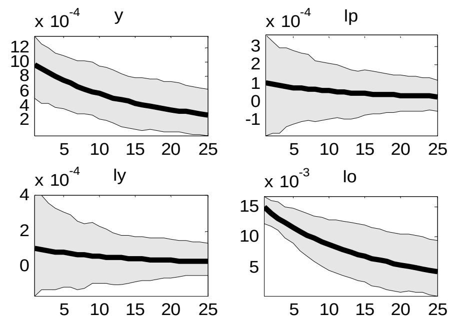
D

Total, yearly, 2nd order polynomial detrended



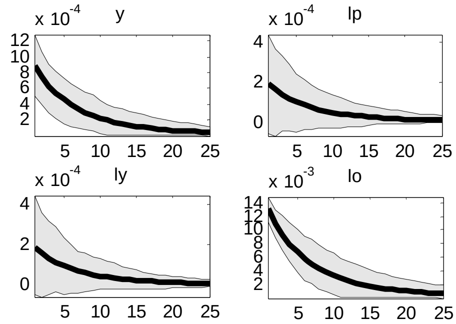
e

Total, quarterly, linear detrended



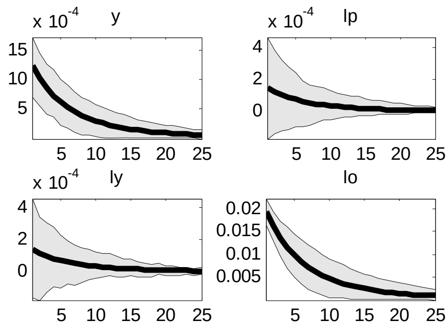
F

Total, quarterly, hp-detrended



g

Total, quarterly, first differenced



H

Total, quarterly, 2nd order polynomial detrended

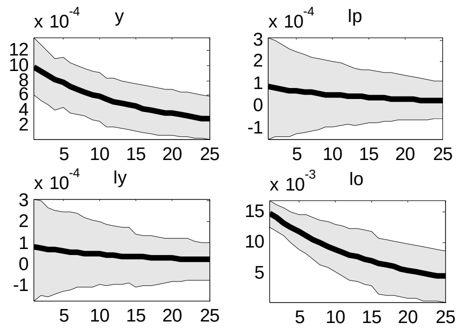


Figure 5: Effect of different specifications on impulse responses and 90% Bayesian confidence intervals for a one standard deviation shock in labor supply of elderly on the other observed variables, basic model, alternative specifications, total employment

Posterior Estimates for the Parameter r of the Basic Model, Alternative Specifications

Since the results crucially depend on the estimated degree of complementarity Table 4 reports the estimated posterior modes, means and confidence bands for the key parameter r which determines the elasticity of substitution. As claimed in section 4.2, a substitution elasticity of one ($r = 0$) is of particular interest since this represents the tipping point between the dominance of complementarity and substitution effects (supposing σ to be roughly one).

As one can see, in the case of males only, the estimates for all specifications lie in the territory of a predominant complementary effect. Considering total employment the results are more mixed and additionally characterized by notably larger confidence intervals.

Model Specification	Post. Mode	Post. Mean	Confidence Interval	
		Male only		
yearly, linear detrended	-1.5065	-1.4891	-2.7375	-0.4683
yearly, hp-detrended	-0.1565	-0.5755	-1.8357	0.7026
yearly, 1st difference	-0.0430	-0.6991	-2.0894	0.8830
yearly, 2nd order polynomial	-0.5043	-0.6587	-1.7225	0.6203
quarterly, linear detrended	-0.6196	-0.9213	-2.4079	0.4814
quarterly, hp-detrended	-0.8083	-1.1489	-2.6998	0.2345
quarterly, 1st yearly difference	-0.0924	-0.7202	-2.1197	0.9259
quarterly, 2nd order polynomial	-1.0503	-1.2226	-2.7813	0.1216
		Total		
yearly, linear detrended	0.0205	-0.2637	-1.2757	0.8582
yearly, hp-detrended	0.3401	-0.5529	-2.0044	0.9790
yearly, 1st difference	0.4277	-0.6490	-2.1268	0.9749
yearly, 2nd order polynomial	0.8940	0.2291	-0.6058	0.9993
quarterly, linear detrended	-0.4711	-0.8059	-2.3846	0.6954
quarterly, hp-detrended	-1.4946	-1.4253	-2.9672	-0.1206
quarterly, 1st yearly difference	-1.6742	-1.3368	-2.9811	0.0795
quarterly, 2nd order polynomial	-0.4024	-0.7599	-2.2330	0.7409

Table 4: Posterior estimates for r depending on different specifications

Summing up, all investigated specifications point to extraordinary small effects of shocks in elderly labor supply on other labor groups. Considering males only, the point estimates show a small positive effect (crowding-in) which in some specifications is even significant. Taking total employment into account, point estimates remain positive in most cases but very small negative effects (crowding-out) are also possible. It remains unclear whether the somewhat different results are due to the statistical problems mentioned above or are actually reflecting a lower complementarity among female employees.

In our view, the main implication of our analysis is as follows: As even the largest plausible negative consequence on youth employment (and primary employment) amounts only to a maximum effect of a seventieth of the originally change in the elderly employment[16], this effect is neglectable by any means and could therefore hardly serve as a convincing argument against increasing actual retirement age.

6.3 Model evaluation

Since the presented results are clearly dependent on the particular model we have to check whether the basic model is capable of reproducing the main features of actual data, i.e. moments and correlation structure. Under the assumption that the basic model is the true data generating process and actual data are only a subsample of these data, we have to deal with the sampling error when comparing moments of actual data with model generated data. Therefore, the following procedure has been applied: Using the estimated model, we generate 400 samples of the same length as actual data, compute moments and corresponding confidence intervals and assess whether the moments of actual data are in accordance with the moments of simulated data, i.e. whether the empirical moments of actual data lie within the confidence intervals.

STD	Actual Data	Simulated Data	Confidence Intervals	
y	0.0253	0.0282	0.0117	0.0447
l_p	0.0106	0.0086	0.0048	0.0125
l_o	**0.1671***	0.0909	0.0262	0.1555
l_y	0.0236	0.0202	0.0131	0.0274
prod	0.0341	0.0248	0.0110	0.0386

Table 5: Comparison of standard deviation for actual and simulated data, 2SE confidence intervals are for simulated data, significant differences are bold with asterisks

Table 5 reports estimated standard deviations for actual data and model generated data for the standard specification males, yearly linear detrended data[17]. Note that we also consider productivity calculated by $\log(Y) - \log(L)$ although this productivity variable is not used for estimation purposes as it is perfectly collinear with the other used variables. Our model passes this first compatibility test quite well as most moments of actual data lie within the 2SE confidence intervals of model generated data. Only the variation of l_o is actually larger than the model predicts.

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[16] This statement is in percentage terms. Presuming shares of primary, old and young on total employment to be 0.8, 0.09 and 0.11, respectively, one concludes in terms of persons, that one additionally employed elderly would *in the worst case* lead to about 0.02 less employed young and 0.13 less employed primary. The expected effects are slightly positive in most cases.

[17] We only present our standard specification. All other specifications yield very similar results. By construction, our model could not reproduce hump shaped impulse responses for quarterly data but nonetheless for the majority of the cases, the impulse responses estimated with actual data lie within the confidence bands of the impulse responses estimated with model generated data.

Table 6 reports the correlation between variables which were actually used for estimation. Here once again nearly all cross correlation could be reproduced by the model or at least lie within the 2SE confidence intervals. Only the correlation between the youth employment rate and productivity is statistically incompatible.

Correlation			Actual Data	Simulated Data	Confidence Intervals	
y	<=>	l_p	0.3228	0.5569	0.1364	0.9774
y	<=>	l_o	-0.0492	0.2421	-0.5138	0.9981
y	<=>	l_y	-0.0566	0.2611	-0.2831	0.8054
y	<=>	*prod*	0.6890	0.8456	0.6000	1.0913
l_p	<=>	l_o	0.7561	0.4755	-0.0690	1.0199
l_p	<=>	l_y	0.6935	0.4300	-0.0347	0.8948
l_p	<=>	*prod*	-0.4095	0.1553	-0.4642	0.7749
l_o	<=>	l_y	0.7017	0.1940	-0.3845	0.7725
l_o	<=>	*prod*	-0.7386	-0.1780	-0.9844	0.6284
l_y	<=>	*prod*	**-0.613***	0.0263	-0.5626	0.6153

Table 6: Comparison of correlation between variables for actual and simulated data, 2SE Confidence Intervals are for simulated data, Significant Differences are bold with asterisks

Next, we examine a standard regression as presented in Gruber and Wise (2010) and Uhl (2012) with youth employment as a dependent variable and gdp per capita, employment rates of primaries and olds as explanatory variables. The results of this regression are reported in the first column of Table 7. The second column shows the average results obtained by our 400 model generated data sets and the last two columns the corresponding 2SE confidence intervals. The inspection of Table 7 clearly reveals that regression results of the actual data are compatible with the model generated ones.

