Jacek Cukrowski and Manfred M. Fischer

Efficient Organization of Collective Data-Processing

Paper

WSG Diskussionspapiere

WSG 64/98

Efficient Organization of Collective Data Processing

Jacek Cukrowski and Manfred M. Fischer

Institut für Wirtschafts- und Sozialgeographie

Wirtschaftsuniversität Wien

Department of Economic and Social Geography

Vienna University of Economics and Business Administration

WSG Discussion Papers

Abteilung für Theoretische und Angewandte Wirtschafts- und Sozialgeographie
Institut für Wirtschafts- und Sozialgeographie
Wirtschaftsuniversität Wien

Vorstand: o.Univ.-Prof. Dr. Manfred M. Fischer
A - 1090 Wien, Augasse 2-6, Tel. ++43-(0)1-31336-4836

Redaktion: Univ.-Ass. Dr. Petra Staufer

WSG 64/98

Efficient Organization of Collective Data Processing

Jacek Cukrowski and Manfred M. Fischer

WSG-Discussion Paper 64

November 1998

Gedruckt mit Unterstützung
des Bundesministerium
für Wissenschaft und Verkehr
in Wien

WSG Discussion Papers are interim
reports presenting work in progress
and papers which have been submitted
for publication elsewhere.

ISBN 3 85037 079 8

Efficient Organization of Collective Data-Processing

Jacek Cukrowski[*a] , Manfred M. Fischer[b]

[a] Economics Institute, Academy of Sciences of the Czech Republic,
P.O. Box 882, Politickych veznu 7, 11121 Prague 1, Czech Republic

and Center for Social and Economic Research,
Bagatela 14, 00-585 Warsaw, Poland

[b] Institute for Urban and Regional Research,
Austrian Academy of Sciences,
Postgasse 7/4, A-1010 Vienna, Austria

and Department of Social and Economic Geography,
Vienna University of Economic and Business Administration,
Augasse 2-5, A-1090 Vienna, Austria

Abstract: *The paper examines the application of the concept of economic efficiency to organizational issues of collective information processing in decision making. Information processing is modeled in the framework of the dynamic parallel-processing model of associative computation with an endogenous set-up cost of the processors. The model is extended to include the specific features of collective information processing in the team of decision makers which could cause an error in data analysis. In such a model, the conditions for efficient organization of information processing are defined and the architecture of the efficient structures is considered. We show that specific features of collective decision making procedures require a broader framework for judging organizational efficiency than has traditionally been adopted. In particular, and contrary to the results presented in economic literature, we show that in human data processing (unlike in computer systems), there is no unique architecture for efficient information processing structures, but a number of various efficient forms can be observed. The results indicate that technological progress resulting in faster data processing (ceteris paribus) will lead to more regular information processing structures. However, if the relative cost of the delay in data analysis increases significantly, less regular structures could be efficient.*

Keywords: decision making, information processing, associative computation, technological progress, efficient organizational forms

JEL Classification: D79.

[*] Corresponding author, e-mail: Jacek.Cukrowski@cerge.cuni.cz, fax: +420 2 242 11 374.

The first author gratefully acknowledge the grant provided by the Institute for Urban and Regional Research at the Austrian Academy of Sciences.

1. Introduction

Several leading economists have recognized the importance of informational structures in decision making (see, e.g., Aoki, 1986; Milgrom and Roberts, 1990). In particular, decision making systems of large multinational companies or public institutions are widely perceived to have complex informational structures (see, Radner, 1992, 1993; Bolton and Dewatripont, 1994). The architecture of these structures is often considered as a factor affecting the efficiency of decision processes as a whole (see, e.g., Milgrom and Roberts, 1990; or Radner, 1992).

There exists a significant body of the literature focusing on the role and the importance of informational processes in decision making in the firm (see, e.g., Arrow, 1985; Milgrom and Roberts, 1990; Radner 1992 and 1993; Bolton and Dewatripont, 1994; Prat, 1996). Marschak and Radner (1972), for instance, consider the organization of decision making in a network of processors, and explore the implications of the delay in information processing on the value of decisions. Radner (1972a, 1972b) analyzes information structures and resource allocation in teams of decision makers. Returns to scale in information processing and its implications on the firm's size are studied by Keren and Levhari (1979, 1983), and Radner and Van Zandt (1992). Efficient organization of data processing is investigated by Radner (1992, 1993), Radner and Van Zandt (1992), Van Zandt (1995), and Bolton and Dewatripont (1994). Necessary and sufficient conditions for decentralization of data processing in enterprises is studied by Cukrowski (1997). An overview of the contributions made by recent research to understanding the information processing in economics is presented by Lipman (1995).

Most of decision processes in large enterprises base on the results of data analysis which is usually performed in the team of individuals (boundedly rational agents), so that in the present work we focus on organizational aspects of collective information processing in real-time decision making systems[1]. To concentrate on the time aspects of data processing, we assume that the task of the decision making system is to analyze cohorts of data coming from the system controlled (the region) in order to identify its current state, and, to produce decisions used to correct the performance of the system under control. Decisions are made based on the recognized state of the system controlled. If the state is identified precisely and a decision is generated instantaneously then the system controlled can be transformed to the desirable state (to focus exclusively on information processing we assume that always an action undertaken corresponds to the state identified).[2]

Since the aggregation of data (to identify a current state of the system under control)[3] and also a number of other commonly used decision making paradigms,[4] can be represented as a sequence of associative operations, in the analysis which follows we focus solely on the computation of associative operations, and, therefore our analysis of informational processes is based on a dynamic parallel-processing model of associative computation (see, Radner, 1992, 1993; Radner and Van Zandt, 1992, Van Zandt, 1995; Lipman 1995; or Prat 1997).

To present the model consider, without loss of generality, a data aggregation

---

[1] Note that 'real-time' is a relational concept, i.e., a system is real-time or not with respect to the temporal constraints of its environment. In our analysis by a 'real-time decision making system' we refer to the decision-making system which operates quickly enough to influence the process it is controlling.

[2] It is assumed that in order to produce a flow of control signals (decisions), the same computational procedure is repeated again and again. Consequently, the analysis is restricted to the organization of the single cohort of data processing (one-shot mode).

[3] Aggregation of information (in particular, the computation of the sum of numerical data) is the simplest example of associative operations and it is used there for the sake of clarity (in practice, the items aggregated may not be just numbers, but large vectors or matrices). Computations of such a kind are commonly used in the methods of statistical prediction or statistical control (see Marschak and Radner, 1972; Aoki, 1986; or Radner and Van Zandt, 1992). In Section 4 we present another frequently used in decision making example of associative operations - a project selection.

