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**Using Non-technological Factors to Explain Changes in  
Unemployment**

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## Abstract

The main research question of this dissertation is whether factors other than labor productivity can help to explain short-run fluctuations and medium-run trends in unemployment in Austria respectively Western Continental Europe.

In the part on analyzing short-term-fluctuations I will set up a New Keynesian DSGE model with a richly specified labor market. This model will be used to compare how different labor market specifications fit to Austrian quarterly data. Most importantly, the Bayesian model comparison indicates an important role for nominal wage rigidities and for a timely response of employment to changes in vacancies. Furthermore, models with consensual determination of working hours ('efficient bargaining') tend to perform relatively well. The best model can reproduce the relative volatility of labor market tightness compared to labor productivity comparatively well. Moreover, shock decompositions show that fluctuations in Austrian labor market tightness are mainly driven by demand shocks and to a much smaller extent by productivity shocks.

In the part on explaining medium-term-trends in unemployment I will set up a theoretical model and demonstrate that certain stylized facts can also be generated by an increase in international trade (and not only by skill-biased technological change). Furthermore I will show that a differential response of different industrial economies ('US' versus 'Continental Europe') might be due to characteristics of sectors which are not directly exposed to globalization.

## Abstract

Die zentrale Forschungsfrage dieser Dissertation ist, ob andere Faktoren als Entwicklungen in der Arbeitsproduktivität helfen können, kurzfristige Fluktuationen und mittelfristige Trends der Arbeitslosenquote in Österreich beziehungsweise Westeuropa zu erklären.

Im Teil über kurzfristige Fluktuationen werde ich ein neukeynesianisches DSGE-Modell aufsetzen, in dem der Arbeitsmarkt ausführlich spezifiziert wird. Dieses Modell wird dann verwendet, um unterschiedliche Arbeitsmarktspezifikationen bezüglich ihres Fits mit österreichischen Quartalsdaten zu vergleichen. Der bayesianische Modellvergleich zeigt eine große Bedeutung von nominellen Lohnrigiditäten und von einer schnellen Reaktion der Beschäftigung auf Änderungen in der Anzahl der offenen Stellen auf. Außerdem werden in den besten Spezifikationen die Arbeitsstunden konsensual zwischen Arbeitgeber und Arbeitnehmer bestimmt. Das beste Modell kann die relative Volatilität der inversen Stellenandrangsziffer im Vergleich zur Arbeitsproduktivität gut reproduzieren. Zudem zeigen Schockzerlegungen an, dass die Fluktuationen der inversen Stellenandrangsziffer vor allem von Nachfrageschocks generiert werden und Produktivitätsschocks eine weitaus geringere Rolle spielen.

Im Teil zur Erklärung mittelfristiger Trends in der Arbeitslosigkeit werde ich ein theoretisches Modell spezifizieren und damit demonstrieren, dass gewisse stilisierte Fakten auch durch internationalen Handel (und nicht nur durch verzerrenden technischen Fortschritt) generiert werden können. Zudem werde ich zeigen, dass die unterschiedliche Reaktion der Arbeitsmärkte verschiedener Volkswirtschaften ('USA' versus 'Kontinentaleuropa') auch durch unterschiedliche Charakteristika jener Sektoren erklärt werden könnten, die der Globalisierung nicht direkt ausgesetzt sind.

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# Chapter 1

## Introduction

### 1.1 Modelling unemployment in macroeconomics

The rate of unemployment is one of the most important macroeconomic variables (together with GDP and the CPI) and a vast amount of economic literature is dedicated to explaining its existence and analyzing differences in unemployment rates across time, space and cohorts.

When one wants to model involuntary unemployment in a macroeconomic<sup>1</sup> setting, one needs a deviation from the neoclassical benchmark case of frictionless markets. In frictionless markets there would be only voluntary unemployment; e.g. people who are able to work but do not want to at the market wage rate.

The currently most popular kind of explaining short- and medium-run changes of unemployment in macroeconomics are so-called 'search models' (or 'search and matching models'); the probably most important references for search unemployment in general are the paper by Mortensen and Pissarides (1994) and the book by Pissarides (2000).

The idea of this approach is as follows: Each period a certain number of jobs is destroyed. Firms post vacancies (for which they usually have to pay a certain fee) and the unemployed search for jobs (in some models there is also the possibility of costly on-the-job-search).

A matching function determines the number of newly created jobs for given numbers of vacancies and job searchers. If – which is the case in most papers in this literature – the matching function has constant returns to scale, then the probabilities of filling a vacancy and of finding a job is determined by the so-called 'labor market tightness' which is the number of vacancies divided by the number of searchers (which often are, but need not be, identical to the number of unemployed). Typically there will be some time for an average labor force participant between losing a job and finding a new one. This implies that there is a positive unemployment rate at all points of a whole business cycle. Unemployment of this kind is often called 'frictional unemployment' and the involved friction is usually referred to as 'search friction'.

The probability of finding a job in a given period (and so the unemployment rate) depends on aggregate conditions of the economy (like the level of productivity), fluctuates over the business cycle and might change permanently after certain (permanent) institutional changes or shocks. The rate of job destruction (fraction of destroyed jobs in a given period) could depend on

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<sup>1</sup>This stands in contrast to explanations for why certain individuals are (more likely to be) unemployed, where factors like home region, (the level and type of) education or age might play a role.

macroeconomic conditions (and economic policy) too or might as well be exogenous.

Two of the most important reasons for the attractiveness of search models are that a law of motion for unemployment comes out naturally (which is a very useful feature for analyzing short-run fluctuations of labor market variables) and that several of its relevant parameters have observable counterparts (unemployment benefits, job finding probabilities ...). Furthermore, in certain cases the before mentioned labor market tightness is the only state variable.

Other approaches to model unemployment emphasize incentive problems (efficiency wage theory of unemployment) or certain institutional factors (like minimum wages, unionized wage bargaining, hiring and firing laws, ...). They are typically used to explain why there is unemployment at all and possibly to discuss certain efficiency aspects like the (in)efficiency of certain labor market regulations/institutions which could lead to involuntary unemployment in otherwise frictionless markets, or considerations why it might be optimal for employers to pay more than the reservation wage (efficiency wage theory).<sup>2</sup> To some extent they are also applied for cross-country-comparisons or explaining medium-run-trends, for example for comparing the differing labor market performances of Continental Europe and the US from the early 1980s to the mid-2000s.<sup>3</sup>

## 1.2 Short-run-fluctuations of unemployment

As said before, one advantage of search unemployment is that there is an explicit law of motion for unemployment (or employment) which comes up naturally when formulating a model. This aspect makes it very attractive for the analysis of short-run-fluctuations of unemployment (and related variables like employment or GDP). A typical analysis in this field sets up a search model with 'some special features', calibrates it, defines certain shocks and then compares the implied moments with empirical data.

### 1.2.1 State of the field

Important stylized facts in this context are that the standard deviation of labor market tightness is typically relatively high compared to the one of labor productivity and that the correlation between unemployment and vacancies is strongly negative (see for example Shimer, 2005, for the case of the US).

Most of the contributions in this literature are in the Real-Business-Cycle (RBC) tradition where the only source of aggregate shocks is productivity. The starting point for this literature is Mortensen and Pissarides (1994), whose framework has been widely used since. Another seminal contribution in this context has been made by den Haan et al. (2000) who showed how including endogenous job destruction in a standard RBC model with search unemployment can help match certain other stylized facts like the high persistence of the response of employment and output to a temporary productivity shock.

Starting with Walsh (2005), dozens of papers have come out which integrate New-Keynesian features, namely nominal (most prominently sticky prices) and additional real frictions (imperfect

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<sup>2</sup>See Cahuc and Zylberberg (2004) for detailed discussions of these various explanations for unemployment in macroeconomic models.

<sup>3</sup>See chapters 8 and 13 in Borjas (2005) or chapter 10 in Cahuc and Zylberberg (2004).

competition, adjustment costs of investment ...), into models of search unemployment.<sup>4</sup> In this kind of models explanations for changes in unemployment typically do not solely rely on technological factors but also on 'exogenous' changes in monetary policy, in preferences, in the extent of competition in product markets, and so on. From now on I will simply refer to them as DSGE (Dynamic Stochastic General Equilibrium) models.<sup>5</sup>

At the same time, papers like Walsh (2005) can also be thought of as part of the vast literature extending the simple monetary three-equations-New-Keynesian-model.<sup>6</sup> Notable contributions to this literature include for example Christiano et al. (2005) and Smets and Wouters (2003). The latter were the first to estimate a medium scale DSGE model with Bayesian inference methods and started a wave of other estimated medium-to-large-sized DSGE models (see for example Christiano et al. (2011), Christoffel et al. (2008) and Fenz et al. (2012)). So far, the main focus of this literature has been on better understanding the transmission of monetary policy on inflation and the real economy and to match certain stylized facts in this context (like the persistent response of GDP and inflation to a monetary policy shock).<sup>7</sup>

Among others, there are two substantial problems in the literature mentioned above.

First, Shimer (2005) claims that models of search unemployment where productivity shocks are the only source of economic fluctuations (RBC models) are incapable of accounting for the high relative volatility of labor market tightness compared to labor productivity.<sup>8</sup>

One possibility to overcome this problem would be to find other sources of labor market fluctuations which do not have such a strong influence on labor productivity. This could be done – still consistent with a typical RBC-setting – via productivity shocks which do not have a strong immediate influence on labor productivity. For example, Faccini and Ortigueira (2010) show that investment-specific technology shocks can help increase the relative volatility of the unemployment rate compared to labor productivity.

One could also try to incorporate different real and monetary shocks on the demand side in a New Keynesian DSGE setting. However, only very few of the contributions in this field try to explain short-term fluctuations of unemployment, the main focus seems to be on the implications of search unemployment for fluctuations of inflation and output in the context of monetary policy shocks.<sup>9</sup> Notable exceptions to that rule are the calibrated models of Sveen and Weinke (2008) and Christoffel and Kuester (2008), who claim that demand and monetary policy shocks can contribute to explaining US unemployment fluctuations when choosing certain labor market specifications.

Second, the disclaimer 'when choosing certain labor market specifications' in the last sentence

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<sup>4</sup>Other early contributions in this context were the working paper version of Trigari (2009) and Christoffel and Linzert (2005).

<sup>5</sup>For a nice overview over the conflicts between the New-Classical and the New-Keynesian school see Snowdon and Vane (2005) and Mankiw (2006). The latter also discusses the combination of these two schools into what is sometimes called 'the new neoclassical synthesis'; Walsh (2005) and most papers on short-run-fluctuations mentioned below fall under this category.

<sup>6</sup>These 3 equations are a forward looking consumption Euler equation (which is occasionally called 'New Keynesian IS curve'), a New Keynesian Phillips curve describing the relationship between inflation and real marginal costs (or the output gap) and a Taylor Rule for monetary policy. A detailed description of this baseline is provided in chapter 5 of Walsh (2003).

<sup>7</sup>This also explains why a large part of the work in this field has been done by central banks.

<sup>8</sup>For an alternative discussion of this problem see Costain and Reiter (2008). For discussions of possible alternative modelling approaches to get a higher relative volatility when having only productivity shocks see for example Hagedorn and Manovskii (2008) and Mortensen and Nagypal (2007).

<sup>9</sup>An important example for this fact is the survey of Christoffel et al. (2009), who compare how different labor market specifications can contribute to a persistent response of inflation to a monetary policy shock (which is a standard VAR result for the euro area).

already indicates another major issue: In the literature on search frictions in New Keynesian DSGE models there is a significant variation in specifications of the labor market along several dimensions. Among others these concern the timing of hiring, (the degree of) wage rigidities, the degree of convexity of hiring costs, and the determination of job destruction and working hours.<sup>10</sup> They all have different implications for the co-movement of observable macroeconomic variables (obviously especially for labor market variables):

- A higher degree of wage rigidity tends to – if it also concerns newly formed matches – lead to a relatively stronger response of employment to demand and supply shocks. In the New Keynesian literature there are also different approaches concerning whether the real (for example in Sveen and Weinke, 2008) or the nominal wage (like in Gertler et al., 2008) is rigid.
- The way of determination of working hours is crucial for the role of the extensive (employment in persons) versus the intensive (hours worked per person) margin of employment when firms react to changes in economic conditions. In the literature, there are the possibilities of choosing hours via maximizing the surplus of the match ('efficient bargaining') or of the employer deciding on the amount of working time ('right-to-manage') by profit maximization. Typically the first option implies a relatively lower volatility of working hours.
- When job destruction becomes endogenous,<sup>11</sup> it means on the one hand a stronger reaction of unemployment to certain shocks: In this case an upswing not only means a higher probability of finding a job for the unemployed, but also a lower probability of losing a job for the employed. On the other hand, endogenous job destruction tends to induce a positive correlation between unemployment and vacancies, which is at odds with empirical evidence. The latter point has been extensively discussed in the RBC literature on search unemployment (see for example Ramey, 2008).
- The timing of hiring, namely whether newly hired workers can start working in the same or only in the next period, is crucial for the speed of reaction to shocks of the extensive margin opposed to the intensive one. In the latter case, firms can adjust employment in  $t$  after a shock in  $t$  only along the intensive margin (working hours), while they can raise employment in persons only in  $t + 1$ . In the other case, firms can immediately adjust along both margins.
- Convex costs of posting vacancies can lead to a relatively smaller response of the number of vacancies to demand and supply shocks (compared to the standard specification of linear costs).

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<sup>10</sup>Other dimensions along which labor market specifications differ in the literature (see for example Christoffel et al., 2009) include for example whether on-the-job-search is possible. Such additional alternatives will not be further discussed in this dissertation.

<sup>11</sup>Job destruction is usually made endogenous via assuming that matches are dissolved when the expected utility of continuing the match is lower than that of the outside option (posting a vacancy or being unemployed).

## 1.2.2 Dissertation I: Explaining short-run-fluctuations of unemployment in Austria

### Dissertation Ia: Model comparison as prerequisite

In the part on short-term-fluctuations I will extend the DSGE model of Fenz et al. (2012), which describes a small open economy without own monetary policy (building on the seminal contribution by Smets and Wouters, 2003). There I will replace the simple labor market which is modeled in the style of Erceg et al. (2000)<sup>12</sup> by a richly specified labor market with search unemployment and endogenous working hours.

The main distinctive feature of this part of my dissertation will be an empirical comparison of different possibilities to model the labor market in a DSGE model.<sup>13</sup> For this I will use Austrian data and compare several models which differ only in their labor market specification along the before mentioned dimensions:

- determination of working hours,
- rigidity of wages,
- the timing of hiring,
- degree of convexity in vacancy posting costs, and
- whether job destruction is exogenous or endogenous.

I will use quarterly data from 1995 on, so there will be close to 70 observations. 1995 is the starting point of a reliable data series for working hours, which are a crucial observable variable in the context of this part of the dissertation. The model comparison will be conducted using system estimation by Bayesian methods.

Comparisons of specifications in this context have been conducted before, but they were either not comprehensive (like Christiano et al., 2009) or they did not look at the overall fit with macroeconomic data (like Christoffel et al., 2009, who compared different specifications with regard to their implications for the response of inflation to a monetary policy shock).

This part of the dissertation will be presented in chapter 2.

### Dissertation Ib: Assessing the impact of demand shocks on Austrian unemployment

Furthermore, I will analyze how well the model with the best information criterion can reproduce certain stylized facts of the Austrian labor market and I will try to investigate which kind of shocks (productivity shocks, mark-up shocks, domestic or foreign demand shocks or monetary policy shocks) contributed most to fluctuations in Austrian unemployment. The latter will be done via a forecast error variance decomposition and a historical shock decomposition (both are byproducts of the Bayesian estimation). These decompositions show which shocks (i.e. which parts of the vector  $\epsilon_t$ ) can be attributed to the observed changes in unemployment (given the model setting). Hereby, I will contribute to the literature which investigates the ability of search

<sup>12</sup>In their widely used modelling approach labor is heterogeneous; the different labor inputs are combined via a CES aggregator and workers are in monopolistic competition to each other. In this kind of setting adjustment of labor inputs is only at the intensive margin (working hours) and (partial) unemployment is of a voluntary nature.

<sup>13</sup>Christoffel et al. (2009) also provide a fairly detailed discussion of different labor market specifications. However, they do not estimate their different models; they only compare the impulse response of inflation to a monetary policy shock.

models to generate certain stylized facts of labor markets and complement contributions like Sveen and Weinke (2008) and Christoffel and Kuester (2008).

This part of the dissertation will be presented in section 2.6 of chapter 2.

### 1.3 Medium-term trends in unemployment

Search models are also usable for explaining medium-term trends in unemployment as important institutional features are easy to integrate (like firing costs) or even show up 'automatically' (unemployment benefits, hiring costs) in such models (this can be seen for example in the application of Mortensen and Pissarides, 1999, where the impact of different labor market regulations is analyzed).

In principle, medium term changes in unemployment patterns can be explained by

1. changes in one or more institutional features for an unchanged macroeconomic environment, or
2. imperfect adjustment of one or more institutional features to changes in the macroeconomic environment (skill biased technological change, globalization ...).

#### 1.3.1 State of the field

The probably most important medium-term trend in labor markets from the early 1980s until the mid-2000s is the relative increase in unemployment rates of low skilled<sup>14</sup> and the decrease of their relative wages, where in the US the decrease in relative wages was more pronounced while in Western Europe the increase in the relative unemployment rate was stronger.<sup>15</sup>

This naturally raises two questions:

1. Which factors contributed to the losses of unskilled labor in Western Europe and the US?
2. Which factors can be made responsible for the differential response of (Continental) Western Europe and the US?

Concerning the first question, the different (but partially interconnected) possible reasons which are discussed most prominently in the literature are: migration, institutional changes, international trade and (skill-)biased technological progress (see for example Cahuc and Zylberberg, 2004, chapter 10).

Like in the literature on short-term-fluctuations, here again technology seems to be the most popular explanation for observed trends in labor markets; there are also several empirical studies claiming a dominant role for skill-biased technological progress. However, a comparatively smaller role of international trade is usually acknowledged too.<sup>16</sup> Contributions to the literature on the effects of trade on unskilled unemployment include Krugman (1995), Davis (1998), Sener (2003), Moore and Ranjan (2005), Egger and Kreckemeier (2008) and Keuschnigg and Ribi (2009), where the latter three papers use search unemployment.

<sup>14</sup>The use of the words 'skilled' and 'unskilled' in this paper is not meant to indicate that people without a university (or high school) degree are less able than people who have a diploma; it is just following a convention in the literature which refers to worse prospects at the job market.

<sup>15</sup>See for example chapter 13 in Borjas (2005) or chapter 10 in Cahuc and Zylberberg (2004).

<sup>16</sup>There is also the reasoning that both factors are closely interrelated as for example globalization might enforce (skill-biased) technological progress (see again Cahuc and Zylberberg, 2004, chapter 10).

One common argument (for example made by Moore and Ranjan, 2005) against a strong contribution of international trade to the relative decrease in unskilled wages and employment is the following: Relative prices of skilled goods and relative employment in skill-intensive sectors did not increase too much over the last decades which goes contrary to predictions of the Heckscher-Ohlin-model for the effects of opening up trade with (unskilled-)labor-abundant ('Southern') countries.