	Actual Data	Simulated Data	Confidence Intervals	
constant	-0.0011	-0.0006	-0.0273	0.0262
y	-0.2135	0.0197	-0.4759	0.5154
l_p	1.277	1.0677	-0.4958	2.6313
l_o	0.0364	-0.0064	-0.1659	0.1531

Table 7: Regression Results of l_y on constant, y, l_p and l_o, 2SE confidence intervals are for simulated data, significant differences are bold with asterisks

As a final check, we estimate a VAR with gdp and primary, elderly and young employment rates as variables to capture the correlation and autocorrelation structure of the data. The special aspect of that procedure is not to compare the model IRFs with the VAR-IRFs based on actual data but using model generated data instead, estimate an equivalent VAR and compare those IRFs with the VAR-IRFs estimated with the actual data. As we are only interested in the correlation structure a big advantage of this method is that we do not have to care about identification, omitted variables or other issues usually encountered with VAR analysis. This approach is in line with the arguments put forward in Kehoe (2006) and Chari, Kehoe, McGrattan (2008).

Figure 6 reveals that the impulse responses of the VAR estimated with actual data are in generally in accordance with the IRFs obtained by the simulated data. The only clear exception is $y \Rightarrow l_o$.[18]

We want to stress that the impulse responses shown in Figure 6 should not be interpreted as causal effects since the errors in this VAR are not identified as primitive shocks. But as Kehoe (2006) has shown even a 'well identified' structural VAR could lead to very misleading results and should only be interpreted with great caution.

All in all, it can be stated that our estimated model is not in an obvious conflict with the main features of actual data and we are therefore quite confident of the relevance of our results.

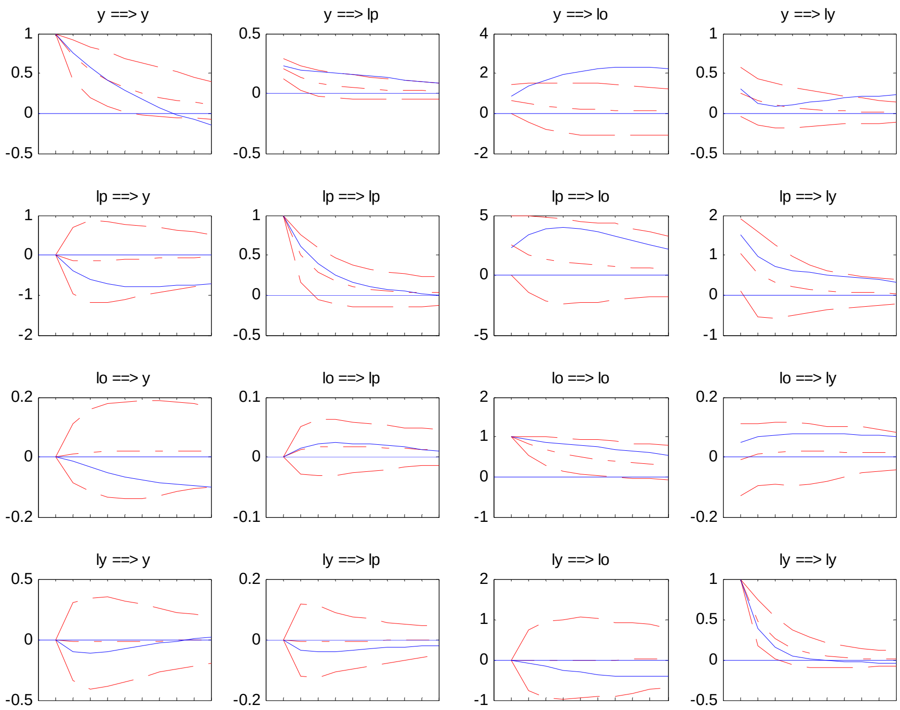

Figure 6: Comparison of VAR impulse responses estimated with actual data and model generated data. VAR estimation was carried in the order y, l_p, l_o, l_y; impulse responses are calculated by the usual Cholesky decomposition. Solid lines correspond to impulse responses estimated with actual data, dotdashed lines are the means of the impulses responses estimated with model simulated data and dashed lines correspond to the 90% confidence intervals obtained by simulated data.

[18] The 'fit' with the other detrending methods is even better than for the basic specification with linear detrending. As an example the case for hp-filter detrending is presented in Appendix D, Figure 13.

6.4 Exogenous demand model

In the last sections we have presented our main results and have shown that the basic model is generally compatible with actual data. Of course, the results are clearly model dependent and therefore the question arises, whether alternative models exist which would lead to other conclusions and whether these models are also compatible with the data. In our context, a natural candidate for such an alternative model is a solely demand-driven, ultra-Keynesian model which reflects the idea of a fixed amount of labor which is divided among several types of labor groups. In such a framework a labor supply shock on the elderly would lead to a decline in employment rates of other groups. In section 4.2 we already have introduced such an exogenous demand model which is a variant of the basic model with only two equations altered.

Prior Specifications and Posterior Estimates of the Exogenous Demand Model

However, in estimating this alternative model several difficulties have been encountered. Firstly, many model parameters and shocks are only extremely weak identified. So, as a first consequence we dropped the general demand shock d from equation (12) as we already have specified general demand y as exogenous shocks, see equation (18). This tactic does not change the results because the estimation procedure has set the variance of d to zero anyway. Secondly, the fit of the model compared to the basic model was extremely bad. The reason is the assumed labor demand function of this alternative model, equation (19), which is just the inverse of the production function. With the parameter α lying in the normal range (approximately 0.33) this labor demand function would imply a larger relative variation in employment than in output which, for the possible reasons of labor hording or varying capital utilization, is clearly counterfactual, especially in Austria. Therefore the 'Keynesian' model would have a severe handicap. To overcome this difficulty, α is allowed to becoming negative by specification of a suitable prior[19]. But here an additional problem arises; especially in the variants estimated with quarterly data, if a very diffuse prior is used, the estimation procedure sets α to extremely low values, like -11. This would lead to a nearly complete decoupling of output and employment variations which is not the basic idea of a Keynesian model. Further, shocks in the unobserved total factor productivity a, which only appear in the labor demand equation in this exogenous demand model, could serve to fit aggregate employment data perfectly well, without any model restrictions coming from y or the different labor groups. Inspecting the standard deviation of detrended log output and log employment in Austrian data, the variation in output is at most three times larger than the variation in employment, depending on the particular detrending method. Consequently, a lower bound for α needs to be set. The basic idea is that employment is mainly driven by aggregate demand shocks in a Keynesian type model. If aggregate demand happens to be the only source of variation in employment then, according to equation (19) α would be equal to $1 - \sigma_y / \sigma_l$. Now we assume that at least half of the variation in employment is due to aggregate demand (variance

[19] However, as α represents profit share in equation (11) we fixed α at a value of 0.33 in this equation.

in a is not larger than the variance in y) and therefore the lower bound of α is equal to $1 - \sqrt{2}\,\sigma_y / \sigma_l$. If we further take into account that variations in capital utilization lead to a correlation between productivity and demand shock and make the extreme assumption of a prefect correlation between a and y we reach to $1 - 2\sigma_y / \sigma_l$.[20]

Apart from that lower bound, the used prior for α resembles nearly a uniform distribution. We think, this continues to be a very loose prior and is only intended to prevent a slipping away of the parameter estimates, especially α, to very implausible ranges. All other prior distributions are assumed to be the same in the basic and exogenous demand model. In the following, the estimation results for the exogenous demand model using the standard specification are presented, i.e. yearly linear detrended data for males only. Figure 7 shows the used priors together with the estimated posterior distributions. Identification, especially in the case of r, seems to be an issue.