[4] See Radner (1993), for details.

process.[5] Suppose that data processing is done by managers and each manager performs similarly to the processor in the computer system. In particular assume that each manager has an external memory for information storage, and can perform a simple operations with data. Each particular operation consists in retrieving a single data item from the memory, analysis, and either keeping the value in the "brain" of the manager or aggregating the value with the actual contents of the "brain". The duration of any operation is assumed to be independent on the values of the data used. Moreover, for the sake of simplicity we assume that a manager can send the result computed (contents of its "brain") to an output or to the external memory of any other manager in zero time (since time of data transfer is negligible comparing with the time needed for the analysis and processing of large data structures).

Note, however, that the result of the aggregation process (the state recognized) could not correspond to the current state of the system controlled. This could happen (1) due to non-instantenous data processing (i.e., due to the delay in information processing, D), or (2) due to the error in computation E which is natural in human data processing activities. In both cases the decision made and the action undertaken cannot properly correct the performance of the system controlled. Consequently, in both cases the value of the decision is obviously lower than in the perfect case (when neither the error nor the delay in data processing is observed). Since, under our simplifying assumptions, the value of the decision made can be used to measure the value of informational service in the decision making process, in the formal analysis below the value of informational service V will be considered as a function of these two variables,[6] i.e., $V(D,E)$. Thus, in the simplest case, the value of informational service can be

[5] In the analysis which follows we use the terminology from data aggregation (addition, multiplication, etc.) for expositorial clarity, but we have in mind much more complex associative operations that are not easily performed by digital computers.

[6] See, e.g., Radner and Van Zandt (1992) for detail analysis of the relationship between the value of the decision and the delay of the decision making process.

represented, as

$$V(D_N, E) = V_{\max} - V^{o}_{\max} - (r D_N + gE), \qquad (1)$$

where V_{max} is the maximum value of the decision with the informational service, V^{o}_{max} is the maximum value of the decision if no information is processed, D_N is the delay in information processing, E is the error in data analysis, ? and ? denote the unit costs of the delay in information processing and the error in data analysis, respectively.

The paper is organized as follows. Section 2 shows the architecture of information processing structures minimizing delay in data processing. Section 3 examines information processing structures in which the error in data processing is minimized. In Section 4 the consolidated efficiency criterion is defined and organizational forms of the efficient informational structures are discussed. The implication of technological progress in data processing on the architecture of the efficient structures is presented in Section 5. The concluding Section summarizes some of the major findings of the study.

2. Structures Minimizing Delay in Data Analysis

To analyze time characteristics of human data processing we assume that the speed of information processing of each individual manager depends upon costly resources allocated to him. In other words we assume that the processing power of the manager[7] is determined by costly resources (e.g., computer equipment) assigned to him, i.e., productivity of each particular manager depends on the resources he uses. To formalize this link denote the vector of the resources the manager needs to perform operations required as

[7] Processing power is understood as a number of operations that can be performed by a single

$\boldsymbol{x}=(x_1,x_2,...,x_J)$, where $x_j \in R_+$ is the quantity of the resource j $(j=1,2,\ldots,J)$ and assume that the duration of a single operation d is determined by the technology of data processing and depends on the quantity of the resources used, i.e., $d(x_1,x_2,...,x_J)$. Therefore, the number of such operations that can be performed sequentially in a unit of time is a function of the resources assigned to each particular manager and can be represented as $f(\boldsymbol{x})=1/d(\boldsymbol{x})$. The function $f(\boldsymbol{x})$ is called an information processing function and can be understood as a "production function" in data processing. Since all resources are costly, the duration of a single operation can be considered as a function of the cost of resources m allocated to the manager, i.e., for each $m \geq 0$ we can define a function

$$\begin{aligned} D(m) = \underset{x_1,\ldots,x_J}{Min} \quad & d(x_1,\ldots,x_J) \\ s.t. \quad & \textstyle\sum_{j=1}^{J} p_j x_j \leq m \end{aligned} \qquad (2)$$

where p_j denotes fixed price of the resource j. Function $D(m)$ specifies the duration of a single operation performed by a single manager when the resources of the value m are assigned to him. Consequently, a function $F(m)=1/D(m)$ specifies the maximum number of operations that can be performed sequentially in a unit of time by a single manager with the help of the resources which cost m. In the simplest case when only a single resource is allocated to data-processing, total cost m corresponds to its volume, if price is normalized to 1. Consequently, without loss of generality, in the analysis below the information processing function $f(\boldsymbol{x})$ will be replaced by a single argument function $F(m)$.

In such a model the duration of a single operation d is determined as $1/F(m)$. Moreover, if the total information processing budget available equals to M, all managers are identical and the cost of each manager (his wage) is m, then the duration of each individual operation can be specified as

---

manager in a unit of time.

$$d_m(M,P) = \frac{1}{F(\frac{M - mP}{P})}, \qquad (3)$$

where P is the number of the managers employed in data processing.

Each manager aggregates data items in a serial fashion. Thus, to speed up this process, data processing can be organized in a decentralized way with the help of more than one manager, i.e., in decentralized information processing structure[8] (we shall call a one-manager structure *centralized* and more-than-one-manager structure *decentralized*). However, in more decentralized structure, the total budget available M has to be distributed among more managers, and, consequently, the processing power of each individual manager decreases. This means while decentralization reduces the length of the longest sequence of operations needed for the derivation of the result, it also increases the duration of each individual operation.

The delay in the analysis of the cohort of N items of data D_N, in any structure with P identical managers, is proportional to the duration of individual operations, $d_m(M,P)$, and, consequently, is a decreasing function of the total budget M. Thus, the information processing structure is said to be *time minimizing* if, for a given number of data items processed N, it is not possible to get the same delay in information processing D_N with the smaller budget M or vice-versa (i.e., for a given information workload and the budget M, it is not possible to get smaller delay in data processing).

In the model under study, the duration of individual operations is not constant but is endogenously determined in the model, i.e., it depends on the budget spent on information processing and the number of managers. Since the minimum delay in the analysis of N data items in the structure with P identical managers (with fixed data processing power and

[8] An information processing structure is defined to be a directed graph with the managers at nodes, and the directed link from one manager to another if and only if the first sends the results computed to the second.

duration of individual operations $d=1$) is determined by the time of $C_N(P)$ operations, where $C_N(P)$ is given as follows[9] [10]

$$C_N(P) = \left\lfloor \frac{N}{P} \right\rfloor + \left\lceil \log_2 (P + N \bmod P) \right\rceil \;, \qquad (4)$$

and the duration of each individual operation d is given by (3), for given budget M, the minimum delay $D_N(M)$ can be determined as $D_N(M)=C_N(P)d_m(M,P)$.

