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# Deriving Consensus Ratings of the Big Three Rating Agencies

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### Abstract

This paper introduces a model framework for dynamic credit rating processes. Our framework aggregates ordinal rating information stemming from a variety of rating sources. The dynamic of the consensus rating captures systematic as well as idiosyncratic changes. In addition, our framework allows to validate the different rating sources by analyzing the mean/variance structure of the rating deviations.

In an empirical study for the iTraxx Europe companies rated by the big three external rating agencies we use Bayesian techniques to estimate the consensus ratings for these companies. The advantages are illustrated by comparing our dynamic rating model to a naïve benchmark model.

**Keywords:** Bayesian estimation, consensus information, credit ratings, external rating agencies, rating validation.

# 1 Introduction

The importance of credit ratings provided by the big three external rating agencies Standard&Poor's, Moody's and Fitch has increased because modern credit risk pricing requires individual risk parameters, like rating implied probabilities of default (PDs; [Kliger and Sarig, 2000](#)). Despite the fact that all three raters express forward-looking opinions about the creditworthiness of firms on an ordinal scale, their ratings are rather incomparable because different rating systems with different granularity as well as different labels (typically, a combination of letters, numbers and/or modifiers). Nevertheless, the agencies consider the likelihood of default to be a centerpiece of creditworthiness and therefore consistent with the goal of an ordinal rating scale, where firms with a lower rating should have a higher PD than firms with a higher rating (e.g., [Cantor and Packer, 1997](#)). Obviously, the raters do not always agree on the creditworthiness of the firms (e.g., [Cantor and Packer, 1995](#); [Jewell and Livingston, 2002](#)). This resulting rating heterogeneity raises questions regarding the (1) nature, (2) quality and (3) interpretation of the ratings and the corresponding PDs. Are there consistent differences in their rating behavior? Does one agency have somewhat better information than the others regarding the creditworthiness? Or, does the rating heterogeneity just evince the very subjective and probabilistic nature of ratings ([Ederington, 1986](#))? Along with different definitions of ratings, do they measure different quantities representing the creditworthiness? Hence, rating heterogeneity nourishes the hypothesis that the rating processes of the agencies are not absolute and the differences in the published ratings may be a result of different sources of information, of different opinions about the obligors or of different discriminative focuses in the rating process, e.g., one agency might give more weight to the balance sheet leverage than the other. In addition, unsystematic or random errors may occur in a rating process. [Cantor and Packer \(1997\)](#) assess the problem whether observed rating heterogeneity reflects different rating scales or is simply the result of selection bias. [Morgan \(2002\)](#) analyzes the occurrence of split ratings and finds that split ratings are more frequent in the banking and insurance industry than in other industries. [Löffler \(2004\)](#) uses a structural model of default to derive rating characteristics if ratings are meant to give through the cycle evaluations as opposed to being based on the borrower's current condition only.

There is a growing literature on the analysis of credit ratings as well as their providers in the context of validation, regulation and information of the credit market ([Moon and Stotsky, 1993](#); [Cantor and Packer, 1995](#); [Krahen and Weber, 2001](#); [Jewell and Livingston, 2002](#); [Altman and Rijken, 2004](#); [Stolper, 2009](#)), but to the best of our knowledge there is no literature discussing how to combine different (heterogeneous) ratings of a company into a common rating. Especially in the area of financial modeling, where ratings play an inevitable role, it is essential to be able to deal with rating heterogeneity. For example, ratings serve as input parameters in industry models, e.g., CreditMetrics, and they are used for regulatory issues, like in the Basel II framework (see [Bank for International Settlements, 2004](#)). [Treacy and Carey \(2000\)](#) present the internal rating systems in use at the 50 largest US banking organizations. They state that "US regulatory agencies already use internal ratings in supervision" (see [Treacy and Carey, 2000](#), p. 168). A further example is the European Central Bank which extends collateralized loans to European banks. The decision whether to accept a collateral or not is mainly based on agency ratings and – if available – reported internal ratings which have to be aggregated. All these needs to validate and extend existing rating systems require to be able to cope with rating heterogeneity in order to combine ordinal ratings from different sources. Our framework addresses these needs.

In order to aggregate information of different raters a measure of "consensus" is required. [Zarnowitz and Lamnros \(1987\)](#) define "consensus" as the degree of agreement among point predictions aimed at the same target by different individuals. It can be computed as the median ([Su and Su, 1975](#)) or the mean of all the predictions in the sample ([Zarnowitz and Lamnros, 1987](#)). Alternative strategies for the aggregation of predictions are discussed by [Cook and Seiford \(1982\)](#); [Schnader and Stekler \(1991\)](#) and [Kolb and Stekler \(1996\)](#). In the context of forecasting the PDs of individual firms, [Hornik et al. \(2010\)](#) use a static mixed-effects model ([Pinheiro and Bates, 2000](#)) to model the consensus PDs with rater-specific

fixed effects and a random effect for firms. They refer to their proposed approach as the *latent trait model*.

The aim of this paper is to solve the information problem of combining different rating information stemming from different rating sources by deriving appropriate *consensus* information, i.e., consensus ratings which incorporate the information of several rating sources. Our approach takes the perspective of a rating user, i.e., an investor or financial regulator, where the respective decision making process has to deal with the existence of split ratings as opposed to the perspective of the rated firms (see [Bongaerts et al., 2012](#)). In a non-technical sense, any aggregation method has to solve two fundamental problems: (i) What is the common economic meaning of the ratings that have to be aggregated? (ii) How can the information be aggregated given the ratings are measured only on an ordinal scale? Our approach aims at aggregating the information concerning the PD of the rated entity. We solve the problem of ordinal information by modeling an underlying metric variable of creditworthiness which can be inferred from observed ordinal ratings. The consensus rating is then derived in a way fully consistent with the assumed model.

We claim that from a general point of view any suitable constructed consensus rating is *more informative* than the use of a single rating source. Ratings solve an information problem. Each rating agency uses certain information which might only be available to this rating agency and derives a rating without disclosing the specific information used to the public at large. The published rating is an estimate about the creditworthiness of the underlying *conditional* on the private information provided to the rating agency. Different raters in general have access to different sources of private information. In estimating a consensus rating, the ideal approach would be to pool all these sets of private information into one information set and estimate a rating based on the complete information. However, this is not possible because only the published ratings are available. Thus, if only ratings but not the underlying information sets are available the best choice is to derive a consensus measure based on the published ratings. Such a measure incorporates the different information sets indirectly via the published ratings and therefore is more informative.