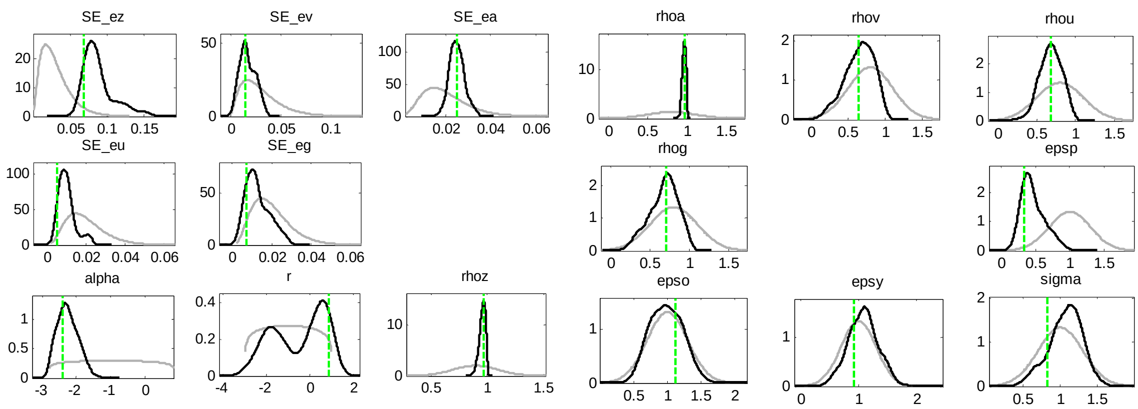

Figure 7: Priors and estimated posterior distributions, exogenous demand model

Model Impulse Responses for the Exogenous Demand Model

The effects of the standard elderly labor supply shock z are presented in Figure 8 for male employment only and in Figure 9 for total employment and should be compared to the corresponding Figure 3 and Figure 4 which show the basic model effects.

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[20] We are searching for a lower bound of α which is compatible with observed variances in y and l given the goods demand driven labor demand function (19). Starting with a modified production function $Y = A \cdot (L \cdot U_L)^{1-\alpha}(K \cdot U_K)^{\alpha}$ with unobserved utilization rates U_L and U_K and constant K the demand for effective labor is: $l + u_L = (-a + y + \alpha u_K)/(1-\alpha)$. This implies: $(1-\alpha)^2 = \left(\sigma_a^2 + \sigma_y^2 + \alpha^2\sigma_{u_K}^2 + 2\alpha\,\mathrm{cov}(y,u_K)\right)/\sigma_{l+u_L}^2$. As we have no observations on u_K, we assign the effect of u_K to productivity a which then leads to a correlation between a and y with correlation coefficient ρ. So we have: $(1-\alpha)^2 = \left(\sigma_a^2 + \sigma_y^2 + 2\rho\,\sigma_a\sigma_y\right)/\sigma_{l+u_L}^2$. For the calculation of a lower bound for α we assume that σ_a is not lager than σ_y and that ρ has the largest possible value of one. So we get $(1-\alpha)^2 = 4\sigma_y^2 / \sigma_{l+u_L}^2$. As we only observe l and not effective labor l+u, a lower bound of α which is compatible with σ_y and σ_l is: $\alpha = 1 - 2\sigma_y / \sigma_l$.

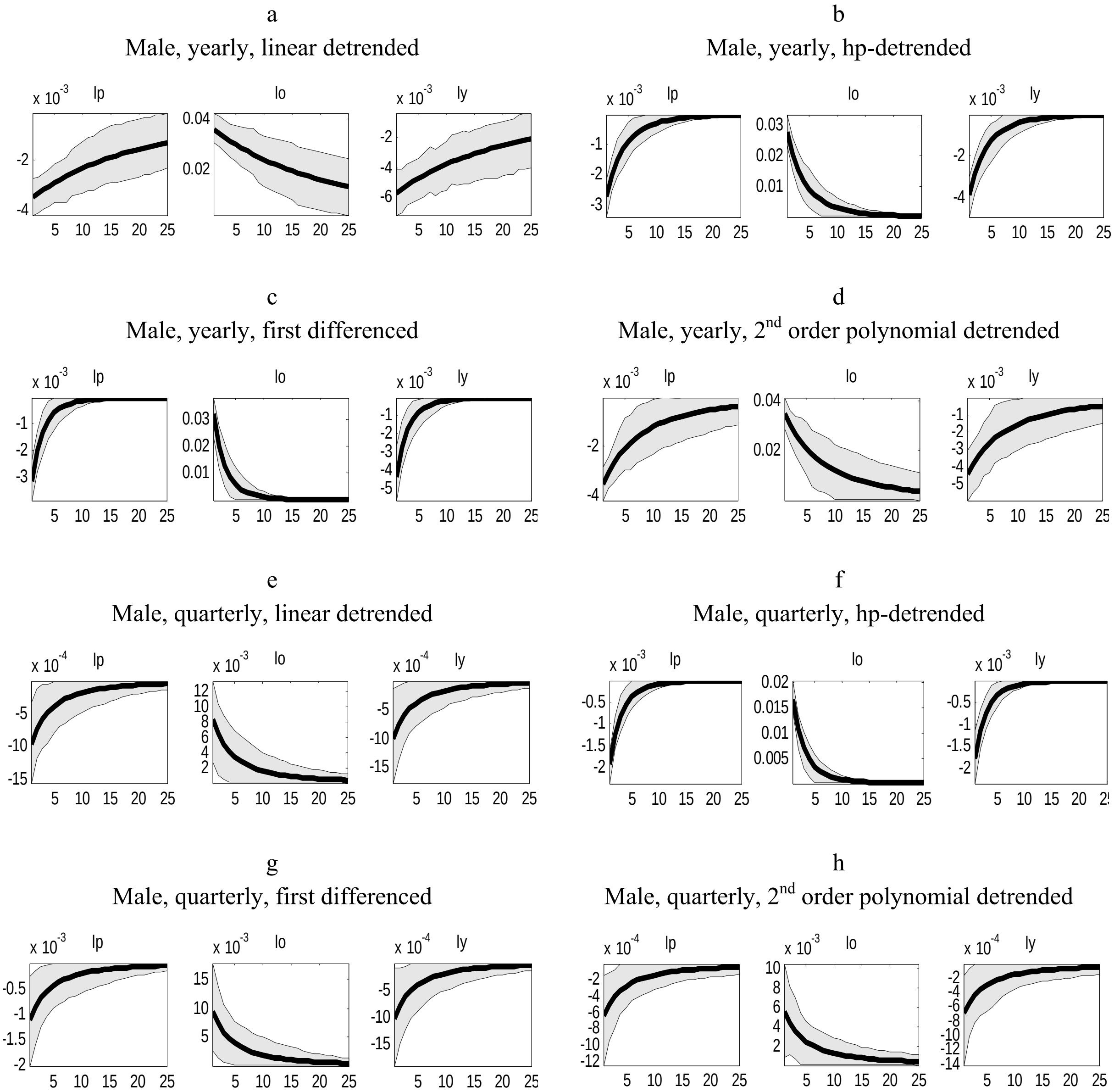

Figure 8: Impulse responses and 90% Bayesian confidence intervals for a one standard deviation shock in labor supply of elderly on the other observed variables, male employment, exogenous demand model

The negative responses of the other employment groups in the exogenous demand model are exactly what one would expect with an exogenous output process. Nevertheless, the numerical effects, although relevant for sure, are still relatively low; one percent more employment in the elderly cohort leads to approximately 0.15% less youth employment. The other variants, especially those with quarterly data even point to slightly lower effects of about 1:10. In absolute terms, this means that – according to our estimates – for every additional employed elderly approximately one person is employed less. But this effect is essentially proportionally distributed among the other groups and so the effect on the young is only 1:10 to 1:15.