Radner (1992,1993) shows that the minimum time (number of cycles) needed to add N items of data with the help of P managers is attained by so-called *"skip-level reporting"* structures with as-equally-as-possible-loaded managers (if $1<P<\lfloor N/2 \rfloor$)[11], or by a fully centralized structure (if $P=1$)[12]. The term *skip-level reporting* refers to the practice in an organization whereby a manager in level l sends results to an output or to a manager in level $l+L$ ($L \geq 1$). That is, the manager in level l can skip one or several levels in reporting to its direct hierarchical superior. An example of the skip-level reporting structure (with $P=8$ managers, designed for the aggregation of $N=40$ items of data) is presented in Figure 1. In this process each manager receives five data items. All the managers spend periods 1 through 5 aggregating data. At this point, four of the managers send their total to the other four, with each manager receiving one data item. This is aggregated with the manager's previous total in period 6. At the end of this period, two of the managers send their partial results to the other two. These data items are aggregated with previous totals in period 7, after which one manager sends its total to the other. Finally, the result is computed in period 8. The time

---

[9] Gibbons and Rytter (1988).

[10] Brackets $\lfloor\,\rfloor$ and $\lceil\,\rceil$ denote rounding down and up to the nearest integer, respectively.

[11] The number of managers P in any skip-level reporting structure is limited ($P \leq \lfloor N/2 \rfloor$) because at least two data items have to be assigned to each of them.

[12] Note that a centralized structure is a very particular case of a skip-level reporting structure, i.e., the number of operations is determined by expression (3) when the number of managers $P=1$.

diagram describing this process is shown in Figure 2.

Figure 1: The skip-level reporting structure (P=8, N=40, managers are represented as ellipses, data items are represented by octagons)

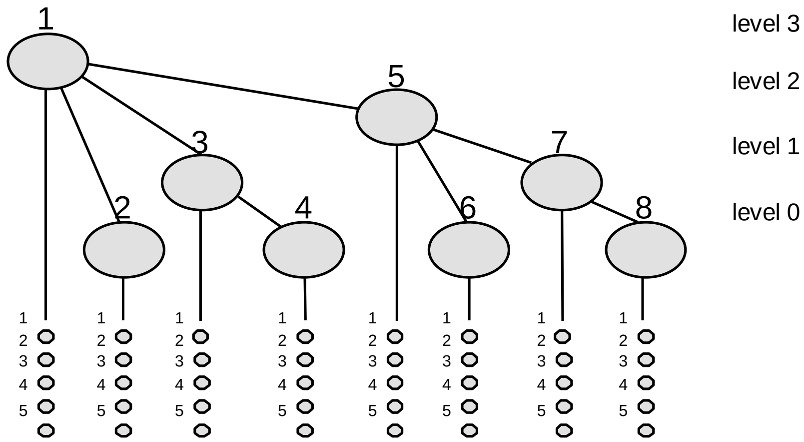
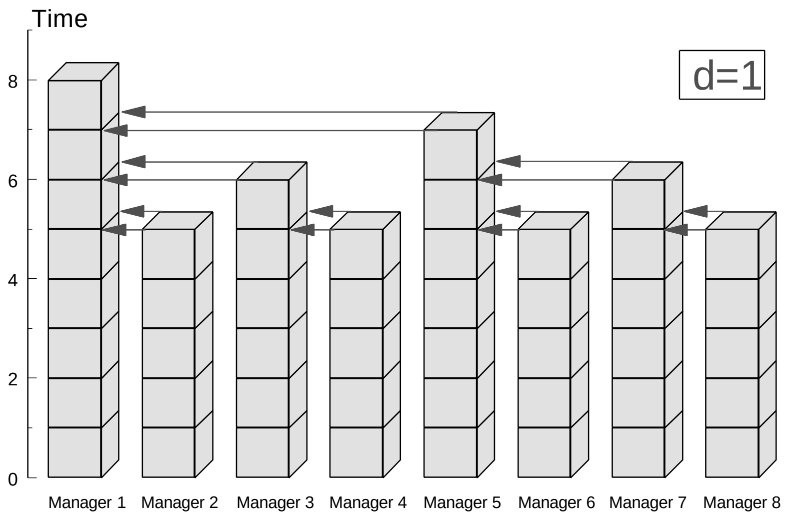


Figure 2. The time diagram corresponding to the information processing structure presented in Figure 1



3. One-shot Structures Minimizing Error in Data Processing

The analysis of data processing in the team of managers requires that a number of features of human information processing such as, for instance, capacity limitations, loyalty, opportunism, distribution of goals in data analysis, preferences to atmosphere,[13] or situational factors that affect human work, should be taken into account (see, e.g., Williamson, 1973).

All these characteristics are summarized by the concept of *bounded rationality* (i.e., limited human rationality in information processing) defined as "rational choice that takes into account the cognitive limitations of decision-maker – limitations of both knowledge and computational capacity"[14] (for the overview of the theories of bounded rationality, see Simon, 1972).

To incorporate the concept of bounded rationality into the model[15] we introduce the possibility of misinterpretation of information (i.e., possibility of the error in data processing). To do this, however, we need to focus on more complex example of associative computation (called "a project selection") where individuals involved in data analysis have some freedom of choice based on individual judgement (informational processes of this kind are frequently used, for example, for the evaluation and selection of investment projects).

In the analysis below we will focus on the process of selecting the best project (out of N projects submitted) in a team of P individuals (managers). Assume that each project can be described by the set of characteristics which can be evaluated in numerical form. Consequently, each project can be formally characterized by a vector with numerical components which can be used to determine the 'goodness' of the project, i.e., how the project fits to the requirements. Thus, without loss of generality, we can assume that project n ($n=1,2,...,N$) can be fully characterized by the value of one (aggregated) numerical attribute

[13] Williamson (1973), for example, points: 'preferences to atmosphere may induce individuals to forego material gains for nonpecuniary satisfactions if the modes or practices are regarded as oppressive or otherwise repugnant'.

[14] See Simon (1987).

[15] In the original dynamic parallel processing model of associative operations (Radner, 1992, 1993; and Radner and Van Zandt 1992) the bounded rationality is treated as limited information processing in the sense that each individual processor (or manager) can process a fixed (limited) number of data items in unit of time.

Q_n determined on the basis of analysis of the entire project.[16] Note that the "goodness" of the project can be positively correlated with some components of the vector, and negatively correlated with the other (i.e., the best project should not be characterized by the greatest or the smallest value of the parameter Q_n, but the "goodness" of the best project should be the closest to the corresponding indicator of the ideal project Q^O). Thus, the purpose of information processing is to select the project, say n^*, so that

$$n^* = arg\{min|Q^O - Q_n|\}, \qquad (5)$$

where Q_n is a numerical characteristic of the project n ($n=1,2,...,N$), and Q^O is an attribute of the project wanted by the individual (or a board of individuals) who specifies the characteristics of the required project.

To select the best project from N projects submitted, a single manager would perform the following procedure (starting from $i=1$): (1) Assign the value i to the indicator of the best project (set $*=i$) selected; (2) If $i+1 \leq N$, then retrieve and analyze the next project (i.e., set i equal to $i+1$ and derive the numerical attribute of the project Q_i), otherwise, finish the analysis; (3) Compare the numerical attribute of project i with the attribute the ideal project (i.e., determine $|Q^O - Q_i|$), if the project analyzed is closer to the ideal project than the best project out of the projects analyzed before (i.e., if $|Q^O - Q_i| < |Q^O - Q_*|$), then assign the value i to the indicator of the best project (set $*=i$) and execute step 2.