We show that our method to construct a consensus rating is superior to a naïve “benchmark” model. Based on a transformation of ordinal ratings into metric variables one could easily construct a “simple consensus rating” by taking the arithmetic mean of the transformed values. The main advantage of our approach compared to this naïve benchmark model is that it accounts for cases where only ratings from different subsets of raters are available and that it is based on a consistent framework by modeling the underlying data-generating process. Lacking such a framework any naïve averaging is very likely to produce biased estimates given rater-specific differences in rating behavior. The empirical results using statistical information criteria presented in this paper support this claim. In addition, based on the consensus ratings and the rating deviations, we assess the *precision* and the *agreement* of the different rating sources which may serve as the basis for validating different rating systems. But also from a purely economic perspective our proposed consensus model outperforms the naïve averaging model: due to the ability to handle missing ratings appropriately, the proposed consensus model captures the companies’ conflicts of purchasing only the most favorable ratings (see [Bolton et al., 2012](#)). Furthermore, in pricing complex financial products ratings are inevitable. As mentioned, many financial decisions are based on creditworthiness estimates and inappropriate handling of rating heterogeneity does not make matters easier.

The model framework presented in this paper is related to other studies on credit rating systems (e.g., [McNeil and Wendin, 2007](#); [Stefanescu et al., 2009](#); [Hornik et al., 2010](#)). In contrast to [Hornik et al. \(2010\)](#) our model framework estimates the consensus rating on an ordinal scale and in a *dynamic* way. In addition, we make use of a *latent* market variable, describing the overall level of “creditworthiness”, which induces a correlation structure between the estimated consensus ratings. This is a well accepted strategy in the credit risk literature (e.g., [Nickell et al., 2000](#); [McNeil and Wendin, 2006, 2007](#); [Stefanescu](#)

et al., 2009). Therefore we refer to our model setup as the *dynamic latent trait model*.

In order to illustrate the potential of our dynamic model framework, we apply it to the iTraxx Europe (Series 10) companies rated by the big three external rating agencies. In particular, we use all available ordinal rating information of these companies by the three raters over a time period from 2007-02 to 2009-01. Using these data, we estimate the consensus ratings and analyze the three raters according to their rating deviations and their agreement with the consensus ratings.

The remainder of this paper is organized as follows: Section 2 describes the model specification and estimation of the consensus ratings. In Section 2.1 we discuss our dynamic latent trait model and Section 2.2 explains the benchmark approach which is used to validate our dynamic model. Section 3 provides a data description of the iTraxx Europe (Series 10) index and the agency ratings of the firms within this index. Section 4 applies the models described in Section 2 to the data. Bayesian estimation techniques are used to estimate the parameters of interest. The benchmark as well as the dynamic model are fitted to the data. The appropriateness of the dynamic model is confirmed by the deviance information criterion (DIC; Spiegelhalter et al., 2002). Section 5 concludes and summarizes the main results and the implications of our framework.

## 2 Consensus modeling

In this section we develop a model framework to derive consensus ratings from raters providing ordinal rating information, e.g., external agency ratings. Our model is designed for a dynamic framework capturing a time dependent rating process. Despite the fact that the raters publish ordinal ratings, we assume that they estimate a numerical variable – representing the creditworthiness of the firm – in an internal rating process. Each firm is then assigned to a particular rating class if this variable lies within a certain interval (e.g., McNeil and Wendin, 2007; Stefanescu et al., 2009). In general, the specific rating process including for example the estimation method is unknown. In the literature, modeling the creditworthiness was first discussed by Altman (1968) who introduces the Z-score. Z-scores are used to predict corporate defaults and are an easy-to-calculate control measure for the financial distress status of companies. The Z-score uses multiple corporate income and balance sheet values to measure the financial health of a company. Furthermore, Merton (1974) assumes that the creditworthiness can be reflected by the distance-to-default (DD) capturing the distance of the firm’s asset value to its default threshold on the real line. Alternatively, the creditworthiness variable can also be the result of an ordered probit or logit regression model (e.g., Altman and Rijken, 2004). To obtain ordinal ratings, the estimated DD, the Z-score, or any other numerical variable representing the creditworthiness – which is in the following referred to as “rating score” – is mapped onto an ordinal rating scale by the raters.

Let  $\{1, \dots, K_j\}$  be the set of possible non-default rating classes of rater  $j$  in descending creditworthiness. That is, 1 denotes the best credit quality and  $K_j$  the worst non-default rating class of rater  $j$ . Further,  $S_{ij}(t)$  denotes the estimated rating score (e.g., negative DD, Z-score) and  $r_{ij}(t)$  the associated observed ordinal rating of firm  $i$  by rater  $j$  at time  $t$ . The relationship between  $r_{ij}(t)$  and  $S_{ij}(t)$  is given by

$$r_{ij}(t) = k \Leftrightarrow S_{ij}(t) \in [\lambda_{k-1,j}, \lambda_{k,j}), \tag{1}$$

for a monotonically increasing sequence  $\lambda_{k,j}$  with  $k = 0, \dots, K_j$ . The class boundaries are assumed to be constant over time. The data consists of observations for  $J$  raters,  $T$  time points and  $I$  companies. Observing rating  $k$  for a firm by rater  $j$  means that its rating score lies somewhere in the interval  $[\lambda_{k-1,j}, \lambda_{k,j})$ .

In general, the thresholds  $\lambda_{k,j}$  are not provided by the raters. One possibility to obtain  $\lambda_{k,j}$  is to relate the ratings to the observable empirical default rates. In particular, the thresholds can be computed

by using the empirical default rates on an appropriate scale<sup>1</sup>. Assuming that the scores of empirical default rates,  $S_{ij}(t)$ , are defined on the real line we have to fix the lower as well as the upper threshold ( $\lambda_{0,j} = -\infty$  and  $\lambda_{K_j,j} = +\infty$ , respectively). The length of the intervals need not be equal and may differ from rater to rater. Nevertheless, it is expected that firms within the same interval will exhibit roughly the same creditworthiness (Stefanescu et al., 2009).

Due to general informational asymmetry between firm owners and raters<sup>2</sup> which can be due to limited access to the existing information, such as incomplete accounting information (Duffie and Lando, 2001), or delayed observations of the driving risk factors (Guo et al., 2008) the raters cannot estimate the “true” score (reflecting the creditworthiness) of a firm. Assuming that the rating deviations can be modeled additively<sup>3</sup> the relationship between the estimated rating score  $S_{ij}(t)$  and the latent score  $S_i(t)$  on the score scale is given by

$$S_{ij}(t) = S_i(t) + \epsilon_{ij}(t), \quad (2)$$

where  $\epsilon_{ij}(t)$  denotes the rating deviation for firm  $i$  by rater  $j$  at time  $t$ . In the following, the latent score  $S_i(t)$  is also referred to as the *consensus* score.

