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Paper

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Abstract. This paper presents a theoretical growth model that extends the Mankiw-Romer-Weil [MRW] model by accounting for technological interdependence among regional economies. Interdependence is assumed to work through spatial externalities caused by disembodied knowledge diffusion. The transition from theory to econometrics leads to a reduced-form empirical spatial Durbin model specification that explains the variation in regional levels of per worker output at steady state. A system of 198 regions across 22 European countries over the period from 1995 to 2004 is used to empirically test the model. Testing is performed by assessing the importance of cross-region technological interdependence, and measuring direct and indirect (spillover) effects of the MRW determinants on regional output.

Keywords: Economic growth, augmented Mankiw-Romer-Weil model, spatial externalities, spatial Durbin model, European regions

JEL Classification: C31, O18, O47, R11

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1 Introduction

Models of economic growth may be split into two broad categories: neoclassical and endogenous growth models. Neoclassical growth models[1] postulate that physical capital accumulation contributes to the growth in the short-run, but long-run growth is totally determined by technological progress which is exogenous to the models so that there is no explicit role for knowledge and spillovers (Stiroh 2003). In contrast, new growth theory has focused renewed attention on the role of knowledge capital in aggregate economic growth, with a prominent role for knowledge spillovers (see Romer 1986; Grossman and Helpman 1991). Knowledge is inherently non-rival in its use and thus its creation and diffusion most likely leads to spillovers and increasing returns. It is this property of knowledge which is at the centre of endogenous growth models that characteristically treat technological knowledge as completely diffused within an economy[2], and implicitly or explicitly assume that knowledge does not diffuse across economies.

Empirical evidence suggests that technological knowledge spillovers[3] are to a substantial degree geographically localized, in the sense that the productivity effects of knowledge decline with the geographic distance between sender and recipient locations (see Keller 2002, and Fischer et al. 2009). At the same time these studies indicate that there are no good reasons to believe that the flow of technological knowledge stops because it hits national or regional boundaries. The rate at which knowledge diffuses outward from the geographical location in which it is created has important implications for the modelling of technological change and economic growth.

In this paper we consider the role of cross-region technological knowledge spillovers in economic growth and focus on the neoclassical growth model as

[1] Neoclassical growth models are characterized by three central assumptions. *First*, the level of technology is considered as given and, thus, exogenously determined. *Second*, the production function shows constant returns to scale in the production factors for a given, constant level of technology. *Third*, the production factors have diminishing marginal products. This assumption is central to neoclassical growth theory.

[2] There are numerous channels through which knowledge might diffuse. It may be disseminated at conferences, seminars and workshops. It can also be part of the human capital that R&D personnel take with them when changing jobs, or it can be the by-product of mergers and acquisitions, or other forms of interfirm cooperation. It may also be uncovered through reverse engineering and other purposive search processes (Fischer and Varga 2003).

[3] We will use the terms spillovers and externalities in this paper interchangeably, even though they are not synonymous. Knowledge spillovers should be distinguished from rent or pecuniary spillovers that are closely linked to knowledge embodied in traded capital or intermediate goods.

augmented by Mankiw et al. (1992), henceforth called the Mankiw-Romer-Weil (MRW) model. This model has become an important tool for understanding the proximate factors that determine interregional differences in output levels and growth. The objective is to extend the MRW model by explicitly accounting for technological interdependence among the economies, caused by disembodied knowledge diffusion, and to test the implied reduced-form empirical spatial Durbin model (SDM). Testing is performed by assessing the technological interdependence among regions and measuring direct and indirect (spillover) effects of the MRW determinants, in terms of the LeSage and Pace (2009a) approach.

The paper draws on some earlier contributions in different ways. In particular, it models technological progress along the lines suggested by Ertur and Koch (2007), but departs from this work in a number of important directions. *First*, the focus is on an MRW rather than a Solow world of economies in which output is produced from physical capital, human capital and consumption[4]. *Second*, the study shifts attention from countries to regions as a more appropriate arena for analyzing growth processes. *Finally*, the paper makes use of the very rich own- and cross-partial derivatives of the implied empirical spatial Durbin model to quantify the magnitude of direct and indirect (spillover) effects of the MRW determinants and hence to test the model predictions.

With López-Bazo et al. (2004) we share the ambition to extend the MRW model by incorporating spatial externalities, but depart from this study in two major respects. Our focus is on levels rather than on rates of growth. This focus is important because – as Hall and Jones (1999) point out – levels capture the differences in long-run performance which are more directly relevant to welfare as measured by consumption of goods and services. Second, in the light of the recurring criticism in the literature that theoretical models are only loosely connected with empirical evidence (see Levine and Renelt 1991; Durlauf 2001), our study attempts to provide a more explicit and closer link between theory and empirical testing, in an analytical rather than a discursive manner[5].

The remainder of the paper consists of four sections. Section 2 presents the theoretical MRW model that accounts for technological interdependence among the regional economies. Section 3 describes the transition from the reduced-form theoretical model to the spatial econometric model specification along with the relevant methodology for estimating and correctly interpreting the model. In Section 4 we use a system of 198 regions across 22 European countries over the

[4] Economists have long stressed the importance of human capital to the process of economic growth, and ignoring human capital could lead to incorrect conclusions (Mankiw et al. 1992).

[5] In the study by López-Bazo et al. (2004) the predictions of their spatial MRW model are only partially empirically tested, in the sense that the MRW determinants are left out of consideration in the testing exercise.

period from 1995 to 2004, to investigate whether the data support the predictions suggested by the model. Section 5 offers some closing comments.

2 The theoretical model

Consider a world of N regional economies, indexed by i=1, …, N. These economies are similar in that they have the same production possibilities. They differ because of different endowments and allocations. Within a regional economy, all agents are identical. The economies evolve independently in all respects except for technological interdependence.