Note that the selection process described above involves only computation of associative operations (in each operation two data items are compared and one is selected as the best of the two). Since, both the derivation of the attribute of the project and the comparison with the attribute of the project selected are time consuming all considerations

[16] It could be determined as, for example, the weighted average or the weighted geometric mean of all components of the characteristic vector.

concerning the time component of informational service presented in the previous section remain valid, and, consequently, in order to reduce the time of data analysis, the process of selecting the best project should be decentralized (i.e., organized in hierarchical structure).

Note, however, that each manager in the team determines (and compares) the absolute values of the differences: $|Q_j^O - Q_{j,n}|$, where $Q_{j,n}$ denotes his subjective evaluation of the project n ($n \in \mathbf{N_j}$, $\mathbf{N_j}$ is a set of projects analyzed by manager j), Q_j^O is an attribute of the project considered to be the best by manager j. Divergences between managers[17] in attitude,[18] in perceptual ability (or ability to concentrate) and random factors such as the emotions, frustrations or stresses imply that subjective evaluations of the same project (say, n), by different members of the team, $Q_{j,n}$ ($j=1,2,...,P$), could not possibly be exactly the same (i.e., $Q_{1,n} \neq Q_{2,n} \neq ... \neq Q_{P,n} \neq Q_n$). Moreover, the possibility of misinterpreting the target of data analysis (i.e., of the understanding the characteristics of the project required Q^O) and divergences among the managers' individual goals imply that the understanding of which project is considered best could be different for each individual manager, i.e., $Q_1^O \neq Q_2^O \neq ... \neq Q_P^O \neq Q^O$. The divergences between managers imply that if all the projects submitted would be considered by all the members of the team, then each manager could choose a different project (also different from the project which would be selected by the board). The decentralization of the process of project selection therefore implies that the result of data analysis could be determined with error, measured by the absolute value of the difference between the numerical characteristics of the project wanted Q^O, and the project selected Q_{n^*} ($|Q^O - Q_{n^*}|$).

To represent the divergences between managers and describe the possible variability

[17] See, for example, O'Reilly III (1990) for a detailed analysis of differences in information use between managers.

[18] An attitude consists of feelings, beliefs and predispositions to behave in certain ways. It is understood as "an unseen force that people presume exists in order to explain certain behavior" (Organ and Bateman, 1986).

in subjective evaluations of the information analyzed in the framework of the dynamic parallel- processing model of associative computation, assume that each particular manager j ($j=1,2,...,P$) derives results according to his individual understanding of the goal of the analysis, Q_j^* (the possibilities of random mistakes in evaluations of projects can be represented as random shifts in Q_j^*). Thus, a numerical characteristic of the projects considered to be best by the members of the team ($Q_1^O, Q_2^O, ..., Q_P^O$) can be described by the random variables distributed around the numerical characteristic of the project wanted Q^O. For the sake of simplicity, assume that this distribution is normal, with mean Q^O and variance s_o^2.

Assuming that all the projects submitted are not identical, but all of them satisfy more or less the expectations of the board, we can presume that numerical characteristics of these projects, Q_n ($n=1,2,...,N$), are distributed around the numerical characteristic of the project wanted Q^O. For simplicity, assume that this distribution is normal with mean Q^O and variance s^2.

The possibility of the error in data analysis implies that the selection process should be organized in decentralized structure (in order to minimize the delay in information processing) which minimizes the expected value of the error in data analysis, $E=\boldsymbol{E}(|Q^o - Q_n^*|)$, where $\boldsymbol{E}$ denotes the expectation operator. If both Q_n ($n=1,2,...N$) and Q_j^* ($j=1,2,...,P$) are normally distributed random variables with the same mean Q^O and variances s^2 and s_o^2 respectively, then the random variable characterizing the project selected Q_n^* in the arbitrary decision making structure can be represented as

$$Q_{n^*} = \sum_{n=1}^{N} Q_n p_n , 1 \qquad (6)$$

where p_n denotes the probability that the project n will be selected ($n=1,2,...,N$). It follows that the random variable Q_{n^*} is distributed normally with mean Q^* and variance $s_{n^*}^2$.

Moreover, the random variable Q^o- Q_{n^*} is normally distributed, with zero mean and variance $s_{n^*}^{\ 2}$. Consequently, the expected value of the error in data analysis can be determined as

$$= -\sqrt{\frac{2}{p}}\, s_{n^*}\, e^{-\frac{x^2}{2 s_{n^*}^2}} \Big|_0^{\infty} = \sqrt{\frac{2}{p}}\, s_{n^*}\,. \qquad (7)$$

2

$$E(|\,Q^o - Q_{n^*}|) = \int_0^{\infty} x\, f_{|Q^o - Q_{n^*}|}(x)dx = \int_0^{\infty} \frac{2x}{s_{n^*}\sqrt{2p}}\, e^{-\frac{x^2}{2 s_{n^*}^2}}\, dx = 3$$

This means that the information processing structure which minimizes the variance of the random variable Q_n* also minimizes the expected value of the error in data processing.

In an arbitrary information processing structure the variance of the random variable Q_n* can be computed as

$$s_{n^*}^2 = s^2 \sum_{n=1}^{N} p_n^2\ , \qquad (8)$$

where p_n denotes the probability that the project n ($n=1,2,...,N$) will be selected. Therefore, the values of the probabilities p_n^* ($n=1,2,...,N$) minimizing variance s_n*^2, and consequently minimizing the expected value of the error in data analysis can be determined by finding the solution to the following optimization problem:

$$\underset{p_1, p_2, ..., p_N}{Min}\ s^2 \sum_{n=1}^{N} p_n^2\ \ 4 \qquad (9)$$

where

$$\sum_{n=1}^{N} p_n = 1 \qquad (10)$$

The probabilities p_n^* ($n=1,2,...,N$) equal $p_1^*=p_2^*= \ldots =p_N^*=1/N$, and, consequently, the minimum expected value of the error in data analysis equals

$$E_{min} = s\sqrt{\frac{2}{N\,p}}\,. \qquad (11)$$

If the characteristics of the projects considered to be the best by each individual manager Q_j^* ($j=1,2,...,P$), are normally distributed random variables with mean Q^O, then the values of the probabilities p_n ($n=1,2,...,N$) are determined only by the architecture and the information workload of the data processing structure. It has the following implications concerning the form of the structures minimizing the expected error in data analysis:

1. The structures minimizing the expected value of the error in data analysis are regular[19] with equally loaded managers (it ensures that all the projects analyzed are selected with the same probability).