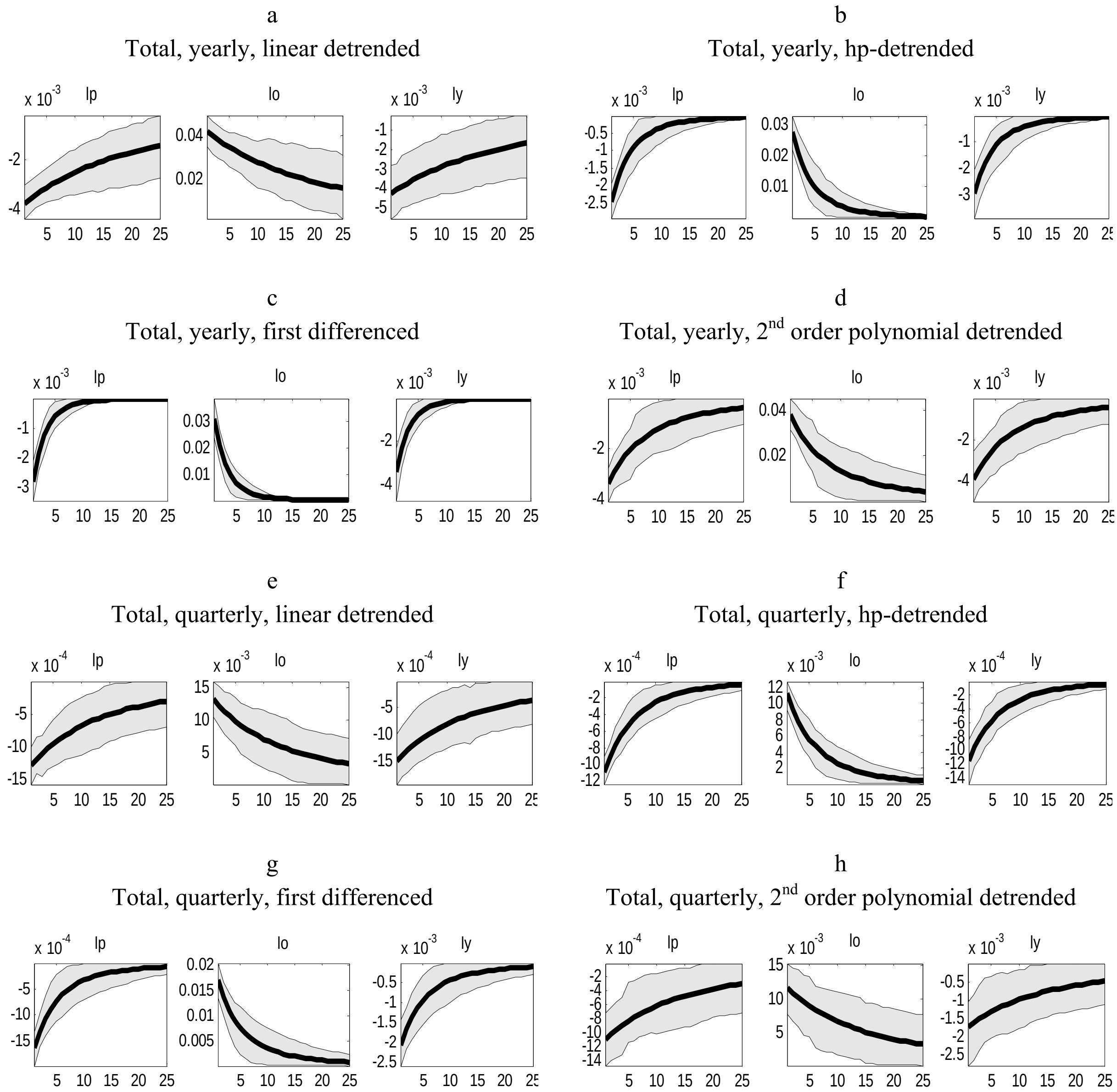

Figure 9: Impulse responses and 90% Bayesian confidence intervals for a one standard deviation shock in labor supply of elderly on the other observed variables, total employment, exogenous demand model

Comparison of Basic and Exogenous Demand Model

The applied Bayesian framework enables a natural way to directly compare models using the estimated posterior distributions, see Griffoli (2013: pp. 80). We have followed this procedure for all investigated data variants. Results are presented in Table 8.

It is a striking fact that, conditional on our prior specifications, the basic model is more probable than the exogenous demand model in every specification considered. For the standard case, male only, yearly linear detrended data, the basic model is an astonishing 140 thousand times more probable than the exogenous demand model. Generally, for yearly data the basic model is at least 421 times more probable than the alternative one.

Model Specification	Probability of basic model	Probability of alternative model	Bayes Ratio
		Male only	
yearly, linear detrended	0.999993	0.000007	140022.9
yearly, hp-detrended	0.997632	0.002368	421.4
yearly, 1st difference	0.999373	0.000627	1595.0
yearly, 2nd order polynom	0.999971	0.000029	34025.0
quarterly, linear detrended	1.000000	0.000000	7687571.4
quarterly, hp-detrended	1.000000	0.000000	2252768.5
quarterly, 1st yearly difference	1.000000	0.000000	19995056.8
quarterly, 2nd order polynom	1.000000	0.000000	42711402.2
		Total	
yearly, linear detrended	1.000000	0.000000	3237700.8
yearly, hp-detrended	0.999566	0.000434	2304.1
yearly, 1st difference	0.999902	0.000098	10240.3
yearly, 2nd order polynom	0.999914	0.000086	11680.7
quarterly, linear detrended	1.000000	0.000000	22421936.6
quarterly, hp-detrended	1.000000	0.000000	247327751.9
quarterly, 1st yearly difference	1.000000	0.000000	442351829929.9
quarterly, 2nd order polynom	1.000000	0.000000	2021629855353.5

Table 8: Comparison of basic and exogenous demand model, using posterior probabilities and posterior odds ratios of the models. The prior probability for each model is assumed to be 0.5.

Considering the variants with quarterly data reveals the same picture. Actually, the dominance of the basic model is even more pronounced than for the yearly variants. This is a little bit surprising. In analyzing quarterly detrended data we are mainly concentrating on within year effects and for very short term effects the notion of a somewhat fixed aggregate output and employment seems to be more plausible than for a longer term perspective. From this point of view it is indeed surprising that the basic neoclassical equilibrium model is much more probable than the exogenous demand model. This more pronounced dominance might be due to the different sample period or simple because we have more observations for the quarterly data case.

In some sense both models are polar cases, especially in the short term, so for within year effects the 'truth' probable lies somewhere in between. Since we are basically confronted with model uncertainty the now popular concept of model averaging seems natural.[21] However, as the basic model is at least 421 times more probable than the exogenous demand driven model, we will abstain from this exercise.

7. Conclusion

In the introduction we posed the question, whether there is a connection between the employment rates of young, primaries and olds and how this connection looks like. There exists a bunch of literature dealing with this topic and the main results point toward a positive effect of elderly employment on youth employment. But some skepticism is appropriate regarding this work because severe identification issues are typically not dealt with. Further, the claimed positive effects are generally not explained in those papers. In this study, we followed the rather natural approach by employing standard economic theory. A concrete general equilibrium model was estimated by Bayesian methods and the effects of interest have been deduced by studying the relevant model impulse responses. By following this procedure not only the identification problem is addressed. We were also able to clarify that, besides a positive output effect due to decreasing wages, the degree of complementarity of different age groups in the production process is responsible for possible positive effects of elderly employment on youth employment.

Several data variants have been examined, i.e. total versus male only employment, different detrending methods and quarterly versus yearly data. Our main result is that shocks in elderly labor supply, for instance due to legal measures in retirement laws, only lead to neglectable effects on the other age groups. For male employment we typically get an even positive effect because of a relatively high estimated complementarity. Results for total employment were a little bit different with substantially wider confidence bands. This might simply be due to statistical problems regarding female employment data or might be due to a higher degree of substitutability among women or, more probable, among jobs which are typically occupied by women. So it remains an open question whether these differences in male and female employment effects are indeed of a systematic nature. One possible way to deal with this question would be to investigate male/female employment effects in other countries which we left for future research. However, for Austrian data the estimated effects are simply neglectable, regardless of the used employment concept.

The reasons for these only small effects are threefold. Firstly, output is not fixed and changing labor supply clearly has effects on economic activity. For supporting this line of argument one could think of demographic effects on equilibrium output. Secondly, old and young are not perfect substitutes and to some degree a broad variety of different skills is demanded by firms. But apparently, these different skills are unevenly distributed among distinct age groups and so positive externalities are possible and probable. Thirdly, even if these complementarities

[21] For an early contribution see for instance Leamer (1978).

are too small the resulting crowding-out is nonetheless expected to be neglectable. The reason is that hiring or retiring of the old surely do not constitute a dominant part of labor separation or finding rates for the whole labor market. Furthermore, substitution certainly does not occur exclusively among young and old but most often involves primaries because these are the main part of the labor force.

The presented results are undoubtedly model dependent. First of all, a neoclassical equilibrium model was presumed which obviously constitutes a severe simplification. In addition, the CES-specification of combined labor could also be problematic as it assumes symmetric substitution elasticities among the different labor groups. Consequently, model evaluation was an important aspect of our work. It was demonstrated that the basic model, although highly stylized, is generally compatible with actual data. We additionally considered an alternative model which captures the lump of output idea with exogenously given aggregate demand determining aggregate employment. The exogenous demand model turned out to be extremely unlikely compared to the basic model for yearly as well as for quarterly data.

As all examined specifications only lead to tiny, mostly positive effects of elderly labor supply on the other labor groups, the argument that reforms in pension's law and increasing actual retirement age have relevant negative effects on youth employment is simply not supported by this study.