On the right hand side of Equation (2) we find two terms, which have to be specified: (1) The latent score  $S_i(t)$  which describes the consensus creditworthiness and (2) the deviation term  $\epsilon_{ij}(t)$  which captures the bias as well as the accuracy of the rating system of a specific rater. In the following those terms are specified for both the dynamic latent trait model and the benchmark approach.

Despite the fact that the scores  $S_{ij}(t)$  are unknown, the latent scores  $S_i(t)$  and the bias/variance structure of the rating deviations can be estimated in our framework by specifying the distribution of the rating deviations and using the interval thresholds  $\lambda_{\cdot,j}$  along with the relationship of Equation (1). The estimated consensus scores  $S_i(t)$  can then be mapped on the rater-specific ordinal scale to derive the consensus ratings  $r_{ij}^*(t)$  which obviously depend on the used rating system (of rater  $j$ ). Since  $r_{ij}(t)$  and  $r_{ij}^*(t)$  for all  $i$  and  $j$  are on the same rating scale one can easily compare these ratings and derive inference about the quality of the observed ratings  $r_{ij}(t)$ .

## 2.1 Dynamic latent trait model

**Latent consensus score.** In order to specify the latent scores  $S_i(t)$ , we follow the lines of McNeil and Wendin (2007) and Stefanescu et al. (2009) and assume that the scores are driven by market-specific (systematic risk) as well as firm-specific effects (idiosyncratic risk). We define a time-dependent process  $m_i(t)$  capturing the idiosyncratic changes and a latent market factor  $f(t)$  capturing the systematic development of the latent scores  $S_i(t)$ . The idiosyncratic changes  $m_i(t)$  track the firm-specific risk and can be modeled as an adequate time series process to cope with repeated observations. The latent market  $f(t)$ , modeling the development of the market, implies a correlation structure between the different firms and can also be modeled by an adequate time-dependent process, e.g., a stationary auto-regressive process or a random walk. Let  $\nu_i$  be the long-term mean of firm  $i$  which can be interpreted as the historical average creditworthiness of the firm. The development of the latent scores  $S_i(t)$  on the score scale is given by

$$S_i(t) = \nu_i + m_i(t) + \alpha f(t), \quad (3)$$

where the factor loading  $\alpha$  captures the dependence of  $S_i(t)$  on  $f(t)$ .

<sup>1</sup>Beside this, we assume that raters do not change their rating technology during the desired time period, i.e, they are always measuring creditworthiness on the same rating scale. This assumption justifies time independent  $\lambda_{k,j}$ .

<sup>2</sup>The general informational asymmetry between firm owners and raters constitutes the cornerstone of modern corporate finance (e.g., Leland and Pyle, 1977; Berk and DeMarzo, 2007).

<sup>3</sup>This is in line with Duffie and Lando (2001) who build their model on a Merton-type log normal firm value process and assume that the error in the observation of the log firm value is normal and additive.

In order to estimate the consensus scores  $S_i(t)$  we have to specify the underlying processes and distributions of this framework. We specify the development of the firm-specific changes  $m_i(t)$  and the latent market factor  $f(t)$  by AR(1) processes as

$$m_i(t) = \beta_i m_i(t-1) + \omega_i(t), \quad (4)$$

$$f(t) = \gamma f(t-1) + \xi(t). \quad (5)$$

$\omega_i(t)$  is a normal distributed error term with mean zero and a constant variance across time and firms, and  $\xi(t)$  is a standard normal distributed error term.  $\beta_i$  ( $|\beta_i| < 1$ ) and  $\gamma$  ( $|\gamma| < 1$ ) reflect the dependence on period  $t-1$  (inter-temporal correlation).

**Rating deviation.** We assume that the  $\epsilon_{ij}(t)$  are independent of the firms and their characteristics (in particular, their creditworthiness itself) and that the general rating process does not change over time  $t$  (see [Hornik et al., 2010](#)). Assuming that  $\mu_j$  and  $\sigma_j$  denote the mean and standard deviation of the rating deviations  $\epsilon_{ij}(t)$ , respectively, the rating deviations  $\epsilon_{ij}(t)$  are given by

$$\epsilon_{ij}(t) = \mu_j + \sigma_j Z_{ij}(t), \quad (6)$$

where  $Z_{ij}(t)$  is assumed to be independent standard normal distributed over  $i, j$  and  $t$ . Thus, the rating deviation of each rater consists of a systematic bias  $\mu_j$  which captures fundamental differences in rating methodology and  $\sigma_j$  which accounts for the variability of the rating deviation due to, e.g., asymmetric information. Our model therefore is able to distinguish and account for both of these types of rating deviations<sup>4</sup>.

## 2.2 Benchmark model

In addition to the dynamic latent trait model, we define a naïve benchmark approach and compare it with our dynamic latent trait model. Being conservative, one could consider to take the companies' worst rating as the benchmark. This is inappropriate for two reasons. Firstly, such an approach disregards the information contained in the other available rating sources. Secondly, from an economic point of view a rated company must be convinced that its credit-quality lies somewhere *between* its ratings and is not represented by the worst rating. Otherwise there would be little reason to obtain several ratings ([Hsueh and Kidwell, 1988](#)). Hence, without accounting for any rater-specific differences in rating deviations, the "mean" of the observed ratings could serve as a consensus benchmark.

**Latent consensus score.** Our benchmark model follows the idea that for any time  $t$ , the consensus score  $S_i(t)$  of a company is simply the *mean* over rating scores  $S_{ij}(t)$ . In doing so, we do not assume any time-dependent process driving the development of  $S_i(t)$ , i.e., for any time  $t$ ,  $S_i(t)$  is independent of  $S_i(t-1)$ .

**Rating deviations.** For the rating deviations, we assume that there are no rater-specific deviation terms  $\mu_j$  and  $\sigma_j$ , but a constant standard deviation  $\sigma$  of the rating deviations between the raters. This implies that all raters are weighted equally in the estimation process. Within our model framework the relationship between consensus score  $S_i(t)$  and the estimated scores  $S_{ij}(t)$  is given for the benchmark

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<sup>4</sup>Extensions of the consensus model could include a time varying rating error  $\mu(t)$ . Since we did not expect any systematic changes in the rating process and of the rating scale over the observation period from any of the raters, constant  $\mu_j$  and  $\sigma_j$  are assumed over time. However, the specification of these parameters in a time varying way might be necessary in other situations.



model by

$$S_{ij}(t) = S_i(t) + \sigma Z_{ij}(t), \tag{7}$$

with  $Z_{ij}(t)$  distributed as in the dynamic case.

For a discussion on deriving rater-specific biases and imprecisions based on the consensus estimated using this benchmark model in a post-hoc approach see Appendix D. It is shown that in the presence of cases, where ratings are not provided by all raters, this approach leads to biased estimates.