The answer of authors like Feenstra and Hanson (2001)<sup>17</sup> is that one has to look at relative employment shares and prices of different fragments/subsectors instead of more aggregated sectors – when doing this, the fit with the theoretical implications of the Heckscher-Ohlin-model tends to become much better.

This is one substantial argument for the theoretical and empirical importance of trade in intermediate goods (see also OECD, 2007). Another relevant aspect of international outsourcing is the so-called scale effect – namely that relative productivity of outsourcing sectors increases which may even lead to an overall increase of employment in this sector (compared to the pre-outsourcing situation). These effects are empirically very hard to distinguish from the ones of skill-biased-technological change.<sup>18</sup>

Concerning the second question, most of the literature refers to factors like relatively higher unemployment benefits, employment protection and union power in Continental Europe (see for example Cahuc and Zylberberg (2004), Pierrard and Sneessens (2008) and Mortensen and Pissarides (1999)). All 3 factors are thought to lead to a relatively smaller reaction of wages to a negative shock, making a stronger reaction of employment 'necessary'.<sup>19</sup>

### 1.3.2 Dissertation II: The role of international fragmentation

In the part on explaining medium-term-trends I will show that the observed relative loss of unskilled labor in industrialized countries can also be generated by international trade and that the differential response of the US (decrease in relative wages) and Continental Europe (increase in unemployment) could be generated by certain characteristics of non-trading sectors (and not necessarily by differences in unemployment benefits or employment protection).

While a few papers in the literature (for example Egger and Kreickemeier, 2008) look explicitly at the effect of international outsourcing (and not at trade in final goods) on unskilled unemployment, these papers do not account for the role of non-trading sectors which can be of crucial importance in this context. These sectors tend to be dominated by consumer services which are presumably relatively intensive in unskilled labor compared to manufacturing. When the share of unskilled labor used in manufacturing decreases after an increase in international fragmentation, there will be a shift of part of the unskilled labor force from manufacturing to the non-trading sector. If the 'ability to absorb labor' of the non-trading sector differed across industrial economies,<sup>20</sup> it could be one of the causes of the differential response to an increase in international trade.

In this part of the dissertation I will set up a model which shows the potential importance

<sup>17</sup>This paper provides a much longer discussion on these arguments. In addition it contains a very detailed empirical analysis of possible effects of outsourcing on the share of low skilled in the overall wage bill (with mixed results).

<sup>18</sup>Grossman and Rossi-Hansberg (2008) show in a theoretical framework that – under certain assumptions – these productivity effects can be so large that the factor affected by outsourcing can even benefit from it.

<sup>19</sup>It should be noted, however, that the effects of employment protection on the response of unemployment to a negative shock can be ambiguous (see for example Mortensen and Pissarides, 1999).

<sup>20</sup>For example, there might be a trade-off between the probability of finding a job and the wage rate in the different sectors.

of international outsourcing and non-trading sectors for explaining the observed increase in unskilled unemployment over the last decades, where the main distinctive feature will be the emphasis on the interaction of outsourcing and the non-trading sectors. Hereby I will complement contributions like Pierrard and Sneessens (2008) who claim that these stylized facts are driven by an interaction of skill-biased technological change and wage rigidities. The model integrates elements of the Heckscher-Ohlin-setting of Feenstra and Hanson (1996) with a skill-abundant 'North' and an unskilled-labor-abundant 'South' into a typical model of search unemployment (which will be relatively similar to Mitra and Ranjan, 2010).

Within this framework, I will show that relocation of parts of the value added chain in manufacturing (due to a relative increase in Southern productivity) makes Northern production more skill-intensive. This causes a shift of part of the unskilled labor force to the non-trading sectors which are assumed to be highly intensive in this factor. Furthermore sectoral unemployment rates of the low skilled will increase. The overall effect on unskilled unemployment will depend heavily on the characteristics of the non-trading sectors, meaning that they are a potential reason for a differential response of different Northern economies to the increase in outsourcing to Southern economies (instead of the usual suspects like unemployment benefits or employment protection in the manufacturing sector).

The model will be formulated in continuous time and will include 2 types of labor (skilled and unskilled), 2 economies (unskilled-labor-abundant 'South' and skilled-labor-abundant 'North') and 2 sectors. One sector produces a homogeneous non-tradable good using unskilled labor only, and the other consists of a continuum of subsectors producing tradable inputs (intermediate goods) for one final consumption good using both skilled and unskilled labor (in varying intensities). In both regions there is a continuum of workers who provide one unit of labor each and who are not mobile across regions (they cannot migrate from North to South or vice versa).

Skilled workers are always employed, they can switch from one firm to the other without any frictions<sup>21</sup> and get paid their marginal revenue product which then has to be equal in all subsectors of the respective region. Unskilled workers are subject to search-unemployment; there is no on-the-job-search. The unemployed have to choose between searching in the (non-tradable) consumer services sector or in the manufacturing sector.

The before-mentioned crucial differences in characteristics of the non-trading sectors will be represented by the costs which firms have to pay for posting vacancies. As the unemployed have to be indifferent between searching in the manufacturing and the service sector, they induce a sectoral trade-off between the job finding probability and the wage in case of having found a job.

After some substitutions it will be possible to derive how the steady state of several important variables (especially wages and unemployment rates) changes after an increase in productivity in the unskilled-labor-abundant economy, which will lead to an increase in outsourcing from the 'North' to the 'South' (as some marginal intermediate goods are then cheaper to produce in the South). The results of this part will be qualitative only.<sup>22</sup>

This part of the dissertation will be presented in chapter 3.

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<sup>21</sup>One could think of them being employed by competitive temp agencies.

<sup>22</sup>The empirical part of this section will be limited to a comparison of main predictions of the model with important stylized facts (especially the increase in unskilled unemployment in Western Europe).



## 1.4 Methodological aspects

In the end, this dissertation is located in the field of the empirical sciences. It is about the relationship between empirical objects (like unemployment rate, GDP ...). So these relationships can (and very likely will) change over time. This (unfortunately) also means that if some setting fits best with the data observed so far, I cannot conclude that it will continue to do so in the future.

In line with what is called 'methodological individualism' in economics,<sup>23</sup> most equations describing relationships between empirically observable data stem from solutions to optimization problems and imposed equilibrium conditions of an artificial model. This part of the work is deductive but serves just as an intermediate step to get to the hypothesized relationships between certain empirical objects.

Furthermore, this thesis mainly stands in an instrumentalist tradition. Its methodology is in line with the approach advocated by Friedman (1953), which can be summarized by 'A model is good not because it is true, but because the world behaves as if it were true' (Hoover, 2001, p. 141).<sup>24</sup> In some sense I could say that 'I do not take my models too literally'. There are plenty of unrealistic assumptions made in these models. For example in both parts certain functional forms for the utility function are imposed, there are no people who do not want to work at all, and heterogeneity of agents is very stylized. So I have to clearly state which variables the model has to be able to predict and which not. For example, the DSGE model for the analysis of short-run-fluctuations of unemployment should be able to make predictions about (observable) macroeconomic aggregates (like GDP, unemployment rate, average working hours, ...). However, it will not be suitable for making predictions concerning objects like the distribution of income (as workers are assumed to be ex ante identical).

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<sup>23</sup>This 'methodological individualism' has been heavily (and successfully) advocated by the New-Classical school. For a discussion of this approach and a critique of common assumptions made there see chapter 3 of Hoover (2001).

<sup>24</sup>This approach is wide-used in macroeconomics but is also subject to substantial criticism. Hoover (2001) states that 'Of course, there is a serious question about why the world should be tractable if the model were not in fact true in some sense' (p. 141). He furthermore calls the methodology of 'as if' the 'best-used tool in the kit of unreflective rationalizations with which economists support their practices' (p. 142).



## Chapter 2

# Modelling the labor market in a DSGE Model for the Austrian economy

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JEL codes: E32, E24, C52, J64

### Abstract

In this paper I integrate a richly specified labor market with search unemployment into the New Keynesian DSGE model for the Austrian economy of Fenz et al. (2012). Within this setting, I compare different labor market specifications which are currently used in the literature concerning their fit with Austrian quarterly data on important macroeconomic aggregates (including several labor market variables).

Most importantly, the Bayesian model comparison indicates an important role for nominal wage rigidities and contemporaneous hiring. Furthermore, determination of working hours via 'efficient bargaining' performs better than via 'right-to-manage'.

The best-performing model can reproduce the relative volatility of labor market tightness compared to labor productivity relatively well (but absolute volatilities are too high). Furthermore, shock decompositions show that fluctuations in Austrian labor market tightness are mainly driven by external demand shocks and to a much smaller extent by productivity shocks.

## 2.1 Introduction

Over the last years, there have been dozens of papers which integrate search unemployment into New-Keynesian DSGE (Dynamic Stochastic General Equilibrium) models.<sup>2</sup> One typical

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<sup>2</sup>Early contributions in this context have been made by Walsh (2005), Christoffel and Linzert (2005) and the working paper version of Trigari (2009).

rationale for doing that is to improve the understanding of the impulse responses of certain non-labor-market-variables to non-labor-market-shocks (like the persistent response of output and inflation to monetary policy shocks). At the same time, these papers are complementing the early contributions in the literature on short-term-fluctuations in search unemployment where the only source of aggregate shocks is productivity<sup>3</sup> by including more rigidities and providing additional possible explanations for changes in unemployment like 'exogenous' changes in monetary policy, in preferences, in the extent of competition in product markets, and so on.

In this literature on search frictions in New Keynesian DSGE models there is a significant variation in specifications of the labor market along several dimensions. Among others, there is variation in the degree of convexity of hiring costs and of wage rigidities, the timing of hiring, and the determination of job destruction and working hours. They all have different implications for the co-movement of observable macroeconomic variables (especially for labor market variables):<sup>4</sup>

- A higher degree of wage rigidity tends to – if it also concerns newly formed matches – lead to a relatively stronger response of employment to demand and supply shocks. In the New Keynesian literature there are also different approaches concerning whether the real (for example in Sveen and Weinke, 2008) or the nominal wage (like in Gertler et al., 2008) is rigid.
- The way of determination of working hours is crucial for the role of the extensive (employment in persons) versus the intensive (hours worked per person) margin of employment when firms react to changes in economic conditions. In the literature, there are the possibilities of choosing hours via maximizing the surplus of the match ('efficient bargaining') or of the employer deciding on the amount of working time ('right-to-manage'). Typically the first option implies a relatively lower volatility of working hours.
- The timing of hiring, namely whether newly hired workers can start working in the same or only in the next period, is crucial for the speed of reaction to shocks of the extensive margin opposed to the intensive one. In the latter case, firms can adjust employment in  $t$  after a shock in  $t$  only along the intensive margin (working hours), while they can raise employment in persons only in  $t + 1$ . In the other case, firms can immediately adjust along both margins.
- When job destruction becomes endogenous, it means on the one hand a stronger reaction of unemployment to certain shocks: In this case an upswing not only means a higher probability of finding a job for the unemployed, but also a lower probability of losing a job for the employed. On the other hand, endogenous job destruction tends to induce a positive correlation between unemployment and vacancies, which is at odds with empirical evidence. The latter point has been extensively discussed in the RBC literature on search unemployment (see for example Ramey, 2008).
- Convex costs of posting vacancies can lead to a relatively smaller response of the number of vacancies to demand and supply shocks (compared to the standard specification of linear costs), while concave costs may make them larger.

<sup>3</sup>See for example Mortensen and Pissarides (1994) and den Haan et al. (2000).

<sup>4</sup>Other dimensions along which labor market specifications differ in the literature include for example whether on-the-job-search is possible (see for example Christoffel et al., 2009). Such additional alternatives will not be further discussed in this paper.

In this paper I will extend the DSGE model of the Oesterreichische Nationalbank (Fenz et al., 2012), which describes a small open economy without own monetary policy. There I will replace the simple labor market which is modeled in the style of Erceg et al. (2000) by a richly specified labor market with search unemployment and endogenous working hours. I will then compare several models which differ only in their labor market specification along the before mentioned dimensions using Bayesian estimation methods.

To my knowledge, this is the first comprehensive comparison of labor market specifications in a DSGE model using system estimation; previous comparisons were either not comprehensive or not based on system estimation. For example, Christoffel et al. (2009) provide a comprehensive comparison of different labor market specifications, but they do not estimate their different models – they only compare the impulse response of macroeconomic variables (with a focus on inflation) to a monetary policy shock.<sup>5</sup> So their approach only accounts for trade-offs concerning the explanation of different variables with different models to a very limited extent. For example, Krause and Lubik (2007) show that in their model real wage rigidities help to generate certain stylized facts on labor markets but not on inflation. Furthermore, economic fluctuations are not solely due to surprise changes in monetary policy but also to changes in other factors like real demand or productivity.

Model comparisons based on (Bayesian) system estimation have been for example conducted by Gertler et al. (2008), Mandelman and Zanetti (2010), Riggi and Tancioni (2010), Christiano et al. (2009), Ichiue et al. (2009) and Cheremukhin (2011); however, all of them compare only a very limited number of specifications (mostly using US data).<sup>6</sup>

A first application of the results will be to analyze how the best-performing model can reproduce stylized facts of Austrian labor market fluctuations and to which kind of shocks (productivity, mark-ups, domestic or foreign demand, monetary policy, ...) they can be attributed. The latter will be done by a forecast error variance decomposition and a historical shock decomposition (which are by-products of the Bayesian estimation). Hereby, I contribute to the literature which investigates the ability of search models to generate certain stylized facts of labor markets and complement contributions like Shimer (2005), Sveen and Weinke (2008) and Christoffel and Kuester (2008).

Sections 2.2 (baseline model without the labor market) and 2.3 (labor market specifications) describe the model framework. Section 2.4 shows the linearized version of the model. Section 2.5 discusses the model comparison (and all related estimation issues) and section 2.6 shows the shock decompositions of Austrian unemployment. Section 2.7 concludes.

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<sup>5</sup>Less comprehensive comparisons of model specifications using impulse responses have been for example conducted by de Walque et al. (2009) and Trigari (2006).

<sup>6</sup>Cheremukhin (2011) investigates the role of training costs and endogenous job destruction in an RBC setting (productivity shock as only shock with economic interpretation; the remaining shocks are measurement errors) and compares it to the calibrations of the simpler models of Shimer (2005) and Hagedorn and Manovskii (2008). Mandelman and Zanetti (2010) compare different specifications of hiring costs, Gertler et al. (2008) compare flexible and nominally rigid wages, Riggi and Tancioni (2010) compare nominal and real wage rigidities, and Christiano et al. (2009) compare exogenous and (two types of) endogenous job destruction with a labor market in the style of Erceg et al. (2000). Ichiue et al. (2009) compare (using Japanese data) specifications with only the intensive margin, only the extensive margin and both margins.

## 2.2 The baseline model without the labor market

The DSGE model of the Oesterreichische Nationalbank (see Fenz et al., 2012), describes a small open economy without own monetary policy (building on the seminal paper by Smets and Wouters, 2003). It combines neoclassical/new classical features like forward-looking optimizing agents with rational expectations and ‘Keynesian’ features like nominal (rigid prices and wages) and real (monopolistic competition, investment adjustment costs, ...) rigidities.

The labor market in their model builds on the contribution of Erceg et al. (2000): Labor is heterogeneous, the different labor inputs are combined via a CES aggregator and workers are in monopolistic competition to each other. In this kind of setting adjustment of labor inputs occurs only at the intensive margin (working hours) and (partial) unemployment is of a voluntary nature. The non-labor-market-parts of Fenz et al. (2012) are described in the rest of this section, while in section 2.3 the specification of the labor market will be discussed in detail.

### 2.2.1 Households

The economy is populated by a continuum of households, indexed by  $h \in [0, 1]$ . Each household consists of infinitely many agents (again a unit interval) which pool their income. Households supply labor (see section 2.3) and accumulate capital. They maximize their intertemporal utility function which is given by<sup>7</sup>

$$Utility_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s e_{t+s}^c \left( \ln(C_{h,t+s} - \kappa C_{t+s-1}) - \int_0^{N_t} \frac{e_t^L \zeta_L}{1 + \sigma_L} H_{h,i,t}^{1+\sigma_L} di \right), \quad (2.1)$$

where  $C_{h,t}$  is the consumption of household  $h$ ,  $H_{h,i,t}$  are working hours supplied by household  $h$  in firm  $i$ ,<sup>8</sup>  $C_{t-1}$  denotes the average consumption of the economy in the previous period, and  $N_t$  is employment in persons.  $\beta$  is the subjective discount factor and  $\kappa$  the degree of (external) habit formation.<sup>9</sup>  $e_t^L = (1 - \rho_L) + \rho_L e_{t-1}^L + \epsilon_t^L$  is a negative labor supply shock and  $e_t^c = (1 - \rho_c) + \rho_c e_{t-1}^c + \epsilon_t^c$  is a positive consumption shock. The budget constraint for the representative household is given by

$$\begin{aligned} C_{h,t} + I_{h,t} + T_t &= \frac{B_{h,t-1}^f}{P_t} - \frac{B_{h,t}^f}{R_t^f \tilde{\phi}_{rp} (nfa_t, e_t^{rp}) P_t} + \int_0^{N_t} W_{i,t} H_{h,i,t} di + A_t b(1 - N_t) + A_t c_{V,t} V_t \\ &+ Div_t + (R_t^K Z_{h,t} - \Psi(Z_{h,t})) K_{h,t-1} + \int_0^1 \Psi(Z_{h,t}) K_{h,t-1} dh + N_t A_t \Phi_L, \quad (2.2) \end{aligned}$$

where  $I_t$  is investment,  $T_t$  is a lump-sum-tax,  $B_{h,t}^f$  are foreign bonds held in period  $t$ ,<sup>10</sup>  $P_t$  is the price level of final (consumption and investment) goods,  $R_t^f$  is the (gross) foreign interest rate paid on bonds,  $\tilde{\phi}_{rp} (nfa_t, e_t^{rp})$  denotes a risk premium on foreign net bond holdings (see section 2.2.4),  $R_t^K$  is the real rate of return on physical capital,  $W_{i,t}$  is the real wage paid in firm  $i$ ,  $Z_{h,t}$

<sup>7</sup>I choose log-utility to ensure that working hours are stationary in spite of the unit root in aggregate productivity.

<sup>8</sup>Note that my notation somehow deviates from conventions. Namely, indices referring to a continuum (like  $h$  and  $i$  above) are written as sub-indices (e.g.  $H_{h,i,t}$ ) and not in parentheses (e.g.  $H_t(h,i)$ ).

<sup>9</sup>Variables without group subindex ( $h, i, j, k$ ) denote averages, variables without time subscripts denote steady state values (or parameters) and variables with hats denote log deviations from the steady state.

<sup>10</sup>Bonds are zero-coupon bonds, i.e. a bond that pays 1 in period  $t + 1$  is bought in period  $t$  for  $\frac{1}{R_t^f \tilde{\phi}_{rp} (nfa_t, e_t^{rp})}$ .

is the utilization rate of the capital stock,  $\Psi(Z_{h,t})$  is the cost of utilization of capital,<sup>11</sup>  $K_{h,t}$  is the stock of physical capital held by household  $h$ ,  $Div_t$  denote received dividend payments,  $c_{V,t}V_t$  are overall vacancy posting costs, and  $N_t A_t \Phi_L$  are overall fixed costs in the labor service sector.<sup>12</sup> The endogenous capital utilization rate introduces an intensive margin of capital; while an increase in the rental rate of capital can only lead to a sluggish response of the extensive margin of the capital stock (via higher physical investment),  $Z_{h,t}$  can jump up immediately.

