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Estimates and inferences of knowledge capital impacts on regional
total factor productivity

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Abstract. This paper explores the contribution of knowledge capital to total factor productivity differences among regions within a regression framework. We provide an econometric derivation of the relationship and show that the presence of latent/unobservable regional knowledge capital leads to a model relationship that includes both spatial and technological dependence. This model specification accounts for both spatial and technological dependence between regions, which allows us to quantify spillover impacts arising from both types of interaction. Sample data on 198 NUTS-2 regions spanning the period from 1997 to 2002 was used to empirically test the model, to measure both direct and indirect effects of knowledge capital on regional total factor productivity, and to assess the relative importance of knowledge spillovers from spatial versus technological proximity.

Keywords: Total factor productivity, knowledge spillovers, technological proximity, spatial Durbin model, European regions

JEL classification: C11, C21, O47, O52, R11
1 Introduction

Recent empirical literature on economic growth has shown that differences in income levels, both at country and regional levels, are mainly due to disparities in total factor productivity (TFP). Easterly and Levin (2001), for example, report that more than 90 percent of the differences in growth rates among countries are explained by total factor productivity rather than traditional factor accumulation. Moreover, a strong stylized fact that emerges from the empirical literature is that regional disparities are longer and more persistent when compared to cross-country differences, at least within the industrialized countries (see Magrini 2004 for a review).

Since the differences in productivity turn out to depend on the efficiency levels, the attention of economists has been increasingly devoted to search for additional factors that may contribute to account for such disparities. Several explanations for the total factor productivity gap have been put forward, but knowledge capital appears to play a key role (see, Grossman and Helpman 1994).

The focus of this paper is to show how the latent unobservable nature of regional knowledge capital in the context of a standard total factor productivity regression relationship can lead to an econometric model specification that contains both spatial and technological dependence. Total factor productivity is defined as output (in terms of gross value added) per unit of labour and physical capital combined. The paper constructs patent stock measures of regional knowledge endowments using data on patent applications. These regional stocks represent a proxy for predetermined knowledge (outputs) generated from past R&D investments (inputs)\(^1\).

It seems plausible to assume that there exist some unobservable regional stocks of knowledge capital since many authors note the limitations of patents as a measure of useful/effective knowledge/innovation especially at the regional level (Autant-Bernard and LeSage 2010, Parent and LeSage 2008). This assumption in conjunction with positive spatial dependence in unobserved regional knowledge leads to an interesting econometric result. Positive spatial

\(^1\) In using patent stocks to proxy regional knowledge stocks, this paper builds on previous work by Robbins (2006) and Fischer et al. (2009), but departs from this prior work. These studies – using fixed or random effects panel data models – provide evidence that knowledge spillovers and their productivity effects are to a substantial degree geographically localized.
dependence means that large unobserved knowledge capital for a region such as Ile de France implies high levels of unobservable knowledge in neighbouring regions as well. Similarly, lower levels of unobservable knowledge for a French region such as the Corse imply low levels of unobservable knowledge in neighbouring regions. Since many studies have found evidence of positive spatial dependence in measured regional knowledge/innovation, it seems plausible that unmeasured regional knowledge also exhibits positive spatial dependence. The conjunction of these two phenomena leads to an extension of the standard spatial Durbin model (SDM) specification that includes both spatial as well as technological dependence structures.

We show how this extended model can be estimated and how to quantify the knowledge spillover impacts arising from spatial and technological proximity between regions. The empirical application illustrates how to correctly assess the relative importance of spatial versus technological connectivity between regions in determining direct and indirect effects of knowledge capital on regional factor productivity, using a simple extension of the LeSage and Pace (2009) approach.

It seems plausible that the econometric issues we raise here also apply to a wider range of models that have been used to explore how innovation/knowledge explains variation in regional growth and income levels. However, we focus our attention on the regional total factor productivity relationship. There are a number of motivations for this focus. First, the TFP relationship can be viewed as using residuals from a regional production function as the dependent variable in a (log) linear regression, where the dependent and candidate explanatory variables are less controversial than in the regional growth literature. In fact, we develop our reasoning using a single measured explanatory variable reflecting regional knowledge stocks and posit the existence of a single latent unobservable variable reflecting unmeasured regional knowledge stocks. A second motivation for the TFP relationship is that it has received less attention in the regional science literature (see Smith, 1999 and Fischer et al. 2006 for exceptions).

The remainder of the paper is organized as follows. Section 2 outlines the framework for assessing the contribution of knowledge capital to regional total factor productivity when faced with unmeasured regional knowledge that exhibits spatial dependence. Section 2.1 starts with an expanded version of the standard regional Cobb-Douglas production function as an accounting
format – and not as an estimation framework – in order to isolate the contribution of knowledge capital to total factor productivity, and leads to a simple log-linear non-spatial relationship. Section 2.2 shows how unobserved or unobservable forms of knowledge capital (such as tacit knowledge) in conjunction with observed proxies measured by patent stocks in this study, will lead to a spatial regression model when both types of regional knowledge capital exhibit spatial dependence and non-zero covariance.

The resulting spatial Durbin model form is extended in Section 2.3 to the case where the latent unobservable knowledge exhibits positive technological dependence in addition to spatial dependence. This extension is in accordance with the increasing evidence that interregional knowledge flows tend to follow particular technological trajectories (see, for example, Fischer et al. 2006, and LeSage et al. 2007, Parent and LeSage 2008). The extended model produces a type of SDM specification that includes both spatial as well as technological connectivity among regions, and we discuss empirical tests that can be used to determine the significance as well as the relative magnitudes of both types of knowledge spillover effects, arising from regional knowledge stocks on regional total factor productivity. Another important methodological contribution of this paper is correct assessment of spillover effects, based on the approach suggested by LeSage and Pace (2009).