2.1 The production function and knowledge externalities

Each regional economy is characterized by a (Hicks-neutral) Cobb-Douglas production function, exhibiting constant returns to scale

$$Y_{it} = A_{it}\ K_{it}^{\alpha_K}\ H_{it}^{\alpha_H}\ L_{it}^{1-\alpha_K-\alpha_H} \tag{1}$$

where i denotes the economy and t the time period. Y is output, K the level of reproducible physical capital, H the level of reproducible human capital, L the level of raw labour and A the level of technological knowledge. Moreover, we assume that the same production function applies to physical capital, human capital, and consumption, so that one unit of consumption can be transformed costlessly into either one unit of human capital or one unit of physical capital. The exponents α_K and α_H represent the output elasticities with respect to physical and human capital, respectively. As in Mankiw et al. (1992) we assume $\alpha_K, \alpha_H > 0$ and $\alpha_K + \alpha_H < 1$ which implies that there are decreasing returns to both types of capital.

All variables are supposed to evolve in continuous time. The level of labour in economy i grows at rate n_i. Each economy augments its physical and human capital stocks at constant investment rates, s_i^K and s_i^H respectively, while both stocks depreciate at the same rate δ. This induces capital accumulation equations of the form

$$\dot{K}_{it} = s_i^K\ Y_{it} - \delta K_{it} \tag{2a}$$

$$\dot{H}_{it} = s_i^H\ Y_{it} - \delta H_{it} \tag{2b}$$

where the dots over K_{it} and H_{it} represent the derivatives with respect to time. According to Eqs. (2a)-(2b), the change in the capital stocks of region *i*, $\dot{K}_{it}$ and $\dot{H}_{it}$, is equal to the amount of gross investment, $s_i^K\, Y_{it}$ and $s_i^H\, Y_{it}$ respectively, less the amount of depreciation that occurs during the production process.

The final factor in the production of output is the level of technological knowledge available in region *i* at time *t*. In accordance with Ertur and Koch (2007) we model A_{it} as

$$A_{it} = \Omega_t\ k_{it}^{\theta}\ h_{it}^{\phi} \prod_{j \neq i}^{N} A_{jt}^{\rho W_{ij}}. \tag{3}$$

Several aspects of modelling the aggregate level of technology deserve mentioning. *First*, the term Ω_t should be understood as an expression reflecting the common stock of knowledge in the world of regions. This stock of knowledge is exogenous to the model: $\Omega_t = \Omega_0 \exp(\mu t)$ where μ is its constant rate of growth.

Second, we assume that technology is embodied in physical and human capital per worker and that region's *i* aggregate level of technology increases with both the aggregate level of physical capital per worker, $k_{it} = K_{it} / L_{it}$, and the aggregate level of human capital per worker, $h_{it} = H_{it} / L_{it}$. The associated technical parameters θ with $0 \le \theta < 1$ and ϕ with $0 \le \phi < 1$ reflect spatial connectivity of k_{it} and h_{it} within region *i*, respectively[6].

Finally, we assume non-embodied knowledge to cause the technological progress of region *i* to positively depend on the technological progress of other regions $j \neq i$, for $j = 1, ..., N$. The last term on the right hand side of Eq. (3) formalizes the spatial extent of this dependence by means of so-called spatial weight terms W_{ij} that represent the spatial connectivity between regions *i* and *j*, for $j = 1,...,N$. These terms are assumed to be non-negative, non-stochastic and finite, with the properties $0 \le W_{ij} \le 1$, $W_{ij} = 0$ if $i = j$, and $\sum_{j=1}^{N} W_{ij} = 1$ for $i = 1,...,N$. The parameter ρ with $0 \le \rho < 1$ reflects the degree of regional interdependence[7]. Regions neighbouring region *i* are defined as those regions *j* for which $W_{ij} > 0$. The more a region *i* is connected to region *j*, the higher W_{ij} is, and the more region *i* benefits from knowledge spilling over from region *j*.

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[6] We assume hereby that each unit of capital investment increases not only the stock of capital, but also generates externalities which lead to knowledge spillovers that increase the level of technology for all firms in the region.

[7] Even though ρ is a global parameter characterizing the degree of technological interdependence in the system of regions, it is important noting that the net effect of this dependence on the productivity level of the firms in region *i* depends on the spatial connectivity relationship incorporated in the model (see LeSage and Fischer 2008).

Rewriting the log-version of Eq. (3) in matrix form at time t yields

$$\boldsymbol{A} = \boldsymbol{\Omega} + \theta \boldsymbol{k} + \phi \boldsymbol{h} + \rho \boldsymbol{W}\boldsymbol{A} \tag{4}$$

where $\boldsymbol{A}$ is the N-by-1 vector of the level of knowledge for the N regions, $\boldsymbol{\Omega}$ is the N-by-1 vector of the exogenous part of technology, $\boldsymbol{k}$ and $\boldsymbol{h}$ are the N-by-1 vectors of per worker physical and human capital respectively. $\boldsymbol{W}$ denotes the N-by-N matrix of spatial weights representing the spatial connectivity structure between the N regions. If $\rho \neq 0$ and if ρ^{-1} is not an eigenvalue of $\boldsymbol{W}$, we can resolve Eq. (4) for $\boldsymbol{A}$ (see Ertur and Koch 2007), yielding

$$\boldsymbol{A} = (\boldsymbol{I} - \rho \boldsymbol{W})^{-1} \boldsymbol{\Omega} + \theta (\boldsymbol{I} - \rho \boldsymbol{W})^{-1} \boldsymbol{k} + \phi (\boldsymbol{I} - \rho \boldsymbol{W})^{-1} \boldsymbol{h} \, . \tag{5}$$