### 3 Data

**Ordinal ratings of the iTraxx Europe companies.** We use historical long-term issuer ratings of the constituents of the iTraxx Europe index (Series 10) from February 2007 to January 2009 provided by the big three external rating agencies Standard&Poor’s, Fitch and Moody’s. The iTraxx Europe index series consists of the 125 most-liquid credit default swaps (CDS) referencing European investment-grade entities and a new series is determined by dealer liquidity poll every six months. Most of the 125 entities in the index are large multinationals and have traded equity. We choose the iTraxx Europe index because it forms a representative contingent of the overall European credit derivative market and its constituents have a high number of co-ratings (occurrences of ratings of a single firm by two different raters) from the big three rating agencies. The time series is constructed using historical ordinal rating announcements taken from Reuters Credit Views. We exclude all companies for which we do not have rating information of at least two agencies for the complete time period, i.e., those with withdrawn ratings and entities which acquire a rating for the first time within the selected time frame. This process yields a sample of 5616 monthly ratings for 95 companies over 24 months (February 2007 to January 2009). Table 1 shows the co-ratings structure of the three raters. The average number of ratings for each firm per month is 2.46.

	Fitch	Moody’s	S&P
Fitch	88	44	88
Moody’s	44	51	51
S&P	88	51	95

Table 1: Co-ratings structure for 95 out of the 125 iTraxx Europe (Series 10) companies of the big three external rating agencies Fitch, Moody’s and Standard&Poor’s (S&P).

As described in Section 1, the three rating agencies use different rating systems. Moody’s rating system for global corporates contains 20 non-default rating categories, ranging from *Aaa* to *C* and is so in the near default ratings more granular than the rating systems of Fitch and Standard&Poor’s (Emery and Ou, 2009). These two agencies assign 17 non-default rating categories (*AAA* to *CCC/C*) to global corporates (Needham and Verde, 2009; Vazza et al., 2009). Table 2 shows the number of ratings (per rating category and rater) of the monthly ratings from February 2007 to January 2009 for the rating agencies Fitch, Moody’s and Standard&Poor’s. According to the three rating distributions, only one firm is once rated as a non-investment firm (ContinentalAG) and this only by Standard&Poor’s (see Crouhy et al., 2001, for a description of investment grades and speculative grades). The distributions show also that the granularity of the three rating systems is equal in the relevant segment of this rating data.

The rating history of 57 firms (60%) changed over the considered time period. Fitch changed the ratings of 35 firms, where 29 firms were downgraded and 4 firms were upgraded. The remaining two companies experienced a downgrade as well as an upgrade. Moody’s changed the ratings of 17 firms, where 8 firms were downgraded and 8 firms were upgraded (the remaining company experienced two

	Fitch		Moody's		S&P	
	Label	#	Label	#	Label	#
1	AAA	6	Aaa	18	AAA	0
2	AA+	85	Aa1	176	AA+	45
3	AA	148	Aa2	41	AA	167
4	AA-	193	Aa3	54	AA-	233
5	A+	226	A1	79	A+	170
6	A	243	A2	153	A	251
7	A-	410	A3	225	A-	473
8	BBB+	454	Baa1	231	BBB+	576
9	BBB	315	Baa2	183	BBB	292
10	BBB-	30	Baa3	64	BBB-	72
11	BB+	2	Ba1	0	BB+	0
12	BB	0	Ba2	0	BB	1
13	worse	0	worse	0	worse	0

Table 2: Number of ratings (per rating category and rater) of the 95 out of the 125 iTraxx Europe companies for the big three rating agencies Fitch, Moody's and Standard&Poor's (S&P).

upgrades as well as two downgrades). Standard&Poor's changed the ratings of 45 firms, where 29 firms were downgraded and 12 firms were upgraded (the remaining four company experienced upgrade(s) as well as downgrade(s)). Hence, a clear tendency of downgrading is observable in this period.

According to [Morgan \(2002\)](#) we should find an excess of split ratings within financial and insurance companies in our data. In fact, using the same mapping to a single numeric scale as [Morgan \(2002\)](#) we observe 57.7% split ratings between S&P and Moody's for the whole sample, but 80.3% when only considering financial and insurance companies. The same pattern is true for Fitch and Moody's with 56.9% for the whole sample and 88.3% for financial and insurance companies. The lowest split rate is observed between S&P and Fitch with 38.8% overall split ratings and 55.6% for financial and insurance companies.

In order to model the consensus ratings (Equation 2), each ordinal rating is identified with a numerical interval reflecting the upper and lower bound of the creditworthiness on the real line (see Equation 1). Here, we estimate the thresholds for the ordinal ratings using the empirical default rates (1990–2006) provided by the external raters ([Needham and Verde, 2009](#); [Emery and Ou, 2009](#); [Vazza et al., 2009](#)). A detailed description of this estimation is given in Appendix B.

**Dow Jones EURO STOXX 50.** By way of comparison we use the Dow Jones EURO STOXX 50 as a representative market development of the iTraxx Europe portfolio from February 2007 to January 2009 (see Figure 2). The Dow Jones EURO STOXX 50 is the leading stock (price) index for the Eurozone and covers 50 stocks from 12 Eurozone countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, and Spain. At January 2009, stocks of 30 out of the 95 companies are contained in the EURO STOXX 50.

## 4 Analysis of the big three rating agencies using their ratings for the iTraxx Europe companies

### 4.1 Model estimation

Using the available ordinal ratings  $r_{ij}(t)$  for each company  $i = 1, \dots, 95$  (out of the 125 iTraxx Europe companies) and external rating agency  $j = \{F, M, SP\}$  from  $t = 1, \dots, 24$  (February 2007 to January 2009) and the associated thresholds  $\lambda_{j,k}$  for  $k = 0, \dots, K_j$  with  $K_F = 17$ ,  $K_M = 20$ , and  $K_{SP} = 17$  we estimate the model parameters of our dynamic latent trait model as well as the parameters of our benchmark model. For the estimation frequentist as well as Bayesian techniques can be used. E.g., [Hornik et al. \(2010\)](#) estimated their model by standard maximum likelihood estimation. Here, we follow [McNeil and Wendin \(2007\)](#) and [Stefanescu et al. \(2009\)](#) and choose a Bayesian estimation approach using Markov chain Monte Carlo (MCMC) methods. Such an approach requires prior distributions to be chosen for the parameter set. In order to minimize the influence of the prior distributions on the posterior distribution we have specified non-informative priors for all our parameters.