Section 3 uses a six-year sample of 198 NUTS-2 regions over the period 1997 to 2002 to empirically implement the models. Section 3.1 provides details on the construction of the total factor productivity and the patent stock measures. Bayesian Markov Chain Monte Carlo estimates and log-marginal likelihood model comparison tests are presented in Section 3.2 to identify the extent of the knowledge diffusion process among regions, in terms of both geographical and technological proximity. Section 3.3 discusses scalar summary measures of direct and spillover impacts associated with changes in knowledge stocks that allows a comparison of the magnitude of spatial versus technological spillovers.

2 The analytical framework for assessing the contribution of knowledge capital

2.1 The production function and total factor productivity
The analytical framework for the study considered here is the regional Cobb-Douglas production function augmented by including knowledge capital as an extra input that yields the following basic relationship between output and knowledge capital

\[ y_{im} = \alpha x_{im} + \beta k_{im} \]  

where \( y_{im} \) is the log of output \( Y \) of region \( i = 1, \ldots, N \) at time \( m = 1, \ldots, M \), \( x \) the log of an index \( X \) of conventional inputs such as physical capital and labour, and \( k \) a measure of cumulated knowledge or knowledge capital \( K \) in log form. \( \beta \) is the elasticity of output with respect to knowledge capital, and \( \alpha \) the elasticity with respect to the index of conventional inputs. We follow the convention that lower case letters denote logs and upper case letters levels, and focus on a value-added specification to simplify the exposition. The functional form of this equation, linear in the logarithms of the variables (that is, Cobb-Douglas) is to be taken as a first approximation to a potentially much more complex relationship.

For the purpose of this paper we define the total input index as 

\[ X_{im} = L_{im} s - C_{im} \]

where \( L \) denotes labour, \( C \) physical capital, and \( s \) is the observed factor share of labour. Assume that \( s \) is observed correctly and proportional to the true coefficient of labour [that is, \( s = \alpha_1 / (\alpha_1 + \alpha_2) \) and \( \alpha_1 + \alpha_2 = 1 \)], and that there is no error in computing the true relative shares of labour and physical capital. Then the log of region’s \( i \) total factor productivity at time \( m \) can be defined in the usual way as 

\[ \text{tfp}_{im} = y_{im} - \alpha_1 l_{im} - (1 - \alpha_2) c_{im} \]

Measured total factor productivity then depends on the contribution of knowledge capital, but not on the level of other inputs:

\[ \text{tfp}_{im} = \beta k_{im} \]  

where \( \text{tfp} \) denotes the log of total factor productivity. Of course, Eq. (2) represents a rather simplistic relationship based upon a whole string of untenable assumptions, the major ones being a Cobb-Douglas production technology with constant returns to scale with respect to physical capital and labour. Nevertheless, this simple relationship is a convenient departure point to show how the presence of latent unobserved regional knowledge will lead to a spatial regression model when both types of knowledge capital, observed and unobserved, exhibit spatial dependence and
non-zero covariance. Omitting spatial lags of the dependent and explanatory variables from this relationship will then result in biased and inconsistent estimates for the parameters relating the impact of knowledge stocks to regional total factor productivity.

The stocks of knowledge capital are proxies for the state of knowledge. The knowledge created by an agent is added to the stock of existing knowledge, to which other agents potentially have access. Even if the benefits of research are fully appropriated by an agent, in the sense that an agent acquires a monopoly right to use the results of research efforts, the knowledge created may diffuse across regions through various transmission channels such as publications, seminars, personal contacts, reverse engineering, joint ventures, and other means (Park 1995). Hence knowledge is viewed as non-rival, but partially excludable (Romer 1990).

In a world of regions with exchange of information and dissemination of knowledge a region’s productivity depends not only on its own knowledge capital, but also on its capacity to attract and assimilate knowledge produced elsewhere. There are different approaches to account for cross-region knowledge spillovers. In this study we follow Fischer et al. (2009), by assuming that regions have greater access to the knowledge resources of neighbouring than non-neighbouring regions. Thus, we can express the relationship between knowledge capital and total factor productivity in matrix form as follows:

\[ tfp = \beta k \]  
\[ k = \phi W k + u \]

where \( tfp \) and \( k \) are the \( N \)-by-1 vectors reflecting (logged) cross-sectional observations on total factor productivity and knowledge capital, respectively, in a world of \( N \) regions, and the knowledge capital vector \( k \) follows a spatial autoregressive process so that

\[ u \sim \mathcal{N}(0, \sigma_u^2 I_N). \]

\( W \) is the \( N \)-by-\( N \) spatial weight matrix with \( W_{ij} > 0 \) when observation \( j \) is a spatial neighbour to observation \( i \), and \( W_{ij} = 0 \) otherwise. We also set \( W_{ii} = 0 \), and assume that \( W \) has row-sums of unity. Note that each element of \( Wk \) represents a linear combination of elements from the
vector $k$ associated with neighbouring locations. The $i$th row of $W_k$ captures region’s $i$ external stock of knowledge capital, for $i = 1, \ldots, N$. The scalar parameter $\phi$ reflects the strength of spatial dependence in $k$, and $u$ is an $N$-by-1 vector of disturbances distributed $\mathcal{N}(0, \sigma_u^2 I_N)$. Of course, there is a great deal of empirical support in the literature for this type of specification where measured regional knowledge depends positively on that from neighbouring regions (see, for example, Anselin, Varga and Acs 1997, Autant-Bernard 2001, Fischer and Varga 2003).

2.2 From the basic relationship to a spatial model relationship

Patent stock measures have several advantages over alternative measures of knowledge capital\(^2\) but miss those parts of the knowledge stock that are not codified in the form of patent documents. Let $K^*$ represent knowledge not captured by the patent stock measure $K$. For convenience, we call $K$ the observed and $K^*$ the unobserved or unobservable (regional) stocks of knowledge capital. We show how $K^*$ in conjunction with $K$ will lead to a spatial regression relationship if both exhibit spatial dependence, and are correlated by virtue of common (correlated) shocks to the spatial autoregressive processes governing these variables.