Households own the capital stock. The law of motion of capital is given by

$$K_{h,t} = (1 - \tau)K_{h,t-1} + \left(1 - S\left(e_t^i \frac{I_{h,t}}{\mu^a I_{h,t-1}}\right)\right) I_{h,t}, \quad (2.3)$$

where  $\tau$  is the rate of depreciation,  $S(\cdot)$  are investment adjustment costs ( $S(1) = S'(1) = 0$  and  $S''(1) = \frac{1}{\phi} > 0$ ),  $\mu_a$  denotes the trend growth rate of the economy and  $e_t^i = (1 - \rho_i) + \rho_i e_{t-1}^i + \epsilon_t^i$  is a negative investment shock. The investment adjustment costs – similar to the external habit formation in consumption – lead to hump-shaped responses of investment to changes in macroeconomic conditions; otherwise investment would be a ‘pure’ jump variable.

The households maximize their utility by choosing the level of consumption, bond holdings, investment and the capital utilization rate subject to (2.2) and (2.3), where dividends, economy-wide capital-utilization costs and vacancy posting costs are taken as given. The Lagrangian for this problem is:

$$\Omega_{h,t} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[ \begin{array}{l} e_{t+s}^c \left( \ln(C_{h,t+s} - \kappa C_{t+s-1}) - \int_0^{N_{t+s}} \frac{e_{t+s}^L \epsilon_L}{1 + \sigma_L} H_{h,i,t+s}^{1 + \sigma_L} di \right) \\ - \Lambda_{h,t+s} \left( \begin{array}{l} C_{h,t+s} + I_{h,t+s} + T_{t+s} + \frac{B_{h,t+s}^f}{R_{t+s}^f \bar{\phi}_{rp}(nfa_{t+s}, e_{t+s}^{rp}) P_{t+s}} - \frac{B_{h,t+s-1}^f}{P_{t+s}} \\ - \int_0^{N_{t+s}} W_{i,t+s} H_{h,i,t+s} di - A_{t+s} b(1 - N_{t+s}) - A_{t+s} c_{V,t+s} V_{t+s} \\ - Div_{t+s} - \left( R_{t+s}^K Z_{h,t+s} - \Psi(Z_{h,t+s}) \right) K_{h,t+s-1} \\ - \int_0^1 \Psi(Z_{h,t+s}) K_{h,t+s-1} dh - N_{t+s} A_{t+s} \Phi_L \\ - \Lambda_{h,t+s} Q_{h,t+s} \left( K_{h,t+s} - K_{h,t+s-1} (1 - \tau) - \left(1 - S\left(e_{t+s}^I \frac{I_{h,t+s}}{\mu^a I_{h,t+s-1}}\right)\right) I_{h,t+s} \right) \end{array} \right) \end{array} \right], \quad (2.4)$$

where  $Q_{h,t}$  is the real value of one unit of capital in household  $h$ .

Differentiating with regard to  $C_{h,t}$ ,  $B_{h,t}^f$ ,  $I_{h,t}$ ,  $Z_{h,t}$  and  $K_{h,t}$  yields the following set of first order

<sup>11</sup>  $Z_i$  is normalized as such that in equilibrium  $Z = 1$ . So  $\Psi(1) = 0$  and  $\Psi'(1) = \frac{1}{\beta} - 1 + \tau = R^K$ .

<sup>12</sup> To ensure consistency of budget constraints and GDP equations, it is assumed that overall utilization costs of capital ( $\int_0^1 \Psi(Z_{h,t}) K_{h,t-1} dh$ ), overall vacancy posting costs ( $c_{V,t} V_t$ ) and overall fixed costs in the labor service sector ( $N_t A_t \Phi_L$ ) are paid to households.

conditions (FOCs):

$$\frac{\partial \Omega_{h,t}}{\partial B_{h,t}^f} = 0 \quad \Rightarrow \quad \mathbb{E}_t \left[ \beta \frac{\Lambda_{h,t+1}}{\Lambda_{h,t}} \frac{R_t^f \tilde{\phi}_{rp} (nfa_t, e_t^{rp}) P_t}{P_{t+1}} \right] = 1, \quad (2.5)$$

$$\frac{\partial \Omega_{h,t}}{\partial C_{h,t}} = 0 \quad \Rightarrow \quad \Lambda_{h,t} = e_t^c (C_{h,t} - \kappa C_{t-1})^{-1}, \quad (2.6)$$

$$\frac{\partial \Omega_{h,t}}{\partial K_{h,t}} = 0 \quad \Rightarrow \quad Q_t = \mathbb{E}_t \beta \frac{\Lambda_{h,t+1}}{\Lambda_{h,t}} \left[ Q_{h,t+1} (1 - \tau) + Z_{h,t+1} R_{t+1}^K - \Psi(Z_{h,t+1}) \right], \quad (2.7)$$

$$\begin{aligned} \frac{\partial \Omega_{h,t}}{\partial I_{h,t}} = 0 \quad \Rightarrow \quad & 1 + Q_{h,t} \left( S' \left( e_t^i \frac{I_{h,t}}{\mu^a I_{h,t-1}} \right) e_t^i \frac{I_{h,t}}{\mu^a I_{h,t-1}} - 1 + S \left( e_t^i \frac{I_{h,t}}{\mu^a I_{h,t-1}} \right) \right) \\ & = \beta \mathbb{E}_t Q_{h,t+1} \frac{\Lambda_{h,t+1}}{\Lambda_{h,t}} S' \left( e_{t+1}^i \frac{I_{h,t+1}}{\mu^a I_{h,t}} \right) e_{t+1}^i \frac{I_{h,t+1}^2}{\mu^a I_{h,t}^2}, \end{aligned} \quad (2.8)$$

$$\frac{\partial \Omega_{h,t}}{\partial Z_{h,t}} = 0 \quad \Rightarrow \quad R_t^K = \Psi'(Z_{h,t}). \quad (2.9)$$

Due to the assumption of income pooling, all members in a household consume they same, whether they are unemployed or employed. As all households are ex ante identical and infinitely large, unemployment rates and (capital and wage) income are therefore the same across all of them. This implies that the marginal utilities of consumption, subjective discount factors and the real value of capital are also identical across households.

## 2.2.2 Domestic firms

The production structure in this model is highly stylized with four stages of production:

1. Labor service firms transform a labor input into a homogeneous output  $L$ . They are subject to perfect competition and have to face search frictions when hiring labor. This sector replaces the CES-aggregator of heterogeneous labor inputs in Smets and Wouters (2003) or Fenz et al. (2012) and is discussed in greater detail in section 2.3.
2. Intermediate goods producing firms transform labor services (of 1st stage) and capital (which is rented from households) into a differentiated good. They are subject to monopolistic competition, cannot freely adjust prices every period (Calvo pricing) and face transitory and permanent shocks to their productivity.
3. Domestic good assembling firms transform the differentiated inputs from the second stage of production into a homogeneous good (via a CES aggregator). They are subject to perfect competition and their production is equal to domestic GDP.
4. Final good firms assembly domestic goods and imports into final goods which are used for consumption (private + government), investment and export.<sup>13</sup> This sector is also subject to perfect competition and faces real adjustment costs like in Christoffel et al. (2008).

<sup>13</sup>The assumption that imports are necessary for producing the export goods is made to account for the high import content of Austrian exports.



### Domestic good assembling firms

The domestic good is assembled by assembling firms which buy differentiated intermediate goods from a continuum of domestic intermediate goods producers and transform them into a homogeneous domestic good. They are subject to perfect competition, their overall production is given by:

$$Y_t = \left[ \int_0^1 Y_{j,t}^{\frac{1}{1+\lambda_{p,t}}} dj \right]^{1+\lambda_{p,t}}, \quad (2.10)$$

where  $Y_t$  denotes production of the domestic good,  $Y_{j,t}$  is the production of differentiated intermediate good  $j$  and  $\lambda_{p,t}$  is a time-varying mark-up ( $\lambda_{p,t} = \lambda_{p,t} + \epsilon_t^p$ ). Cost minimization of the domestic goods assembling firms yields demand for output of firm  $j$ :

$$Y_{j,t} = \left( \frac{P_{j,t}^d}{P_t^d} \right)^{\frac{-(1+\lambda_{p,t})}{\lambda_{p,t}}} Y_t, \quad (2.11)$$

where  $P_{j,t}^d$  denotes the price of the differentiated good  $j$ . The aggregate price  $P_t^d$  of the domestic good (the GDP deflator) is given by:

$$P_t^d = \left[ \int_0^1 (P_{j,t}^d)^{\frac{-1}{\lambda_{p,t}}} dj \right]^{-\lambda_{p,t}}. \quad (2.12)$$

### Firms producing domestic intermediate goods

There is a continuum  $j \in [0,1]$  of intermediate goods producers that transform homogeneous input from labor service firms and capital (rented from households) into a differentiated output. The production function is given by

$$Y_{j,t} = A_t^{1-\alpha} e_t^a \check{K}_{j,t}^\alpha L_{j,t}^{1-\alpha} - A_t \Phi, \quad (2.13)$$

where  $A_t$  is a non-stationary global technology process,  $e_t^a$  is a stationary domestic technology process,  $L_{j,t}$  and  $\check{K}_{j,t}$  denote labor services bought from first-stage-firms and effective capital rented from households by firm  $j$ .  $A_t \Phi$  are fixed real costs of production, which allow for relatively larger reactions of output to changes in employment or the (effective) capital stock. The levels of the technology shocks evolve according to  $\frac{A_t}{A_{t-1}} =: \mu_t^a = (1 - \rho_{\mu^a}) \mu^a + \rho_{\mu^a} \mu_{t-1}^a + \mu^a \epsilon_t^{\mu^a}$  and  $e_t^a = (1 - \rho_a) + \rho_a e_{t-1}^a + \epsilon_t^a$ . The capital stock  $\check{K}_{j,t}$  employed by firms in period  $t$  is related to the households' capital stock as follows:

$$\int_0^1 \check{K}_{j,t} dj = \int_0^1 Z_{h,t} K_{h,t-1} dh. \quad (2.14)$$

The intermediate goods producers maximize their profits from selling their products to the domestic goods assembling firm. The cost-minimizing condition for all firms is given by

$$\frac{\check{K}_{j,t}}{L_{j,t}} = \frac{\alpha}{1-\alpha} \frac{P_{L,t}}{R_t^K}, \quad (2.15)$$

where  $P_{L,t}$  is the price of labor services bought from labor service firms. This leads to the following equation for real marginal costs:

$$MC_{j,t} = MC_t = \frac{(R_t^K)^\alpha \left(\frac{P_{L,t}}{A_t}\right)^{1-\alpha}}{e_t^\alpha \alpha^\alpha (1-\alpha)^{1-\alpha}}, \quad (2.16)$$

which are identical over firms as input prices  $R_t^K$  and  $P_{L,t}$  are the same for all firms. Furthermore, from firm  $j$ 's perspective, the size of marginal costs is independent of the quantity it produces. Nominal profits of firm  $j$  are given by  $Profit_{j,t} = (P_{j,t}^d - P_t MC_t) Y_{j,t} - P_t MC_t A_t \Phi$ . Firm  $j$  sells its differentiated products to the domestic good assembling firms on a market with monopolistic competition. This form of competition is needed to be able to model nominally rigid prices and is also useful to introduce a wedge between inflation and real marginal costs via variations in  $\lambda_{p,t}$ . Plugging the demand function of the domestic good assembling firms (2.11) into this equation yields an expression for nominal profits of firm  $j$  in period  $t$ :

$$Profit_{j,t} = \left(P_{j,t}^d - P_t MC_t\right) \left(\frac{P_{j,t}^d}{P_t^d}\right)^{\frac{-(1+\lambda_{p,t})}{\lambda_{p,t}}} Y_t - P_t MC_t A_t \Phi. \quad (2.17)$$

Firms face nominal frictions a la Calvo (1983) when maximizing their profits. In each period, only a fraction  $1 - \zeta_p$  of firms is allowed to adjust their prices. These firms set the price  $\tilde{P}_t^j$  to maximize their expected profits. The remaining  $\zeta_p$  firms are assumed to follow a simple partial indexation rule based on past developments of the price of their goods  $P_{j,t}^d = (\Pi_{t-1}^d)^{\gamma_p} P_{j,t-1}^d$ . This indexation leads to hump-shaped responses of inflation to macroeconomic shocks.

So when a firm is able to maximize its profits in period  $t$ , it has to take into account that it might not be able to do adjustments in the following periods:

$$\max_{\tilde{P}_{j,t}^d} \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \frac{Profit_{j,t+s}}{P_{t+s}}. \quad (2.18)$$

Plugging the indexation rule and (2.17) into (2.18) and rearranging yields:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \beta^s \Lambda_{t+s} Y_{j,t+s} \left[ \left(\frac{P_{t+s-1}^d}{P_{t-1}^d}\right)^{\gamma_p} \frac{\tilde{P}_{j,t}^d}{P_{t+s}} - (1 + \lambda_{p,t}) MC_{t+s} \right] = 0. \quad (2.19)$$

Using (2.12), the price of the domestic good  $P_t^d$  can be obtained as a CES aggregate over the prices of adjusters and non-adjusters:

$$P_t^d = \left[ \zeta_p \left(P_{t-1}^d (\Pi_{t-1}^d)^{\gamma_p}\right)^{-\frac{1}{\lambda_{p,t}}} + (1 - \zeta_p) \left(\tilde{P}_{j,t}^d\right)^{-\frac{1}{\lambda_{p,t}}} \right]^{-\lambda_{p,t}}. \quad (2.20)$$

### Firms assembling final goods

For the sake of simplicity I assume that there is only one final good in the domestic economy ( $F_t$ ), that is used for private consumption, investment, exports and for government consumption. This final good is assembled by a continuum of final good assembling firms, which work under perfect competition and use domestically produced and imported goods as inputs. The production

function of final good assembling firm  $k$  has the following CES form:

$$F(D_{k,t}, M_{k,t}) = \left[ \mu^{\frac{\sigma_m}{1+\sigma_m}} D_{k,t}^{\frac{1}{1+\sigma_m}} + (1-\mu)^{\frac{\sigma_m}{1+\sigma_m}} (\phi_{k,t} M_{k,t})^{\frac{1}{1+\sigma_m}} \right]^{1+\sigma_m}, \quad (2.21)$$

where  $\mu$  is a parameter for a home bias for domestically produced goods, and  $\frac{1+\sigma_m}{\sigma_m}$  is the elasticity of substitution between domestically produced and imported intermediate goods. There is an adjustment cost (represented by the function  $\phi_{k,t}$ ) when firm  $k$ 's ratio of imported over domestic inputs deviates from the previous period's economy-wide average:

$$\phi_{k,t} = \left[ 1 - \phi_m \left( \tilde{e}_t^m - \frac{M_{k,t}/D_{k,t}}{M_{t-1}/D_{t-1}} \right)^2 \right], \quad (2.22)$$

with  $\tilde{e}_t^m = (1 - \rho_m) + \rho_m \tilde{e}_{t-1}^m + \tilde{e}_t^m$ . This specification ensures that changes in relative prices do not lead to 'too fast' responses of relative demand (again creating hump-shaped responses). Final good firm  $k$  decides on its input demand for domestic and imported goods by maximizing profits:

$$\max_{D_{k,t}, M_{k,t}} \left[ P_t F(D_{k,t}, M_{k,t}) - P_t^d D_{k,t} - P_t^m M_{k,t} \right], \quad (2.23)$$

where  $P_t$  is the price of the final good. In the aggregate it holds that  $F_t = \int_0^1 F(D_{k,t}, M_{k,t}) dk$ , where  $\int_0^1 D_{k,t} dk = Y_t$  and  $\int_0^1 M_{k,t} dk = M_t$ .

### 2.2.3 The foreign economy

Austria is linked to the foreign economy (= rest of the world) via trade and financial flows. The foreign economy is modelled in a parsimonious way. It is infinitely large compared to Austria which implies that the share of imports from and exports to Austria tend to zero (and it is not affected by shocks occurring just in Austria). I denote foreign variables with superscript  $f$  (e.g.  $Y_t^f$ ).

#### Three-equation model for output, inflation and the interest rate

The core model for the rest of the world consists of a simple New Keynesian model with three equations for foreign output ( $Y_t^f$ ), foreign inflation ( $\Pi_t^f$ ) and the foreign interest rate ( $R_t^f$ ).<sup>14</sup> It is assumed that the unit-root-productivity process  $A_t$  (which was described before) also enters the production function of the rest of the world. While this 3-equation-model is probably not the best way to explain the behavior of foreign (Euro area)<sup>15</sup> output, prices and interest rates, it should be sufficient to generate meaningful (e.g. demand, cost-push/supply and interest rate) external shocks for the Austrian economy and reasonable expectations for future values of foreign variables.

The foreign economy is populated by a continuum of households, indexed by  $h \in [0, 1]$ . They

<sup>14</sup>A more detailed description of such a simple monetary three-equations-New-Keynesian-model is for example provided in chapter 5 of Walsh (2003).

<sup>15</sup>In the context of this paper, the terms 'foreign', 'rest of the world' and '(rest of) Euro area' are used synonymously.

maximize their intertemporal utility function given by

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s e_t^{yf} \left( \ln(C_{h,t+s}^f - \kappa^f C_{t+s-1}^f) - \frac{(H_{h,t+s}^f)^{1+\sigma_l^f}}{1+\sigma_l^f} \right),$$

where  $e_t^{yf} = (1 - \rho_{yf}) + \rho_{yf} e_{t-1}^{yf} + \epsilon_t^{yf}$  is a positive demand shock. The budget constraint for the representative household is given by

$$C_{h,t}^f + \frac{B_{h,t}^f}{R_t^f P_t^f} = \frac{B_{h,t-1}^f}{P_t^f} + W_t^f H_{h,t}^f + Div_t^f,$$

where wages are assumed to flexible and taken as given by households.