Appendix A

To clarify the argument, consider the following data generating process where the endogenous variables like l_o, l_p, l_y, y, etc. are driven by some exogenous factors:

$$\begin{bmatrix} l_y \\ l_o \\ y \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \boldsymbol{B} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{20}$$

Thereby u, v and w are unobserved and uncorrelated shocks, like global or specific preference shocks, productivity shocks, general demand shocks, etc. and the x_is are some other observed exogenous factors. We want a representation with l_y, l_o and y on the left hand side of each equation which is obtained by:

$$\boldsymbol{Adj} \begin{bmatrix} l_y \\ l_o \\ y \end{bmatrix} = |\boldsymbol{A}| \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \boldsymbol{Adj} \cdot \boldsymbol{B} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad with\ \boldsymbol{A}^{-1} = \frac{\boldsymbol{Adj}}{|\boldsymbol{A}|} \tag{21}$$

This could be normalized to the following form:

$$\begin{aligned} l_y &= \gamma_{11} l_o + \gamma_{12} y + f^1(u,v,w,x_1,x_2,x_3) \\ l_y &= \gamma_{21} l_o + \gamma_{22} y + f^2(u,v,w,x_1,x_2,x_3) \\ l_y &= \gamma_{31} l_o + \gamma_{32} y + f^3(u,v,w,x_1,x_2,x_3) \end{aligned} \tag{22}$$

Here, the γ_{ij} are functions of the coefficients of the adjoint of $\boldsymbol{A}$. All three equations of (22), which are the basis of the single equation approach, and any linear combination of them are of the same structure. So clearly, the γs are not identified without further assumptions. Moreover the right hand side variables l_o and y are correlated with the error terms. The standard procedure for identification and estimation is to assume a structure of the following form,

$$\begin{aligned} l_y &= \delta_{11} l_o + \delta_{12} y + \delta_{12} x_1 + g^1(u,v,w) \\ l_o &= \delta_{21} l_y + \delta_{22} y + \delta_{22} x_2 + g^2(u,v,w) \\ y &= \delta_{31} l_o + \delta_{32} l_y + \delta_{32} x_3 + g^3(u,v,w) \end{aligned} \tag{23}$$

with zero restrictions for the x_is. Now, the equations are unambiguous and x_2 and x_3 could be used as instruments for l_o and y in the first equation. Note, that at least two (extern) instruments are necessary for consistent estimation in this particular case.

Therefore, using a single equation approach with a left hand variable youth employment and a right hand variable elderly employment automatically leads to an identification problem.

Appendix B

The problem of the representative firm

We postulate a standard Cobb-Douglas production function:

$$Y = A\,\overline{L}^{(1-\alpha)} . \tag{24}$$

Or in log-deviations around the steady-state:

$$y \;=\; a \;+\; (1-\alpha)\;\overline{l}\;. \tag{25}$$

Here, lower case letters denote log-deviations from the corresponding steady-state value, and $\overline{L}$ denotes combined labor which is aggregated by the following CES-function. Note, that the parameter r is a measure of the degree of substitutability between the different types of labor.

$$\overline{L} \;=\; \left(b_o^{\;1-r} L_o^{\;r} \;+\; b_p^{\;1-r} L_p^{\;r} \;+\; b_y^{\;1-r} L_y^{\;r}\right)^{1/r} . \tag{26}$$

The b_is are defined as volume shares at steady-state and sum to one:

$$b_i \;=\; \frac{\hat{L}_i}{\hat{L}_o + \hat{L}_p + \hat{L}_y} . \tag{27}$$

The hats over the variables now indicate the corresponding steady-state values.

Calibrating a CES-function could be tricky, as the share parameters are generally not independent from the exponent r. See for instance Cantore, Levine (2012) or Temple (2012). We follow an approach suggested by Senhadji (1997) and use $b_i^{\,1-r}$ instead of b_i in the CES-aggregator. As it will be shown, this actually allows us to use volume labor shares for calibrating the b_is.

Turning to the problem of log-linearization of combined labor, the general formula for doing this is given by[22]:

$$\overline{l} \;=\; \varepsilon_{L_o} l_o \;+\; \varepsilon_{L_p} l_p \;+\; \varepsilon_{L_y} l_y . \tag{28}$$

Here, as above, lower case letters denote log-deviations from the corresponding steady-state value and the ε_is represent elasticities and are defined as $\varepsilon_i \equiv \frac{\partial \overline{L}}{\partial L_i}\frac{L_i}{\overline{L}} = \frac{\partial \log(\overline{L})}{\partial L_i} L_i$ which are calculated at their steady-state values. So we have:

$$\log(\overline{L}) \;=\; \frac{1}{r}\log\left(b_o^{\;1-r} L_o^{\;r} \;+\; b_p^{\;1-r} L_p^{\;r} \;+\; b_y^{\;1-r} L_y^{\;r}\right). \tag{29}$$

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[22] See for instance Hülsewig 2011, p. 32

Calculating the elasticity for L_o as a representative example we get:

$$\varepsilon_{L_o} = \frac{\partial \log(\overline{L})}{\partial L_o} L_o = \frac{1}{r} \cdot \frac{1}{\left(b_o^{1-r} L_o^{\,r} + b_p^{1-r} L_p^{\,r} + b_y^{1-r} L_y^{\,r}\right)} \cdot b_o^{1-r} \cdot r \cdot L_o^{\,r-1} \tag{30}$$

$$= \frac{b_o^{1-r} \cdot \hat{L}_o^{\,r}}{b_o^{1-r} \hat{L}_o^{\,r} + b_p^{1-r} \hat{L}_p^{\,r} + b_y^{1-r} \hat{L}_y^{\,r}} . \tag{31}$$

In (31) we have taken into account that the ε_is are evaluated at the steady-state values, therefore the hats. Considering the definition of the b_is (27) we get

$$\varepsilon_{L_o} = \frac{\left(\frac{\hat{L}_o}{\hat{L}_o + \hat{L}_p + \hat{L}_y}\right)^{1-r} \cdot \hat{L}_o^{\,r}}{\left(\frac{\hat{L}_o}{\hat{L}_o + \hat{L}_p + \hat{L}_y}\right)^{1-r} \hat{L}_o^{\,r} + \left(\frac{\hat{L}_p}{\hat{L}_o + \hat{L}_p + \hat{L}_y}\right)^{1-r} \hat{L}_p^{\,r} + \left(\frac{\hat{L}_y}{\hat{L}_o + \hat{L}_p + \hat{L}_y}\right)^{1-r} \hat{L}_y^{\,r}} \tag{32}$$

$$= \frac{\hat{L}_o^{\,1-r} \cdot \hat{L}_o^{\,r}}{\hat{L}_o^{\,1-r} \hat{L}_o^{\,r} + \hat{L}_p^{\,1-r} \hat{L}_p^{\,r} + \hat{L}_y^{\,1-r} \hat{L}_y^{\,r}} = \frac{\hat{L}_o}{\hat{L}_o + \hat{L}_p + \hat{L}_y} = b_o . \tag{33}$$

Note that by calibrating the b_is with the corresponding volume shares the elasticities are now independent of r. Finally, by combining equations (28) and (33) we get the following surprisingly simple log-linearization for equation (26):

$$\overline{l} = b_o l_o + b_p l_p + b_y l_y . \tag{34}$$

Solving the profit maximization problem of the firm, we follow a two-step procedure. In the first step, combined labor is obtained by maximizing profits

$$\Pi = Y - \overline{W}\,\overline{L} \tag{35}$$

subject to the production function (24). So we have:

$$\begin{aligned} \Pi &= A\overline{L}^{(1-\alpha)} - \overline{W}\,\overline{L} \\ \frac{\partial \Pi}{\partial \overline{L}} &= (1-\alpha)\, A\overline{L}^{-\alpha} - \overline{W} \\ \Rightarrow \quad (1-\alpha)\, A\overline{L}^{-\alpha} &= \overline{W} . \end{aligned} \tag{36}$$

Here, $\overline{W}$ denotes the aggregate wage rate which is defined later. As we log-linearize around the steady-state constants drop out and we therefore get:

$$a - \alpha \overline{l} = \overline{w} . \tag{37}$$

Now we turn to the cost minimization problem which optimally allocates the different input factors. Costs are defined as

$$\overline{\overline{W}}\,\overline{L} \equiv W_o L_o + W_p L_p + W_y L_y . \tag{38}$$