Consistent with our assumption that regions have greater access to the knowledge resources of neighbouring regions captured by $K$, we assume that the unobserved components of knowledge capital exhibit spatial dependence of the type assigned to $K$. Specifically, we assume that

\begin{equation}
\log k^* = \theta W k^* + v
\end{equation}

\begin{equation}
v \sim \mathcal{N}(0, \sigma_v^2 I_N)
\end{equation}

where $k^* = \log K^*$ is an $N$-by-1 vector representing the unobserved elements of knowledge endowment, for each of the $N$ regions. The scalar parameter $\theta$ reflects the strength of spatial dependence in $k^*$, $W$ is defined as above and $v$ is a zero mean, constant variance disturbance

\(^2\) One problem with the R&D input measure widely used in the literature is that some double counting occurs because R&D labour and capital are counted twice, once in the available measures of physical capital and labour, and again in the measure of R&D capital stocks (see Griliches and Mairesse 1984). By using patents we avoid this problem. But patents have their own well-known weaknesses. To the extent that patents document inventions, an aggregation of patents is arguably more closely related to a stock of knowledge than is an aggregation of R&D expenditures (see Robbins 2006).
term. Moreover, we assume that $k$ and $k^*$ are correlated by virtue of common (correlated) shocks to the spatial autoregressive processes governing these variables:

$$v = u \gamma + \epsilon$$

(8)

$$\epsilon \sim N(0, \sigma_\epsilon^2 I_N).$$

(9)

The relationship in Eq. (8) reflects simple (Pearson) correlation between shocks $u$ and $v$ to knowledge capital stocks $k$ and $k^*$ when the scalar parameter $\gamma \neq 0$. $\epsilon$ is a zero mean, constant variance disturbance term. We note that correlation in the shocks implies non-zero covariance between $k$ and $k^*$.

If we begin with the relationship between knowledge capital and total factor productivity that captures the influence of unobserved knowledge elements,

$$tfp = \beta k + k^*$$

(10)

and apply the definitions given in Eqs. (4), (6) and (8) we arrive at

$$tfp = \theta W tfp + k \delta_1 + W k \delta_2 + \epsilon$$

(11)

where

$$\delta_1 = \beta + \gamma$$

(12)

$$\delta_2 = -(\theta \beta + \phi \gamma).$$

(13)

The model relation given by Eqs. (11)-(13) represents what has been labelled a spatial Durbin model (SDM) by Anselin (1988). This model subsumes the spatial error model (SEM):

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3 See LeSage and Pace (2009) for a more general and detailed exposition of this type of result.
\[(I_N - \theta^TW)tfp = (I_N - \theta^W)k \delta_1 + \varepsilon\] as a special case when, first, \(k\) and \(k^*\) are not correlated, and, second, the parameter restriction \(\delta_2 = -\theta \delta_1\) holds\(^4\).

Three implications are worth noting. First, a spatially dependent omitted variable that is correlated with the stock of knowledge measure included in the model will invalidate the parameter restriction and lead to a spatial regression model that must contain a spatial lag of the \(tfp\) variable. This is true whenever \(\gamma\) is not equal to zero, which rules out the parameter restriction \(\delta_2 = -\theta \delta_1\).

Second, if the spatial Durbin model relation between knowledge capital and total factor productivity is consistent with the sample data, but not the SEM model relation, omitting spatial lags of the \(tfp\) and knowledge capital variables from the empirical model will result in biased and inconsistent estimates for the parameters (see Pace and LeSage 2010) relating the impact of knowledge capital to total factor productivity, the focus of this study.

A third implication is that calculation of the response of total factor productivity to knowledge capital, \(\frac{\partial tfp}{\partial k}\), will differ depending on which model is appropriate. For the case of the SEM model, the coefficient estimates have the usual least-squares regression interpretation, where the log-form of the relationship leads directly to elasticity estimates for the response of \(tfp\) to variation in the levels of knowledge capital across the regions. For this case, there are no spatial spillover impacts that arise from changes in knowledge stocks.

In the case of the SDM model, \(\frac{\partial tfp}{\partial k}\) takes a much more complicated form and allows for spatial knowledge spillover impacts. These measure the effect arising from a change in knowledge capital in region \(i\) on total factor productivity in other regions \(j \neq i\). Specifically, Eq. (14) shows the partial derivatives which take the form of an \(N\)-by-\(N\) matrix.

\(^4\) Note that \(\gamma = 0\) is a necessary, but not sufficient condition. That is, we might have no correlation between \(k\) and \(k^*\), and still find the restriction \(\delta_2 = -\theta \delta_1\) inconsistent with our sample data. This would point to the need for the SDM model specification for reasons other than the omitted variable motivation set forth in this paper. A simple likelihood-ratio test of the SEM versus SDM model can be carried out using the log-likelihoods from the two model specifications.
\[
\frac{\partial \text{tp}}{\partial k} = (I_N - \theta W)^{-1}(I_N \delta_1 + W \delta_2).
\] (14)

LeSage and Pace (2009) have proposed scalar summary measures for the \(N\)-by-\(N\) matrix of direct and cumulative spatial spillover impacts arising from changes in the explanatory variable \(k\) on the dependent variable vector representing regional total factor productivity. By cumulative we mean that spillovers falling on all neighbours are summed. They point out that the main diagonal of the matrix \((I_N - \theta W)^{-1}(I_N \delta_1 + W \delta_2)\) represents own partial derivatives, which they label direct effects, and summarize using an average of these elements of the matrix. The off-diagonal elements correspond to cross-partial derivatives, which can be summarized into scalar measures of the cumulative spillovers using the average of the row-sums of the matrix elements. LeSage and Pace (2009) provide an approach to calculating measures of dispersion that can be used to draw inferences regarding the statistical significance of direct or indirect effects. These are based on simulating parameters from the normally distributed parameters \(\delta_1, \delta_2, \theta\) and \(\sigma^2\), using the estimated means and variance-covariance matrix. The simulated draws are then used in computationally efficient formulas to calculate the implied distribution of the scalar summary measures.