In particular, we run four parallel Markov chains, each initialized with a different seed and a different random number generator. The Gibbs sampler ran for 50,000 iterations, using a thinning of 10 whereby the first 5,000 iterations were discarded as burn-in period. This yields 4,500 draws from the posterior for each parameter for each chain. Trace plots as well as the Geweke diagnostic and the Gelman Rubin's convergence diagnostic indicated satisfactory convergence (e.g., [Gelman and Rubin, 1992](#); [Plummer et al., 2008](#)).

**Model selection.** To take a decision in favour of one of the considered models is no trivial choice. Beside the fit of the models, one should also bear in mind the complexity of the single models. Intuitively, if two models fit the data equally well, the model with lower model complexity should be favored. In order to compare our dynamic latent trait model with the benchmark model we follow [Stefanescu et al. \(2009\)](#) and use the *deviance information criterion* (DIC; [Spiegelhalter et al., 2002](#)). DIC is in frequent use in Bayesian hierarchical settings and it can easily be derived during the MCMC simulations (see [Claeskens and Hjort, 2008](#)). The DIC is a generalization of the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) for hierarchical models and does not require nested models. In contrast to the AIC and BIC, DIC allows to compare Bayesian hierarchical models where the effective number of parameters is not clearly defined.

A lower DIC value indicates a better model fit. According to [Spiegelhalter et al. \(2002\)](#), if the difference in DIC is greater than 10, then the model with the larger DIC value has considerably less support than the model with the lower DIC value. For our models, the lower DIC value of our dynamic latent trait model (DIC = 9577.49) indicates that this model dominates in the terms of model fit as well as model complexity the naïve benchmark model (DIC = 16399.92).

### 4.2 Results for the dynamic latent trait model

**Rating deviations.** We begin our analysis of the estimation results with the rating deviations. Our dynamic latent trait model provides estimates for the rating bias  $\mu_j$  and the standard deviation  $\sigma_j$  of the rating deviation of the big three external rating agencies on the score scale. [Table 3](#) shows the results for the estimated posterior distribution of the parameters  $\mu_j$  and  $\sigma_j$  for the three raters. The posterior distributions of the parameters are characterized by the mean values (Mean) and the standard deviations (SD) of the 18,000 ( $4 \times 4,500$ ) posterior draws.

We infer from [Table 3](#) that Fitch has the smallest absolute rating bias from the consensus on the score scale with respect to the posterior mean (0.0157). Moody's clearly seems to be too optimistic in its credit assessment yielding a posterior mean for the rating bias  $\mu$  of  $-0.089$  on the score scale. Note, that our

	$\mu_j$		$\sigma_j$	
	Mean	SD	Mean	SD
Fitch	0.0157	0.0018	0.0753	0.0021
Moody's	-0.0891	0.0024	0.1002	0.0028
S&P	0.0734	0.0017	0.0642	0.0018

Table 3: Estimated bias  $\mu_j$  and standard deviations  $\sigma_j$  for the rating deviations (on the score scale) of the big three external rating agencies Fitch, Moody's and Standard&Poor's (S&P). The posterior distributions of the parameters are characterized by the mean values (Mean) and the standard deviations (SD) of the 18,000 ( $4 \times 4,500$ ) posterior draws.

model is based on the thresholds  $\lambda_{j,k}$  (and therefore PD equivalents) which are clearly lower for Moody's than the other two raters. Despite the high average difference between the investment grades (on the score scale: 0.139) in the PD equivalents of Moody's and Standard&Poor's indicated in the Appendix (see Table 8), Moody's is still more optimistic when rating investment-grade firms than Standard&Poor's. In this study, Standard&Poor's is with a posterior mean of the rating bias of 0.073 the most conservative rater out of the three considered rating agencies.

In addition to the rating biases, our model captures the standard deviation (which is indirectly proportional to the precision) of the rating deviations of the three raters (Table 3). Whereas the posterior mean of the standard deviation  $\sigma$  of the rating deviations is rather similar for Fitch and Standard&Poor's (0.075, 0.064), Moody's has a higher posterior mean of the standard deviation (0.100), indicating that its ratings deviate more strongly from the consensus ratings.

**Consensus score.** In addition to the analysis of the bias/variance structure of the rating deviations, we analyze the estimated consensus scores of our dynamic latent trait model. Instead of showing the consensus scores of all iTraxx Europe companies, Figure 1 shows the estimated consensus rating scores of four sample companies (ENELSPA, NESTLE, GLENCORE INT. AG, ROYAL BANK OF SCOTLAND) and compares them with the original ratings (mapped onto the score scale) of the three raters Fitch, Moody's and Standard&Poor's as well as with the mean rating scores of the three raters derived with the benchmark model.

Due to the fact that the companies ENELSPA and NESTLE are rated by all three raters, the consensus score (solid line) is very similar to the mean score (dashed line). In the case of the two other companies GLENCORE INT. AG and ROYAL BANK OF SCOTLAND where for each company ratings of only two raters are available, Figure 1 shows remarkable differences between the consensus and the mean score. Due to rater specific deviation terms, our latent consensus score is able to incorporate the non-availability of ratings.

A justification of the latent market  $f(t)$  in our framework can be found looking at the correlation between its estimated values and an empirical benchmark market. In fact, the correlation between  $f(t)$  and the Dow Jones EURO STOXX 50 index is  $-0.947$ . The negative sign is due to the estimation of  $f(t)$  on the score scale. Additionally we can compare the trend of both markets, when looking at Figure 2 in the paper. E.g., both markets show the calm period around 2007-07 and the massive shocks by the end of 2007. Even though the result is not surprising, it cannot be expected. The comparison of the two serves as a post-modeling check and the similarity between these two enhances the face validity of our proposed model.

**Consensus rating.** In addition to the analysis of the consensus scores, we can use the consensus ratings derived by mapping the scores onto the raters' rating scales to analyze the rating agreement of the raters.