The FOCs for bonds and consumption are both similar to the domestic economy and can be combined with  $Y_t^f = C_t^f$  to get to an Euler equation for output:

$$\mathbb{E}_t \left[ \beta \frac{\Lambda_{t+1}^f R_t^f P_t^f}{\Lambda_t^f P_{t+1}^f} \right] = 1, \quad (2.24)$$

$$\Lambda_t^f = e_t^{yf} (C_t^f - \kappa^f C_{t-1}^f)^{-1}. \quad (2.25)$$

Wages are flexible and hours are set optimally such that the marginal rate of substitution between leisure and consumption equals the real wage rate:

$$e_t^{yf} (H_{h,t}^f)^{\sigma_l^f} = \Lambda_t^f W_t^f. \quad (2.26)$$

Aggregate production is a simple function of aggregate working hours and the global technology process:  $Y_t^f = A_t H_t^f$ . So real marginal costs can be expressed as follows:

$$MC_t^f = \frac{W_t^f}{A_t} = \frac{(H_t^f)^{\sigma_l^f}}{(C_t^f - \kappa^f C_{t-1}^f)^{-1} A_t} = \frac{\left( \frac{Y_t^f}{A_t} \right)^{\sigma_l^f} (C_t^f - \kappa^f C_{t-1}^f)}{A_t} = \left( \frac{Y_t^f}{A_t} \right)^{\sigma_l^f + 1} \left( 1 - \kappa^f \frac{Y_{t-1}^f}{Y_t^f} \right). \quad (2.27)$$

Optimal price setting and the law of motion of the foreign price level are given by:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_p^f)^s \beta^s \Lambda_{t+s}^f Y_{j,t+s}^f \left[ \frac{\tilde{P}_{j,t}^f}{P_t^f} \left( \frac{P_{t+s-1}^f}{P_{t-1}^f} \right)^{\gamma_p^f} \frac{P_t^f}{P_{t+s}^f} - (1 + \lambda_{p,t}^f) MC_t^f \right] = 0, \quad (2.28)$$

$$P_t^f = \left[ \zeta_p^f \left( P_{t-1}^f (\Pi_{t-1}^f)^{\gamma_p^f} \right)^{-\frac{1}{\lambda_{p,t}^f}} + (1 - \zeta_p^f) \left( \tilde{P}_t^f \right)^{-\frac{1}{\lambda_{p,t}^f}} \right]^{-\lambda_{p,t}^f}. \quad (2.29)$$

The last 3 equations can be combined to get to a simple New Keynesian Phillips curve.

Finally, monetary policy can be described by the following log-linear rule:

$$\hat{R}_t^f = \rho_r R_{t-1}^f + (1 - \rho_r) (\psi_\pi^f \hat{\Pi}_t^f + \psi_y^f \hat{y}_t^f) + \epsilon_t^r, \quad (2.30)$$

where  $\rho_r$  is introduced to account for the persistence of interest rates and it is assumed that deviations of output from its steady state are taken as proxy for the output gap (therefore  $\hat{y}_t^f$  enters the last equation).

### Trade with Austria

Exports to Austria are assembled out of the foreign good (which has price  $P_t^f$ ) and are sold at price  $P_t^m$  to Austrian final good firms:

$$P_t^m = e_t^{\pi m} P_t^f, \quad (2.31)$$

where the wedge  $e_t^{\pi m}$  between  $P_t^f$  and  $P_t^m$  can be interpreted as a mark-up process ( $e_t^{\pi m} = \rho_{\pi m} e_{t-1}^{\pi m} + (1 - \rho_{\pi m}) e^{\pi m} + e_t^{\pi m} \epsilon_t^{\pi m}$ ).

Imports from Austria (which are bought at price  $P_t$ ) and domestic production are assembled to foreign final goods with the following production function:

$$F_{k,t}^f = \int_0^1 \left[ \mu_f^{\frac{\sigma_{mf}}{1+\sigma_{mf}}} \left( D_{k,t}^f \right)^{\frac{1}{1+\sigma_{mf}}} + (1 - \mu_f)^{\frac{\sigma_{mf}}{1+\sigma_{mf}}} \left( \phi_{k,t}^f M_{k,t}^f \right)^{\frac{1}{1+\sigma_{mf}}} \right]^{1+\sigma_{mf}} dk, \quad (2.32)$$

where  $\mu_f \rightarrow 1$ . Therefore  $F_t^f \rightarrow Y_t^f = D_t^f$  and – as assemblers are perfectly competitive – the price of the foreign final good  $P_t^f$  equals the price of the foreign intermediate good.  $\phi_{k,t}^f$  represents import adjustment costs as in (2.22) with shock process  $\tilde{e}_t^{mf}$ .

### 2.2.4 Aggregate output and net foreign assets

In addition to the equations presented above, a set of market clearing conditions is needed to complete the model. The aggregate production function can be derived by putting (2.13) into (2.10):

$$Y_t = A_t^{1-\alpha} e_t^a \left( \int_0^1 \tilde{K}_{j,t}^{\frac{\alpha}{1+\lambda_{p,t}}} L_{j,t}^{\frac{1-\alpha}{1+\lambda_{p,t}}} dj \right)^{1+\lambda_{p,t}} - A_t \Phi. \quad (2.33)$$

The market clearing condition for the final goods market relates supply ( $F_t$ ) to total demand, given by the sum of private consumption, investment, government consumption and exports:<sup>16</sup>

$$F_t = C_t + I_t + X_t + A_t e_t^g, \quad (2.34)$$

where  $A_t e_t^g$  denotes government consumption, which evolves according to  $A_t e_t^g = (1 - \rho_g) A_t g c + \rho_g A_t e_{t-1}^g + A_t g c e_{g,t}$ . As final goods firms are competitive, it holds that  $P_t F_t = P_t^d Y_t + P_t^m M_t$ , which – using (2.34) – can be reformulated as follows:

$$P_t^d Y_t = P_t F_t - P_t^m M_t = P_t (C_t + I_t + X_t + A_t e_t^g) - P_t^m M_t. \quad (2.35)$$

As indicated before, monetary policy (and so the euro area interest rate) are treated as exogenous. Therefore I use a (risk) premium on foreign bond holdings to ensure stationarity of net foreign

<sup>16</sup>Since there is only one final good, the price index cancels out of the equation.

assets and other macroeconomic aggregates.<sup>17</sup>

The risk-adjusted interest rate is similar to Adolfson et al. (2007) and given by  $R_t^f \tilde{\phi}_{rp} (nfa_t, e_t^{rp})$ , where  $\tilde{\phi}_{rp}$  is related to the ratio of real net foreign assets over domestic production valued at export prices ( $nfa_t := \frac{B_t^f}{P_t Y_t}$ ) via the following functional form:

$$\tilde{\phi}_{rp} (nfa_t, e_t^{rp}) = \exp \left( -\phi_{rp} nfa_t + e_t^{rp} \right). \quad (2.36)$$

The budget constraint of the government is given by

$$A_t e_t^s + A_t (1 - N_t) b + \frac{B_{g,t}^f}{R_t^f \tilde{\phi}_{rp} (nfa_t, e_t^{rp}) P_t} = T_t + \frac{B_{g,t-1}^f}{P_t},$$

where  $B_{g,t}^f$  are bonds held by the government in  $t$  (so in reality  $B_{g,t}^f$  would be negative in most industrialized countries). Aggregating the budget constraints of the domestic households and the domestic government ( $B_t^f = \int_0^1 B_{h,t}^f dh + B_{g,t}^f$ ), one gets to the law of motion of foreign bond holdings

$$\frac{B_t^f}{R_t^f \tilde{\phi}_{rp} (nfa_t, e_t^{rp})} = B_{t-1}^f + P_t X_t - P_t^M M_t. \quad (2.37)$$

Dividing (2.37) by  $P_t Y_t$  and using the definition of  $nfa_t$  leads to:

$$\frac{nfa_t}{R_t^f \tilde{\phi}_{rp} (nfa_t, e_t^{rp})} = \frac{nfa_{t-1}}{\Pi_t \frac{Y_t}{Y_{t-1}}} + \frac{X_t}{Y_t} - \frac{P_t^M}{P_t} \frac{M_t}{Y_t}. \quad (2.38)$$

## 2.3 Specifications of the labor market

In the model of Fenz et al. (2012), on which the previous chapter is based upon, there are just two observable labor market variables: real wages  $W$  and employment in persons  $N$ . They are linked to the rest of the model by one equation relating wages to the marginal rate of substitution between consumption and leisure and one equation relating unobservable working hours to observable employment in persons.<sup>18</sup>

In this paper, I will additionally include average working hours of employed persons  $H$  and the number of posted vacancies  $V$  as observables (as the size of the labor force is normalized to 1, employment  $N$  implies the number of unemployed  $U$ ).

### 2.3.1 Baseline specification

The baseline specification in the model is a combination of features which are widely used in the literature on search unemployment in DSGE models; for example, there are many similarities with the baseline model in Christoffel et al. (2009).

Entrepreneurs in the 1st stage of production (the linkages to the other stages of production are

<sup>17</sup>See Schmitt-Grohe and Uribe (2003) for an overview over options to close small-open-economy-models.

<sup>18</sup>The latter has been done as data on employment in persons is thought to be more reliable than data on working hours.

described in section 2.2.2) are ex ante symmetric.<sup>19</sup> They search for workers in the labor market. The production of firm  $i$  is given by:

$$L_{i,t} = \zeta_x N_{i,t} H_{i,t}^\gamma,$$

where  $\gamma < 1$ . This specification ensures that – like with capital – there is both an extensive (number of employees) and an intensive (working hours) margin of employment.

All of these firms employ a unit interval of workers (such that every firm employs members from all households in the economy), so  $N_{i,t} = 1 \forall i$ . Due to perfect competition they all charge the same price  $P_{L,t}$  for their good. The relationship between demand (of intermediate good producing firms) and supply of labor services is as follows:

$$L_t = \int_0^1 L_{j,t} dj = \int_0^{N_t} L_{i,t} di = \int_0^{N_t} \zeta_x H_{i,t}^\gamma di. \quad (2.39)$$

To hire labor, entrepreneurs post vacancies  $V$ . They are matched to the unemployed  $U$  by a linear homogeneous matching function; the number of newly hired workers who start working in  $t$  is given by:

$$Match_t = \zeta_m e_t^\theta V_{t-1}^\eta U_{t-1}^{1-\eta},$$

where  $e_t^\theta = (1 - \rho_\theta) + \rho_\theta e_{t-1}^\theta + \epsilon_t^\theta$  is time-varying matching efficiency. The exact timing in each period is as follows:

1. A fraction  $\delta$  of existing matches separates exogenously.
2. Matches are formed out of vacancies from  $t - 1$  and the unemployed from  $t - 1$  (where matching efficiency depends on the realization of  $e_t^\theta$ ). People having lost their job at the beginning of the period are excluded in this stage.
3. At the same time the other aggregate shocks (productivity, external demand ...) materialize.
4. Entrepreneurs post vacancies and the unemployed (including the ones who just lost their job) search.
5. At the same time existing matches (including the ones formed just at the beginning of the period) are negotiating wages and working hours, and are producing labor services.

People who are unemployed at the end of period  $t - 1$  can search in the labor market in the beginning of  $t$ . Their probability  $q_U$  of starting to work in  $t$  is given by:

$$\frac{Match_t}{U_{t-1}} =: q_U(\theta_{t-1}, e_t^\theta) = \zeta_m e_t^\theta \theta_{t-1}^\eta,$$

where  $\theta$  denotes labor market tightness  $\theta_t = \frac{V_t}{U_t}$ . There is no on-the-job-search. Firms have to pay a fee  $c_{V,t}$  for posting vacancies; the probability  $q_V$  of filling a vacancy and start working in  $t$  is

<sup>19</sup>Kuester (2007) combines the search friction (here in the first stage of production) and the price-setting friction (here in the second stage) into one stage of production. I do not use this specification as then the model comparison would become very difficult computationally.

given by:

$$\frac{Match_t}{V_{t-1}} =: q_V(\theta_{t-1}, e_t^\theta) = \zeta_m e_t^\theta \theta_{t-1}^{\eta-1}.$$

The employed in period  $t$  are given by the employed of the previous period whose job was not destroyed  $((1 - \delta)N_{t-1})$  plus the previously unemployed who found a job  $(q_U(\theta_{t-1}, e_t^\theta)U_{t-1})$ :

$$N_t = (1 - \delta)N_{t-1} + q_U(\theta_{t-1}, e_t^\theta)U_{t-1}. \quad (2.40)$$

As there is a unit mass of agents in each household, unemployment is simply given by:

$$U_t = 1 - N_t. \quad (2.41)$$

Shocks to matching efficiency  $e_t^\theta$  drive a wedge between (un)employment in  $t$  and (un)employment and vacancies in  $t - 1$ . As all these variables are observable, it is assumed that the matching efficiency shock is realized only at the beginning of the period where new matches are starting to work (i.e. in period  $t$ ); otherwise there would have been problems when estimating the model.

Let  $J_F$  and  $J_V$  denote the value of a match and of posting one more vacancy to the employer and let  $J_W$  and  $J_U$  be the values of being employed at firm  $i$  and of being unemployed.<sup>20</sup> The corresponding asset equations are as follows:

$$J_{F,i,t} = P_{L,t} \zeta_x H_{i,t}^\gamma - W_{i,t} H_{i,t} - A_t \Phi_L + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} (1 - \delta) J_{F,i,t+1}, \quad (2.42)$$

$$J_{V,t} = -A_t c_{V,t} + \beta \mathbb{E}_t q_V(\theta_t, e_{t+1}^\theta) \frac{\Lambda_{t+1}}{\Lambda_t} J_{F,new,t+1}, \quad (2.43)$$

$$J_{W,i,t} = W_{i,t} H_{i,t} - \frac{e_t^L e_t^c \zeta_L}{(1 + \sigma_L) \Lambda_t} H_{i,t}^{1+\sigma_L} + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} ((1 - \delta) J_{W,i,t+1} + \delta J_{U,t+1}), \quad (2.44)$$

$$J_{U,t} = A_t b + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left( q_U(\theta_t, e_{t+1}^\theta) J_{W,new,t+1} + (1 - q_U(\theta_t, e_{t+1}^\theta)) J_{U,t+1} \right), \quad (2.45)$$

where  $W_{i,t}$  is the hourly wage rate paid in firm  $i$ ,  $A_t \Phi_L$  are fixed costs (paid to households),<sup>21</sup>  $A_t c_{V,t}$  is the cost of posting a vacancy (in terms of the final good),  $P_{L,t}$  is the price of one unit of labor service in  $t$ ,  $\delta$  is the exogenous job-breakup rate and  $A_t b$  is the unemployment benefit.<sup>22</sup> The subscript *new* means that an average is taken over newly formed matches only; if no subscript is used, then the respective expressions refer to economy-wide averages.

### 2.3.2 Variations within this baseline

As said before, I will compare several different specifications of the labor market in this paper. Alternative rules for hiring costs, wage determination and hours determination can be specified within the above stated framework (in contrast to endogenous job destruction and contemporaneous hiring, which will be discussed separately in subsections 2.3.3 and 2.3.4).

<sup>20</sup>The difference between  $J_W$  and  $J_U$  divided by the marginal utility of consumption yields the marginal value of having one more employed member in a household.

<sup>21</sup>For the rationale behind these fixed costs in the spirit of Christoffel and Kuester (2008) see section 2.5.3.

<sup>22</sup>To ensure that hiring costs, fixed costs and unemployment benefits do not become negligible in the long run, they are assumed to grow in line with  $A_t$ .



### Alternative hiring costs

To ensure that firms do not post an infinite number of vacancies, it is assumed that they have to pay fees for that. The exact specification of this fee can influence the co-movements of labor market variables.

**1. Linear hiring costs:** Linear hiring costs are the widely used standard specification in the literature on search unemployment and mean that  $c_{V,t} = c_V$ . As the zero-expected-profit-condition for entrepreneurs ( $J_{V,t} = 0$ ) holds in all periods, the following condition for hiring can be derived:<sup>23</sup>

$$\begin{aligned} \mathbb{E}_t \frac{A_t c_V}{q_V(\theta_t, e_{t+1}^\theta)} &= \\ &= \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[ P_{L,t+1} \zeta_x H_{new,t+1}^\gamma - A_{t+1} \Phi_L - W_{new,t+1} H_{new,t+1} + (1 - \delta) \frac{A_{t+1} c_V}{q_V(\theta_{t+1}, e_{t+2}^\theta)} \right] \end{aligned} \quad (2.46)$$

The left-hand side (LHS) is simply the expected cost of filling a vacancy. The right-hand side (RHS) is given by the discounted flow profit of next period (simply revenue minus wages and fixed costs) plus an expression for the expected value of the match in later periods (multiplied by the survival probability).

This equation is crucial for the determination of the labor market tightness and thereby the level of unemployment: An increase in price  $P_{L,t+1}$  or a decrease in hourly wages  $W_{t+1}$  causes – ceteris paribus – an increase in expected cost of filling a vacancy which is equivalent to an increase in labor market tightness  $\theta_t$  (via a higher number of vacancies this period). This leads to a decrease in unemployment next period.<sup>24</sup>

**2. Non-linear hiring costs:** Several prominent papers in the literature deviate from the standard case of linear hiring costs; for example Gertler et al. (2008) and Thomas (2008) model large firms whose overall vacancy posting costs are quadratic in the number of their posted vacancies and decreasing in its overall workforce.

To keep things simple I assume that for a given entrepreneur the cost of posting one vacancy depends on the aggregate number of posted vacancies with the following functional form:

$c_{V,t} = \frac{A_t c_V V_t^{\psi_v}}{V^{\psi_v}}$ .<sup>25</sup> Then (2.46) becomes:

$$\begin{aligned} \mathbb{E}_t \frac{A_t c_V V_t^{\psi_v}}{q_V(\theta_t, e_{t+1}^\theta) V^{\psi_v}} &= \\ &= \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[ P_{L,t+1} \zeta_x H_{new,t+1}^\gamma - A_{t+1} \Phi_L - W_{new,t+1} H_{new,t+1} + (1 - \delta) \frac{A_{t+1} c_V V_{t+1}^{\psi_v}}{q_V(\theta_{t+1}, e_{t+2}^\theta) V^{\psi_v}} \right] \end{aligned} \quad (2.47)$$

<sup>23</sup>Derivation in appendix 2.A.

<sup>24</sup>Note that  $\frac{c_{V,t}}{q_V(\theta_t, e_{t+1}^\theta)}$  enters (2.46) with coefficient 1 while  $\mathbb{E}_t \frac{c_{V,t+1}}{q_V(\theta_{t+1}, e_{t+2}^\theta)}$  enters with coefficient  $\beta(1 - \delta) \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}$  which is significantly smaller than 1 when being close to the steady state.

<sup>25</sup>The scaling by  $V^{\psi_v}$  is done to ensure that in steady state the costs of posting one vacancy are  $c_V$  in both specifications. If  $\psi_v > 0$ , I could interpret this equation as the result from large firms facing overall vacancy posting costs of  $\frac{A_t c_V V_t^{1+\psi_v}}{(1+\psi_v) V^{\psi_v}}$ .

This would imply marginal costs of posting another vacancy of  $\frac{A_t c_V V_t^{\psi_v}}{V^{\psi_v}}$ . Then equations (2.42) and (2.43) would describe the value of employing one more worker and posting one more vacancy, respectively.

If  $\psi_V > 0$ , the number of posted vacancies would likely fluctuate less than in the case of linear vacancy posting costs (while with  $\psi_V < 0$  they would likely fluctuate more), as then it would be relatively cheaper for entrepreneurs to smooth the number of posted vacancies over time.

### Alternative ways of working hours determination

In a competitive neoclassical economy without frictions both firms and workers take the wage rate as given when deciding on labor demand respectively labor supply. In this case it would have to hold that the marginal rate of substitution between leisure and consumption equals the wage rate equals the marginal revenue product of labor, where the first equality is the FOC for labor supply and the second equality the FOC for labor demand.

However, wages are not taken as given by agents in models with search unemployment – they are the result of a negotiation process between the worker and the entrepreneur (see also below in subsection 2.3.2), in which the rent generated by the match is shared. Currently there are two popular ways to determine working hours (in case they are endogenous at all); both assume that the number of hours can be freely adjusted (reoptimized) every period.