### 2.3 Extension of the Model Relationship

It has become increasingly common to recognize that geographical proximity represents only part of the story of the (disembodied) knowledge diffusion mechanism (see Jaffe 1986, Schartinger et al. 2002, Parent and LeSage 2008). Geographical proximity matters, but proximity – reflecting technological networks of connectivity between regions – appears to be prevalent (see Fischer et al. 2006). To account for the technological dimension to the spillover mechanism, we assume that a region’s ability to make productive use of another region’s knowledge depends on the degree of technological similarity between regions. Technological similarity between regions is defined in terms of closeness in a technological space spanned by a number of distinct technological fields, where each field has a somewhat unique set of applications. We continue to assume that those parts of knowledge capital not captured by the measure \(k\) exhibit spatial dependence.
Thus, the dependence process governing measurable knowledge stocks $k$ now indicates that these depend on ‘neighbouring’ regions in technological space rather than conventional ‘neighbours’ in a geographical sense reflected by the spatial weight (connectivity) matrix $W$. The motivation for this specification is that codified knowledge is accessible across greater distances to regions that work in similar production or scientific fields. That is, field-specific knowledge codified in patents ‘travels well’.

Unobserved knowledge stocks $k^*$ are specified to exhibit conventional spatial dependence. A motivation for this specification is that person-to-person communication becomes relatively more important for the diffusion of non-codified forms of knowledge. Patent statistics will necessarily miss that part, because codification is necessary for patenting to occur. We assume that part of the knowledge generated with the idea leading to a patent is embodied in persons, imperfectly codified, and linked to the experience of the inventor(s). This stock of knowledge increases in a region as local inventors discover new ideas. It diffuses mostly via face-to-face interactions. Following Bottazzi and Peri (2003) we think of it as a local public good as it benefits researchers within the region and its neighbourhood, motivating our spatial specification for unmeasured knowledge.

Formally, we assume that

$$\begin{align*}
  k &= \phi T k + u \\
  k^* &= \theta W k^* + v \\
  v &= u \gamma + \epsilon
\end{align*}$$

$$\begin{align*}
  u &\sim \mathcal{N}(0, \sigma_u^2 I_N) \\
  v &\sim \mathcal{N}(0, \sigma_v^2 I_N) \\
  \epsilon &\sim \mathcal{N}(0, \sigma_\epsilon^2 I_N)
\end{align*}$$

where $T$ is an $N$-by-$N$ technological weight matrix with $T_{ij} > 0$ when region $j$ is a neighbour to region $i$ in technological rather than geographical space, and $T_{ij} = 0$ otherwise. We also set
$T_w = 0$, and assume that $T$ has row-sums of unity. Note that each element of $Tk$ represents a linear combination of elements from the vector $k$ associated with technologically similar regions. The $i$th row of $Tk$ captures region’s $i$ external stock of knowledge capital, for $i = 1, ..., N$. The scalar parameter $\phi$ now reflects the strength of technological dependence in $k$. All other vectors, matrices and parameters are defined as in Section 2.2.

Following the same substitutions as in the previous section, applied to Eq. (10), we arrive at the following relationship between knowledge capital and total factor productivity

$$tfp = \theta W tfp + k \delta_1 + W k \delta_2 + T k \delta_3 + \epsilon \quad (21)$$

with

$$\delta_1 = \beta + \gamma \quad (22)$$

$$\delta_2 = -\theta \beta \quad (23)$$

$$\delta_3 = -\phi \gamma. \quad (24)$$

There are a number of points to note here. First, if the parameter $\phi = 0$, so that no technological dependence exists, then $\delta_3 = 0$ and this model has the same reduced form as the simpler model from Eq. (11). But this is not true of the structural forms for the two model specifications. The strength of spatial dependence indicated by the parameter $\theta$ is determined by that of the spatial process assigned to govern unobserved forms of knowledge, as in the simpler model with no technological dependence. This results from the specification choice made in Eq. (16). The specification leads to a reduced form expression for the extended model that nests the simpler model when no technological dependence exists. Second, in this extended version of the model impacts on $tfp$ from changes in $k$ take the form

$$\frac{\partial tfp}{\partial k} = (I_N - \theta W)^{-1} (I_N \delta_1 + W \delta_2 + T \delta_3). \quad (25)$$
Since the parameter $\delta_j$ is significantly different from zero in our empirical application, we can use the nested reduced form interpretation to compare the effects of knowledge capital on total factor productivity that arise from spatial versus technological proximity. This is done by comparing the scalar summary measures proposed by LeSage and Pace (2009) for the model specification where we restrict $\delta_j = 0$ to those from the unrestricted model. Effects associated with the restricted model are purely spatial whereas those for the unrestricted model represent both spatial and technological dimensions of the spillover mechanism.

3 An empirical implementation

3.1 The sample data

Our sample is a cross-section of 198 regions representing the 15 pre-2004 EU member states over the 1997-2002 period. The units of observation are the NUTS-2 regions\(^5\) (NUTS revision, 1999, except for Finland revision 2003). These regions, though varying in size, are generally considered to be appropriate spatial units for modelling and analysis purposes. In most cases, they are sufficiently small to capture subnational variations. But we are aware that NUTS-2 regions are formal rather than functional regions, and their delineation does not represent the boundaries of regional growth processes very well.