2. The expected value of the error in data analysis does not depend upon the number of managers, i.e., it is the same for one manager (centralized structure) as for any decentralized regular structure with an equalized workload of the managers.

3. For any number of projects analyzed N there exists at least one data processing structure which minimizes the expected value of the error in data analysis. (This is a centralized structure.)

The analysis above shows that the expected value of the error in data processing is the same in centralized as in any equally-loaded decentralized regular structure. This means that the specific features of human information processing such as disagreement about the goals of data analysis or the possibility of random errors do not imply a hierarchical organization of management (data processing is organized in decentralized structures in order to reduce the delay in data processing). However, if the value of informational service

[19] The hierarchy is called *regular* if (1) all the immediate subordinates of any manager are at the next lower level, and (2) all managers of the same level have the same number of immediate subordinates.

depends not only upon the delay in information processing, but also upon the error in data analysis, then the forms of information processing structures could be more regular than those derived for the case when managers cannot make errors.

6. Efficient Organization of Data Processing in Decision Making

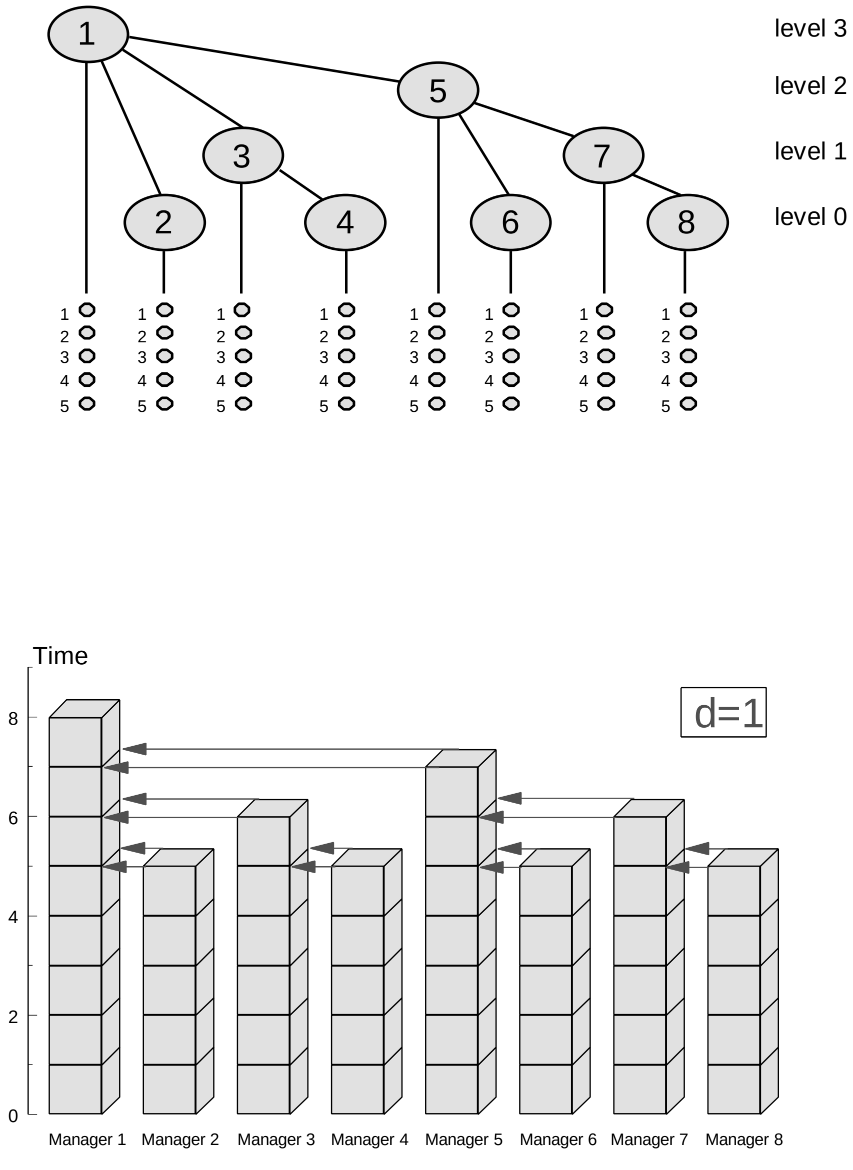
As mentioned in the introduction, the value of informational service is a decreasing function of both the delay in data processing *D* and the error in computation *E* (see expression (1)). Since the error in data analysis is a random variable the expected value of the informational service in decision making can be represented as

$$\boldsymbol{E}(V) = V_{\max} - V^{o}_{\max} - (\, r\, D_N + g\sqrt{\frac{2}{p}}\, s_{n^*}), \qquad (12)$$

where V_{max} is the maximum value of the decision with informational service, V^{o}_{max} is the maximum value of the decision if no information is processed, D_N is the delay in information processing, s_n* is the standard deviation of the random variable describing a numerical characteristic of the project selected, ? and ? denote the unit costs of the delay in information processing and the error in data analysis, respectively; the information processing structure is said to be *efficient* (for a given information workload and the variance of the characteristics of the projects submitted s^2), if it is not possible to get the same expected value of the informational service with the smaller budget (i.e., using less of the resources).

Taking into account that in an arbitrary information processing structure the standard deviation of a random variable describing a numerical characteristics of the selected project can be represented as $s_n*=W_N s$ (where W_N is the coefficient characterizing the structure considered and s is the standard deviation of the numerical characteristics of the projects analyzed), the value of informational service is determined solely by the delay in information processing, if all projects analyzed are identical (i.e., if $s^2=0$) and all members of the decision making team have the same objectives and do not make errors in data analysis (that is, if $Q_1^O=Q_2^O=...=Q_P^O$). In this particular case, the skip-level reporting structures are efficient. Otherwise, the architecture of the efficient information processing structures should be determined taking into account not only the delay in information processing D_N, but also the expected value of the error in data analysis E.

Figure 3: The reduction of the regular structure with P=7 managers (the projects analyzed are represented by octagons, and the managers by ellipses)



To see how the delay in information processing and the expected value of the error in data analysis depend on the architecture of information processing structures, consider the reduction process of the regular structure with $P=7$ managers presented in Figure 3. Assume that the characteristics of the projects analyzed Q_n ($n=1,2,...,8$) and the projects considered to be the best by individual managers Q_j^O ($j=1,2,...,P$) are described by normally distributed random variables with mean Q^O and variances s^2 and s_o^2, respectively. Note that the reduction process under study makes each subsequent structure less regular, and, consequently, increases the expected value of the error in data analysis E,[20] but at the same time decreases the delay in information processing D_N.