Using the Sherman-Morrison formula to develop $(\boldsymbol{I} - \rho \boldsymbol{W})^{-1}$ in its Taylor expansion form and regrouping terms, we get[8] for a region i

$$A_{it} = \Omega_t^{\frac{1}{1-\rho}} \; k_{it}^{\theta} \; h_{it}^{\phi} \prod_{j \neq i}^{N} k_{jt}^{\theta \sum_{r=1}^{\infty} \rho^r (\boldsymbol{W}^r)_{ij}} \; h_{jt}^{\phi \sum_{r=1}^{\infty} \rho^r (\boldsymbol{W}^r)_{ij}} \tag{6}$$

where $(\boldsymbol{W}^r)_{ij}$ denotes the (i, j)th element of $\boldsymbol{W}^r$. Inserting this equation in the per worker production function, given by normalizing Eq. (1) by L_{it}, we obtain the theoretical spatial MRW model[9]

$$y_{it} = \Omega_t^{\frac{1}{1-\rho}} \; k_{it}^{u_{ii}} \; h_{it}^{v_{ii}} \prod_{j \neq i}^{N} k_{jt}^{u_{ij}} \; h_{jt}^{v_{ij}} \tag{7a}$$

$$u_{ii} = \alpha_K + \theta \left(1 + \sum_{r=1}^{\infty} \rho^r \, (\boldsymbol{W}^r)_{ii} \right) \tag{7b}$$

[8] Note that $(\boldsymbol{I} - \rho \boldsymbol{W})^{-1} = \Sigma_{r=0}^{\infty} (\rho \boldsymbol{W})^r = \Sigma_{r=0}^{\infty} \rho^r (\boldsymbol{W}^r)$, $\Sigma_{r=0}^{\infty} \boldsymbol{W}^r$ is row standardized since $\boldsymbol{W}$ is so, $\Sigma_{r=0}^{\infty} \boldsymbol{W}^r \boldsymbol{\Omega} = \boldsymbol{\Omega}$, $\Sigma_{r=0}^{\infty} \rho^r = 1/(1-\rho)$ if $|\rho| < 1$.

[9] It is worth noting that this model would become an endogenous growth model if $\alpha_K + \alpha_H = 1$. Then there are constant returns to scale in the reproducible factors. In this case, there is no steady state for the model. Regions that invest more would grow faster indefinitely.

$$u_{ij} = \theta \sum_{r=1}^{\infty} \rho^r \, (\mathbf{W}^r)_{ij} \quad \text{for } i \neq j \tag{7c}$$

$$v_{ii} = \alpha_H + \phi \left(1 + \sum_{r=1}^{\infty} \rho^r \, (\mathbf{W}^r)_{ii} \right) \tag{7d}$$

$$v_{ij} = \phi \sum_{r=1}^{\infty} \rho^r \, (\mathbf{W}^r)_{ij} \quad \text{for } i \neq j. \tag{7e}$$

Equation (7a) relates per worker output, $y_{it} = Y_{it} / L_{it}$, in region *i* to physical and human capital intensities in the same region and its neighbours *j*, with $j \neq i$. Note that if $\theta = \phi = 0$, then the model collapses to the MRW model with $Y_{it} = \Omega_t \, K_{it}^{\alpha_K} \, H_{it}^{\alpha_H} \, L_{it}^{1-\alpha_K-\alpha_H}$, which is characterized by a world of closed economies.

We can evaluate the social elasticity of output per worker in region *i* at time *t* with respect to both types of capital per worker. From Eqs. (7a) to (7e) it is evident that when region *i* increases its own stocks of per worker physical and human capital, it receives a social return of

$$\frac{\partial y_{it}}{\partial k_{it}} \frac{k_{it}}{y_{it}} + \frac{\partial y_{it}}{\partial h_{it}} \frac{h_{it}}{y_{it}} = u_{ii} + v_{ii} \tag{8a}$$

whereas this return increases to

$$\begin{aligned} &\frac{\partial y_{it}}{\partial k_{it}} \frac{k_{it}}{y_{it}} + \sum_{j \neq i}^{N} \frac{\partial y_{it}}{\partial k_{jt}} \frac{k_{jt}}{y_{it}} + \frac{\partial y_{it}}{\partial h_{it}} \frac{h_{it}}{y_{it}} + \sum_{j \neq i}^{N} \frac{\partial y_{it}}{\partial h_{jt}} \frac{h_{jt}}{y_{it}} = \\ &= u_{ii} + \sum_{j \neq i}^{N} u_{ij} + v_{ii} + \sum_{j \neq i}^{N} v_{ij} \end{aligned} \tag{8b}$$

if all regions simultaneously increase their per worker stocks as well.

2.2 Transitional dynamics and the steady state

The evolution of output per worker in region *i* is governed by the dynamic equations for *k* and *h* given by

$$\dot{k}_{it} = s_i^K \, y_{it} - (n_i + \delta) \, k_{it} \tag{9a}$$

$$\dot{h}_{it} = s_i^H \, y_{it} - (n_i + \delta) \, h_{it} \tag{9b}$$

where s_i^K is the fraction of output in region i invested in physical capital, s_i^H the fraction of output invested in human capital, n_i the rate of population growth, and δ a constant and identical rate of depreciation.

Since the per worker production function given by Eq. (7a) is characterized by decreasing returns to both types of capital, Eqs. (9a) and (9b) imply that per worker output of region i, for i=1, …, N, converges to a steady state[10] defined by

$$y_{it}^* = \Omega_t^{\frac{1}{(1-\rho)(1-u_{ii}-v_{ii})}} \left[\frac{(s_i^K)^{u_{ii}} (s_i^H)^{v_{ii}}}{(n_i + g + \delta)^{u_{ii}+v_{ii}}} \right]^{\frac{1}{1-u_{ii}-v_{ii}}} \prod_{j \neq i}^{N} \left(k_{jt}^{*\,u_{ij}} \; h_{jt}^{*\,v_{ij}} \right)^{\frac{1}{1-u_{ii}-v_{ii}}} \tag{10a}$$

with the balanced growth rate[11]

$$g = \frac{\mu}{(1-\rho)(1-\alpha_K-\alpha_H)-\theta-\phi} \tag{10b}$$

where the asterisk is used to signify the steady state levels for y, k and h. Hence, the physical capital-output and human capital-output ratios of region i, for i=1, …, N, are constant so that

$$\frac{k_{it}^*}{y_{it}^*} = \frac{s_i^K}{n_i + g + \delta} \tag{11a}$$

[10] Note that the balanced growth path is defined as a situation in which (i) per worker physical and human capital grow at the same rate denoted by g, (ii) the exogenous part of technology grows at the constant rate μ, and (iii) the population growth rate and the investment rates for physical and human capital are constant.