The sample regions include regions located in Western Europe covering Austria (nine regions), Belgium (11 regions), Denmark (one region), Finland (four regions), France (20 regions), Germany (40 regions), Greece (11 regions), Ireland (three regions) Italy (20 regions), Luxembourg (one region), the Netherlands (12 regions), Portugal (five regions), Spain (16 regions), Sweden (eight regions) and United Kingdom (37 regions).

\(^5\) We exclude the Spanish North African territories of Ceuta and Melilla, the Portuguese non-continental territories Azores and Madeira, Corse, the French Départements d’Outre-Mer Guadeloupe, Martinique, French Guayana and Réunion. Two Greek NUTS-2 regions (Ionía Nisia and Voreio Aigaio) that had zero patent stocks were combined with neighbouring NUTS-2 regions to avoid outliers in the spatial and technological lag variables. Since matrix product $W_k$, for example, reflects an average of knowledge stocks from geographical neighbours, the introduction of zero values in the vector $k$ will produce aberrant observations in the spatial lag vector $W_k$. 
Empirical implementation of the two models described in the previous section uses data on total factor productivity and knowledge stocks for each of the \( N \) regional economies at six points in time. Total factor productivity calculations at the regional level require interregionally comparable data on regional outputs and inputs. In this study we define \( \text{tfp} \) in the usual way as 
\[
\ln \left( \frac{Y}{L} \right) - (1 - s) \odot \ln C
\]
and use gross value added data in euro (constant prices of 1995, deflated) as measure of output \( Y \). \( s \) denotes the \( N \)-by-1 vector of regional shares in production costs. Following the approach suggested by Hall (1990), \( s \) is not calculated as the ratio of total labour compensation to value added (the revenue-based regional factor shares), but as cost-based factor shares that are robust in the presence of imperfect factor shares. The symbol \( \odot \) denotes the Haddamard (element-by-element) product of the \( N \)-by-1 vector of shares, and \( L \) regional labour and \( C \) physical capital.

The data for regional labour come from Cambridge Econometrics. They include only employees, not the self-employed for each region. We adjusted these data on labour inputs to account for differences in average annual hours worked across countries. This is important because average annual hours worked in Swedish manufacturing in the year 1997, for example, were almost 14 percent lower than in Greek manufacturing. Without adjusting for differences in input usage, productivity in Greek and Portuguese regions would be overestimated throughout, while in Swedish and Dutch regions underestimated (Fischer et al. 2009).

Physical capital stock data is not available in the Cambridge Econometrics database, but gross fixed capital formation in current prices is. Thus, the stocks of physical capital were derived for each region \( i \) from investment flows, using the perpetual inventory method:
\[
C(t+1) = C(t)(1 - r_c) + I(t+1),
\]
where \( C(t) \) is the stock of physical capital at the end of period \( t \), \( I(t+1) \) gross investment during \( (t, t+1) \). We applied a constant rate \( r_c \) of ten percent depreciation (obsolescence) across space and time. The annual flows of fixed investments were deflated by national gross-fixed capital formation deflators. The mean annual rate of growth, which precedes the benchmark year 1997, covers the period 1990-1997 to estimate initial regional physical capital stocks.

Besides the TFP measure, the models also contain a measure of the knowledge capital stock for each of the \( N \) regions and the six time periods. We use corporate patent applications to proxy
knowledge capital. Corporate patents cover inventions of new and useful processes, machines, manufactures, and compositions of matter. To the extent that patents document inventions, an aggregation of patents is arguably more closely related to a stock of knowledge than is an aggregation of R&D expenditures (Robbins 2006). However, a well known problem of using patent data is that technological inventions are not all patented. This could be because applying for a patent is a strategic decision, so not all patentable inventions are actually patented. Even if this is not an issue, as long as a large part of knowledge is tacit, patent statistics will necessarily miss that part, because codification is necessary for patenting to occur. All of these issues provide a motivation for our approach that posits latent unobservable knowledge stocks.

Patent stocks were derived from European Patent Office (EPO) documents. Each EPO document provides information on the inventor(s), his or her name and address, the company or institution to which property rights have been assigned, citations to previous patents, and a description of the device or process. To create the patent stocks for 1997-2002, the EPO patents with an application date 1990-2002 were transformed from individual patents into stocks by first sorting based on the year that a patent was applied for, and second the region where the inventor resides. In the case of cross-region inventor teams we used the procedure of fractional rather than full counting. Then for each region \( i \), patent stocks were derived from patent data, using the perpetual inventory method:

\[
K(t + 1) = K(t)(1 - r_K) + S(t + 1),
\]

where \( K(t) \) is the patent stock at the end of period \( t \), \( S(t + 1) \) are knowledge production activities during \((t, t + 1)\), measured in terms of corporate patent applications, and \( r_K \) is a constant depreciation rate. Because of evident complications in tracking obsolescence over time, we used a constant depreciation rate \( r_K = 12 \) that corresponds to the rate of knowledge obsolescence in the US over the past century, as found in Caballero and Jaffe (1993). Patent stocks were initialized the same way as physical capital.

### 3.2 Estimates and tests of the model assumptions
For presentation purposes we will consider the two models shown in Eqs. (26) and (27), where we have added an intercept term $\alpha_0$ and associated $N$-by-1 vector of ones, $\iota_N$, to the models introduced in Section 2, to reflect the non-zero mean of the dependent variable $tfp$:

Model 1: 
$$tfp = \alpha_0 \iota_N + \theta W tfp + \delta_1 k + \delta_2 W k + \epsilon$$  \hspace{1cm} (26)

Model 2: 
$$tfp = \alpha_0 \iota_N + \theta W tfp + \delta_1 k + \delta_2 W k + \delta_3 T k + \epsilon.$$  \hspace{1cm} (27)

A pooled model was used because estimates based on a cross-sectional sample for each of the six years produced estimates that were within one standard deviation of each other. These estimates along with an average standard deviation are reported in Table 1. Pooling over the $M$ time periods involves forming a vector \( \tilde{tfp} = \text{vec}(tfp_1, ..., tfp_M) \), where \( \text{vec} \) represents the “vec” operator that stacks the $N$-by-1 column vectors $tfp_m$, $(m = 1, ..., M)$, to create an $MN$-by-1 vector for the dependent variable. Similarly, we can form: \( \tilde{k} = \text{vec}(k_1, ..., k_M) \). The spatial weight matrix $W$ does not change over time, so we can form $\tilde{W} = I_M \otimes W$ to implement the pooled model.