Figure 3 confirms the theoretical result that the expected value of the error in data analysis is minimized in regular structures, while the delay in information processing is minimized in irregular, skip-level reporting structures. Moreover, it implies that the expected value of the error in data analysis and the delay in information processing can be minimized in the single structure only if the decentralization of data analysis doesn't decrease the delay in information processing (in this case a centralized structure is efficient). Otherwise, the forms of the efficient information-processing structures should be determined individually for each particular information processing task taking into account the information workload of the structure N, the information processing function $f(\mathbf{x})$ (alternatively, $F(m)$), the variance of the characteristics of the projects submitted s^2, the form of the relationship between the delay in information processing, the error in data analysis and the function describing the value of informational service $V(D_N,E)$.

The considerations above imply that:

– if the delay in information processing is crucial for the value of the informational process ($r>>0$) and the effect of the error in computation is minor ($g \approx 0$), then the skip-level

[20] The expected values of the errors in data analysis are determined in Appendix.

reporting structures are efficient;

– if the error in data analysis is crucial for the value of the computational service (i.e., if $\gamma > 0$) and the effect of the delay in data processing is minor ($\rho \approx 0$), then the efficient structures are regular with equally loaded managers;

– if ρ and γ are of the same order, then

(1) the relationship between the error in data analysis E and the variability of the numerical characteristics of the projects submitted ($E \sim \sigma_n * \sim \sigma$, where σ is the standard deviation of the numerical characteristics of the projects submitted) implies that, for small values of σ, less regular structures are expected to be efficient (in the extreme case, if $\sigma = 0$ then the skip-level reporting structures are efficient);

(2) the relationships between the delay in data processing D_N, the expected error in data analysis and the information workload ($D_N \sim N$, and $E \sim 1/N$ where N denotes a number of data items analyzed)[21], imply that for large numbers of data items analyzed, less regular structures are expected to be efficient than for small N (*ceteris paribus*).

Moreover, since delay in data processing is inversely related to the budget allocated to data processing ($D_N \sim 1/M$) and the expected value of the error in data analysis does not depend on the budget available, changes in the budget (amount of resources allocated to information processing) affect the value of informational service as well (for higher budget more regular structures are expected to be efficient than for the lower one). It implies that data processing structures could be efficient only for particular range of the budget allocated to data processing (or particular range of quantities of the resources allocated to information processing). To illustrate this result suppose that the technology is summarized by function $F(m)=m^{\alpha}$ (where m denotes the value of the resources allocated to a single manager) and the

[21] Note that the expected value of the error in data analysis does not depend on the number of data items processed N.

budget *M* is assigned to data processing. Furthermore, assume that the expected value of the loss due to non-instantaneous and imprecise information processing *F* is a linear function of (deterministic) delay in data processing and the expected value of the error in data analysis (i.e., *F(D,E)=?D+?E*). Assume that *?=0.01*, *?=1*, *a=0.2* and consider the relationship between the loss due to non-instantaneous and imprecise information processing and the budget available in various data processing structures (designed for the selection of the best project out of *N=8* projects submitted; s^2*=1*).

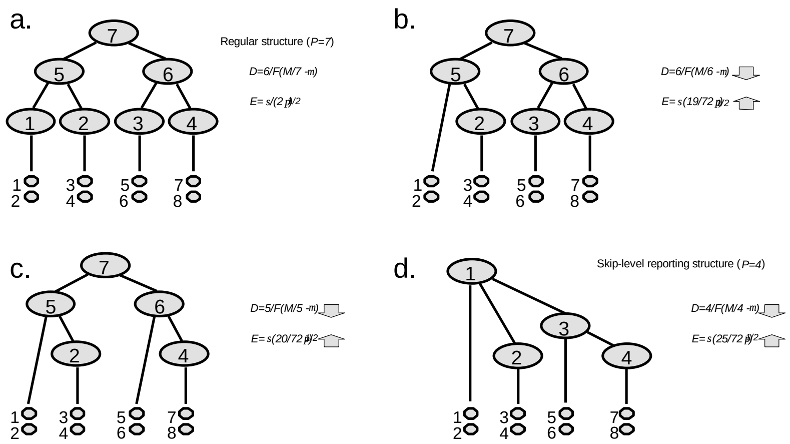
The analyzed relationship for *MÎ[0.008,0.0484]* is presented in Figure 4. The intersections of curves C with F and F with G indicate that in the interval considered there are three efficient information processing structures:

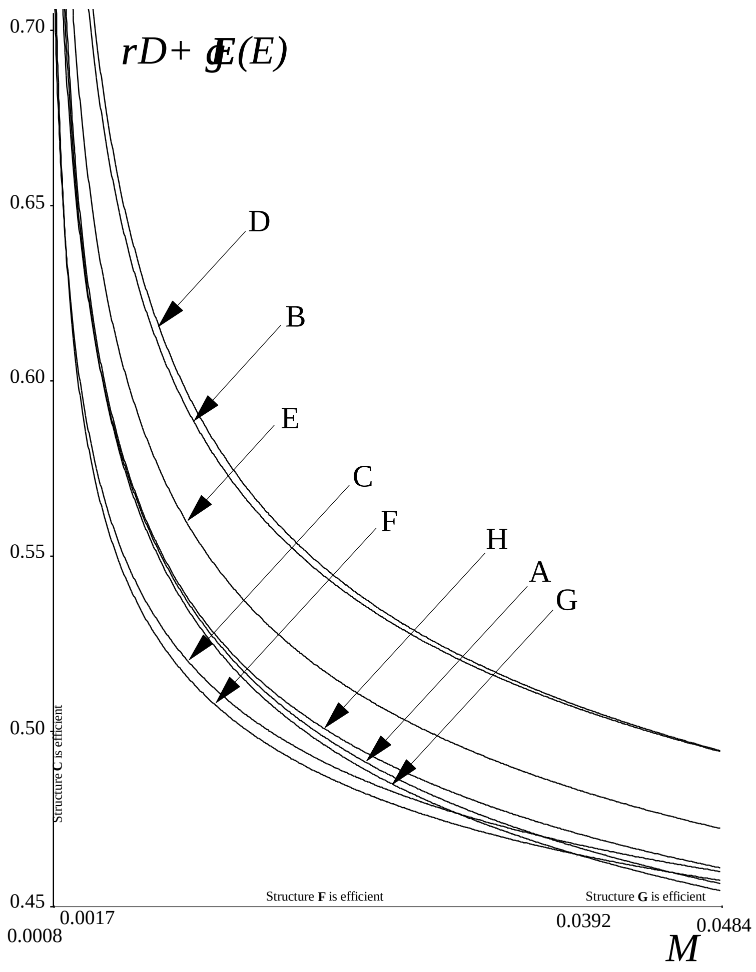
–skip-level reporting structure with *P=2* managers, if *0.0008£M£0.0017*;

–is skip-level reporting structure with *P=4* managers (Figure 3.d), if *0.0017<M£0.0392*;

–regular equally loaded structure with *P=3* managers, if *0.0392<M£0.0484*.