[11] This follows from solving $\partial \ln y_{it} / \partial t = \dot{\Omega}_t / \Omega_t + \rho \, \Sigma_{j \neq i} \, (\dot{y}_{jt} / y_{jt}) \; + (\alpha_K + \theta) \; (\dot{k}_{it} / k_{it}) +$ $+(\alpha_H + \phi) \; (\dot{h}_{it} / h_{it}) - \alpha_K \, \rho \Sigma_{j\neq i} W_{ij} \; (\dot{k}_{jt} / k_{jt}) - \alpha_H \, \rho \Sigma_{j \neq i} W_{ij} \; (\dot{h}_{jt} / h_{jt})$ for g at the balanced growth path.

$$\frac{h_{it}^*}{y_{it}^*} = \frac{s_i^H}{n_i + g + \delta}. \tag{11b}$$

Substituting these expressions of capital-output ratios at steady state into the per worker production function in Eq. (10a) and taking the logarithm, gives an equation of the output per worker of region i at steady state:

$$\begin{aligned}\ln y_{it}^* = {} & \frac{1}{1-\eta} \ln \Omega_t + \frac{\alpha_K + \theta}{1-\eta} \ln s_i^K + \frac{\alpha_H + \phi}{1-\eta} \ln s_i^H - \frac{\eta}{1-\eta} \ln (n_i + g + \delta) \\ & - \frac{\alpha_K}{1-\eta} \rho \sum_{j \neq i}^{N} W_{ij} \ln s_j^K - \frac{\alpha_H}{1-\eta} \rho \sum_{j \neq i}^{N} W_{ij} \ln s_j^H + \\ & + \frac{\alpha_K + \alpha_H}{1-\eta} \rho \sum_{j \neq i}^{N} W_{ij} \ln (n_j + g + \delta) + \frac{1-\alpha_K - \alpha_H}{1-\eta} \rho \sum_{j \neq i}^{N} W_{ij} \ln y_{jt}^* \end{aligned} \tag{12}$$

with $\eta = \alpha_K + \alpha_H + \theta + \phi$. It is important to note that Eq. (12) is valid only if the regions are in their steady states or, more generally, deviations from steady state are random.

This equation states – as the MRW model does – that a region will have higher per worker output at a point in time (in the steady state) the higher are its investment rates in physical and human capital, and the lower are its rates of depreciation and population growth. Per worker output of region i, however, depends also on determinants that lie outside MRW's original theory. This output (at steady state) is negatively influenced by physical and human capital investment rates in neighbouring regions j, for $j \neq i$, those identified by $W_{ij} > 0$, and positively influenced by their population growth rates. Output (per worker) of region i also depends on (per worker) steady state levels in neighbouring regions. These output levels $(\ln y_j^*)$ of neighbouring regions in turn depend on the MRW variables, so that changes in explanatory variables will affect the dependent variable $\ln y_i^*$. We note that if $\theta = \phi = \rho = 0$, Eq. (12) reduces to the conventional MRW steady state equation[12].

[12] It is interesting to note that the spatialized MRW steady state equation collapses to the Ertur-Koch model (Ertur and Koch 2007), if $\alpha_H = \phi = 0$.

3 Model specification, estimation and interpretation

Section 3.1 presents the empirical model associated with the reduced-form of the theoretical model given by Eq. (12), along with the relevant estimation approach. Section 3.2 directs attention to correctly interpret the parameter estimates.

3.1 Model specification and estimation

It is easy to see that the empirical counterpart of model (12) can be expressed at a given time (t=0 for simplicity) in the following form for region i

$$\begin{aligned}\ln y_i = \beta_0 + \beta_1 \ln s_i^K + \beta_2 \ln s_i^H + \beta_3 \ln (n_i + g + \delta) + \gamma_1 \sum_{j \neq i}^{N} W_{ij} \ln s_j^K \\ + \gamma_2 \sum_{j \neq i}^{N} W_{ij} \ln s_j^H + \gamma_3 \sum_{j \neq i}^{N} W_{ij} \ln (n_j + g + \delta) + \lambda \sum_{j \neq i}^{N} W_{ij}\, y_j + \varepsilon_i \qquad (13)\end{aligned}$$

where $(1-\eta)^{-1} \ln \Omega_0 = \beta_0 + \varepsilon_i$ for $i = 1, ..., N$, with β_0 a constant and ε_i an independently and identically distributed random variable[13]. Note that we have the following theoretical constraints between coefficients: $\beta_1 + \beta_2 + \beta_3 = 0$ and $\gamma_1 + \gamma_2 + \gamma_3 = 0$, since the theoretical model predicts not only the signs, but also the magnitudes of the coefficients on the MRW variables and their spatial lags.

Rewriting Eq. (13) in matrix form yields

$$\boldsymbol{y} = \boldsymbol{\iota}_N\, \beta_0 + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{W}\, \boldsymbol{X}\, \boldsymbol{\gamma} + \lambda \boldsymbol{W}\, \boldsymbol{y} + \boldsymbol{\varepsilon} \qquad (14)$$

where

- $\boldsymbol{y}$ — N-by-1 vector of observations on the per worker output level for each of the N regions,
- $\boldsymbol{X}$ — N-by-Q matrix of observations on the Q non-constant exogenous variables [here Q=3], including the vectors of the physical and human capital investment rates and the population growth rate for each of the N regions,
- $\boldsymbol{\beta}$ — Q-by-1 vector of the regression parameters associated with the Q non-constant exogenous variables [here: $\boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3)'$],

[13] Following Mankiw et al. (1992) we view the term Ω_0 to reflect not just technology, but also idiosyncratic regional characteristics such as resource endowments, climate, institutions and so on.