Table 1 about here

The $N$-by-$N$ technological weight matrix $T$ in Model 2 measures the closeness of regional economies in a technological space spanned by 120 distinct technology fields, described by 120 patent classes of the International Patent Code (IPC) classification\(^6\). We utilized corporate patents applied at EPO with an application date in the years 1990 to 1995 to define the technological position of a region, in terms of a 120-by-1 vector with the share of patents filed in each of the six years in the IPC categories. This definition reflects the region’s diversity of inventive activities of its firms. Following Jaffe (1986), a Pearson correlation coefficient was used to measure the technological proximity between any two regions of the sample. A high correlation indicates similarity and a low correlation dissimilarity. The matrices $T_m (m = 1, ..., M)$

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\(^6\) These patent classes refer to the second level of the IPC classification system that is used to classify inventions claimed in the EPO patent documents.
were formed for each year by finding the $m$ regions that exhibited the highest correlation coefficients with each region. A single value of $m$ was used, but separate matrices form the pooled weight matrix $\tilde{T} = \text{diag}(T_1, \ldots, T_m)$ based on the IPC category patenting activities in each of the six years. This allows us to express the pooled models in an identical format as in Model 2 by replacing the $N$-by-$1$ vectors, $tfp, k, Wk, Tk$ with $\tilde{tfp}, \tilde{k}, \tilde{Wk}$, and $\tilde{T}$.

Table 2 about here

Bayesian model comparison methods were used to calculate posterior model probabilities based on the log-marginal likelihood for pooled models with varying numbers $s$ of technological neighbours and spatial neighbours $r$, based on nearest neighboring regions in technological and geographical space respectively. The log-marginal likelihoods and posterior model probabilities reported in Table 2 are based on LeSage and Parent (2007). Since these models all contain the same number of parameters, non-informative priors were used\(^7\). The log-marginal likelihoods and posterior model probabilities in Table 2 used models based on spatial weight matrices containing $r = 5$ to $r = 9$ nearest neighbours, and technological weight matrices constructed using $s = 2$ to $s = 10$ nearest technological neighbours. Estimates of spillover impacts arising from changes in regional knowledge stocks are dependent on the specification of the spatial and technological weight matrices $W$ and $T$, as can be seen from the partial derivative in Eq. (25). This motivated Bayesian model comparison of alternative matrices $W$ and $T$. The posterior model probabilities point to eight nearest technological neighbours and indicate seven spatial neighbours. Empirical results reported in the remainder of the paper were based on $r = 7$ and $s = 8$.

Table 3 about here

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\(^7\) See LeSage (1997) regarding Bayesian MCMC estimation of these models.
Pooled estimates for Model 1 and Model 2 are presented in Table 3. These are Bayesian MCMC estimates based on non-informative priors, which were nearly identical to maximum likelihood estimates. We relied on MCMC estimation to produce a sequence of 5,000 retained draws that could be used to construct the measures of dispersion for the effects estimates discussed in the next section. It is important to keep in mind that the parameter estimates for $\delta_2$ and $\delta_3$ do not represent the impact of spatial spillovers arising from regional knowledge stocks. To accurately assess the magnitude of spatial spillovers we will rely on the scalar summary measures that represent $\partial tfp / \partial k$ discussed in Section 2. This topic will be taken up in Section 3.3.

One point of interest is whether excluded variables reflecting unobserved or unobservable knowledge capital are correlated with the included knowledge stock measure $k$. This can be formally tested by examining the restriction $-\theta \delta_1 = \delta_2$ for Model 1. If this restriction holds, then the SEM model is appropriate and the shocks to observed and unobserved knowledge stocks are uncorrelated. From the posterior mean estimates for Model 1 in Table 3, we see that $-\theta \delta_1 = -0.0689$ with a lower 99% interval of $-0.0460$ and $\delta_2 = -0.0137$, so we can conclude this restriction is not consistent with the estimates.

A likelihood ratio test statistic can be constructed using twice the difference in log-likelihood function values from the SDM and SEM models, which is chi-squared distributed with one degree of freedom reflecting the single restriction. These two log-likelihood values were -159.4, and -181.0, respectively, producing a chi-squared statistic equal to 43.2. Since the 99% critical value for a chi-squared deviate with one degree of freedom is 6.315, we can reject the restriction as being inconsistent with the sample data. Of note, the log-likelihood function value for Model 2 equalled -143.3, which is significantly different from that for Model 1, when subjected to a likelihood ratio test based on the restriction implied by these nested models.

A second issue is whether the (pooled) knowledge stock variable $\tilde{k}$ exhibits spatial dependence, an assumption we made in deriving Model 1. Using the spatial regression model: $\tilde{k} = \alpha_0 + \theta (I_M \otimes W) \tilde{k} + \epsilon$, we find a maximum likelihood estimate $\hat{\theta} = 0.7249$ and an asymptotic $t$-statistic equal to 33.4, allowing us to conclude that (log) knowledge stocks exhibit strong spatial dependence.
For the extended Model 2, we tested whether (pooled) knowledge stocks \( \tilde{k} \) exhibit technological dependence, using \( \tilde{k} = \alpha_0 + \phi \tilde{\kappa} + \epsilon \). The parameter estimate for \( \phi \) is 0.6869 with a \( t \)-statistic of 17.9, so we conclude that the assumptions made in constructing Model 2 appear consistent with the sample data used here.