An intuitive way for this is the Hit-Miss-Match (HMM) matrix which counts how many consensus

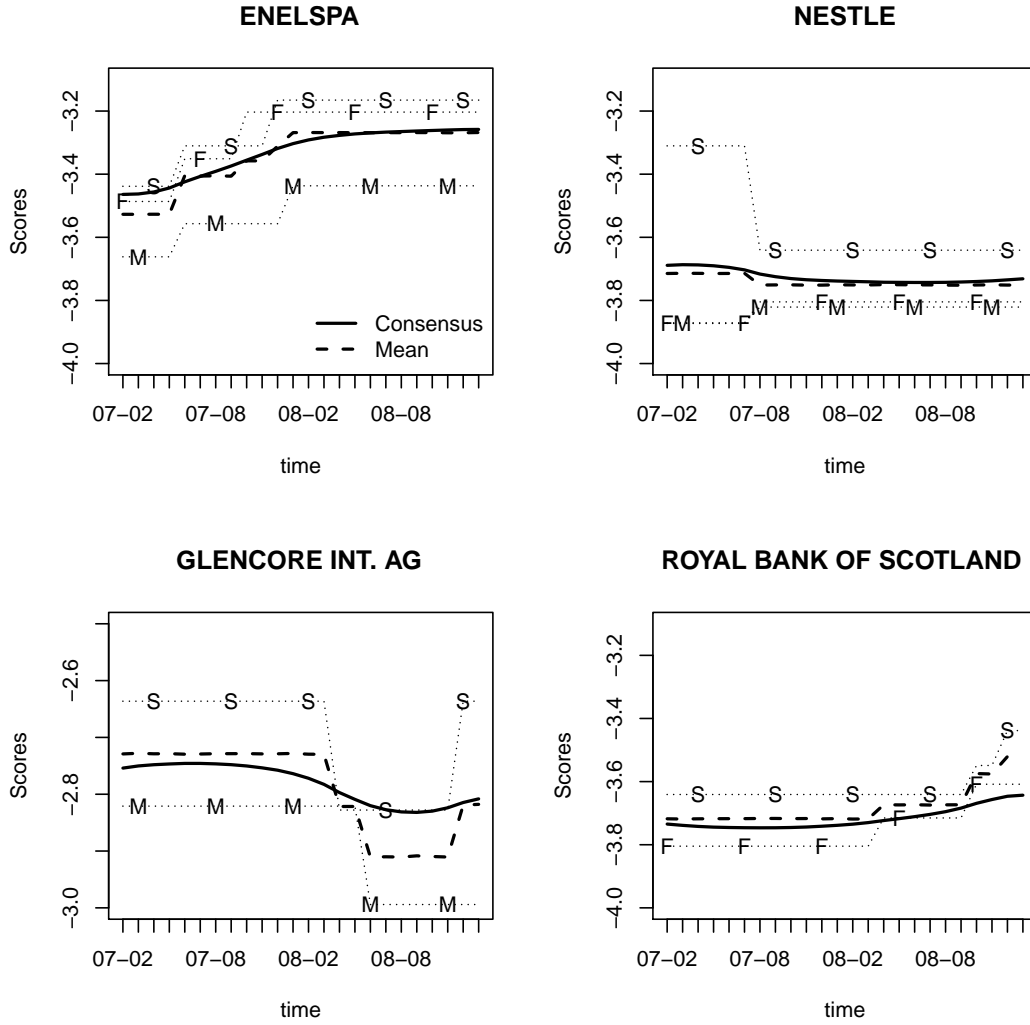


Figure 1: Estimated consensus score (Consensus), the mean score from the benchmark model (Mean), and the original ratings mapped onto the score scale of the big three external rating agencies Fitch (F), Moody's (M) and Standard&Poor's (S).

ratings exactly match the ratings provided by a rater. Appendix A presents the HMM matrix for each rater.

	-4	-3	-2	-1	0	1	2	3
Fitch	0.000	0.000	0.008	0.154	0.725	0.108	0.004	0.000
Moody's	0.000	0.000	0.017	0.028	0.280	0.541	0.114	0.020
S&P	0.003	0.000	0.030	0.432	0.528	0.007	0.000	0.000

Table 4: Proportion of ratings per rating class deviation between the consensus ratings and the original ratings provided by the big three rating agencies Fitch, Moody's and Standard&Poor's (S&P).

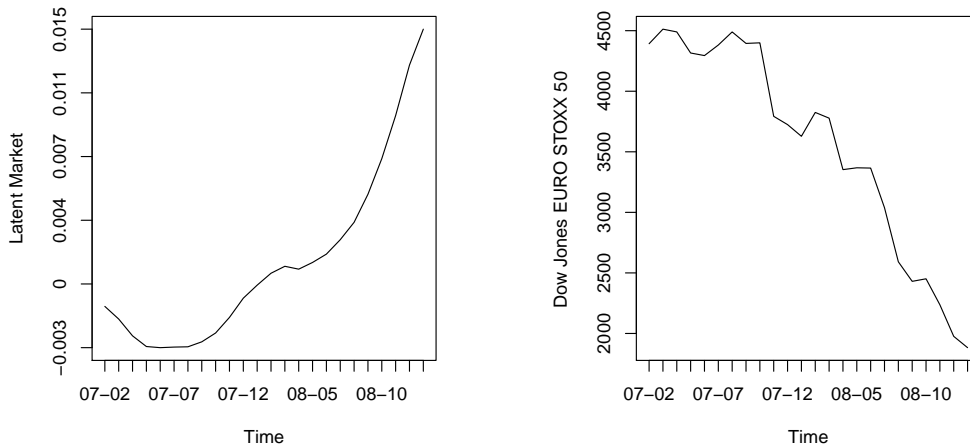


Figure 2: Estimated latent market factor  $f(t)$  and the Dow Jones EURO STOXX 50 index over the full time period (2007-02 to 2009-01).

Furthermore, we can compute the proportion of ratings for each rating deviation (measured in rating notches) between the consensus ratings and the ratings provided by the raters. Table 4 shows that Fitch’s ratings have a very high accordance (72.5%) with the estimated consensus ratings. According to the estimated rating biases (see Table 3) Moody’s is rather more “optimistic” than the other raters. This effect is also seen in Table 4. Only 28.0% of Moody’s ratings exactly hit the consensus rating. 84.9% are within one rating notch and 67.5% are more optimistic, i.e., are at least one rating category better than our estimated consensus rating. For Standard&Poor’s we obtain that 96.7% are within one rating category in comparison to the consensus rating. In contrast to Fitch, Standard&Poor’s even has a few ratings which are 4 rating classes below the estimated consensus rating.

## 5 Discussion

In this paper we investigate a new dynamic framework for aggregating credit-rating information in a multi-rater set-up, i.e., in situations where ordinal ratings from different sources for the same firm are available. In our model we assume that the raters do not directly estimate the ordinal ratings, but they estimate a numerical variable – representing the creditworthiness of the firm – in an internal rating process. We treat the true unobservable numerical variable of a firm as a latent variable and model its dynamic by using systematic as well as idiosyncratic changes. In contrast to other methods, our model class accounts for the fact that not all firms are rated by all raters at all time points in the data and captures the panel structure of the data.

In addition to the solution for the aggregation problem, our model is useful in the validation of the different sources. The analysis of the mean/variance structure of the rating deviations yields rater-specific rating biases as well as the different imprecisions of the rating systems.