$\boldsymbol{WX}$	N-by-Q matrix of the Q spatially lagged non-constant exogenous variables,
$\boldsymbol{\gamma}$	Q-by-1 vector of the regression parameters associated with the Q spatially lagged non-constant exogenous variables [here: $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \gamma_3)'$],
$\boldsymbol{W}\,\boldsymbol{y}$	N-by-1 vector of the dependent spatial lag variable that contains a linear combination of the per worker output levels from neighbouring regions, those identified by $W_{ij} > 0$,
λ	the spatial autocorrelation coefficient, where $\lambda = (1-\alpha_K - \alpha_H)\rho(\eta - 1)^{-1}$,
$\boldsymbol{\iota}_N$	N-by-1 vector of ones with the associated scalar parameter β_0,
$\boldsymbol{\varepsilon}$	N-by-1 vector of errors assumed to be identically and normally distributed with zero mean: $\boldsymbol{\varepsilon} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \boldsymbol{I})$.

All variables are in log form. The variables spanned by $\boldsymbol{X}$ represent the determinants that are suggested by the MRW model, whereas $\boldsymbol{WX}$ represent those that lie outside MRW's original theory, as does $\boldsymbol{W}\,\boldsymbol{y}$ that represents the technological interdependence between the regions and defines the difference to a MRW world of closed regions.

In the spatial econometrics literature, a model specification like Eq. (14) is referred to as a spatial Durbin model. Maximizing the full log likelihood for this model would involve setting the first derivatives with respect to the coefficient vector $\boldsymbol{\delta} = (\beta_0, \boldsymbol{\beta}, \boldsymbol{\gamma})'$ equal to zero and simultaneously solving these first-order conditions for all the parameters. Equivalent ML estimates can be found using the log likelihood function concentrated (with respect to $\boldsymbol{\delta}$ and the noise variance parameter σ^2) which takes the form

$$\ln \mathcal{L}(\lambda) = \frac{N}{2}\ln 2\pi + \ln |\boldsymbol{I} - \lambda \boldsymbol{W}| - \frac{1}{2}\ln (\hat{\boldsymbol{e}}_0 - \lambda \hat{\boldsymbol{e}}_L)'\,(\hat{\boldsymbol{e}}_0 - \lambda \hat{\boldsymbol{e}}_L). \tag{15}$$

The notation $\ln \mathcal{L}(\lambda)$ in this equation indicates that the scalar concentrated log likelihood function value depends on the parameter λ. $\hat{\boldsymbol{e}}_0$ and $\hat{\boldsymbol{e}}_L$ are the estimated residuals in a regression of $\boldsymbol{y}$ on $\boldsymbol{Z}$ and $\boldsymbol{Wy}$ on $\boldsymbol{Z}$, respectively, with $\boldsymbol{Z} = [\boldsymbol{\iota}_N\ \boldsymbol{X}\ \boldsymbol{WX}]$, see LeSage and Pace (2009a) for details.

Optimizing $\ln \mathcal{L}(\lambda)$ with respect to λ permits us to find the ML estimate $\hat{\lambda}$ and to use this estimate in the closed form expressions for $\hat{\boldsymbol{\beta}}(\hat{\lambda})$, $\hat{\boldsymbol{\gamma}}(\hat{\lambda})$ and $\hat{\sigma}^2(\hat{\lambda})$ to produce ML estimates for these parameters. A variety of univariate techniques may be used for optimizing the concentrated log likelihood function. In this study we used the simplex optimization technique.

3.2 Interpreting the model

While linear regression parameters have a straightforward interpretation as the partial derivatives of the dependent variable with respect to the explanatory variables, in the SDM specification given by Eq. (14) interpretation of the parameters becomes more complicated. This comes from introducing regional dependence (on a few neighbouring regions) at the outset in the theoretical model. A logical consequence of the simple dependence on a small number of nearby regions in the initial theoretical specification in Eq. (6) leads to a reduced-form of the model such that each region potentially depends on all other regions (not just the few neighbours that made up our initial model specification). But of course there is a decay of influence as one moves to more distant or less connected regions.

Because of this, partial derivatives take a much more complicated form and allow for measuring direct and indirect (or spatial spillover) impacts. These measure the effect arising from a change in an MRW characteristic variable in region *i* on per worker output in other regions $j \neq i$. Specifically, Eq. (16) shows the partial derivatives that take the form of an *N*-by-*N* matrix

$$\frac{\partial \boldsymbol{y}}{\partial X_q} = (\boldsymbol{I}_N - \lambda \boldsymbol{W})^{-1}(\boldsymbol{I}_N \beta_q + \boldsymbol{W} \gamma_q) \tag{16}$$

where X_q denotes the *q*th MRW characteristic variable, and β_q and γ_q the associated parameter coefficients.

Following LeSage and Pace (2009a) we can actually quantify and summarize the complicated set of non-linear impacts that fall on all regions as a result of changes in the MRW variables in any region, using their scalar summary impact measures for the *N*-by-*N* matrix of direct and cumulative spatial spillover (indirect) impacts. By cumulative we mean that spillovers falling on all neighbours are summed. LeSage and Pace (2009a, b) point out that the main diagonal of the matrix $(\boldsymbol{I}_N - \lambda \boldsymbol{W})^{-1}(\boldsymbol{I}_N \beta_q + \boldsymbol{W} \gamma_q)$ represents own partial derivatives, which they label direct effects, and summarize using an average of these elements of the matrix. The off-diagonal elements correspond to cross-partial derivatives, which can be summarized into scalar measures of the cumulative spillovers using the average of the row-sum of the matrix elements.

To properly interpret the model we rely on LeSage and Pace's (2009a) approach to calculating measures of dispersion to draw inferences regarding the statistical significance of direct or indirect effects. These are based on simulating parameters from the normally distributed parameters β_q, γ_q, λ and σ^2, using the estimated means and variance-covariance matrix. The simulated draws are then used in computationally efficient formulas to calculate the implied distribution of the scalar summary measures.

4 Testing the spatial MRW model

In this section we consider the question whether data for European regions support the predictions suggested by the spatially augmented MRW model. Using the empirical model in Eq. (14), we estimate the direct and indirect effects of the three MRW determinants, and assess the role played by regional technological interdependence in the growth process.