3.3 Spillover impacts from knowledge capital on total factor productivity

As indicated in Section 2.2, it is necessary to properly calculate the direct, indirect and total effects associated with changes in knowledge stocks on total factor productivity in our spatial regression framework. For Model 1 the direct and spillover effects reflect an average of diagonal and off-diagonal elements of: 

\[
\frac{\partial \text{tfp}}{\partial k} = [I_M \otimes I_N - \hat{\theta} (I_M \otimes W)]^{-1} (I_M \otimes I_N) \hat{\delta}_1 + (I_M \otimes W) \hat{\delta}_2
\]

which correspond to scalar summary measures of the own and cross-partial derivatives. The set of 5,000 retained MCMC draws from estimation were used to construct upper and lower 99% credible intervals for these effects estimates, allowing us to test for their statistical significance.

Table 4 about here

Table 4 shows the posterior mean effects estimates along with 99% credible intervals, which indicate that the direct, indirect and total effects for the two models are positive and different from zero based on the credible intervals. The indirect effects reported in the table are what economists usually refer to as spatial spillovers. We emphasize that it would be a mistake to interpret the coefficient estimate \( \hat{\delta}_2 \) as representing spatial spillover magnitudes in spatial regression models that involve spatial lags of the dependent variable. To see how inaccurate this is, consider the difference between the coefficient estimates for \( \hat{\delta}_2 \) in Table 3 and the true indirect effects correctly calculated from the partial derivatives of the spatial regression model. Using Model 1 as an example we see that \( \hat{\delta}_2 \) is not statistically significantly different from zero, whereas the true indirect effect estimate is 0.1631 in Table 4, with a lower 0.01 bound of 0.0729 making it clearly a positive and significant effect.
Model 2 allows for both spatial as well as technological spillover effects, and produces the largest indirect effects, based on \( \frac{\partial tfp}{\partial k} = [I_M \otimes I_N - \hat{\theta}(I_M \otimes W)]^{-1}[(I_M \otimes I_N) \delta N + (I_M \otimes W) \delta W + \text{diag}(T_1, ..., T_M) \delta_1]. \)

The interpretation of these partial derivative effects estimates is that changes in knowledge stocks would lead to a move from one steady-state equilibrium to a new steady-state (see LeSage and Pace 2009). The effects estimates in Table 4 reflect the cumulative impact of knowledge stock changes that would arise in the movement between equilibrium steady-states. Since we have a cross-sectional model, there is no information regarding the time required for the move between steady-states. Given the log-transformation of both the dependent and independent variables in our models, the effects estimates have an elasticity interpretation. For Model 1, a 10% increase in regional patent stocks is associated with a 2.7% increase in factor productivity, composed of a 1.1% direct effect and 1.6% indirect effect. For Model 2, a 10% increase in regional patent stocks would lead to a 3.7% increase in factor productivity in the new steady-state equilibrium. Of this, 2.7% represents indirect effects and less than one percent a direct effect.

To better understand the scalar summary measures of cumulative direct, indirect and total effects over space reported in Table 4, we can carry out a spatial decomposition of the effects estimates following LeSage and Pace (2009). This is based on the profile of marginal indirect effects associated with each order of the matrix \( W \). Note that we can rely on the asymptotic expansion:

\[
[I_M \otimes I_N - \hat{\theta}(I_M \otimes W)]^{-1} = I_M \otimes I_N + \hat{\theta}W + \hat{\theta}^2 W^2 + \hat{\theta}^3 W^3 \ldots
\]

to produce effects estimates for first-order neighbours \( (\tilde{W}) \), second-order neighbours, \( (\tilde{W}^2) \), third-order neighbours \( (\tilde{W}^3) \), etc., which is how the marginal indirect effects associated with each order of the matrix \( \tilde{W}^r \) \( (r = 1, ..., 10) \) were produced. Table 5 shows the marginal indirect effects, which were cumulated (to order \( r=100 \)) to produce the numbers reported in Table 4. The table also reports...
lower and upper 99% credible intervals constructed from the 5,000 retained MCMC draws, allowing us to pass judgement on the statistical significance of the marginal effects estimates.

From the table, we see that the Model 1 indirect (spillover) effects are significantly different from zero beginning with the first-order neighbours where \( \hat{W} = \hat{\theta} \). They decay to less than one-half of the \( r = 2 \) magnitude by \( r = 4 \). There are seven first-order neighbours, and the average number of second-order neighbours in \( \hat{W}^2 \) equals 18, whereas the average number of third-order neighbours in \( \hat{W}^3 \) is 30. The spillover impacts decline rapidly as we move to regions that are ‘neighbours to the first-order neighbours’ (\( \hat{W}^3 \)), and ‘neighbours to the neighbours of the first-order neighbours’ (\( \hat{W}^3 \)), etc., which seems to indicate geographic localization of the productivity effects. From the table we see that Model 1 indirect effects are still positive and significantly different from zero for \( \hat{W}^{10} \), which encompasses around 130 regions on average for our sample. However, given our elasticity interpretation of the impacts, the effects for tenth-order neighbours equal to 0.0029 are not likely to be economically significant in terms of their impact on total factor productivity.

The indirect effects for Model 2 show a large and significant impact when \( r = 2 \), and as in the case of Model 1, there is a rapid decay as we move to higher-order neighbours. For \( r = 4 \), the effects are less than one-half of those for \( r = 2 \).