**I. 'Efficient bargaining':** In case of 'efficient bargaining', which is for example used by Thomas (2008), the joint surplus of entrepreneur and worker  $J_{F,i,t} + J_{W,i,t} - J_{U,t}$  is maximized by taking the first derivative with regard to working hours.<sup>26</sup> The optimality condition 'marginal revenue product = marginal rate of substitution between leisure and consumption' leads to:

$$P_{L,t}\zeta_x\gamma H_{i,t}^{\gamma-1} = \frac{e_t^c e_t^L \zeta_L H_{i,t}^{\sigma_L}}{\Lambda_t}. \quad (2.48)$$

**II. 'Right-to-manage' bargaining:** When assuming efficient bargaining, there is no direct relationship between wages and consumer prices as wages then do not enter the Price Phillips curve.

This has led to the suggestion of an alternative specification in which, given a wage schedule, firms maximize their value of the match  $J_{F,i,t}$ . Among others, Trigari (2006) and Christoffel and Kuester (2008) employ this setting. Taking the first derivative of the RHS of (2.42) with regard to working hours leads to the optimality condition that 'marginal revenue product = hourly wage'.<sup>27</sup>

$$P_{L,t}\zeta_x\gamma H_{i,t}^{\gamma-1} = W_{i,t}. \quad (2.49)$$

Assuming that  $\sigma_L$  is relatively large (meaning a relatively high marginal disutility when working slightly more), this specification can lead to relatively large reactions of working hours to certain shocks as adjusting the intensive margin of employment is typically much 'cheaper' from the firm's perspective than in the efficient-bargaining-case.

Note that the terms 'efficient bargaining' and 'right-to-manage' have been used in a somewhat older literature in a slightly different context, namely the negotiations between a union and a

<sup>26</sup>As hours can be freely adjusted every period, all derivatives of terms of  $t + 1$  with regard to working hours in  $t$  are zero.

<sup>27</sup>To ensure that firms can cover the costs of posting vacancies in the case of 'right-to-manage', one either needs decreasing returns to labor or a downward sloping demand curve for each firm. I choose the former as merging all the involved frictions (search friction, Calvo pricing, possibly rigid wages) into the differentiated sector would be very complicated (see for example Christoffel et al., 2009).

firm on employment in persons and wages. Furthermore the term (weakly)<sup>28</sup> efficient bargaining is only used there in case of Nash bargaining on both wages and employment (see for example Cahuc and Zylberberg, 2004, chapter 7), while in this paper ‘efficient bargaining’ on working hours does not necessarily mean that there is Nash bargaining on wages too (see below).

### Alternative ways of incorporating wage rigidities

As indicated before, in models with search frictions wage rates are determined in negotiations between workers and entrepreneurs. In principle, every wage rate which makes both better off than their respective outside option (posting a vacancy/being unemployed) is a possible solution to the bargaining problem (see for example Hall, 2005).

The standard assumption is that wages are renegotiated every period via Nash bargaining between entrepreneur and worker where  $J_{F,i}^v(J_{W,i} - J_U)^{1-v}$  is maximized with regard to the (hourly) wage. Making this assumption can lead to a significant volatility of nominal and real wages. While the level of wages is relevant for the determination of working hours only in case of right-to-manage, the wages of newly hired employees are crucial for job creation. The more rigid they are, the larger will be the response of the extensive margin of employment to shocks to productivity and/or demand. So one could suppose that wages move only slowly within the possible range (which makes both negotiators better off than their outside option), as for example suggested by Hall (2005).<sup>29</sup>

For the rest of the paper I will assume that, in absence of wage rigidities, there will be Nash bargaining on wages between the entrepreneur and workers. They maximize  $J_{F,i,t}^v(J_{W,i,t} - J_{U,t})^{1-v}$  with respect to the wage rate; this yields:

$$v(J_{W,i,t} - J_{U,t}) \frac{\partial(J_{W,i,t} - J_{U,t})}{\partial W_{i,t}} + (1-v)J_{F,i,t} \frac{\partial J_{F,i,t}}{\partial W_{i,t}} = 0.$$

In case of efficient bargaining<sup>30</sup> this reduces to:

$$v(J_{W,i,t} - J_{U,t}) = (1-v)J_{F,i,t},$$

which means that the entrepreneur will get a share of  $v$  of the overall surplus (with the residuum going to the worker). In case of Nash bargaining every period in firm  $i$  (but not

<sup>28</sup>As the negotiations in my model are just between one worker and one employer, there is no distinction between ‘weakly efficient bargaining’ and ‘strongly efficient bargaining’ as in chapter 7 of Cahuc and Zylberberg (2004), where due to risk-aversion of workers there is over-employment (and therefore the outcome only being ‘weakly efficient’) when unemployment benefits and/or severance payments for the non-employed union members are not part of the bargaining process.

<sup>29</sup>In both variants of rigid wages I actually would have to constrain the wages to lie in the bargaining set of worker and employer – to avoid cases in which the surplus of one or both agents is negative. In the steady state both surpluses are positive and in my analysis of out-of-steady-state-dynamics I linearize around this point. According to Hall (2005)[p. 64] ignoring this constraint is ‘unlikely to have any practical effect’ in such cases.

<sup>30</sup>Here it holds that  $\frac{\partial(J_{W,i,t} - J_{U,t})}{\partial W_{i,t}} = H_{i,t}$  and  $\frac{\partial J_{F,i,t}}{\partial W_{i,t}} = -H_{i,t}$ .

necessarily in the other firms), the real wage rate would be given by:<sup>31</sup>

$$\begin{aligned} W_{i,t}^{nb} H_{i,t} &= (1 - \nu) \left( P_{L,t} \zeta_x H_{i,t}^\gamma - A_t \Phi_L \right) + \nu \left( A_t b + \frac{e_t^L e_t^c \zeta_L}{(1 + \sigma_L) \Lambda_t} H_{i,t}^{1 + \sigma_L} \right) \\ &\quad + \nu \beta \mathbb{E}_t q_U(\theta_t, e_{t+1}^\theta) \frac{\Lambda_{t+1}}{\Lambda_t} (J_{W,new,t+1} - J_{U,t+1}). \end{aligned} \quad (2.50)$$

In case of right-to-manage the number of hours depends directly on the wage rate,<sup>32</sup> which makes things more complicated:<sup>33</sup>

$$\nu \frac{\frac{MRS_{i,t}}{W_{i,t}} - \gamma}{1 - \gamma} (J_{W,i,t} - J_{U,t}) = (1 - \nu) J_{E,i,t},$$

where  $MRS_{i,t} = \frac{e_t^L e_t^c \zeta_L H_{i,t}^{\sigma_L}}{\Lambda_t}$ . In this specification the share of workers in the overall surplus increases in the number of hours worked, which dampens the effects of an increase in the price of the labor service good on the number of hours worked (see for example Christoffel et al., 2009). Solving for the wage rate yields:<sup>34</sup>

$$\begin{aligned} &\left( 1 + \frac{1 - \nu}{\nu} \frac{1 - \gamma}{\frac{MRS_{i,t}}{W_{i,t}^{nb}} - \gamma} \right) W_{i,t}^{nb} H_{i,t} = \\ &= \frac{1 - \nu}{\nu} \frac{1 - \gamma}{\frac{MRS_{i,t}}{W_{i,t}^{nb}} - \gamma} (P_{L,t} H_{i,t}^\gamma - A_t \Phi_L) + A_t b + \frac{e_t^L e_t^c \zeta_L}{(1 + \sigma_L) \Lambda_t} H_{i,t}^{1 + \sigma_L} \\ &\quad + \beta \frac{1 - \nu}{\nu} (1 - \delta) \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} J_{E,i,t+1} \left( \frac{1 - \gamma}{\frac{MRS_{i,t}}{W_{i,t}^{nb}} - \gamma} - \frac{1 - \gamma}{\frac{MRS_{i,t+1}}{W_{i,t+1}^{nb}} - \gamma} \right) \\ &\quad + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} q_U(\theta_t, e_{t+1}^\theta) (J_{W,new,t+1} - J_{U,t+1}). \end{aligned} \quad (2.51)$$

The combination of Nash bargaining on wages and right-to-manage-determination of working hours leads to Pareto-inefficient contracts. In case of ‘classical’ right-to-manage (between unions and employers) this can be justified by – implicitly – assuming that wages and employment are being determined on different levels (i.e. wages are determined between unions and federations of employers while employment is determined by the aggregation of decisions of individual employers) and by the fact that wages are much easier to enforce at court than employment in persons (see for example Cahuc and Zylberberg, 2004, chapter 7). However, these arguments do apply to a much smaller extent in this case as it is clearly about negotiation of working hours and wages on the firm level. Therefore I will also use simple ‘surplus sharing’ (equation (2.50), i.e. the solution to the wage determination problem in case of efficient bargaining) in combination with right-to-manage. This will be useful to see how the empirical performance of models with right-to-manage is influenced separately by the different setting of working hours and the more

<sup>31</sup>Derivation in appendix 2.A. The superscript *nb* is used as (see below) this wage rate might be only a hypothetical reference value in case of real or nominal wage rigidities.

<sup>32</sup>Rearranging (2.49) yields  $H_{i,t} = \left( \frac{P_{L,t} \zeta_x \gamma}{W_{i,t}} \right)^{\frac{1}{1-\gamma}}$ . This implies  $\frac{\partial (J_{W,i,t} - J_{U,t})}{\partial W_{i,t}} = \frac{H_{i,t}}{1-\gamma} \left( \frac{MRS_{i,t}}{W_{i,t}} - \gamma \right)$  and  $\frac{\partial J_{E,i,t}}{\partial W_{i,t}} = -H_{i,t}$ .

<sup>33</sup>Derivations in appendix 2.A.

<sup>34</sup>Derivation in appendix 2.A.

complicated formula for wage determination.

**A. Flexible wages:** In case of absence of wage rigidities, the wage is simply given by:<sup>35</sup>

$$W_{i,t}H_{i,t} = W_{i,t}^{nb}H_{i,t}. \quad (2.52)$$

**B. Real rigidity:** A relatively simple possibility to introduce real wage rigidities is used in Krause and Lubik (2007), where the actual wage is a convex combination of the hypothetical Nash bargaining wage and the steady state wage.<sup>36</sup>

$$W_{i,t} = (1 - \zeta_{wr})W_{i,t}^{nb} + \zeta_{wr}A_t w, \quad (2.53)$$

where  $w$  is the steady state value of  $\frac{W_t}{A_t}$ . Note that in this case as well as in A the  $i$ -index can be dropped as there 1st-stage-firms are also completely symmetric when out of the steady state.

To some extent, one could think of this specification as a special case of flexible wages with a relatively lower bargaining power of the worker. However, in contrast to a high value of  $\zeta_{wr}$ , a lower bargaining power of workers would also have a strong influence on steady state ratios, which can be important for reactions to permanent shocks and in some cases also for driving short-run-fluctuations.

**C. Nominal rigidity (Calvo wages):** Another popular (and slightly more complicated) variant to introduce wage rigidity is a Calvo-type rigidity of nominal wages; among others, Thomas (2008) and Gertler et al. (2008) make this assumption.<sup>37</sup>

In each period, the wage of a match can be freely adjusted with probability  $1 - \zeta_w$ ; if adjusted, they are determined by Nash bargaining (taking into account that there is a chance that the wage cannot be readjusted the following periods). The remaining fraction  $\zeta_w$  of existing matches does full indexation to the growth in  $A_t$  and partial indexation to previous inflation which means that the previous nominal wage is multiplied by  $\frac{A_t}{A_{t-1}} \left( \frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w}$ .

Firms and employees which are optimizing in  $t$  agree on a nominal wage  $P_t W_t^*$  which solves the following condition:<sup>38</sup>

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s} (1 - \delta)^s \zeta_w^s \left( \frac{A_{t+s}}{A_t} \left( \frac{P_{t+s-1}}{P_{t-1}} \right)^{\gamma_w} \frac{P_t}{P_{t+s}} W_{i,t}^* H_{i,t+s} - W_{i,t+s}^{nb} H_{i,t+s} \right) = 0. \quad (2.54)$$

For a better understanding of (2.54) one can think in the following steps:

- $P_t W_t^*$  is the nominal wage agreed on in  $t$ .

<sup>35</sup>If in each periods all matches can vary their wages freely according to the proposed surplus sharing rule, one could actually reduce (2.50) to  $W_{i,t}^{nb}H_{i,t} = (1 - \nu) \left( P_{L,t} \zeta_x H_{i,t}^\gamma + c_{V,t} \theta_t \right) + \nu \left( A_t b + \frac{e_t^\gamma \zeta_L}{(1 + \sigma_L) \Lambda_t} H_{i,t}^{1 + \sigma_L} \right)$ , which might be a more familiar expression for many readers. Note again that (2.50) is computed as if there were Nash bargaining/surplus sharing every period in match  $i$ , but not necessarily in all the other matches.

<sup>36</sup>Hall (2005) proposes to use a convex combination of the hypothetical Nash bargaining wage and last period's actual wage.

<sup>37</sup>This looks similar to the nominal wage rigidities used in Smets and Wouters (2003) and Fenz et al. (2012). However, the search-friction allows me to incorporate such rigidities without the assumption that workers have all the bargaining power in negotiation with firms.

<sup>38</sup>I follow here the approach of Thomas (2008) to express the derivation of Calvo wages.

- If not adjusted in the following periods, due to indexations the nominal wage in period  $t + s$  is given by  $\frac{A_{t+s}}{A_t} \left( \frac{P_{t+s-1}}{P_{t-1}} \right)^{\gamma_w} P_t W_t^*$ .
- Dividing the later expression by  $P_{t+s}$  yields the implied real wage in period  $t + s$ .
- The net present value of this expression has to be equal to the net present value of Nash bargaining wages  $W_{t+s}^{nb}$  of the respective periods (weighted by working hours  $H_{i,t+s}$ ). The standard discount factor  $\beta^s \Lambda_{t+s}$  is multiplied by  $\zeta_w^s$  as there can be a renegotiation of wages and by  $(1 - \delta)^s$  as the match may break up.

I assume that newly matched employees receive the average wage of the others.<sup>39</sup> So the law of motion for average real wages  $W_t$  of employed workers is as follows:

$$W_t = (1 - \zeta_w) W_t^* + \zeta_w \frac{A_t}{A_{t-1}} \frac{P_{t-1}}{P_t} \left( \frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} W_{t-1}. \quad (2.55)$$

Main differences to the specification in B are the forward-looking-nature and the possible importance of inflation; the latter point being more important in the model selection procedure (which is based on a backward-looking policy function).

In both the specification on rigid real wages and rigid nominal wages I assume that the rigidities affect all workers. Note that Haefke et al. (2009) provide micro-evidence which shows that in the US the wages of newly hired workers tend to be less rigid than the ones of 'old' workers (see also Pissarides, 2009). However, Gertler and Trigari (2009) argue that this is at least partly driven by composition effects, namely that in economically bad times job creation is higher in low earning occupations (they also provide some empirical evidence for this argument).

### 2.3.3 Endogenous job destruction

In the specifications discussed so far, variations in employment are driven by fluctuations in hiring (via changes in vacancies and shocks to the exogenous matching efficiency). This is not consistent with stylized facts on business cycles as involuntary (from the employee's viewpoint) job destruction tends to be higher in bad times.

This is why plenty of papers also let job destruction vary over the business cycle, contributions in the New Keynesian DSGE context include Krause and Lubik (2007) and Trigari (2009). However, endogenous job destruction tends to induce a counterfactual positive correlation between unemployment and vacancies, which somehow limits its popularity for modelling search unemployment.<sup>40</sup>

To make things tractable, I will follow the approach used by Trigari (2009), in which workers receive iid shocks to their disutility of work at the beginning of each period.<sup>41</sup> This means that the flow utility of household  $h$  is now given by (the old flow utility is visible in equation (2.1)):

$$e_t^c \left( \ln(C_{h,t} - \kappa C_{t-1}) - \int_0^{N_t} \left( \frac{e_t^L \zeta_L}{1 + \sigma_L} H_{h,i,t}^{1+\sigma_L} + \frac{b_{i,t}}{\Lambda_{h,t}} \right) di \right),$$

<sup>39</sup>The same assumption is for example also made by Kuester (2007).

<sup>40</sup>See for example Shimer (2005) and Ramey (2008). However, Ramey (2008) shows that this 'undesired' positive correlation can disappear when also including on-the-job-search.

<sup>41</sup>The approach of Mortensen and Pissarides (1994), in which workers receive idiosyncratic shocks to their productivity, is more popular in the literature. However, implementing this way of endogenous destruction in a setting with endogenous working hours is very complicated as working hours would not be symmetric across matches in the steady state (see for example Christoffel et al., 2009).

where  $b_{i,t}$  has a cumulative distribution function  $G(b)$  and a probability density function  $g(b) = G'(b)$ ; the scaling of  $b_{i,t}$  via dividing by  $\Lambda_{i,t}$  is done for notational convenience. If the realization of  $b_i$  is large enough to make the value of the match for the employer negative (via higher wages which he has to pay), match  $i$  is resolved. So it is assumed that the worker has no influence on the job destruction decision (i.e. the worker would even stay in the job if he were better off when being unemployed).

The exact timing in each period is as follows:

1. A fraction  $\delta$  of existing matches separates exogenously.
2. Matches are formed out of vacancies from  $t - 1$  and the unemployed from  $t - 1$  (where matching efficiency depends on the realization of  $e_t^\theta$ ).
3. At the same time the other aggregate shocks (productivity, external demand ...) materialize.
4. The idiosyncratic shock to the disutility of work materializes; jobs above the threshold are destroyed.
5. Entrepreneurs post vacancies and the unemployed (including the ones who just lost their job) search.
6. At the same time existing matches are negotiating wages and working hours, and are producing labor services.

### Determination of employment and vacancies

The new law of motion of employment is given by:<sup>42</sup>

$$N_t = G(\tilde{b}_t)((1 - \delta)N_{t-1} + q_U(\theta_{t-1}, e_t^\theta)U_{t-1}), \quad (2.56)$$

where  $\tilde{b}_t$  is the time-varying threshold for the disutility of work.

The new asset equations are given by:

$$J_{F,i,t} = P_{L,t}\zeta_x H_{i,t}^\gamma - A_t\Phi_L - W_{i,t}H_{i,t} + \beta\mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} (1 - \delta)G(\tilde{b}_{t+1})J_{F,i,t+1}, \quad (2.57)$$

$$J_{V,t} = -A_t c_{V,t} + \beta\mathbb{E}_t q_V(\theta_t, e_{t+1}^\theta) \frac{\Lambda_{t+1}}{\Lambda_t} G(\tilde{b}_{t+1})J_{F,new,t+1}, \quad (2.58)$$

$$J_{W,i,t} = W_{i,t}H_{i,t} - \frac{e_t^I e_t^C \zeta_L}{(1 + \sigma_L)\Lambda_t} H_{i,t}^{1+\sigma_L} - A_t \zeta_b b_{i,t} + \beta\mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left( (1 - \delta)G(\tilde{b}_{t+1})J_{W,i,t+1} + (1 - (1 - \delta)G(\tilde{b}_{t+1}))J_{U,t+1} \right), \quad (2.59)$$

$$J_{U,t} = A_t b + \beta\mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left( q_U(\theta_t, e_{t+1}^\theta)G(\tilde{b}_{t+1})J_{W,new,t+1} + (1 - q_U(\theta_t, e_{t+1}^\theta)G(\tilde{b}_{t+1}))J_{U,t+1} \right) \quad (2.60)$$

<sup>42</sup>Exogenous job destruction  $\delta$  and the flow value of being unemployed  $b$  are scaled down in this setting, such that overall job destruction and the sum of the flow value of being unemployed plus average disutility of work correspond to their exogenous-job-destruction-counterparts in the previous section. For further details see appendix 2.B.