The suggested framework for modeling consensus of a multi-rater panel is very general and allows for a variety of possible enhancements. We could aim at employing more flexible models for the distributions of the rating scores and rating deviations, e.g., via suitable mixtures of normals. We could also allow more flexibility in the specification of the factor loading  $\alpha$  capturing the dependence between the latent scores

and the latent market (see Equation 3) by using a firm- or industry-specific factor loading. In addition, it would be interesting to allow for industry-specific parameters for the rating bias, the standard deviation of the rating deviation and the long-term mean (see Hornik et al., 2010). We could also try to use an external market factor (e.g., the Dow Jones EURO STOXX 50) instead of a latent market factor to describe the systematic changes of the latent scores. The use of Bayesian estimation techniques allows very flexible specification of models, so that we intend to explore these possible enhancements in our future research.

By using the ratings for the iTraxx Europe companies (Series 10) provided by the big three rating agencies Fitch, Moody’s and Standard&Poor’s we compute a more informative rating, the consensus rating for each company, and show that there are remarkable differences in the rating behavior and rating systems of the three raters. In particular, we infer from our results, that Moody’s is the most favorable and Standard&Poor’s the most pessimistic rater.

## Computational details

All computations were carried out in the R system (version 2.15.1) for statistical computing (R Development Core Team, 2009). In particular, the R package `rjags` (Plummer, 2009) was used for Gibbs sampling and model selection, and the R package `coda` (Plummer et al., 2008) was used for the output diagnostic.

### A Hit-Miss-Match matrices for the raters

Consensus rating	Fitch rating										
	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+
AAA	0	0	0	0	0	0	0	0	0	0	0
AA+	0	33	0	0	0	0	0	0	0	0	0
AA	6	52	124	17	0	0	0	0	0	0	0
AA-	0	0	21	157	44	14	0	0	0	0	0
A+	0	0	3	19	149	50	0	0	0	0	0
A	0	0	0	0	33	166	33	0	0	0	0
A-	0	0	0	0	0	13	309	82	3	0	0
BBB+	0	0	0	0	0	0	68	350	93	0	0
BBB	0	0	0	0	0	0	0	22	218	4	0
BBB-	0	0	0	0	0	0	0	0	1	26	2
BB+	0	0	0	0	0	0	0	0	0	0	0

Table 5: Hit-Miss-Match matrix between the estimated consensus ratings and the ratings provided by Fitch, measured on the Fitch rating scale.

In Table 5 most ratings are on the main diagonal or one rating notch below or above indicating a high agreement between Fitch’s ratings and the consensus ratings. Table 6 shows that Moody’s ratings are rather one or more rating notches below the consensus ratings, confirming the negative rating bias shown in Table 3. In contrast to Moody’s ratings, Standard&Poor’s ratings are rather one or more rating notches above the consensus ratings (see Table 7), confirming the positive rating bias shown in Table 3.

Consensus rating	Moody's rating										
	Aaa	Aa1	Aa2	Aa3	A1	A2	A3	Baa1	Baa2	Baa3	Ba1
Aaa	0	0	0	0	0	0	0	0	0	0	0
Aa1	1	0	0	0	0	0	0	0	0	0	0
Aa2	10	80	7	2	0	0	0	0	0	0	0
Aa3	7	96	31	3	0	0	0	0	0	0	0
A1	0	0	3	16	33	24	0	0	0	0	0
A2	0	0	0	33	19	3	0	0	0	0	0
A3	0	0	0	0	3	126	74	0	0	0	0
Baa1	0	0	0	0	24	0	149	77	4	20	0
Baa2	0	0	0	0	0	0	2	154	101	4	0
Baa3	0	0	0	0	0	0	0	0	77	40	0
Ba1	0	0	0	0	0	0	0	0	1	0	0

Table 6: Hit-Miss-Match matrix between the estimated consensus ratings and the ratings provided by Moody's, measured on the Moody's rating scale.

Consensus rating	Standard&Poor's rating											
	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB
AAA	0	43	46	0	0	0	0	0	0	0	0	0
AA+	0	2	93	7	0	6	0	0	0	0	0	0
AA	0	0	28	137	3	0	0	0	0	0	0	0
AA-	0	0	0	89	109	9	0	0	0	0	0	0
A+	0	0	0	0	58	121	2	0	0	0	0	0
A	0	0	0	0	0	99	159	0	0	0	0	0
A-	0	0	0	0	0	16	311	185	0	0	0	0
BBB+	0	0	0	0	0	0	1	391	91	0	0	0
BBB	0	0	0	0	0	0	0	0	201	47	0	0
BBB-	0	0	0	0	0	0	0	0	0	25	0	1
BB+	0	0	0	0	0	0	0	0	0	0	0	0
BB	0	0	0	0	0	0	0	0	0	0	0	0

Table 7: Hit-Miss-Match matrix between the estimated consensus ratings and the ratings provided by Standard&Poor's, measured on the Standard&Poor's rating scale.

## B Estimation of the rating thresholds

In order to map the ordinal ratings provided by the three external rating agencies to PD ratings (PD equivalents) we follow the approach proposed by [Neagu et al. \(2009\)](#). They relate empirical PDs to ratings on an appropriate score scale. The score variable represents a rank ordering of risk of default over some future time horizon (we use a one year future time period). The task is to find a transformation of the score variable into an empirical PD. In other words, this method aims at finding a function  $F$  such that:

$$\text{PD} = F(\text{score}),$$

which can be written by using a default indicator as:

$$\text{Prob}(\text{default indicator} = 1) = F(\text{score})$$



and gives the base formulation for the class of binary response models. Different types of models, utilizing different forms for the function  $F$ , can be fit. [Neagu et al. \(2009\)](#) suggest to try the three most commonly used binary response models: logit, probit, and complementary log-log (CLL) models. These models can be applied directly to the score data, but in real-world applications the score data tends to exhibit a high degree of skewness. In this case it is recommended that a transformation of the score variable is made: a Box-Cox power transformation ([Box and Cox, 1964](#)) or a Box-Tidwell transformation ([Granger and Newbold, 1977](#)).

In particular, we use the published historical empirical global corporate default rates of the three external rating agencies from 1990 to 2006 ([Emery and Ou, 2009](#); [Needham and Verde, 2009](#); [Vazza et al., 2009](#)). In order to yield one-year empirical default rates we compute the averages over this time period. We then fit all combinations of binary response models (probit, logit, and CLL) and transformations (Box-Cox power and Box-Tidwell) to the average default rates. A probit score model with Box-Tidwell transformation is selected as the best method according to the Hosmer-Lemeshow statistic ([Hosmer and Lemeshow, 2000](#)).