4.1 Sample data and the spatial weight matrix

Our sample is a cross-section of 198 regions belonging to 22 European countries over the 1995-2004 period. The units of observation are the NUTS-2 regions. These regions, though varying in size, are generally considered to be the most appropriate spatial units for modelling and analysis purposes (Fingleton 2001). In most cases, they are sufficiently small to capture subnational variations. But we are aware that NUTS-2 regions are formal rather than functional regions, and their delineation does not represent the boundaries of regional growth processes very well. The choice of the NUTS-2 level might also give rise to a form of the modifiable areal unit problem, well known in geography (see, for example, Getis 2005).

The sample regions include regions located in *Western Europe* as well as in *Eastern Europe*. Western Europe is represented by 159 regions[14] covering Austria (nine regions), Belgium (11 regions), Denmark (one region), Finland (four regions), France (21 regions), Germany (40 regions), Italy (18 regions), Luxembourg (one region), the Netherlands (12 regions), Norway (seven regions), Portugal (five regions), Spain (15 regions), Sweden (eight regions) and Switzerland (seven regions). *Eastern Europe* is covered by 39 regions including the Baltic states (three regions), the Czech Republic (eight regions), Hungary (seven regions), Poland (16 regions), Slovakia (four regions) and Slovenia (one region). The main data source is Eurostat's Regio database[15]. The data for Norway and Switzerland were provided by Statistics Norway and the Swiss Office Fédéral de la Statistique, respectively.

[14] We exclude the Spanish North African territories of Ceuta y Melilla, the Spanish Balearic islands, the Portuguese non-continental territories Azores and Madeira, the French Départements d'Outre-Mer Guadaloupe, Martinique, French Guayana and Réunion, and, moreover, Åland (Finland), Corse, Sardegna and Sicilia from consideration.

[15] The data used for labour stem from the Cambridge Econometrics database.

The data cover the period from 1995 to 2004 when economic recovery in Eastern Europe gathered pace. The time period is relatively short[16] due to a lack of reliable figures for the regions in Eastern Europe (Fischer and Stirböck 2006). The political changes since 1989 have resulted in the emergence of new or re-established states (the Baltic states, the Czech Republic, Slovakia and Slovenia) with only a very short history as sovereign national entities. In most of these states historical data series simply do not exist. Even for states such as Hungary and Poland that existed for much longer time periods in their present boundaries, the quality of data referring to the period of central planning imposes serious limitations on analyzing regional growth. This is closely related to the change in accounting conventions, from the material product balance system to the European system of accounts 1995. Cross-region comparisons require internationally comparable regional data which are not only statistically consistent but also expressed in the same numéraire. The absence of market exchange rates in the former centrally planned economies is a further impediment.

We focus on an output based measure and use gross value added, *gva*, rather than gross regional product at market prices as a proxy for regional output. *gva* is the net result of output at basic prices less intermediate consumption valued at purchasers' prices, and measured in accordance with the European system of accounts 1995. The dependent variable is *gva* divided by the number of workers in 2004[17]. We measure n as the growth rate of the working age population, where working age is defined as 15-64 years, and use gross fixed capital formation per worker as a proxy for physical capital investment. There is no clear-cut definition on how human capital should be represented and measured. In this study we measure human capital in terms of educational attainment[18] based on data for the active population aged 15 years and older that attained the level of tertiary education, as defined by the International Standard Classification of Education (ISCED) 1997 classes five and six. n_i, s_i^K and s_i^H are averages for the period 1995-2003. We suppose[19] that $g+\delta=0.05$, which is a fairly standard assumption in the literature (see among others, Mankiw et al. 1992; Islam 1995; Temple 1998; Durlauf and Johnson 1995; Ertur and Koch 2007; Fingleton and Fischer 2009).

[16] Islam (1995), and Durlauf and Quah (1999) emphasize growth regressions of the type considered in this paper are also valid for shorter time spans since they are steady state regressions.

[17] To implement the model we have been assuming that the regions were in their steady state in 2004 (or more generally, that the deviations from steady state were random).

[18] This variable is clearly imperfect: it completely ignores primary and secondary education, and on-the-job training, and does not account for the quality of education.

[19] There are no strong reasons to expect g and δ to vary greatly across regions, nor are there any data that would allow us to estimate region-specific balanced growth and depreciation rates.

The definition of the spatially lagged variables in the model depends on the specification of the spatial weight matrix that summarizes the spatial connectivity structure between the regions. Different spatial weight matrices may be chosen[20]. In this study, we employ a binary first-order contiguity matrix, constructed on the basis of digital boundary files in a GIS and implemented in row-standardized form in order to assign equal weight to all contiguous neighbouring regions. Two regions are defined as neighbours when they share a common boundary. This choice of the spatial weight matrix is well in line with the empirical evidence that knowledge spillovers and their productivity effects are to a substantial degree localized (see Fischer et al. 2009).

4.2 Estimation results

We begin by briefly considering the ML-parameter estimates and associated implied parameter values from our spatial MRW model version. Table 1 summarizes these estimates along with some diagnostics and performance measures. Diagnostic tests were carried out for heteroskedasticity, using the spatial Breusch-Pagan test[21], and for normality, using the Jarque-Bera test[22]. Performance of the model is expressed in terms of conventional statistical measures of goodness of fit, such as the log likelihood value divided by N, the noise variance sigma square, and R^* defined as the correlation between the fitted and observed values of the dependent variable.

Table 1 about here

The first three columns of the table present the results imposing the theoretical constraints $\beta_1+\beta_2+\beta_3=0$ and $\gamma_1+\gamma_2+\gamma_3=0$ on estimating the model, the final three columns report those without imposing the constraints. The parameter estimates are given in the first and fourth column, followed by the standard errors in the second and fifth, and the p-values in the third and sixth column. The parameters obtained by constrained or unconstrained estimation allow us to calculate output elasticity parameters α_K and α_H, and implied values of the

[20] For extensive reviews see Cliff and Ord (1981), Anselin (1988), Anselin and Bera (1998), and Griffith (1995). The latter provides some guidelines for specifying the weight matrix.

[21] This test points to homogeneity in the unconstrained estimation of the spatially augmented MRW model, but reveals heterogeneity in the constrained estimation.