The direct effect magnitudes are not presented in Table 5 because they die down very quickly to zero. Since these reflect the main diagonal elements of the matrix measuring \( \frac{\partial tfp}{\partial k} \), we note that although the spatial weight matrix \( W \) contains zeros on the main diagonal, the matrices \( W^2, W^3, \ldots \), do not have zero diagonals. This is because a region is a second-order neighbour to itself, which has the implication that even the ‘direct effect’ estimates reflect some spatial feedback in any model that contains spatial lags of the dependent variable. Despite this, the amount of feedback is small for our sample data, as can be seen by the closeness of the direct effect estimates for the two models reported in Table 4 and the parameter estimates for \( \delta_1 \) in Table 3. For example, in Model 1, the coefficient estimate for \( \delta_1 \) is equal to 0.1029 and the direct effect estimate in Table 4 equals 0.1106, with the small difference between these two magnitudes reflecting feedback effects from neighbours. Similarly, we see small magnitudes
separating the estimates for $\delta_1$ from Model 2 in Table 3 and the direct effects estimates reported in Table 4, suggesting very little feedback effect.

Having explained issues related to interpreting the direct, indirect and total effects estimates, we can consider the magnitudes of these estimates from the two models shown in Table 4. The indirect effects or cross-region spillovers from knowledge stocks arising from spatial connectivity of the regions are captured by Model 1 as magnitudes around 1.5 times the direct effects. In contrast, Model 2 that includes technological connectivity between regions increases the knowledge spillover (indirect effects) estimates to nearly triple that of the direct effects. Comparing spatial spillovers from Model 1 with spatial and technological spillovers (indirect effects) arising from Model 2, we see almost a doubling in spillovers (0.16 versus 0.27). These Model 2 indirect effects appear significantly larger than those from Model 1, since the mean for the indirect effects from Model 2 fall outside the 95% interval for the Model 1 indirect effects. From this, we conclude that both spatial as well as technological proximity of regions is important when attempting to measure the impact of knowledge spillovers on regional total factor productivity. Our empirical results suggest that both types of interaction play a role, with a larger role for technological than spatial connectivity. Specifically, we find spatial spillovers (on average, cumulated over all regions) having a magnitude of 1.5 times the direct/own-region impact for a total impact of 2.5, whereas spatial plus technological spillovers (on average, cumulated over all regions) have 3 times the spillover impact leading to a total impact of 4.

A policy implication is that setting spatial spillovers to zero (as is done in ordinary regression models) would lead to a four-fold underestimate (25 percent of the true value) of positive spatial spillover benefits that accrue when cumulating over all other regions. This would of course severely bias any cost/benefit study of programs that target or promote regional knowledge capital accumulation. Further, ignoring/excluding technological dependence (through the use of a spatial regression model alone) would also lead to a less severe (62.5 percent of the true value) underestimate of positive spillover benefits accruing to other regions.

Programs that target specific regions will benefit neighbouring regions and the (cumulative) magnitude of these benefits can be estimated. In addition, we show how a profile of decay in spillovers across neighbouring regions (what we refer to as ‘marginal effects’) can be estimated.
We note that if interest is on benefits for a specific region, the methods described here can be used to produce measurements/estimates for specific regions rather than the scalar summary average over the entire sample. This would involve use of a single row from the matrix of partial derivatives shown in Eq. (25). The main diagonal (row) element from this row measures the direct effect whereas the sum of off-diagonal (row) elements reflects spillovers to other regions (see LeSage and Pace 2009 for additional details). Here again, the spatial profile of benefits falling on individual neighbouring regions could be calculated using the same approach as illustrated in Table 5.

4 Closing remarks

Despite the possible measurement difficulties and reservations with our simple reduced-form regression model framework for assessing the contribution of knowledge capital to total factor productivity, our study has produced a number of interesting empirical results. First, evidence suggests that total factor productivity of a region not only depends on its own knowledge capital (direct impact), but also on other regions’ knowledge capital (indirect effects). Second, direct impacts are important, but knowledge spillover effects are more important. In fact, indirect effects are three times the magnitude of the direct effects. Third, while the beneficial productivity effects from geographically neighbouring knowledge stocks have been established in earlier empirical literature (see Smith 1999, Robbins 2006, Fischer et al. 2009), evidence for the importance of the technological dimension in the spillover-productivity nexus is new. Finally, it is worth noting that indirect productivity effects from knowledge capital arising due to spatial connectivity of the regions are to a substantial degree geographically localized, and this result is consistent with the findings in Fischer et al. (2009).

Diffusion of knowledge takes time, sometimes a considerable period of time. The price paid for the simplicity of our framework is abstraction from any explicit time lag structure for the effects of knowledge capital on regional total factor productivity. Further explorations with disaggregated data and an explicit treatment of the dynamics involved using a space-time panel data methodology to explore the knowledge-productivity nexus would undoubtedly provide additional insights.
Acknowledgements. The authors gratefully acknowledges the grant no. P19025-G11 provided by the Austrian Science Fund (FWF).

References

Table 1 Annual model estimates

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Note: *The Model 2 estimates reported are based on seven nearest spatial neighbours and eight technological neighbours. Determination of the number of neighbours is described in the running text.*
Table 2 Posterior model probabilities for numbers of spatial and technological neighbours

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Table 3 Estimates for models pooled over 1997 to 2002: (a) Model 1 and (b) Model 2

(a) Model 1: \( \tilde{t} = \alpha_0 + \theta \tilde{W} \tilde{t} + \delta_1 \tilde{k} + \delta_2 \tilde{W} \tilde{k} + \varepsilon \)

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(b) Model 2: \( \tilde{t} = \alpha_0 + \theta \tilde{W} \tilde{t} + \delta_1 \tilde{k} + \delta_2 \tilde{W} \tilde{k} + \delta_3 \tilde{T} \tilde{k} + \varepsilon \)

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Table 4  Cumulative direct, indirect and total impact estimates

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Table 5 Marginal knowledge spillover and total impact estimates: (a) Model 1 and (b) Model 2

(a) Model 1

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(a) Model 2

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