Figure 4. The relationship between the expected value of the loss due to non-instantaneous and imprecise data processing $\rho D+\gamma E(E)$ and budget M ($N=8$, $\rho=0.01$, $\gamma=1$, $\sigma^2=1$, $\mu=0.001$, $F(m)=m^{0.25}$) A: Centralized structure, $P=1$, $D=8/F(M-m)$, $E=\sigma/(2\pi^{0.5})$; B: Irregular structure $P=6$ (Figure 3.b), C: Skip-level reporting structure, $P=2$, $D=5/F(M/2-m)$, $E=\sigma\,[17/(50\pi)]^{0.5}$; D: Regular structure $P=7$ (Figure 3.a); E: Non-equally loaded regular structure, $P=4$, $D=6/F(M/4-m)$, $E=\sigma[21/(81\pi)]^{0.5}$; F: Skip-level reporting structure P=4 (Figure 3.d); G: Equally loaded regular structure, $P=3$, $D=6/F(M/3-m)$, $E=\sigma/(2\pi^{0.5})$; H: Irregular structure $P=5$ (Figure 3.c).





5. Technological Progress and Restructuring of Efficient Organizational Forms

Following the analysis presented in the previous section assume that the expected value of informational service can be represented by the expression (12) and consider the effects of the technological progress in data processing on the form of the efficient structures.

Technological change in data processing can be represented as a modification of information processing function $f(x)$, and, consequently, also $F(m)$. In particular, for any given value of the resources allocated to single manager m, technological change (progress in data processing technology) increases the value of the function $F(m)$. Consequently, for any given budget M (or vector of resources allocated to data processing) technological progress in information processing technology decreases delay in data analysis. Thus, other things constant, the time component in the expression (12) becomes less important, and, consequently, the shift of the efficient forms towards more regular information processing structures can be observed.

On the other hand, a change in the environment, due to the global technological progress, which speeds up all processes, changes also the relationship between the unit costs of the delay in information processing ? and the error in data analysis ?. In particular, global technological change could make the time component in the expression (12) relatively more important, i.e., could increase the ratio ?/?. In this case the shift of efficient forms towards less regular information processing structures can be watched.

In real world, however, the technological change in data processing is always accompanied by the overall technological progress. In other words, the two processes described above are observed simultaneously. Consequently, the resulting change in the efficient organizational forms of information processing structures in decision making systems depends on the total effect of the overall technological change on the value of informational service. In particular, if the ratio $?/F(m)$ decreases, then the shift of the efficient forms towards more regular information processing structures should be expected. If the ratio $?/F(m)$ does not change, the shape of the efficient structures does not change as well, and, finally, if the ratio $?/F(m)$ increases, the shift of the efficient forms towards less regular information processing structures should be expected.

6. Concluding Remarks

The analysis of data processing in decision making presented in this paper shows that organizational forms of the efficient information processing structures strongly depend on functional forms describing the value of the informational services. In particular, we show that if the value of informational service depends mostly on the delay in information processing and relatively less on the error in data analysis then the irregular, skip-level reporting, structures are efficient. On the other hand, if the value of informational service depends much more on the error in data analysis than on the delay in information processing, then the efficient structures are regular with equally loaded managers. Moreover, changes in the budget (or the resources) allocated to various information processing structures affect the value of informational service as well, and, consequently, that the architecture of the efficient structures could be different for various ranges of the resources allocated to data processing. Consequently, contrary to the results presented in the literature (see, e.g., Radner, 1993; or Radner and Van Zandt, 1992), in human data processing (unlike in computer systems) there is no single form of an efficient information processing structure. Instead a number of various architectures of efficient structures can be observed.

Furthermore, the paper shows that organizational restructuring of the efficient information processing structures in response to the technological progress depends on the effect of the overall technological change on the value of informational service. In particular, if the unit cost of the delay in data processing increases more than the speed of data processing then the efficient structures would be less regular (close to skip-level reporting). On the other hand, if the unit cost of the delay in data processing increases less than the speed of data processing then more regular structures would be efficient.

References

Aoki, M. 1986. "Horizontal vs. Vertical Information Structure of the Firm." *American Economic Review* 76: 971-983.

Arrow, K.J. 1985. "Informational Structure of the Firm." *American Economic Review* 75: 303-107.

Bolton, P. and M. Dewatripont. 1994. "*The Firm as a Communication Network.*" *Quarterly Journal of Economics*, 59: 809-840.

Cukrowski, J., 1997. Parallel Data Processing in Decision Making: Necessary and Sufficient Conditions. *Central European Journal for Operations Research and Economics*, Vol. 5, No. 2, 99-110.

Gibbons, A. and W. Rytter. 1988. *Efficient Parallel Algorithms*, Cambridge University Press, Cambridge.

Keren, M. and D. Levhari. 1979. "The Optimum Span of Control in a Pure Hierarchy." *Management Science* 11: 1162-1172.

Keren, M. and D. Levhari. 1983. "The Internal Organization of the Firm and the Shape of Average Cost." *Bell Journal of Economics* 14: 474-486.

Lipman, L.B. 1995. "Information Processing and Bounded Rationality: A Survey." *Canadian Journal of Economics* 28: 42–67.

Marschak, T. and R. Radner. 1972 *Economics theory of teams*, Yale University Press, New Haven, CT.

Milgrom, P. and J. Roberts. 1990. "The Economics of Modern Manufacturing: Technology, Strategy, and Organization." *American Economic Review* 80: 511-528.

Organ, D.W. and T. Bateman. 1986. *Organizational Behavior. An Applied Psychological Approach*, Homewood, Illinois: Business Publications, Inc.

O'Reilly III, Ch.A. 1990. “The Use of Information in Organizational Decision Making.” in Cummings, L.L. and B.M. Staw, Eds., *Information and Cognition in Organizations.* JAI Press Inc., London.

Prat, A., 1997. “Hierarchies of Processors with Endogeneous Capacity.” *Journal of Economic Theory*, 77:214-222.

Radner, R. 1972a. “Teams.” in Mc Guire, C.B. and R. Radner, Eds., *Decision and Organization*: 189-216. North Holland, Amsterdam.

Radner, R. 1972b. “Allocation of scarce resource under uncertainty: an example of team.” in Mc Guire, C.B. and R. Radner, Eds., *Decision and Organization*: 217-136. North Holland, Amsterdam.

Radner, R. 1992. “Hierarchy: The Economics of Managing.” *Journal of Economic Literature* 30: 1382-1415.

Radner, R. 1993. “The Organization of Decentralized Information Processing.” *Econometrica*: 61: 1109-1146.

Radner, R. and T. Van Zandt. 1992. “Information Processing in Firms and Return to Scale.” *Annales d'Economie et de Statistique* 25/26: 265-298.

Simon, H. 1972. “Theories of bounded rationality.” in Mc Guire, C.B. and R. Radner, Eds., *Decision and Organization*: 161-176. North Holland, Amsterdam.