Building the Lagrange-function

$$\mathcal{L} = W_o L_o + W_p L_p + W_y L_y + \lambda \left[\overline{L} - \left(b_o^{1-r} L_o^{\ r} + b_p^{1-r} L_p^{\ r} + b_y^{1-r} L_y^{\ r} \right)^{1/r} \right] \tag{39}$$

and differentiating with respect to L_i yields the following first order conditions:

$$W_o = \frac{1}{r} \cdot \lambda \left(b_o^{1-r} L_o^{\ r} + b_p^{1-r} L_p^{\ r} + b_y^{1-r} L_y^{\ r} \right)^{(1/r)-1} \cdot b_o^{1-r} \cdot r \cdot L_o^{\ r-1} \tag{40}$$

$$W_p = \frac{1}{r} \cdot \lambda \left(b_o^{1-r} L_o^{\ r} + b_p^{1-r} L_p^{\ r} + b_y^{1-r} L_y^{\ r} \right)^{(1/r)-1} \cdot b_p^{1-r} \cdot r \cdot L_p^{\ r-1} \tag{41}$$

$$W_y = \frac{1}{r} \cdot \lambda \left(b_o^{1-r} L_o^{\ r} + b_p^{1-r} L_p^{\ r} + b_y^{1-r} L_y^{\ r} \right)^{(1/r)-1} \cdot b_y^{1-r} \cdot r \cdot L_y^{\ r-1} . \tag{42}$$

Dividing (40) by (41)

$$\frac{W_o}{W_p} = \frac{b_o^{1-r}\, L_o^{\ r-1}}{b_p^{1-r}\, L_p^{\ r-1}} \tag{43}$$

and further log-linearizing around steady-state the constants involving $\log(b_i)$ drop out and we get

$$w_o - w_p = (r-1)\,(l_o - l_p), \tag{44}$$

and with the same procedure

$$w_o - w_y = (r-1)\,(l_o - l_y). \tag{45}$$

Rearranging equations (40) to (42) results in factor demand. Thereby we exploit the fact that the Lagrange multiplier λ measures the change in the objective function $\overline{\overline{W}}\,\overline{L}$ if the constraint $\overline{L}$ is a little bit relaxed. This implies $\lambda = \overline{\overline{W}}$.

$$L_o^{\ r-1} = \frac{W_o}{\overline{\overline{W}}} \cdot \left(b_o^{1-r} L_o^{\ r} + b_p^{1-r} L_p^{\ r} + b_y^{1-r} L_y^{\ r} \right)^{(r-1)/r} \cdot b_o^{\ r-1} \tag{46}$$

$$\Rightarrow \quad L_o = \left(\frac{W_o}{\overline{\overline{W}}} \right)^{1/(r-1)} \cdot b_o \cdot \overline{L} . \tag{47}$$

And further:

$$L_p = \left(\frac{W_p}{\overline{W}} \right)^{1/(r-1)} \cdot b_p \cdot \overline{L} \tag{48}$$

$$L_y = \left(\frac{W_y}{\overline{W}} \right)^{1/(r-1)} \cdot b_y \cdot \overline{L}\,. \tag{49}$$

Now we want to show that L_is could only take their steady-state values if $W_i = \overline{W}$. First we have to show that $\overline{L} = \hat{L}_o + \hat{L}_p + \hat{L}_y$ at the steady state:

$$\begin{aligned}
\overline{L} &= \left(b_o^{\,1-r} L_o^{\,r} + b_p^{\,1-r} L_p^{\,r} + b_y^{\,1-r} L_y^{\,r} \right)^{1/r} \\
&= \left(\frac{\hat{L}_o}{\hat{L}_o + \hat{L}_p + \hat{L}_y}^{1-r} \hat{L}_o^{\,r} + \frac{\hat{L}_p}{\hat{L}_o + \hat{L}_p + \hat{L}_y}^{1-r} \hat{L}_p^{\,r} + \frac{\hat{L}_y}{\hat{L}_o + \hat{L}_p + \hat{L}_y}^{1-r} \hat{L}_y^{\,r} \right)^{1/r} \\
&= \left(\left(\hat{L}_o + \hat{L}_p + \hat{L}_y \right) \left(\hat{L}_o + \hat{L}_p + \hat{L}_y \right)^{r-1} \right)^{1/r} \\
&= \left(\hat{L}_o + \hat{L}_p + \hat{L}_y \right)^{1/r} \left(\hat{L}_o + \hat{L}_p + \hat{L}_y \right)^{(r-1)/r} \\
&= \left(\hat{L}_o + \hat{L}_p + \hat{L}_y \right)^{(1/r)+(r-1)/r} \\
&= \left(\hat{L}_o + \hat{L}_p + \hat{L}_y \right).
\end{aligned} \tag{50}$$

This fact explains the simple log-linearization (34). Evaluating (47) at the steady state yields:

$$\begin{aligned}
\hat{L}_o &= \left(\frac{W_o}{\overline{W}} \right)^{1/(r-1)} \cdot \underbrace{\frac{\hat{L}_o}{\hat{L}_o + \hat{L}_p + \hat{L}_y}}_{b_o} \cdot \underbrace{\hat{L}_o + \hat{L}_p + \hat{L}_y}_{\overline{L}} \quad \Rightarrow \\
&\left(\frac{W_o}{\overline{W}} \right)^{1/(r-1)} = 1 \quad \Rightarrow \\
&W_o = \overline{W}.
\end{aligned} \tag{51}$$

Therefore the steady state values of L_i do not depend on the parameter r. This is the big advantage of the Senhadji (1997) CES-aggregator used. Employing a standard CES-aggregator with share parameters b_i instead of $b_i^{\,1-r}$, the steady state values (baseline values) of L_i would depend on r even if wages are all equal and remain constant (Kamien, Schwartz (1968), p. 12, or Klump, Saam (2006), p. 2ff).

Finally, we have to derive a formula for the implied wage aggregator. We use equations (38), (47), (48) and (49):

$$\begin{aligned} \overline{W}\,\overline{L} \;=\; & W_o \cdot W_o^{1/(r-1)} \cdot W^{1/(1-r)} \cdot b_o \cdot \overline{L} \;+ \\ & W_p \cdot W_p^{1/(r-1)} \cdot W^{1/(1-r)} \cdot b_p \cdot \overline{L} \;+\; W_y \cdot W_y^{1/(r-1)} \cdot W^{1/(1-r)} \cdot b_y \cdot \overline{L} \end{aligned} \tag{52}$$

$$\Rightarrow \quad W^{r/(r-1)} \;=\; W_o^{\,r/(r-1)} \cdot b_o \;+\; W_p^{\,r/(r-1)} \cdot b_p \;+\; W_y^{\,r/(r-1)} \cdot b_y \tag{53}$$

$$\Rightarrow \quad \overline{W} \;=\; \left(W_o^{\,r/(r-1)} \cdot b_o \;+\; W_p^{\,r/(r-1)} \cdot b_p \;+\; W_y^{\,r/(r-1)} \cdot b_y \right)^{(r-1)/r} . \tag{54}$$

Log-linearizing this wage aggregator we once again use the formula:

$$\overline{w} \;=\; \varepsilon_{W_o} w_o \;+\; \varepsilon_{W_p} w_p \;+\; \varepsilon_{w_y} w_y \,. \tag{55}$$

$$\varepsilon_{W_o} \;=\; \frac{r-1}{r} \cdot \frac{1}{W_o^{\,r/r-1} \cdot b_o \;+\; W_p^{\,r/r-1} \cdot b_p \;+\; W_y^{\,r/r-1} \cdot b_y} \cdot b_o \cdot \frac{r}{r-1} \cdot W_o^{\,(r/r-1)-1} \cdot W_o \tag{56}$$

$$\varepsilon_{W_o} \;=\; \frac{1}{W_o^{\,r/r-1} \cdot b_o \;+\; W_p^{\,r/r-1} \cdot b_p \;+\; W_y^{\,r/r-1} \cdot b_y} \cdot b_o \cdot W_o^{\,(r/r-1)} \tag{57}$$

$$\varepsilon_{W_o} \;=\; \left(\frac{W_o}{\overline{W}} \right)^{(r/r-1)} \cdot\; b_o \tag{58}$$

As $W_i = \overline{W}$ in the steady-state we finally get:

$$\overline{w} \;=\; b_o w_o \;+\; b_p w_p \;+\; b_y w_y \,. \tag{59}$$

The problem of the representative consumer of group i

Maximizing utility

$$U_i(C,L) \;=\; \frac{C_i^{1-\sigma}}{1-\sigma} \;-\; X_i \frac{L_i^{1+\varphi_i}}{1+\varphi_i} \tag{60}$$

subject to the budget constraint

$$C_i \;=\; W_i L_i \;+\; b_i \,(\Pi \;-\; T). \tag{61}$$

Thereby, we assume that each consumer group earns the share b_i of aggregate profits and pays the share b_i of aggregate taxes.