[22] The Jarque-Bera test indicates a lack of normality. Because of the large sample, the test is very powerful, detecting significant deviations from normality which have, however, little significance in practice.

parameters θ, ϕ and ρ. Standard deviations and *p*-values were computed based on the simulation technique with 10,000 random draws.

The following aspects of the results presented in the table support our spatial MRW model. *First*, the coefficients of all the determinants have the predicted signs and are highly significant. The only exception is the β_3 parameter estimate for population growth that has the correct sign, but is insignificant.

Second, the estimates of the output elasticities implied by the SDM parameter estimates are empirically plausible. The elasticity of output with respect to the stock of physical capital is close to one-third. The implied value of α_H is significant, but smaller by a factor of about two. It is interesting to recognize that the implied parameter values obtained from constrained and unconstrained estimation are strikingly similar.

Third, the spatial autocorrelation λ is positive and significant. The implied value of ρ that measures the degree of technological interdependence among the regions is 0.63 with a standard deviation of 0.07 (*p*=0.00) in the case of unconstrained estimation, and 0.74 with a standard deviation of 0.05 (*p*=0.00) in the constrained case. This result indicates that regions can not be treated as independent observations and hence growth models should explicitly account for this kind of interdependence.

Fourth, a common factor test using likelihood ratios (see LeSage and Pace 2009a for details) rejects the three non-linear restrictions[23]: $\gamma_1+\beta_1\,\rho=0$, $\gamma_2+\beta_2\,\rho=0$ and $\gamma_3+\beta_3\,\rho=0$. The likelihood is 225.61 for the spatially augmented model specification and 192.69 for the MRW model with spatial error terms, based on the binary first-order contiguity matrix and non-constrained estimation. This leads to a difference of 32.92, and this represents a rejection of the spatial error model in favour of the spatial Durbin model specification using the 99 percent critical value for which $\chi^2(3)$ equals 11.34. This result is in accordance with López-Bazo and Fingleton (2006), questioning the credibility of specifications with dependence structures in the error terms.

Finally, differences in the MRW variables and their spatial lags account for a large fraction of the cross-region variation in per worker output. The measure R^* of the overall fit of the model, defined as the correlation between the fitted and observed values of the dependent variable, ranges from 0.949 (constrained estimation) to 0.966 (unconstrained estimation). Nonetheless, the model is not completely successful, since the joint theoretical restrictions between the parameters are rejected by a likelihood ratio test.

As emphasized in Section 3.2, it is necessary to calculate the direct and indirect effects associated with changes in the MRW determinants on regional output to

[23] The model specification with these restrictions is then the so-called constrained SDM, which is formally equivalent to a MRW model with spatial autoregressive errors that may be written in matrix form as $\boldsymbol{y}=\boldsymbol{X}\,\boldsymbol{\gamma}+\boldsymbol{\varepsilon}_{MRW}$ with $\boldsymbol{\varepsilon}_{MRW}=\rho\,\boldsymbol{W}\,\boldsymbol{\varepsilon}_{MRW}+\boldsymbol{\varepsilon}$ where $\boldsymbol{\varepsilon}_{MRW}$ is the same as before if $\theta=0=\phi$.

arrive at a correct interpretation of the model, in terms of the LeSage and Pace (2009a) approach. Table 2 presents the corresponding impact estimates, along with their associated statistics. A comparison of the direct impact estimates in Table 2 and the SDM coefficient estimates in Table 1 shows that these two sets of estimates are not so dissimilar in magnitude. The direct impact estimate of the human capital variable is slightly lower, while that of the physical capital variable is somewhat larger than one would infer from the SDM coefficient estimates (unconstrained estimation). The difference between these estimates is due to feedback effects.

Table 2 about here

The indirect impact estimates are what economists usually refer to as spatial spillovers. The presence or absence of significant cross-region spillovers depends on whether the indirect effects that arise from changing region *i*'s MRW variables result in statistically significant indirect effects. We emphasize that it would be a mistake to interpret the SDM coefficient estimates γ_q $(q=1, ...,3)$ as representing spatial spillover magnitudes.

To see how incorrect this is, consider the difference between the spatial lag coefficient γ_2 for the human capital investment rate from the SDM model (reported in Table 1) and the indirect impact estimate calculated from the partial derivatives of the model (reported in Table 2). We see that the SDM coefficient associated with the spatial lag variable $W \ln s_j^H$ is -0.135 with *p*=0.001 and a standard deviation of 0.041. The indirect impact is -0.092, but not significantly different from zero, based on the *t*-statistic $(t=-1.08\%)$. If we would incorrectly view the SDM coefficient estimate γ_2 on the spatial lag of $\ln s_j^H$ as reflecting the indirect impact, this would lead to an inference that the human capital variable in neighbouring regions exerts a negative and significant indirect impact on regional output. However, the true impact estimate points to the absence of cross-region human capital spillovers.

The SDM coefficient associated with the spatial lag variable $W \ln s_j^K$ is -0.2768 (standard deviation: 0.062) and statistically significant (*p*=0.000). If we would incorrectly view this SDM coefficient estimate as reflecting the indirect impact, this would lead to an inference that the physical capital variable in neighbouring regions exerts a negative and significant indirect impact on regional output. But, the true impact estimate indicates a positive and significant spillover impact arising from changes in the physical capital variable (see Table 2).

It is also the case that treating the sum of the SDM coefficient estimates from the MRW determinants and their spatial lags as total impact estimates would lead to erroneous results. The total impact of physical capital accumulation on regional output is a positive 0.845 (standard deviation: 0.062) that is statistically significant based on the *t*-statistic (*t*=13.568), whereas the total impact suggested by summing up the SDM coefficients would equal to one third of this magnitude only. This difference is due to the size of indirect impacts which can not be correctly inferred from the SDM coefficient. In the case of the human capital variable, where the indirect impact is zero, and the SDM estimate $(\gamma_2 = 0.154)$ is close to the direct impact estimate of 0.146, the total impact could be inferred from the SDM coefficient without major error.