The new hiring condition is given by (see appendix):

$$\begin{aligned} \mathbb{E}_t \frac{A_t c_{V,t}}{q_V(\theta_t, e_{t+1}^\theta)} &= \\ &= \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} G(\tilde{b}_{t+1}) \left( P_{L,t+1} \zeta_x H_{t+1}^\gamma - A_{t+1} \Phi_L - W_{new,t+1} H_{t+1} + (1 - \delta) \frac{A_{t+1} c_{V,t+1}}{q_V(\theta_{t+1}, e_{t+2}^\theta)} \right) \end{aligned} \quad (2.61)$$

### Determination of wages and working hours

To ensure that working hours are independent of the realization of  $b_i$  and therefore identical across all matches (which holds with efficient bargaining due to the assumed additivity of fixed and variable disutility of work), I exclude the case of 'right-to-manage'-determination of working hours.

Furthermore I exclude the combination of endogenous destruction with Calvo-type nominal wage rigidities such that the realizations of  $b_i$  are the only driver of wage dispersion (as in the case of rigid real or flexible wages). In case of Calvo wages it would also be influenced by the probability to reoptimize nominal wages.<sup>43</sup>

Nash bargaining wages in match  $i$  are given by

$$\begin{aligned} W_{i,t}^{nb} H_{i,t} &= (1 - \nu) \left( P_{L,t} \zeta_x H_{i,t}^\gamma - A_t \Phi_L \right) + \nu \left( A_t b + \frac{e_t^l e_t^c \zeta_L}{(1 + \sigma_L) \Lambda_t} H_{i,t}^{1+\sigma_L} + A_t \zeta_b b_{i,t} \right) \\ &\quad + \nu \beta \mathbb{E}_t q_U(\theta_t, e_{t+1}^\theta) \frac{\Lambda_{t+1}}{\Lambda_t} G(\tilde{b}_{t+1}) (J_{W,new,t+1} - J_{U,t+1}). \end{aligned} \quad (2.62)$$

The average wage bill  $W_t^{nb} H_t$  in case of Nash bargaining wages is derived by integrating over  $b$ :

$$\begin{aligned} W_t^{nb} H_t &= (1 - \nu) \left( P_{L,t} \zeta_x H_t^\gamma - A_t \Phi_L \right) + \nu \left( A_t b + \frac{e_t^l e_t^c \zeta_L}{(1 + \sigma_L) \Lambda_t} H_t^{1+\sigma_L} + A_t \zeta_b \frac{\int_0^{\tilde{b}_t} b g(b) db}{G(\tilde{b}_t)} \right) \\ &\quad + \nu \beta \mathbb{E}_t q_U(\theta_t, e_{t+1}^\theta) \frac{\Lambda_{t+1}}{\Lambda_t} G(\tilde{b}_{t+1}) (J_{W,new,t+1} - J_{U,t+1}). \end{aligned} \quad (2.63)$$

As before, in case of rigid real wages, wage in firm  $i$  is given by  $W_{i,t} = (1 - \zeta_{wr}) W_{i,t}^{nb} + \zeta_{wr} A_t w$ .

### Determination of job destruction

As I excluded the case of Calvo wages, wages are symmetric for a given values of  $b_i$ . So it holds that  $\mathbb{E}_t J_{F,i,t+1} = \mathbb{E}_t J_{F,new,t+1}$ . Combining (2.57) and (2.58) and using that  $J_{V,t} = 0$  yields:

$$J_{F,i,t} = P_{L,t} \zeta_x H_{i,t}^\gamma - A_t \Phi_L - W_{i,t} H_{i,t} + \mathbb{E}_t (1 - \delta) \frac{A_t c_{V,t}}{q_V(\theta_t, e_{t+1}^\theta)}.$$

<sup>43</sup>In the following there will be a state-dependent threshold for the wage rate above which matches are destroyed. In economic downturns (upturns) Calvo wages lead to the situation that matches will (not) be destroyed in spite of relatively low (high) realizations of  $b_i$  because they are not able to reoptimize their wages.



Job  $i$  is directly at the margin of being destroyed when  $J_{F,i,t} = 0$ . Plugging in for the wage rate yields:

$$\begin{aligned} & P_{L,t}\zeta_x H_t^\gamma - A_t\Phi_L - \zeta_{wr}A_t w \\ & - (1 - \zeta_{wr}) \left( (1 - \nu)(P_{L,t}\zeta_x H_{i,t}^\gamma - A_t\Phi_L) + \nu \left( A_t b + \frac{e_t^L e_t^c \zeta_L}{(1 + \sigma_L)\Lambda_t} H_{i,t}^{1+\sigma_L} + A_t \zeta_b b_{i,t} \right) \right) \\ & - (1 - \zeta_{wr}) \nu \beta \mathbb{E}_t q_U(\theta_t, e_{t+1}^\theta) \frac{\Lambda_{t+1}}{\Lambda_t} G(\tilde{b}_{t+1})(J_{W,new,t+1} - J_{U,t+1}) + \mathbb{E}_t(1 - \delta) \frac{A_t c_{V,t}}{q_V(\theta_t, e_{t+1}^\theta)} = 0, \end{aligned}$$

where in case of fully flexible wages  $\zeta_{wr} = 0$ . Rearranging and replacing  $b_{i,t}$  by  $\tilde{b}_t$  yields the job destruction condition:

$$\begin{aligned} & (1 - (1 - \zeta_{wr})\nu) (P_{L,t}\zeta_x H_t^\gamma - A_t\Phi_L) - \zeta_{wr}A_t w \\ & - (1 - \zeta_{wr})\nu \left( A_t b + \frac{e_t^L e_t^c \zeta_L}{(1 + \sigma_L)\Lambda_t} H_t^{1+\sigma_L} + A_t \zeta_b \tilde{b}_t \right) + \mathbb{E}_t(1 - \delta) \frac{A_t c_{V,t}}{q_V(\theta_t, e_{t+1}^\theta)} \\ & - (1 - \zeta_{wr})\nu \beta \mathbb{E}_t q_U(\theta_t, e_{t+1}^\theta) \frac{\Lambda_{t+1}}{\Lambda_t} G(\tilde{b}_{t+1})(J_{W,new,t+1} - J_{U,t+1}) = 0. \end{aligned} \quad (2.64)$$

Note that the choice of the driving force of endogenous destruction makes the interaction with wage rigidities at the firm level somehow counterintuitive. A – from the perspective of the entrepreneur – negative idiosyncratic shock to a match is a higher disutility of work, which leads to higher wage requests of the worker. Rigid wages then mean that the worker gets only compensated for a small part of this additional disutility, making a negative value of the match for the entrepreneur less likely. However, this does not affect aggregate labor market dynamics as the distribution of this idiosyncratic shock is assumed to be time-invariant. So wage rigidities still have the intuitive effect that there is a ‘too small’ adjustment of wages to a negative aggregate shock, leading to a – ceteris paribus (i.e. same parameter values) – higher job destruction. Furthermore, such wage rigidities can lead to inefficient separations, i.e. certain matches are resolved (i.e. jobs destroyed) in spite of having a positive surplus (but due to these rigidities the surplus cannot be distributed such that the entrepreneur is better off than with his outside option).

While wages are symmetric in steady state when assuming exogenous job destruction, they are not in case of endogenous destruction as wages in firm  $i$  depend on the realization of the idiosyncratic shock process  $b_i$ . So the reference value  $A_t w$  is not the steady state wage of a given match, it is only the average over all firms. This means that wage dispersion would become relatively smaller when  $\zeta_{wr} > 0$  (and thereby dampening the endogenous component of job destruction) for a given standard deviation of  $b_i$  given parameter values. As  $g(\tilde{b})$  (and thereby the standard deviation of  $b_i$ ) will be estimated, though, this effect can be compensated.

### 2.3.4 Contemporaneous hiring

The typical textbook version of the search and matching model is formulated in continuous time (see for example Pissarides (2000) or Cahuc and Zylberberg (2004)). However, when one wants to estimate the model, one has to assume that time is discrete. In the case discussed in the previous subsections (from now on referred to as ‘lagged hiring’), the contemporaneous response to changes in demand for labor services is made only via working hours and job destruction (if it

is endogenous), but not via job creation. So in case of exogenous destruction this specification leads automatically to the result that the one-step-ahead-forecast-errors in unemployment are solely attributed to the matching efficiency shock as apart from  $e_t^\theta$  unemployment in  $t$  only depends on variables from  $t - 1$ .

The situation is quite similar with the other factor of production, namely (physical) capital. For production in  $t$ , the capital stock of the end of period  $t - 1$  is used. A shock raising the rental rate (marginal productivity) of capital contemporaneously affects only the utilization rate  $Z_t$  (the analogue to working hours), while it can raise the capital stock only from the next period on (via higher investment – which could be seen as an analogue to vacancies – in  $t$ ).

In this subsection I will let employment in  $t$  depend on vacancies posted in  $t$ , which will lead to a faster reaction of (un)employment to changes in economic conditions. This specification is for example also employed in Ravenna and Walsh (2008) or in one of the variations in Christoffel et al. (2009).

The exact timing in this new setting is as follows in each period:

1. A fraction  $\delta$  of existing matches separates exogenously.
2. Entrepreneurs post vacancies and the unemployed (including the ones who just lost their job) search.
3. Matches are formed out of vacancies from  $t$  and people who are unemployed at the beginning from period  $t^{44}$  (they start working the same period).
4. At the same time the other aggregate shocks (productivity, external demand ...) materialize.
5. The idiosyncratic shock to the disutility of work materializes; jobs above the threshold are destroyed (in case of exogenous job destruction this stage disappears).
6. Existing matches are negotiating wages and working hours, and are producing labor services.

In the following only the equations for the case of endogenous job destruction will be shown. The equations for exogenous job destruction can be simply derived by setting  $G(\tilde{b}_t) = 1$  and neglecting the job destruction condition.<sup>45</sup> Appendix 2.B provides a discussion of changes in job destruction rates and job finding rates to ensure that as many steady state ratios as possible are the same as before.

### Determination of employment and vacancies

The new law of motion of employment is:

$$N_t = G(\tilde{b}_t)((1 - \delta)N_{t-1} + q_U(\theta_t, e_t^\theta)(1 - (1 - \delta)N_{t-1})), \quad (2.65)$$

where the new labor market tightness is given by:

$$\theta_t = \frac{V_t}{1 - (1 - \delta)N_{t-1}}. \quad (2.66)$$

<sup>44</sup>These 'unemployed at the beginning from period  $t$ ' are given by the unemployed at the end from  $t - 1$  ( $= 1 - N_{t-1}$ ) plus the ones who just lost their job by exogenous destruction ( $= \delta N_{t-1}$ ). Note that in the rest of the section the term 'unemployed' will only refer to the subset of them who were not successful in the beginning of  $t$  in finding a job ( $= 1 - N_t$ ).

<sup>45</sup>Intermediate steps for computing the equations below and for the linearized equations are not shown in the appendix, but they are available from the author on demand.

The new asset equations are given by:

$$J_{F,i,t} = P_{L,t}\zeta_x H_{i,t}^\gamma - A_t\Phi_L - W_{i,t}H_{i,t} + \beta\mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} (1-\delta)G(\tilde{b}_{t+1})J_{F,i,t+1}, \quad (2.67)$$

$$J_{V,t} = -A_t c_{V,t} + q_V(\theta_t, e_t^\theta)G(\tilde{b}_t)J_{F,new,t}, \quad (2.68)$$

$$J_{W,i,t} = W_{i,t}H_{i,t} - \frac{e_t^L e_t^c \zeta_L}{(1+\sigma_L)\Lambda_t} H_{i,t}^{1+\sigma_L} - A_t \zeta_b b_{i,t} + \beta\mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left( (1-\delta)G(\tilde{b}_{t+1})J_{W,i,t+1} + \delta q_U(\theta_{t+1}, e_{t+1}^\theta)G(\tilde{b}_{t+1})J_{W,new,t+1} \right) \quad (2.69)$$

$$+ \beta\mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left( \delta(1 - q_U(\theta_{t+1}, e_{t+1}^\theta)) + (1 - \delta + \delta q_U(\theta_{t+1}, e_{t+1}^\theta))(1 - G(\tilde{b}_{t+1})) \right) J_{U,t+1} \quad (2.70)$$

$$J_{U,t} = A_t b + \beta\mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left( q_U(\theta_{t+1}, e_{t+1}^\theta)G(\tilde{b}_{t+1})J_{W,new,t+1} + (1 - q_U(\theta_{t+1}, e_{t+1}^\theta)G(\tilde{b}_{t+1}))J_{U,t+1} \right) \quad (2.71)$$

The new hiring condition is given by:

$$\begin{aligned} \frac{A_t c_{V,t}}{q_V(\theta_t, e_t^\theta)} &= \\ &= G(\tilde{b}_t) \left( P_{L,t}\zeta_x H_{new,t}^\gamma - A_t\Phi_L - W_{new,t}H_{new,t} + \mathbb{E}_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} (1-\delta) \frac{A_{t+1} c_{V,t+1}}{q_V(\theta_{t+1}, e_{t+1}^\theta)} \right). \end{aligned} \quad (2.72)$$

### Determination of wages

The average wage bill  $W_t^{nb}H_t$  in case of Nash bargaining is:

$$\begin{aligned} W_t^{nb}H_t &= (1-\nu)(P_{L,t}\zeta_x H_t^\gamma - A_t\Phi_L) + \nu \left( A_t b + \frac{e_t^L e_t^c \zeta_L}{(1+\sigma_L)\Lambda_t} H_t^{1+\sigma_L} + A_t \zeta_b \frac{\int_0^{\tilde{b}_t} b g(b) db}{G(\tilde{b}_t)} \right) \\ &\quad + \nu \beta \mathbb{E}_t q_U(\theta_{t+1}, e_{t+1}^\theta) (1-\delta) \frac{\Lambda_{t+1}}{\Lambda_t} G(\tilde{b}_{t+1}) (J_{W,new,t+1} - J_{U,t+1}), \end{aligned}$$

where the difference between value of being employed and of being unemployed  $J_{WU}$  is now:

$$\begin{aligned} J_{WU,t} &:= W_t H_t - \frac{e_t^L e_t^c \zeta_L}{(1+\sigma_L)\Lambda_t} H_t^{1+\sigma_L} - A_t b - A_t \zeta_b \frac{\int_0^{\tilde{b}_t} b g(b) db}{G(\tilde{b}_t)} \\ &\quad + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} (1-\delta) (1 - q_U(\theta_{t+1}, e_{t+1}^\theta)) G(\tilde{b}_{t+1}) J_{WU,t+1}. \end{aligned}$$

### Determination of job destruction

Finally, the new job destruction condition is given by:

$$\begin{aligned} 0 &= (1 - (1 - \zeta_{wr})\nu) (P_{L,t}\zeta_x H_t^\gamma - A_t\Phi_L) - \zeta_{wr} A_t w + \beta\mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} (1-\delta) \frac{A_{t+1} c_{V,t+1}}{q_V(\theta_{t+1}, e_{t+1}^\theta)} \\ &\quad - (1 - \zeta_{wr})\nu \left( A_t b + \frac{e_t^L e_t^c \zeta_L}{(1+\sigma_L)\Lambda_t} H_t^{1+\sigma_L} + A_t \zeta_b \tilde{b}_t \right) \\ &\quad - (1 - \zeta_{wr})\nu \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} q_U(\theta_{t+1}, e_{t+1}^\theta) (1-\delta) G(\tilde{b}_{t+1}) (J_{W,new,t+1} - J_{U,t+1}). \end{aligned} \quad (2.73)$$

## 2.4 Linearized version of the model

To be able to estimate the model, the equations of sections 2.2 and 2.3 have to be approximated by a Taylor series around the steady state. In this paper I choose to linearize the model.<sup>46</sup> I use the convention that log-deviations from the steady state are indicated by a hat ( $\hat{y}_t := \ln \frac{y_t}{\bar{y}}$ ). Variables without a time subscript denote steady state values.

### 2.4.1 Efficiency units, relative prices and measurement equations

In this model there are two sources of non-stationarity. They make certain transformations necessary such that the model is saddle-path-stable and can be estimated.

The technology process  $A_t$  has a unit root and thereby induces non-stationarity in all quantities (with the exception of hours worked), the real wage and the price of labor services. So I have to transform the respective variables by dividing them by  $A_t$ . I use the convention that capital letters refer to the original variables and small letters refer to stationarized variables, i.e.  $y_t := \frac{Y_t}{A_t}$ ,  $c_t := \frac{C_t}{A_t}$ ,  $i_t := \frac{I_t}{A_t}$ ,  $k_t := \frac{K_t}{A_t}$ ,  $x_t := \frac{X_t}{A_t}$ ,  $m_t := \frac{M_t}{A_t}$ ,  $y_t^f := \frac{Y_t^f}{A_t}$ ,  $w_t := \frac{W_t}{A_t}$ , and  $p_{L,t} = \frac{P_{L,t}}{A_t}$ . Furthermore, the marginal utility of consumption has to be multiplied with  $A_t$  to become stationary ( $\lambda_t := \Lambda_t A_t$ ). Most of the before mentioned variables are observables (see also section 2.5.1), so the following measurement equations have to be added:

$$\Delta \ln Y_t = \hat{y}_t - \hat{y}_{t-1} + \hat{\mu}_t^a, \quad (2.74)$$

$$\Delta \ln C_t = \hat{c}_t - \hat{c}_{t-1} + \hat{\mu}_t^a, \quad (2.75)$$

$$\Delta \ln I_t = \hat{i}_t - \hat{i}_{t-1} + \hat{\mu}_t^a, \quad (2.76)$$

$$\Delta \ln X_t = \hat{x}_t - \hat{x}_{t-1} + \hat{\mu}_t^a, \quad (2.77)$$

$$\Delta \ln M_t = \hat{m}_t - \hat{m}_{t-1} + \hat{\mu}_t^a, \quad (2.78)$$

$$\Delta \ln W_t = \hat{w}_t - \hat{w}_{t-1} + \hat{\mu}_t^a. \quad (2.79)$$

The other 8 observable variables are all included directly in the model as endogenous variables (see section 2.5.1); so their measurement equations are straightforward. In principle, measurement equations could possibly include measurement errors of observable variables. I decided, however, to give all residuals a structural interpretation and do therefore not use measurement errors.