Simon, H. 1987. “Bounded rationality.” in Eatwell, J., Milgate,M., and P. Newman, Eds., *The New Palgrave,* W.W.Norton, New York.

Van Zandt, T. 1995. “Continuous Approximations in the Study of Hierarchies.” *Rand Journal of Economics (*Winter) 26: 575-590.

Williamson, O.E. 1973. “Markets and Hierarchies: Some Elementary Considerations.” *American Economic Review* 63: 316-325.

Appendix: COMPUTATION OF EXPECTED VALUES OF THE ERROR IN DATA ANALYSIS IN SELECTED DATA PROCESSING STRUCTURES

Suppose, that the numerical attributes of the projects analyzed Q_n ($n=1,2,...,8$) and the projects considered to be the best by individual managers Q_j^O ($j=1,2,...,P$) are described by normally distributed random variables with mean Q^O and variances s^2 and s_o^2, respectively.

Consider the information processing structure presented in Figure 3.a. The analyzed structure is regular. Therefore each project under consideration can be selected as the best one with the same probability ($1/N=1/8$). Consequently, the random variable describing the numerical characteristic of the project selected in this structure $Q*$ can be represented as

$$Q_7* = 1/8\,(Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 + Q_7 + Q_8).$$

The variance of this random variable equals $s_{(7)}*^2=1/8\ s^2$. The expected value of the error in data analysis is: $E = s_{(7)}*\,(2/p)^{1/2} = s/(2p^{1/2})$.

To examine the structure presented in Figure 3.b, consider its two substructures with the managers *5* and *6* at the tops. The substructure with manager *6* at the top is regular. Consequently, projects *5*, *6*, *7* and *8* can be selected by manager *6* with the same probability (*1/4*). The substructure with the manager *5* at the top is irregular (skip-level reporting). Therefore, project *1* is selected by manager *5*

(1) if it is selected by manager *5* from the projects *1,2,* and *3* (random variables Q_1, Q_2 and Q_3 are independent and identically distributed, thus, the probability of this event equals *1/3*), and project *3* is selected by manager *2* (the probability of this event equals *1/2*);

or,

(2) if it is selected by manager *5* from the projects *1*, *2*, and *4* (random variables Q_1, Q_2 and Q_4 are independent and identically distributed, thus, the probability of this event equals *1/3*), and project *4* is selected by manager *2* (the probability of this event equals *1/2*);

Finally, project *1* is selected by manager *5* with the probability: $1/2\ 1/3+1/2\ 1/3=1/3$. Similarly, project *2* is selected by manager *5* with the probability *1/3*.

Project *3* is selected by manager *5* if it is selected by manager *2* (probability of this event equals *1/2*), and if it is selected by manager *5* (the probability of this event equals *1/3*). Consequently, the probability that project *3* is selected by manager *5* equals *1/2 1/3 = 1/6*. Similarly, the probability that project *4* is selected by manager *5* equals *1/6*.

Project *1* is selected as the best (by manager *7*) if it is selected by manager *5* (probability of this event equals *1/3*), and if it is selected by manager *7*. Project *1* is selected by manager *7* if it is better than projects *5*, or *6*, or *7*, or *8*, upon the condition that the corresponding project (i.e., *5*, *6*, *7*, or *8*) is selected by manager *6*. The probabilities that projects *5*,*6*,*7*, or *8* are selected by manager *6* equal *1/4*. Thus, the probability that project *1* is selected by manager *7* equals *4(1/3 1/2 1/4)= 1/6*.

Analogously, the probabilities that projects *2*,*3*,...,*8* are selected by manager *7* equal *1/6*, *1/12*, *1/12*, *1/8*, *1/8*, *1/8*, *1/8*, respectively. Consequently, the random variable characterizing the project selected in the structure under study Q_7* is:

$$Q_7* = Q_1/6 + Q_2/6 + Q_3/12 + Q_4/12 + Q_5/8 + Q_6/8 + Q_7/8 + Q_8/8.$$

The variance of this random variable equals $s_{(7)}*^2=19/144\ s^2$, and the expected value of the error in data analysis is: $E = s_{(7)}*(2/p)^{1/2} = s\ [19/(72p)]^{1/2}$.

The structure presented in Figure 3.c contains two identical irregular (skip-level reporting) substructures with managers *5* and *6* at the tops. The probabilities that projects *1*,*2*,*3* and *4* are selected by manager *5* equal *1/3*, *1/3*, *1/6* and *1/6*, respectively (see the consideration above). Analogously, the probabilities that projects *5*, *6*, *7* and *8* are selected by manager *6* equal respectively *1/3*, *1/3*, *1/6* and *1/6*. Consequently, probabilities that projects *1* through *8* are selected by manager *7* equal respectively *1/3 1/2=1/6*, *1/3 1/2=1/6*, *1/6 1/2=1/12*, *1/6 1/2=1/12*, *1/3 1/2=1/6*, *1/3 1/2=1/6*, *1/6 1/2=1/12*, *1/6 1/2=1/12* (see the analysis above). Thus, the random variable characterizing the project selected in the structure under study Q_7* is:

$$Q_7*= Q_1/6 + Q_2/6 + Q_3/12 + Q_4/12 + Q_5/6 + Q_6/6 + Q_7/12 + Q_8/12.$$

The variance of this random variable equals $s_{(7)}*^2= 20/144\ s^2$, and the expected value of the error in data analysis is: $E = s_{(7)}*(2/p)^{1/2} = s\ [20/(72p)]^{1/2}$.

In the structure presented in Figure 3.d, project *1* is selected as the best one by manager *7* if it is better than

(1) project *2*,

and it is better than

(2) project *3*, if project *3* is selected by manager *2* (the probability that this project is selected by manager *2* equals *1/2*), or project *4*, if project *4* is selected by

manager *2* (probability that this project is selected by manager *2* equals *1/2*);

and it is better than

(3) projects *5*, or *6*, or *7*, or *8*, upon the condition that the corresponding project (i.e., *5*, *6*, *7*, or *8*) is selected by manager *6* (probabilities that these projects are selected by manager *6* equal *1/3*, *1/3*, *1/6* and *1/6*, respectively).

This implies that the probability of project *1* being selected by manager *7* equals *1/4*. Analogously, the probabilities that projects *2* through *8* are selected as the best (by manager *7*) equal *1/4*, *1/8*, *1/8*, *1/12*, *1/12*, *1/24*, *1/24*, respectively. Consequently, the random variable characterizing the project selected in the structure under study Q_7^* is:

$$Q_7^* = Q_1/4 + Q_2/4 + Q_3/8 + Q_4/8 + Q_5/12 + Q_6/12 + Q_7/24 + Q_8/24.$$

The variance of this random variable equals $s_{(7)}^{*2} = 25/144\ s^2$, and the expected value of the error in data analysis equals: $E = s_{(7)}^*/(2/p)^{1/2} = s\ [25/(72p)]^{1/2}$.