Since our empirical model is specified by using a log-transformation of both the dependent and independent variables the total impact estimates can be interpreted as elasticities. Based on the positive 0.845 estimate for the total impact of the physical capital determinant, we would conclude that a 10 percent increase in regional physical capital investment would result in a 8.5 percent (and significant) increase in regional output. Around two thirds of this impact comes from the direct effect magnitude of 0.58, and one third from the indirect or spatial spillover impact based on its scalar impact estimate of 0.2653.

5 Closing comments

In this paper, we have suggested a spatially augmented MRW model for explaining interregional differences in output per worker. Output is produced from physical capital, human capital, and labour, and used for investment in physical capital, investment in human capital and consumption. The economies evolve independently in all respects except for technological interdependence. Technological interdependence is assumed to work through spatial externalities caused by disembodied knowledge diffusion.

The theoretical model and the associated reduced-form empirical SDM model both imply a non-independent relationship between changes in region *j*'s physical and human capital or population growth rates and region *i*. A correct interpretation of the model parameters, in terms of the LeSage and Pace (2009a, b) approach, indicates that the model is consistent with the empirical evidence on cross-region technological knowledge spillovers. Interdependence among regions works through physical capital externalities. The results indicate the existence of cross-region physical capital, but the absence of such human capital externalities. This does not imply, however, that the role of human capital is unimportant. Even using an imprecise proxy for human capital, we find that human capital investment is important. A 10 percent increase in human capital investment would lead to a 1.5 percent increase in regional output and this increase is statistically significant.

Our model rests on the existence of a geographic component to the disembodied knowledge spillover mechanism. Conventional wisdom that geographic distance attenuates spillovers supports this assumption. Regardless of geographic proximity, knowledge spillovers are also believed to be higher between regions with similar technological profiles (see, for example, Fischer et al. 2006). According to this view, the ability to make productive use of another region's knowledge depends on the degree of technological similarity between regions. One avenue for future research would be to explore the importance of the technological dimension to the spillover mechanism in regional growth processes.

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Table 1 The spatial MRW model: Constrained and unconstrained estimation results

	Constrained estimation			Unconstrained estimation		
	Coeffic.	Standard deviation	*p*-value	Coeffic.	Standard deviation	*p*-value
Constant	2.7900	0.5659	0.0000	5.9666	1.0509	0.0000
$\ln s_i^K$	–	–	–	0.5582	0.0377	0.0000
$\ln s_i^H$	–	–	–	0.1535	0.0319	0.0000
$\ln(n_i + 0.05)$	–	–	–	-0.0873	0.1001	0.3829
$\ln s_i^K - \ln(n_i + 0.05)$	0.5421	0.0411	0.0000	–	–	–
$\ln s_i^H - \ln(n_i + 0.05)$	0.1180	0.0341	0.0005	–	–	–
$W \ln s_j^K$	–	–	–	-0.2768	0.0624	0.0000
$W \ln s_j^H$	–	–	–	-0.1353	0.0409	0.0009
$W \ln(n_j + 0.05)$	–	–	–	0.4387	0.1701	0.0099
$W[\ln s_j^K - \ln(n_j + 0.05)]$	-0.3196	0.0602	0.0000	–	–	–
$W[\ln s_j^H - \ln(n_j + 0.05)]$	-0.1248	0.0431	0.0038	–	–	–
$W \ln y_j$	0.7770	0.0456	0.0000	0.6670	0.0584	0.0000
Implied parameters						
α_K	0.2604	0.0357	0.0000	0.2548	0.0394	0.0000
α_H	0.1018	0.0342	0.0029	0.1257	0.0373	0.0007
θ	0.0659	0.0238	0.0056	0.0714	0.0288	0.0133
ϕ	-0.0310	0.0245	0.2063	-0.0361	0.0276	0.1908
ρ	0.7361	0.0548	0.0000	0.6307	0.0660	0.0000
ξ	0.4203	0.0157	0.0000	0.4404	0.0143	0.0000
Test of restrictions (LR)	–	–		46.3978	(p=0.0000)	
Common factor test (LR)	63.8451	(p=0.0000)		29.6094	(p=0.0000)	
Diagnostics						
Heteroskedasticity	12.6332	(p=0.0132)		7.7210	(p=0.2593)	
Normality	231.2504	(p=0.0010)		23.8075	(p=0.0016)	
Performance measures						
Log likelihood/N	0.9308			1.0479		
Sigma square	0.0152			0.0128		
R^*	0.9493			0.9660		

Notes: The rates s^K, s^H and n are averages over the period 1995-2003; LR denotes likelihood ratio; $\xi = \alpha_K + \alpha_H + (\theta + \phi)(1 - \alpha_K - \alpha_H - \theta - \phi)^{-1}$; standard errors and p-values of the implied values of α_K, α_H, θ, ϕ, ρ and ξ are calculated using a simulation method (10,000 random draws); heteroskedasticity is tested using the studentized spatial Breusch-Pagan test, and normality using the Jarque-Bera test; R^* is a measure of the overall fit of the model, defined as the correlation between the fitted and observed values of the dependent variable.

Table 2 The spatial MRW (unrestricted estimation): Direct and indirect impact estimates

	Impact estimates		
	Coeffic.	Standard deviation	p-value
Direct impacts			
$\ln s_i^K$	0.5801	0.0355	0.0000
$\ln s_i^H$	0.1463	0.0309	0.0000
$\ln(n_i + 0.05)$	-0.0022	0.0988	0.9826
Indirect impacts [=*spatial spillovers*]			
$W \ln s_j^K$	0.2653	0.0680	0.0001
$W \ln s_j^H$	-0.0919	0.0849	0.2791
$W \ln(n_j + 0.05)$	1.0717	0.4268	0.0120

Notes: The direct and indirect impact estimates reflect an average of diagonal and off-diagonal elements of $(\boldsymbol{I}_N - \lambda \boldsymbol{W})^{-1}(\boldsymbol{I}_N \beta_q + \boldsymbol{W} \gamma_q)$ which corresponds to scalar summary measures of the own and cross-partial derivatives. A set of 10,000 random draws from estimation was used to construct standard deviations and p-values for these impact estimates.