While inflation rates are pinned down by the foreign monetary policy rule, price levels are not necessarily stationary. I choose the price of the final good as numeraire and express all other price levels in relative terms ( $p_t^d := \frac{P_t^d}{P_t}$ ;  $p_t^m := \frac{P_t^m}{P_t}$ ;  $p_t^f := \frac{P_t^f}{P_t}$ ). The laws of motion of the relative price levels are:<sup>47</sup>

$$\hat{p}_t^d = \hat{p}_{t-1}^d + \hat{\Pi}_t^d - \hat{\Pi}_t \quad (2.80)$$

$$\hat{p}_t^m = \hat{p}_{t-1}^m + \hat{\Pi}_t^m - \hat{\Pi}_t \quad (2.81)$$

$$\hat{p}_t^f = \hat{p}_{t-1}^f + \hat{\Pi}_t^f - \hat{\Pi}_t. \quad (2.82)$$

<sup>46</sup>An appendix to this section which shows intermediate steps for the computationally more demanding linearizations is available on demand.

<sup>47</sup>Here I use that  $p_t^d = \frac{P_t^d}{P_t} = \frac{p_t^d}{p_{t-1}^d} \frac{p_{t-1}^d}{p_{t-2}^d} \frac{p_{t-2}^d}{p_{t-3}^d} = \frac{p_{t-1}^d}{p_{t-1}^d} \frac{p_{t-1}^d}{p_{t-1}^d} \frac{p_{t-1}^d}{p_{t-1}^d} = p_{t-1}^d \frac{\Pi_t^d}{\Pi_t}$ .

## 2.4.2 Non-labor equations

### Households

Log-linearizing the first order conditions (2.5) to (2.9) yields the following set of equations for the households:

The intertemporal Euler equation is computed by linearizing (2.5):

$$\widehat{\lambda}_t - \mathbb{E}_t \widehat{\lambda}_{t+1} = \widehat{R}_t^f - \mathbb{E}_t \widehat{\Pi}_{t+1} - \phi_{rp} d n f a_t - \widehat{\mu}_{t+1}^a + e_t^{rp}, \quad (2.83)$$

where the marginal utility of consumption is given by:

$$\widehat{\lambda}_t = \widehat{e}_t^c - \frac{1}{1 - \frac{\kappa}{\mu_a}} \left( \widehat{c}_t - \frac{\kappa}{\mu_a} (\widehat{c}_{t-1} - \widehat{\mu}_t^a) \right). \quad (2.84)$$

The law of motion for the real value of capital  $Q_t$  is derived from (2.7):

$$\widehat{Q}_t = \mathbb{E}_t \left( \widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \widehat{\mu}_{t+1}^a + \frac{\beta}{\mu_a} \left( (1 - \tau) \widehat{Q}_{t+1} + R^K \widehat{R}_{t+1}^K \right) \right). \quad (2.85)$$

The investment equation is derived from (2.8):

$$\widehat{i}_t = \frac{1}{1 + \beta} \left( \widehat{i}_{t-1} - \widehat{e}_t^I - \widehat{\mu}_{a,t} \right) + \frac{\beta}{1 + \beta} \left( \widehat{i}_{t+1} + \widehat{e}_{t+1}^I + \widehat{\mu}_{a,t+1} \right) + \frac{\varphi}{1 + \beta} \widehat{Q}_t. \quad (2.86)$$

Linearization of (2.9) yields

$$\widehat{Z}_t = \Psi \widehat{R}_t^K, \quad (2.87)$$

where  $\Psi = \frac{\Psi'(1)}{\Psi''(1)}$ . In addition to these first order conditions, the log-linear capital accumulation equation can be obtained by log-linearizing (2.3):

$$\widehat{k}_t = \frac{(1 - \tau)}{\mu_a} \left( \widehat{k}_{t-1} - \widehat{\mu}_t^a \right) + \left( 1 - \frac{1 - \tau}{\mu_a} \right) \widehat{i}_t. \quad (2.88)$$

### Domestic firms

Combining (2.19) with (2.20) and log-linearizing yields the Phillips curve for the price of the domestic good:

$$\widehat{\Pi}_t^d = \frac{\gamma_p}{1 + \beta \gamma_p} \widehat{\Pi}_{t-1}^d + \frac{\beta}{1 + \beta \gamma_p} \mathbb{E}_t \widehat{\Pi}_{t+1}^d + \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p(1 + \beta \gamma_p)} \left( \widehat{m}c_t - \widehat{p}_t^d \right) + \epsilon_t^{\lambda_p}, \quad (2.89)$$

where  $\widehat{m}c_t = \alpha \widehat{R}_t^K + (1 - \alpha) \widehat{p}_{L,t} - \widehat{e}_t^a$  and  $\epsilon_t^{\lambda_p}$  is a rescaled version of the iid shock to  $\lambda_{p,t}$ . Note that here the price of labor services and not the wage rate itself drives real marginal costs and that the relative price  $\widehat{p}_t^d$  enters the equation.

Linearizing (2.15) and then averaging over  $i$  leads to an equation for input demands:

$$\widehat{p}_{L,t} + \widehat{N}_t + \gamma \widehat{H}_t + \widehat{\mu}_t^a = \widehat{R}_t^K + \widehat{Z}_t + \widehat{k}_{t-1}. \quad (2.90)$$

Linearizing and combining the equations that describe the behaviour of the final goods assembling

firms yields the demand equations for domestic goods and imports:

$$\widehat{p}_t^m = \frac{\sigma_m \mu}{1 + \sigma_m} (\widehat{y}_t - \widehat{m}_t) + 2\phi_m (-\widehat{y}_{t-1} + \widehat{m}_{t-1} + \widehat{y}_t - \widehat{m}_t) + \widehat{e}_t^m, \quad (2.91)$$

$$\widehat{p}_t^d = \frac{\sigma_m(1-\mu)}{1 + \sigma_m} (\widehat{m}_t - \widehat{y}_t) - 2\phi_m \frac{(1-\mu)}{\mu} (-\widehat{y}_{t-1} + \widehat{m}_{t-1} + \widehat{y}_t - \widehat{m}_t) - \frac{(1-\mu)}{\mu} \widehat{e}_t^m, \quad (2.92)$$

where  $e_t^m$  is a rescaled version of  $\widehat{e}_t^m$ .

The relationship between import prices and foreign prices is derived by linearizing (2.31):

$$\widehat{p}_t^m = \widehat{p}_t^f + \widehat{e}_t^{\pi m}. \quad (2.93)$$

### Foreign economy

The Euler equation for consumption/output is given by:

$$\widehat{y}_t^f = \frac{\frac{\kappa_f}{\mu_a}}{1 + \frac{\kappa_f}{\mu_a}} (\widehat{y}_{t-1}^f - \widehat{\mu}_t^a) + \frac{1}{1 + \frac{\kappa_f}{\mu_a}} \mathbb{E}_t (\widehat{y}_{t+1}^f + \widehat{\mu}_{t+1}^a) - \frac{1 - \frac{\kappa_f}{\mu_a}}{1 + \frac{\kappa_f}{\mu_a}} \mathbb{E}_t (\widehat{R}_t^f - \widehat{\Pi}_{t+1}^f + \widehat{e}_t^{yf} - \widehat{e}_{t+1}^{yf}). \quad (2.94)$$

The Phillips curve is:

$$\begin{aligned} \widehat{\Pi}_t^f = & \frac{\gamma_{p,f}}{1 + \beta\gamma_{p,f}} \widehat{\Pi}_{t-1}^f + \frac{\beta}{1 + \beta\gamma_{p,f}} \mathbb{E}_t \widehat{\Pi}_{t+1}^f \\ & + \frac{(1 - \beta\zeta_{p,f})(1 - \zeta_{p,f})}{\zeta_{p,f}(1 + \beta\gamma_{p,f})} \left( (1 + \sigma_t^f) \widehat{y}_t^f + \frac{\frac{\kappa_f}{\mu_a}}{1 - \frac{\kappa_f}{\mu_a}} (\widehat{y}_t^f - \widehat{y}_{t-1}^f + \widehat{\mu}_t^a) \right) + \epsilon_t^{\Pi f}, \end{aligned} \quad (2.95)$$

where  $\epsilon_t^{\Pi f}$  is a rescaled version of the shock to  $\lambda_{p,t}^f$ . The monetary policy rule has already been stated in section 2.2:

$$\widehat{R}_t^f = \rho_r \widehat{R}_{t-1}^f + (1 - \rho_r) (\psi_\pi^f \widehat{\Pi}_t^f + \psi_y^f \widehat{y}_t^f) + \epsilon_t^r. \quad (2.96)$$

The demand for imports from Austria is as follows:

$$\widehat{p}_t^f = \frac{\sigma_{mf}}{1 + \sigma_{mf}} (\widehat{y}_t^f - \widehat{x}_t) + 2\phi_{mf} (-\widehat{y}_{t-1}^f + \widehat{x}_{t-1} + \widehat{y}_t^f - \widehat{x}_t) + \widehat{e}_t^{mf}, \quad (2.97)$$

where  $e_t^{mf}$  is a rescaled version of  $\widehat{e}_t^{mf}$ .

### GDP and net foreign assets

The evolution of net foreign assets can be described by:

$$\beta \, d n f a_t = \frac{1}{\mu^a} d n f a_{t-1} + x_y (\widehat{x}_t - \widehat{m}_t - \widehat{p}_t^m), \quad (2.98)$$

where  $x_y$  is the share of exports in GDP ( $\frac{x}{y}$ ). The nominal GDP identity is:

$$\widehat{p}_t^d + \widehat{y}_t = c_y \widehat{c}_t + i_y \widehat{i}_t + (1 - i_y - c_y) \widehat{e}_t^G + x_y (\widehat{x}_t - \widehat{m}_t - \widehat{p}_t^m), \quad (2.99)$$

where  $c_y$  and  $i_y$  are the shares of consumption and investment in GDP. Supply of real GDP is:

$$\hat{y}_t = \left(1 + \frac{\Phi}{y}\right) \left(\hat{e}_t^q + \alpha \left(\hat{Z}_t + \hat{k}_{t-1} - \hat{\mu}_t^q\right) + (1 - \alpha)(\hat{N}_t + \gamma \hat{H}_t)\right). \quad (2.100)$$

### 2.4.3 Labor market equations

The equation for labor market tightness is given by

$$\hat{\theta}_t = \hat{V}_t - \hat{U}_t. \quad (2.101)$$

The linearized law of motion of employment is

$$\hat{N}_t = (1 - q_U - \delta)\hat{N}_{t-1} + \eta\delta\hat{\theta}_{t-1} + \delta e^{\hat{\theta}_t}. \quad (2.102)$$

The linearized relationship between employment and unemployment is

$$\hat{N}_t = -\frac{U}{N}\hat{U}_t. \quad (2.103)$$

#### Hiring costs

**1. Linear hiring costs:** Linearizing (2.46) yields:

$$\begin{aligned} & \frac{c_V}{q_V} \left( (1 - \eta)\hat{\theta}_t - \mathbb{E}_t \hat{e}_{t+1}^\theta + \hat{\lambda}_t - \mathbb{E}_t \hat{\lambda}_{t+1} \right) = \\ & = \beta p_L \zeta_x H^\gamma \mathbb{E}_t \left( \hat{p}_{L,t+1} + \gamma \hat{H}_{t+1} \right) - \beta w H \mathbb{E}_t \left( \hat{H}_{t+1} + \hat{w}_{t+1} \right) \\ & + \beta(1 - \delta) \frac{c_V}{q_V} \mathbb{E}_t \left( (1 - \eta)\hat{\theta}_{t+1} - \hat{e}_{t+2}^\theta \right). \end{aligned} \quad (2.104)$$

**2. Non-linear hiring costs:** Linearizing (2.47) leads to:

$$\begin{aligned} & \frac{c_V}{q_V} \left( (1 - \eta)\hat{\theta}_t - \mathbb{E}_t \hat{e}_{t+1}^\theta + \psi_v \hat{V}_t - \mathbb{E}_t \hat{\lambda}_{t+1} + \hat{\lambda}_t \right) = \\ & = \beta p_L \zeta_x H^\gamma \mathbb{E}_t \left( \hat{p}_{L,t+1} + \gamma \hat{H}_{t+1} \right) - \beta w H \mathbb{E}_t \left( \hat{w}_{t+1} + \hat{H}_{t+1} \right) \\ & + \beta(1 - \delta) \frac{c_V}{q_V} \mathbb{E}_t \left( (1 - \eta)\hat{\theta}_{t+1} - \hat{e}_{t+2}^\theta + \psi_v \hat{V}_{t+1} \right). \end{aligned} \quad (2.105)$$

#### Determination of working hours

**I. Efficient bargaining:** Linearizing (2.48) and averaging over  $i$  yields:

$$\hat{p}_{L,t} + (\gamma - 1)\hat{H}_t = \hat{\epsilon}_t^c + \hat{\epsilon}_t^L + \sigma_L \hat{H}_t - \hat{\lambda}_t. \quad (2.106)$$

**II. Right-to-manage bargaining:** Linearizing (2.49) and averaging over  $i$  leads to:

$$\hat{p}_{L,t} + (\gamma - 1)\hat{H}_t = \hat{w}_t. \quad (2.107)$$

This equation implies that newly hired workers work the average hours as (by assumption) newly hired workers receive the average wage, which is a very important result for deriving the hiring

condition (2.104). The same is also true for the efficient bargaining case, but there hours are symmetric across all firms anyway.

### Wage determination

The equation for the average Nash bargaining wage is derived by loglinearizing (2.50) and then averaging over  $i$ :

$$\begin{aligned} wH(\widehat{w}^{nb}_t + \widehat{H}_t) &= (1-\nu)p_L\zeta_x H^\gamma (\widehat{p}_{L,t} + \gamma\widehat{H}_t) + \nu \frac{\zeta_L}{(1+\sigma_L)\lambda} H^{1+\sigma_L} (\widehat{e}^L_t + \widehat{e}^c_t + (1+\sigma_L)\widehat{H}_t - \widehat{\lambda}_t) \\ &\quad + \nu q_U \beta j_{wu} (\mathbb{E}_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \eta \widehat{\theta}_t + \mathbb{E}_t \widehat{e}^\theta_{t+1} + \mathbb{E}_t \widehat{j}_{wu,t+1}), \end{aligned} \quad (2.108)$$

where  $j_{wu,t} := \frac{J_{W,new,t} - J_{U,t}}{A_t}$  and

$$\begin{aligned} j_{wu} \widehat{j}_{wu,t} &= wH(\widehat{w}_t + \widehat{H}_t) - \frac{\zeta_L}{(1+\sigma_L)\lambda} H^{1+\sigma_L} (\widehat{e}^L_t + \widehat{e}^c_t + (1+\sigma_L)\widehat{H}_t - \widehat{\lambda}_t) \\ &\quad - \beta q_U j_{wu} (\eta \widehat{\theta}_t + \mathbb{E}_t \widehat{e}^\theta_{t+1}) + \beta(1-\delta - q_U) j_{wu} \mathbb{E}_t (\widehat{j}_{wu,t+1} + \widehat{\lambda}_{t+1} - \widehat{\lambda}_t). \end{aligned} \quad (2.109)$$

The expression for the average Nash bargaining wage in case of right-to-manage is given by:

$$\begin{aligned} wH(\widehat{w}^{nb}_{i,t} + \widehat{H}_{i,t}) &= (1-\nu)p_L H^\gamma (\widehat{p}_{L,t} + \gamma\widehat{H}_{i,t}) + \nu \frac{\zeta_L H^{1+\sigma_L}}{(1+\sigma_L)\lambda} (\widehat{e}^L_t + \widehat{e}^c_t - \widehat{\lambda}_t + (1+\sigma_L)\widehat{H}_{i,t}) \\ &\quad + \nu q_U \beta j_{wu} (\mathbb{E}_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \eta \widehat{\theta}_t + \mathbb{E}_t \widehat{e}^\theta_{t+1} + \mathbb{E}_t \widehat{j}_{wu,t+1}) \\ &\quad + \nu \beta (1-\delta) \frac{j_{wu}}{1-\gamma} \mathbb{E}_t (\widehat{mrs}_{i,t+1} - \widehat{w}^{nb}_{i,t+1}) - \nu \frac{j_{wu}}{1-\gamma} (\widehat{mrs}_{i,t} - \widehat{w}^{nb}_{i,t}). \end{aligned} \quad (2.110)$$

**A. No rigidities:** In case of flexible wages, it holds that:

$$\widehat{w}_t = \widehat{w}^{nb}_t. \quad (2.111)$$

**B. Real rigidity:** In case of rigid real wages, linearization of (2.53) yields

$$\widehat{w}_t = (1 - \zeta_{wr}) \widehat{w}^{nb}_t. \quad (2.112)$$

**C. Nominal rigidity (Calvo wages):** In case of Calvo wages, linearization of (2.54) and (2.55) leads to:

$$\begin{aligned} \widehat{w}_t &= \frac{\zeta_w}{1 + \zeta_w^2 \beta (1-\delta)} \widehat{w}_{t-1} + \frac{(1-\zeta_w)(1-\beta(1-\delta)\zeta_w)}{1 + \zeta_w^2 \beta (1-\delta)} \widehat{w}^{nb}_t + \frac{\zeta_w \beta (1-\delta)}{1 + \zeta_w^2 \beta (1-\delta)} \mathbb{E}_t \widehat{w}_{t+1} \\ &\quad + \frac{\zeta_w \gamma w}{1 + \zeta_w^2 \beta (1-\delta)} \widehat{\Pi}_{t-1} - \frac{1 + \beta(1-\delta)\zeta_w \gamma w}{1 + \zeta_w^2 \beta (1-\delta)} \widehat{\Pi}_t + \frac{\beta(1-\delta)\zeta_w}{1 + \zeta_w^2 \beta (1-\delta)} \widehat{\Pi}_{t+1}. \end{aligned} \quad (2.113)$$

In all these 3 cases (A, B, C)  $\widehat{w}^{nb}_t$  can be replaced by  $\widehat{w}^{sh}_t$  to get to the combination of right-to-manage-determination of hours with surplus sharing for wages.