Building the Lagrange-function for each consumer group

$$\mathcal{L}_i \;=\; \frac{C_i^{1-\sigma}}{1-\sigma} \;-\; X_i \frac{L_i^{1+\varphi_i}}{1+\varphi_i} \;-\; \lambda_i \,(C_i \;-\; W_i L_i \;-\; b_i \,(\Pi \;-\; T)) \tag{62}$$

and differentiating with respect to C_i, L_i and λ_i yields the usual first order conditions:

$$C_i^{-\sigma} = \lambda_i \tag{63}$$

$$X_i L_i^{\varphi_i} = \lambda_i W_i \tag{64}$$

$$C_i = W_i L_i + b_i (\Pi - T). \tag{65}$$

Combining equations (63) and (64) yields

$$\frac{X_i L_i^{\varphi_i}}{C_i^{-\sigma}} = W_i \tag{66}$$

and further expressing (66) in log-deviations from steady-state:

$$\chi_i + \varphi_i l_i + \sigma c_i = w_i . \tag{67}$$

Rearranging (67) yields the following equations which could be interpreted as labor supply schedules, determining labor supply as a function of real wage given marginal utility of consumption (see Gali 2008, p. 18). The preference shocks χ_i/φ_i are thereby assumed to split into normed specific components (u, v, z) and a normed general component (g):

$$l_o = \frac{1}{\varphi_o} w_o - \frac{\sigma}{\varphi_o} c_o + z + g \tag{68}$$

$$l_p = \frac{1}{\varphi_p} w_p - \frac{\sigma}{\varphi_p} c_p + u + g \tag{69}$$

$$l_y = \frac{1}{\varphi_y} w_y - \frac{\sigma}{\varphi_y} c_y + v + g . \tag{70}$$

Log-linearization of aggregate consumption $\overline{C} \equiv C_o + C_p + C_y$ yields:

$$\overline{c} = \frac{\hat{C}_o}{\overline{C}} c_o + \frac{\hat{C}_p}{\overline{C}} c_p + \frac{\hat{C}_y}{\overline{C}} c_y . \tag{71}$$

Now, we want to show that in *steady-state* consumption shares equal labor shares $\frac{\hat{C}_o}{\overline{C}} = \frac{\hat{L}_o}{\overline{L}}$. To demonstrate this, consider the ratio of the budget constraints for two consumer groups, evaluated at the steady state.

$$\frac{\hat{C}_i = W_i\hat{L}_i + b_i(\Pi - T)}{\hat{C}_j = W_j\hat{L}_j + b_j(\Pi - T)} \Rightarrow$$

$$\frac{\hat{C}_i = W_i\hat{L}_i + \frac{\hat{L}_i}{\bar{L}}(\Pi - T)}{\hat{C}_j = W_j\hat{L}_j + \frac{\hat{L}_j}{\bar{L}}(\Pi - T)} \tag{72}$$

according to (50). Applying (51) yields:

$$\frac{\hat{C}_i = \hat{L}_i\left(W_i + \frac{1}{\bar{L}}(\Pi - T)\right)}{\hat{C}_j = \hat{L}_j\left(W_i + \frac{1}{\bar{L}}(\Pi - T)\right)} \Rightarrow$$

$$\frac{\hat{C}_i}{\hat{C}_j} = \frac{\hat{L}_i}{\hat{L}_j}. \tag{73}$$

Therefore

$$\hat{C}_i = \hat{C}_j\frac{\hat{L}_i}{\hat{L}_j}, \quad \forall j. \tag{74}$$

Now we want to show that for any z

$$\frac{\hat{C}_z}{\bar{C}} \equiv \frac{\hat{C}_z}{\sum_i \hat{C}_i} = \frac{\hat{L}_z}{\sum_i \hat{L}_i} \equiv \frac{\hat{L}_z}{\bar{L}}. \tag{75}$$

Considering (74) in (75) yields:

$$\frac{\hat{C}_z}{\sum_i \hat{C}_j\frac{\hat{L}_i}{\hat{L}_j}} = \frac{\hat{C}_z}{\frac{\hat{C}_j}{\hat{L}_j}\sum_i \hat{L}_i} = \frac{\hat{C}_z}{\frac{\hat{C}_z}{\hat{L}_z}\sum_i \hat{L}_i} = \frac{\hat{L}_z}{\sum_i \hat{L}_i} = \frac{\hat{L}_z}{\bar{L}}. \tag{76}$$

And so employing (27) and (50), aggregate consumption, equation (71), becomes to

$$\bar{c} = b_o c_o + b_p c_p + b_y c_y. \tag{77}$$

To determine consumption structure we log-linearize equation (65) around the steady-state. For this task we once again use the general formula:

$$c_i = \varepsilon_W w_i + \varepsilon_L l_i + \varepsilon_\Pi \pi_i + \varepsilon_T \tau_i. \tag{78}$$

Thereby, the ε_is once again represent the corresponding elasticities. So we have:

$$\begin{aligned}
\varepsilon_W &= \frac{\partial \log(C_i)}{\partial W_i} W_i = \frac{1}{C_i} L_i W_i = 1-\alpha \\
\varepsilon_L &= \frac{\partial \log(C_i)}{\partial L_i} L_i = \frac{1}{C_i} L_i W_i = 1-\alpha \\
\varepsilon_\Pi &= \frac{\partial \log(C_i)}{\partial \Pi} \Pi = \frac{1}{C_i} b_i \Pi = \frac{\Pi_i}{C_i} = \alpha \\
\varepsilon_T &= \frac{\partial \log(C_i)}{\partial T} T = -\frac{1}{C_i} b_i T = -\frac{T_i}{C_i} = -t\,.
\end{aligned} \tag{79}$$

Here, we interpret $(1 - \alpha)$ as wage income share and t as average tax rate.

Combining equations (78) and (79) yields:

$$c_i = (1-\alpha)\left(w_i + l_i\right) + \alpha\pi - t\tau\,. \tag{80}$$

Building pairwise difference for the particular consumer groups we finally get:

$$c_p - c_y = (1-\alpha)\left[w_p + l_p - w_y - l_y\right] \tag{81}$$

$$c_o - c_p = (1-\alpha)\left[w_o + l_o - w_p - l_p\right] \tag{82}$$

$$c_o - c_y = (1-\alpha)\left[w_o + l_o - w_y - l_y\right]. \tag{83}$$

Given aggregate consumption (77), only two of the above three equations are necessary for determining c_o, c_p and c_y.

Finally, we have to log-linearize the definition of aggregate demand. Aggregate demand consists of consumption and some other demand components D (for instance government expenditures):

$$Y = C + D\,. \tag{84}$$

These other components are a share γ of total demand Y, $D = \gamma Y$. We interpret the demand shock as a shock in this share γ. Log linearization around the steady state then yields:

$$
\begin{aligned}
Y &= \frac{1}{1-\gamma} C \\
\log(Y) &= \log(C) - \log(1-\gamma) \\
y &= \varepsilon_{\gamma} \gamma + \varepsilon_{c} c \\
\varepsilon_{c} &= \frac{\partial \log(Y)}{\partial C} \hat{C} = \frac{1}{\hat{C}} \hat{C} = 1 \\
\varepsilon_{\gamma} &= \frac{\partial \log(Y)}{\partial \gamma} \hat{\gamma} = \frac{1}{1-\hat{\gamma}} \hat{\gamma} \\
\Rightarrow \quad y &= c + \frac{\hat{\gamma}}{1-\hat{\gamma}} \gamma .
\end{aligned}
\tag{85}
$$

As the share shock only occurs in this equation, we can normalize it in any way and finally get to:

$$
y = c + d . \tag{86}
$$

Appendix C

Figure 10 depicts the estimated time series for the innovations of the various shocks. The first pic shows the innovation in elderly labor supply, denoted as *ez*, with a pronounced positive peak in the middle of the series. This corresponds to a large pension reform in 1993. In addition, the cumulated effects of the reforms in 2003 to 2008 are reproduced with a positive deviation from zero.