In order to cleave to the ordinal structure of ratings, thresholds for the mapping PDs derived from the empirical default rates have to be computed. We compute the thresholds by the means of two adjacent mapping PDs on the logit scale for each rater  $j$ . I.e., the upper threshold  $\lambda_k$  of rating class  $k = 1, \dots, K_j - 1$  of rater  $j$  is given by  $\lambda_k = 1/2(\text{logit}(\text{PD}_{k+1}) + \text{logit}(\text{PD}_k))$  and the “lower” threshold of the best rating class is  $-\infty$  and the “upper” threshold of the worst rating class is  $+\infty$  ([Altman and Rijken, 2004](#)).

Table 8 shows the estimated rating thresholds for the three different rating systems of Fitch, Moody’s, and Standard & Poor’s using the empirical default rates from 1990 to 2006. Note, that the rating system of Moody’s is finer on the upper side, i.e., assigning four more rating grades to the high PD segment than the other two raters. Whereas the empirical default rates and the PD mappings of Fitch and Standard&Poor’s seem to be rather similar, Moody’s empirical default rates is clearly below the other two. E.g., on average the difference on the probit scale between the investment grades of Standard&Poor’s and Moody’s is 0.139.

Moody’s scale	Moody’s score	Fitch / S&P scale	Fitch score	S&P score
Aa1	-3.8461	AA+	-3.8388	-3.7372
Aa2	-3.7859	AA	-3.7601	-3.6768
Aa3	-3.7063	AA-	-3.6621	-3.5951
A1	-3.6097	A+	-3.5479	-3.4940
A2	-3.4974	A	-3.4194	-3.3748
A3	-3.3703	A-	-3.2779	-3.2385
Baa1	-3.2295	BBB+	-3.1245	-3.0857
Baa2	-3.0754	BBB	-2.9599	-2.9172
Baa3	-2.9086	BBB-	-2.7849	-2.7334
Ba1	-2.7296	BB+	-2.5998	-2.5348
Ba2	-2.5388	BB	-2.4053	-2.3217
Ba3	-2.3365	BB-	-2.2017	-2.0946
B1	-2.1230	B+	-1.9893	-1.8536
B2	-1.8986	B	-1.7685	-1.5991
B3	-1.6636	B-	-1.5395	-1.3311
Caa1	-1.4180	CCC/C	-1.3025	-1.0500
Caa2	-1.1621			
Caa3	-0.8961			
Ca/C	-0.6200			

Table 8: Estimated class boundaries  $\lambda_{k,j}$  for the three raters on the score scale based on a probit score model with Box-Tidwell transformation using the empirical default rates from 1990 to 2006.

## C Summary of parameter estimates

For the factor loading  $\alpha$  we derive a posterior mean of 0.0094. Analogously, the posterior mean of the AR(1) coefficient of the latent market factor  $\gamma$  is 0.8560. This indicates that the estimated latent market factor is highly persistent. Different AR(1) coefficients  $\beta_i$  are estimated for each single firm for the idiosyncratic changes  $m_i(t)$  and the posterior mean values vary between 0.070 and 0.9936. The mean of the posterior mean values of  $\beta_i$  is equal to 0.3169 and a standard deviation of 0.3188 is observed. For the individual firms the differences in persistence are hence large varying from only a small dependence to a very high dependence.

## D Deriving rater-specific biases and imprecisions from the naïve benchmark

The relationship between the “observed” scores  $S_{ij}(t)$  and the consensus score  $S_i(t)$  is given by

$$S_{ij}(t) = S_i(t) + \mu_j + \sigma_j Z_{ij}(t). \quad (8)$$

For the naïve benchmark model this relationship is simplified to

$$S_{ij}(t) = S_i(t) + \sigma Z_{ij}(t) \quad (9)$$

and the consensus can be estimated in this case by

$$\tilde{S}_i(t) = \frac{1}{\sum_{j=1}^J \delta_{ij}(t)} \sum_{j=1}^J \delta_{ij}(t) S_{ij}(t), \quad (10)$$

where  $J$  is the number of raters and  $\delta_{ij}(t)$  is a binary indicator which is 1 if rater  $j$  rates firm  $i$  at time point  $t$  and 0 otherwise.

Based on these consensus estimates the systematic errors of the single raters  $\mu_j$  can be estimated in a post-hoc approach using

$$\tilde{\mu}_j = \frac{1}{\sum_{i=1}^I \sum_{t=1}^T \delta_{ij}(t)} \sum_{i=1}^I \sum_{t=1}^T \delta_{ij}(t) (S_{ij}(t) - \tilde{S}_i(t)). \quad (11)$$

$T$  denotes the number of time points and  $I$  the number of firms. These estimates are then transformed in order to ensure that they sum to zero.

In similar vein we can estimate the imprecisions of the single raters  $\sigma_j^2$ :

$$\tilde{\sigma}_j^2 = \frac{1}{\sum_{i=1}^I \sum_{t=1}^T \delta_{ij}(t) - IT - 1} \sum_{i=1}^I \sum_{t=1}^T \delta_{ij}(t) (S_{ij}(t) - \tilde{S}_i(t) - \tilde{\mu}_j)^2. \quad (12)$$

Finally, we derive the bias of the estimator  $\tilde{\mu}_j$  by determining its expected value  $E[\tilde{\mu}_j]$ .

$$E[\tilde{\mu}_j] = \mu_j - \frac{1}{\sum_{i=1}^I \sum_{t=1}^T \delta_{ij}(t)} \sum_{k=1}^J \sum_{i=1}^I \sum_{t=1}^T \frac{\delta_{ik}(t) \delta_{ij}(t)}{\sum_{l=1}^J \delta_{il}(t)} \mu_k. \quad (13)$$

The second term,

$$\frac{1}{\sum_{i=1}^I \sum_{t=1}^T \delta_{ij}(t)} \sum_{k=1}^J \sum_{i=1}^I \sum_{t=1}^T \frac{\delta_{ik}(t) \delta_{ij}(t)}{\sum_{l=1}^J \delta_{il}(t)} \mu_k, \quad (14)$$

is the bias of the estimate for the systematic error  $\mu_j$  in the naïve benchmark model which becomes relevant if the data includes unbalanced repetition patterns, i.e., not all firms are rated by all raters at all time points. Otherwise, if  $\delta_{ij}(t) \equiv 1$ , the term becomes  $(1/II) \sum_{j=1}^J (II/J) \mu_j = (1/J) \sum_{j=1}^J \mu_j = 0$ . Unbiased estimates could be obtained by correcting the estimate by the term given in Equation (14). Essentially, our proposed consensus model takes the cases which did not receive ratings from all raters into account and avoids this bias.

Using our dataset and the above presented estimation techniques, we estimated the bias of rating errors  $\mu_j$  to range from  $-0.0164$  to  $0.0086$ . In economic terms this implies an average mis-estimation of PDs up to 15 bps for lower credit quality firms. Additionally we observe that the resulting ratings differ by at least one rating notch in 17% – 23% of the cases.

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