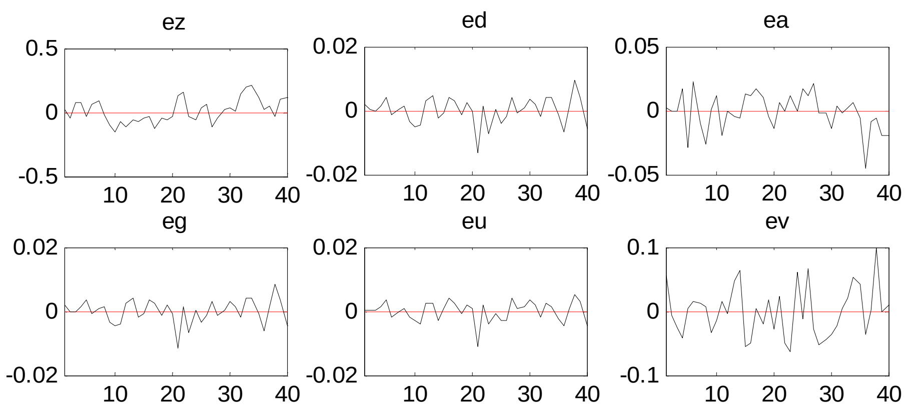
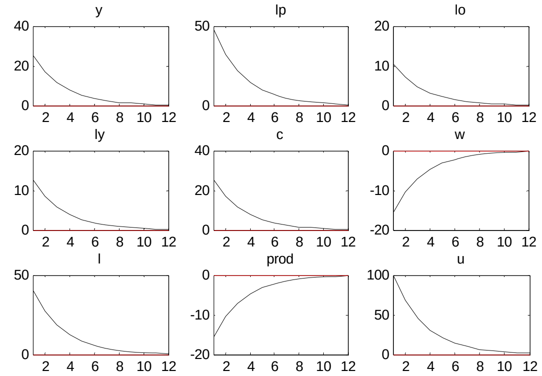


Figure 10: Time series of shocks implied by the basic model

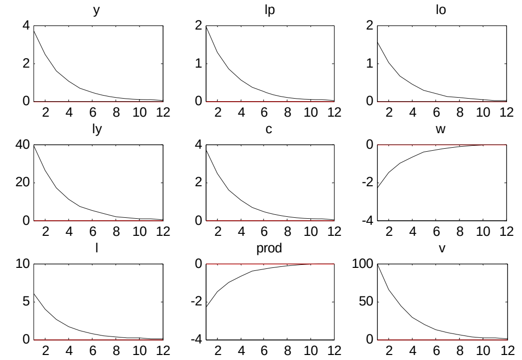
a

primary labor supply shock, *u*



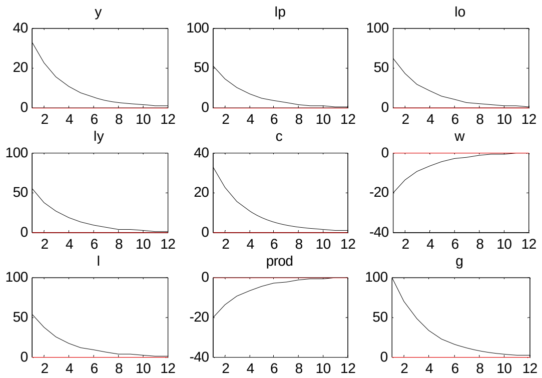
b

youth labor supply shock, *v*



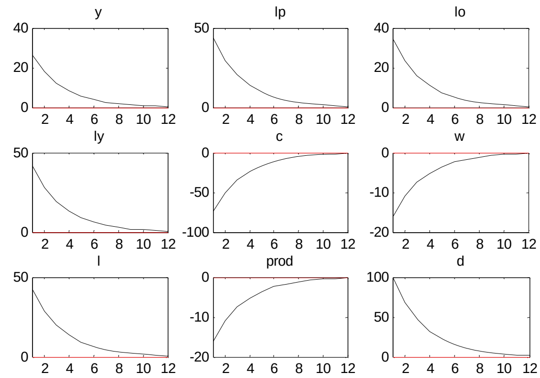
c

general labor supply shock, *g*



d

aggregate demand shock, *d*



e

general productivity shock, *a*

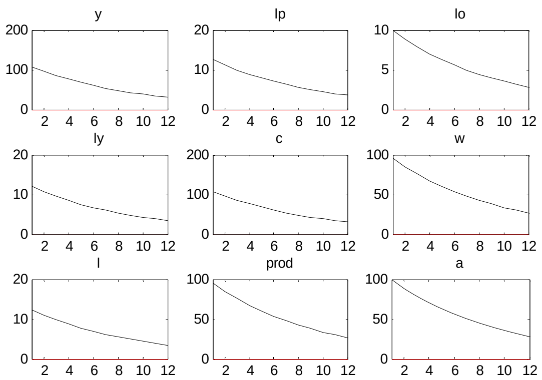


Figure 11: Model impulse responses due to a primary labor supply shock, *u*, youth labor supply shock, *v,* general labor supply shock, *g,* an aggregate demand shock, *d* and due to a general productivity shock, *a,* basic model, basic specification

Panel d in Figure 11 shows the effect of an aggregate demand shock on output and the other variables. Although output rises like in a demand driven model, the mechanism is different. Here, a demand shock, ceteris paribus, leads to lower consumption which in turn increases labor supply through a wealth effect. Panel e in Figure 11 shows a positive effect of a productivity shock on total employment which is due to the estimated value of σ to be less than one. This means that the substitution effect outweighs the wealth effect in labor supply in this case.

a | b

primary labor supply shock, *u* | youth labor supply shock, *v*

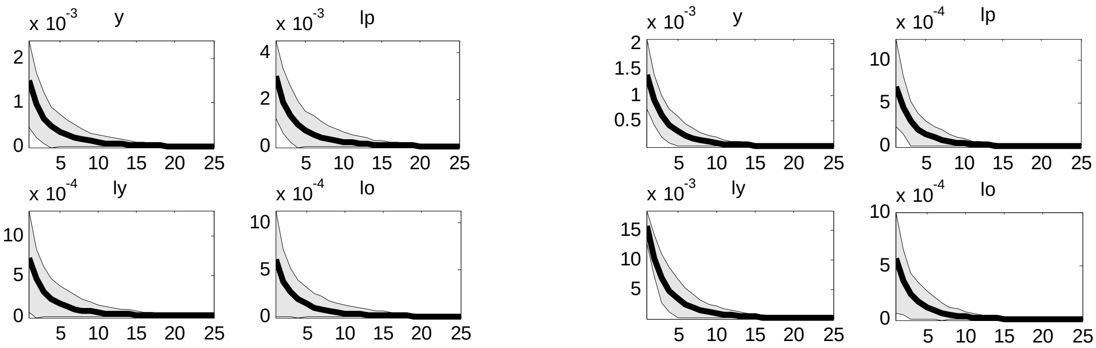


Figure 12: Impulse responses and 90% Bayesian confidence intervals for a one standard deviation shock in labor supply of primaries and youth on the other observed variables, basic model, basic specification

Appendix D

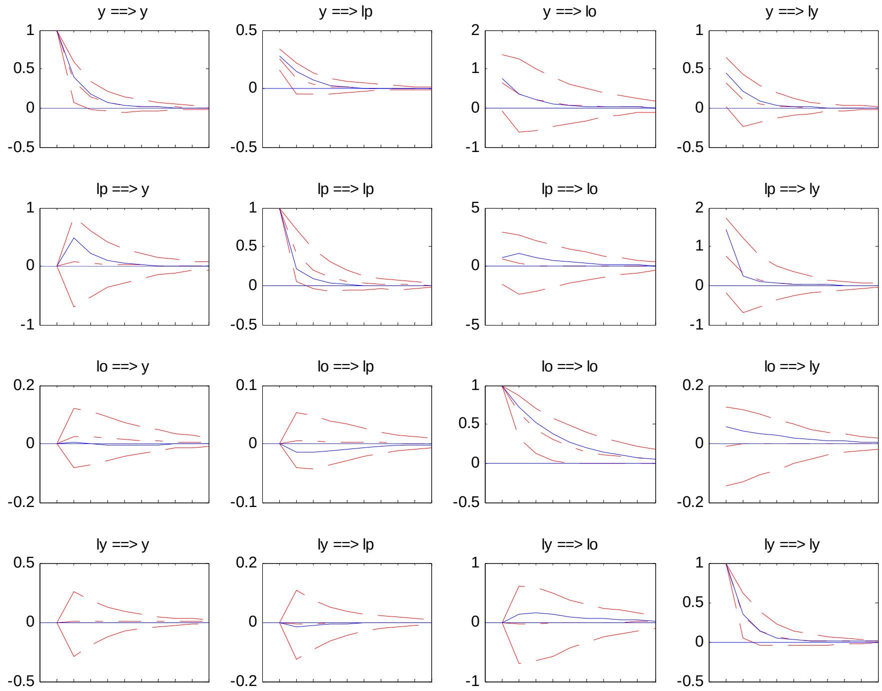


Figure 13: Comparison of VAR impulse responses estimated with actual data and model generated data for the case of hp-filter detrending.. VAR estimation was carried in the order y, l_p, l_o, l_y; impulse responses are calculated by the usual Cholesky decomposition. Solid lines correspond to impulse responses estimated with actual data, dotdashed lines are the means of the impulses responses estimated with model simulated data and dashed lines correspond to the 90% confidence intervals obtained by simulated data.

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