### Endogenous job destruction

Nash bargaining wages are given by

$$\begin{aligned}
wH(\widehat{w}_t^{nb} + \widehat{H}_t) &= (1 - \nu)p_L H^\gamma (\widehat{p}_{L,t} + \gamma \widehat{H}_t) + \nu \frac{H^{1+\sigma_L}}{\lambda(1+\sigma_L)} ((1 + \sigma_L)\widehat{H}_t + \widehat{e}_t^c + \widehat{e}_t^L - \widehat{\lambda}_t) \\
&\quad + \nu \zeta_b \frac{\widetilde{g}\widetilde{b}}{\widetilde{G}} d\widetilde{b}_t - \nu \zeta_b \frac{\int_0^{\widetilde{b}} b g(b) db}{\widetilde{G}^2} \widetilde{g} d\widetilde{b}_t \\
&\quad + \nu q_U \beta j_{wu} \widetilde{G} \left( \widehat{j}_{wu,t+1} + \widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \eta \widehat{\theta}_t + \widehat{e}_{t+1}^\theta + \frac{\widetilde{g}}{\widetilde{G}} d\widetilde{b}_{t+1} \right), \quad (2.114)
\end{aligned}$$

where  $\widetilde{g} := g(\widetilde{b})$ ,  $\widetilde{G} := G(\widetilde{b})$  and

$$\begin{aligned}
j_{wu} \widehat{j}_{wu,t} &= wH(\widehat{w}_t + \widehat{H}_t) - \frac{\zeta_L}{(1 + \sigma_L)\lambda} H^{1+\sigma_L} (e^{\widehat{L}_t} + \widehat{e}_t^c + (1 + \sigma_L)\widehat{H}_t - \widehat{\lambda}_t) \\
&\quad - \zeta_b \frac{\widetilde{g}\widetilde{b}}{\widetilde{G}} d\widetilde{b}_t + \zeta_b \frac{\int_0^{\widetilde{b}} b g(b) db}{\widetilde{G}^2} \widetilde{g} d\widetilde{b}_t - \beta q_U \widetilde{G} j_{wu} (\eta \widehat{\theta}_t + \mathbb{E}_t \widehat{e}_{t+1}^\theta) \\
&\quad + \beta(1 - \delta - q_U) j_{wu} \mathbb{E}_t \left( \widetilde{G} (\widehat{j}_{wu,t+1} + \widehat{\lambda}_{t+1} - \widehat{\lambda}_t) + \widetilde{g} d\widetilde{b}_{t+1} \right). \quad (2.115)
\end{aligned}$$

The law of motion of employment is derived by linearizing (2.56):

$$\widehat{N}_t = \widetilde{G}(1 - \delta - q_U) \widehat{N}_{t-1} + (1 - \widetilde{G} + \delta \widetilde{G}) (\eta \widehat{\theta}_{t-1} + \widehat{e}_t^c) + \frac{\widetilde{g}}{\widetilde{G}} d\widetilde{b}_t. \quad (2.116)$$

Linearizing (2.64) yields the job destruction condition:

$$\begin{aligned}
(1 - (1 - \zeta_{wr})\nu) p_L \zeta_x H^\gamma (\widehat{p}_{L,t} + \gamma \widehat{H}_t) - (1 - \zeta_{wr})\nu \frac{\zeta_L}{(1 + \sigma_L)\lambda} H^{1+\sigma_L} ((1 + \sigma_L)\widehat{H}_t + \widehat{e}_t^c + \widehat{e}_t^L - \widehat{\lambda}_t) \\
- (1 - \zeta_{wr})\nu \zeta_b d\widetilde{b}_t + (1 - \delta) \frac{c_V}{q_V} (\widehat{c}_{V,t} + (1 - \eta)\widehat{\theta}_t - \mathbb{E}_t \widehat{e}_{t+1}^\theta) \\
- (1 - \zeta_{wr})\nu q_U \beta \widetilde{G} j_{wu} \mathbb{E}_t \left( \eta \widehat{\theta}_t + \widehat{e}_{t+1}^\theta + \widehat{\lambda}_{t+1} - \widehat{\lambda}_t + \widehat{j}_{wu,t+1} + \frac{\widetilde{g}}{\widetilde{G}} d\widetilde{b}_{t+1} \right) = 0, \quad (2.117)
\end{aligned}$$

where in case of flexible wages  $\zeta_{wr} = 0$ ,  $\widehat{c}_{V,t} = 0$  in case of linear hiring costs and  $\widehat{c}_{V,t} = \psi_V \widehat{V}_t$  in case of non-linear ones. The hiring condition is derived by linearizing (2.61):

$$\begin{aligned}
\frac{c_V}{\widetilde{G} q_V} \mathbb{E}_t \left( \widehat{c}_{V,t} + (1 - \eta)\widehat{\theta}_t - \mathbb{E}_t \widehat{e}_{t+1}^\theta - \widehat{\lambda}_{t+1} + \widehat{\lambda}_t - \frac{\widetilde{g}}{\widetilde{G}} d\widetilde{b}_{t+1} \right) = \\
= \beta \left( p_L \zeta_x H^\gamma \mathbb{E}_t (\widehat{p}_{L,t+1} + \gamma \widehat{H}_{t+1}) - wH \mathbb{E}_t (\widehat{w}_{t+1} + \widehat{H}_{t+1}) \right) \\
+ \beta(1 - \delta) \frac{c_V}{q_V} \mathbb{E}_t \left( \widehat{c}_{V,t+1} + (1 - \eta)\widehat{\theta}_{t+1} - \widehat{e}_{t+2}^\theta \right). \quad (2.118)
\end{aligned}$$

### Contemporaneous hiring

As in section 2.3.4, just the equations with contemporaneous hiring and endogenous job destruction are shown; the equations with exogenous job destruction can be simply extracted from here by setting  $\widetilde{G} = 1$ ,  $\widetilde{g} = 0$  and  $d\widetilde{b}_t = 0$ , and by neglecting the job destruction condition. The new

law of motion of employment is:

$$\hat{N}_t = \tilde{G}(1-\delta)(1-q_U)\hat{N}_{t-1} + (1-\tilde{G}(1-\delta))\left(\eta\hat{\theta}_t + \hat{e}_t^\theta\right) + \frac{\tilde{g}}{\tilde{G}}\tilde{d}\tilde{b}_t. \quad (2.119)$$

Labor market tightness is given by:

$$\hat{\theta}_t = \hat{V}_t + \frac{(1-\delta)N}{1-(1-\delta)N}\hat{N}_{t-1}. \quad (2.120)$$

The hiring condition becomes:

$$\begin{aligned} \frac{c_V}{q_V\tilde{G}}\left(\hat{c}_{V,t} + (1-\eta)\hat{\theta}_t - \hat{e}_t^\theta - \tilde{g}\tilde{d}\tilde{b}_t\right) &= \\ &= p_L\zeta_x H^\gamma\left(\hat{p}_{L,t} + \gamma\hat{H}_t\right) - \beta\omega H(\hat{w}_t + \hat{H}_t) \\ &\quad + \beta(1-\delta)\frac{c_V}{q_V}\mathbb{E}_t\left(\hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{c}_{V,t+1} + (1-\eta)\hat{\theta}_{t+1} - \hat{e}_{t+1}^\theta\right). \end{aligned} \quad (2.121)$$

Nash bargaining wages are given by

$$\begin{aligned} wH(\hat{w}_t^{nb} + \hat{H}_t) &= (1-\nu)p_L H^\gamma(\hat{p}_{L,t} + \gamma\hat{H}_t) + \nu\frac{H^{1+\sigma_L}}{\lambda(1+\sigma_L)}\left((1+\sigma_L)\hat{H}_t + \hat{\epsilon}_t^c + \hat{\epsilon}_t^L - \hat{\lambda}_t\right) \\ &\quad + \nu\beta q_U\tilde{G}(1-\delta)j_{wu}\left(\hat{j}_{wu,t+1} + \hat{\lambda}_{t+1} - \hat{\lambda}_t + \eta\hat{\theta}_{t+1} + \hat{e}_{t+1}^\theta + \frac{\tilde{g}}{\tilde{G}}\tilde{d}\tilde{b}_{t+1}\right) \\ &\quad + \nu\zeta_b\frac{\tilde{g}\tilde{b}}{\tilde{G}}\tilde{d}\tilde{b}_t - \nu\zeta_b\frac{\int_0^{\tilde{b}} bg(b)db}{\tilde{G}^2}\tilde{g}\tilde{d}\tilde{b}_t, \end{aligned} \quad (2.122)$$

where

$$\begin{aligned} j_{wu}\hat{j}_{wu,t} &= wH(\hat{w}_t + \hat{H}_t) - \frac{\zeta_L}{(1+\sigma_L)\lambda}H^{1+\sigma_L}\left(\hat{\epsilon}_t^L + \hat{\epsilon}_t^c + (1+\sigma_L)\hat{H}_t - \hat{\lambda}_t\right) - \zeta_b\frac{\tilde{g}\tilde{b}}{\tilde{G}}\tilde{d}\tilde{b}_t \\ &\quad + \zeta_b\frac{\int_0^{\tilde{b}} bg(b)db}{\tilde{G}^2}\tilde{g}\tilde{d}\tilde{b}_t + \beta(1-\delta)(1-q_U)\tilde{G}j_{wu}\left(\hat{j}_{wu,t+1} + \hat{\lambda}_{t+1} - \hat{\lambda}_t\right) \\ &\quad + \beta(1-\delta)(1-q_U)j_{wu}\tilde{g}\tilde{d}\tilde{b}_t - \beta(1-\delta)\tilde{G}q_Uj_{wu}\left(\eta\hat{\theta}_{t+1} + \hat{e}_{t+1}^\theta\right). \end{aligned} \quad (2.123)$$

Finally, the job destruction condition is given by:

$$\begin{aligned} (1-(1-\zeta_{wr})\nu)p_L\zeta_x H^\gamma(\hat{p}_{L,t} + \gamma\hat{H}_t) - (1-\zeta_{wr})\nu\frac{\zeta_L}{(1+\sigma_L)\lambda}H^{1+\sigma_L}\left((1+\sigma_L)\hat{H}_t + \hat{\epsilon}_t^c + \hat{\epsilon}_t^L - \hat{\lambda}_t\right) \\ - (1-\zeta_{wr})\nu\beta q_U\tilde{G}(1-\delta)j_{wu}\mathbb{E}_t\left(\eta\hat{\theta}_{t+1} + \hat{e}_{t+1}^\theta + \hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{j}_{wu,t+1} + \frac{\tilde{g}}{\tilde{G}}\tilde{d}\tilde{b}_{t+1}\right) \\ - (1-\zeta_{wr})\nu\zeta_b\tilde{d}\tilde{b}_t + (1-\delta)\frac{c_V}{q_V}\left(\hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{c}_{V,t+1} + (1-\eta)\hat{\theta}_{t+1} - \mathbb{E}_t\hat{e}_{t+1}^\theta\right) = 0. \end{aligned} \quad (2.124)$$

### 2.4.4 Exogenous processes

It is assumed that the foreign interest rate shock  $\epsilon_t^r$  and the mark-up shocks  $\epsilon_t^{\lambda p}$  and  $\epsilon_t^{\tau f}$  are iid. All the other shock processes are assumed to be AR(1):

$$\widehat{e}_t^a = \rho_a \widehat{e}_{t-1}^a + \epsilon_t^a \quad [\text{stationary technology shock}], \quad (2.125)$$

$$\widehat{\mu}_t^a = \rho_{\mu_a} \widehat{\mu}_{t-1}^a + \epsilon_t^{\mu^a} \quad [\text{permanent technology shock}], \quad (2.126)$$

$$\widehat{e}_t^g = \rho_g \widehat{e}_{t-1}^g + \epsilon_t^g \quad [\text{government spending shock}], \quad (2.127)$$

$$\widehat{e}_t^i = \rho_i \widehat{e}_{t-1}^i + \epsilon_t^i \quad [\text{investment shock}], \quad (2.128)$$

$$\widehat{e}_t^c = \rho_c \widehat{e}_{t-1}^c + \epsilon_t^c \quad [\text{consumption preference shock}], \quad (2.129)$$

$$\widehat{e}_t^l = \rho_l \widehat{e}_{t-1}^l + \epsilon_t^l \quad [\text{labor supply shock}], \quad (2.130)$$

$$\widehat{e}_t^\theta = \rho_\theta \widehat{e}_{t-1}^\theta + \epsilon_t^\theta \quad [\text{matching efficiency shock}], \quad (2.131)$$

$$\widehat{e}_t^m = \rho_m \widehat{e}_{t-1}^m + \epsilon_t^m \quad [\text{own import demand shock}], \quad (2.132)$$

$$\widehat{e}_t^{mf} = \rho_{mf} \widehat{e}_{t-1}^{mf} + \epsilon_t^{mf} \quad [\text{foreign import demand shock}], \quad (2.133)$$

$$\widehat{e}_t^{rp} = \rho_{rp} \widehat{e}_{t-1}^{rp} + \epsilon_t^{rp} \quad [\text{risk premium shock}], \quad (2.134)$$

$$\widehat{e}_t^{\pi m} = \rho_{\pi m} \widehat{e}_{t-1}^{\pi m} + \epsilon_t^{\pi m} \quad [\text{import price shock}], \quad (2.135)$$

$$\widehat{e}_t^{yf} = \rho_{yf} \widehat{e}_{t-1}^{yf} + \epsilon_t^{yf} \quad [\text{world demand shock}]. \quad (2.136)$$

## 2.5 Model comparison

The linearized equations in section 2.4 are all of the following form:

$$\mathbb{E}_t (A(\theta) \widehat{z}_{t-1} + B(\theta) \widehat{z}_t + C(\theta) \widehat{z}_{t+1} + F(\theta) \epsilon_t + G(\theta) \widehat{\epsilon}_{t+1}) = 0, \quad (2.137)$$

where  $\widehat{z}_t$  denotes log-deviations of the endogenous variables from their steady state,  $\epsilon_t$  denotes the iid. shock innovations and  $A, B, C, F$  and  $G$  are (typically highly non-linear) matrix-valued functions of the (to-be-) estimated parameters  $\theta$  (for details see for example Ratto and Iskrev, 2011).

As the expectations  $\mathbb{E}_t z_{t+1}$  cannot be directly observed, (2.137) cannot be related directly to empirical data. However, when there is a unique solution, (2.137) can be rewritten into the following policy function:

$$\widehat{z}_t = K(\theta) \widehat{z}_{t-1} + L(\theta) \epsilon_t, \quad (2.138)$$

where  $K$  and  $L$  are again matrix-valued functions of the estimated parameters. Furthermore, a measurement equation relating the observable variables  $\widehat{y}_t$  to the endogenous variables  $\widehat{z}_t$  is needed, which in case of this model has a fairly simple form (see section 2.4.1).

The different model specifications are estimated using Bayesian methods (see for example An and Schorfheide, 2007).<sup>48</sup> The estimated parameters  $\theta$  (which in this case include certain unobserved steady state ratios) are treated as random variables, for which a prior distribution  $p(\theta|M_i)$  is specified (which could be model-specific). The implied prior beliefs are then updated by

<sup>48</sup>For this purpose I have used DYNARE (which runs in MATLAB); for a description of this software see Adjemian et al. (2011).

computing a conditional distribution of the parameters given the data:

$$p(\theta|Y, M_i) = \frac{p(Y|\theta, M_i)p(\theta|M_i)}{p(Y|M_i)}, \quad (2.139)$$

where  $p(Y|\theta, M_i) = L(\theta|Y, M_i)$  is the likelihood function (it is assumed that the data is normally distributed for given parameter values and given past data), and  $p(Y|M_i) = \int p(Y|\theta, M_i)p(\theta|M_i)d\theta$  is the marginal data density of model  $i$ .

This marginal data density can be used for model selection (see for example Del Negro and Schorfheide, 2011).<sup>49</sup> The criterion for choosing the 'best' model is then the overall fit with all observable variables of the model (based on the in-sample-one-step-ahead-forecasts of the model), and not a certain impulse response like in Christoffel et al. (2009).

To reduce computational costs, I use the Laplace approximation of the marginal likelihood for model comparison. This approximation only needs information from the maximization of the posterior likelihood function; otherwise a large number of draws from the posterior would be needed too (see again Del Negro and Schorfheide, 2011).

### 2.5.1 The data

Data availability restricts the number of observations I can use for the estimation. Reliable data on hours worked by employees is only available from 1995, which means that there are 67 observations (from 1995Q2 to 2011Q4).

Observable domestic variables in my model are real GDP ( $Y$ ), real consumption ( $C$ ), real investment ( $I$ ), real exports ( $X$ ), real imports ( $M$ ), working hours of the employed ( $\int_0^N H$ ), compensation of employees ( $\int_0^N PWH$ ), unemployment ( $U$ ), dependent employment ( $N$ ), the GDP deflator (as a proxy for  $P^d$ ), the consumption deflator (as a proxy for  $P$ ) and vacancies ( $V$ ). All of them are taken from the seasonally adjusted quarterly national accounts<sup>50</sup> with the exception of unemployment (which is taken from the Eurostat Labor Force Survey) and vacancies (which come from domestic labor market statistics)<sup>51</sup>. The 3 foreign variables output ( $Y^f$ ), interest rates ( $R^f$ ) and foreign price level ( $P^f$ ) refer to Euro area aggregates and are taken from the database of Fenz et al. (2012).

Note that gross fixed capital formation was used as proxy for investment. This implies that the residuum of equations (2.35) respectively (2.99) contains not only government consumption and statistical discrepancies, but also changes in inventories and net acquisitions of valuables. So the interpretation of the 'government consumption shock'  $\hat{e}_t^G$  has to be done cautiously.

I rescale variables as follows:

- As I restricted the labor force to have a measure of one, I divide employment, vacancies, GDP, consumption, investment, exports and imports by the sum of dependent employment and unemployment.<sup>52</sup> This restriction of the size of the labor force to 1<sup>53</sup> also means that

<sup>49</sup>In principle, one could assign different prior probabilities to different models. I will abstain from that option and assign the same prior probability to all models.

<sup>50</sup>Note that published Austrian quarterly data do not include the error component of the seasonal adjustment (in contrast to the rest of the EU) and therefore tend to be less volatile.

<sup>51</sup>Seasonal adjustment of vacancies is also done by myself.

<sup>52</sup>Self-employed are excluded in this setting as the variable 'compensation of employees' (which is used as a proxy for the overall wage bill) does not include the earnings of self-employed. This also means that there is a discrepancy between the 'unemployment rate' in this model (whose changes are shown in figure 2.2) and published numbers (which are typically based on the Eurostat definition or taken from domestic labor market statistics).

<sup>53</sup>This assumption also implies that the size of the labor force does not react to economic conditions; so factors like

I ‘lose’ one of the observed variables, implying that only 14 variables will be used in the estimation.

- As  $W$  refers to the average hourly real wage, I divide the compensation of employees by overall working hours and the consumption deflator.
- As  $H$  refers to the average working hours per employee, I divide the overall number of working hours by the number of employees.

All variables are demeaned and detrended: A quadratic trend is subtracted from all domestic quantities (except the labor market variables), real wages and all price variables. This implies that the growth rates of these variables are linearly detrended. Vacancies and foreign output are HP-filtered;<sup>54</sup> a linear trend is subtracted from all remaining variables (working hours, unemployment and the foreign interest rate). The measurement equations are discussed in section 2.4.1.

## 2.5.2 Identification analysis

Not all of the parameters showing up in the equations of appendix 2.4 can be estimated using the available data. This has several reasons:

- Linearizing around a steady state means that I lose information on most steady state ratios and parameters which are directly implied by these ratios (like  $\beta$ ). So I have to calibrate most of them (mostly using the raw data).
- In medium-scale DSGE models there are typically also other parameters which do not have an influence on the log-likelihood at all or only when certain other parameters are calibrated.<sup>55</sup>

Therefore I apply the routines discussed in Ratto and Iskrev (2011)<sup>56</sup> to check which parameters are identifiable in the estimation of the model. They try to check whether the Jacobian matrix  $J$  of the mapping from the vector of estimated parameters  $\theta$  to the first two moments of the data<sup>57</sup> has full column rank. As the underlying equation system is highly nonlinear, identification can only be checked locally (see again Ratto and Iskrev, 2011), which is done via taking different draws from the prior distribution of  $\theta$ . As the data is assumed to be normally distributed (conditional on the parameters) in this paper, the first two moments of the observable variables contain all information on the distribution and so a full rank of the Jacobian is both necessary and sufficient for local identification.

To ease the computational burden, Iskrev (2010b) suggests to split up this Jacobian into the Jacobian  $J_1$  of the mapping from the policy function elements (or, to be more precise, to the elements of the matrices  $K$  and  $L$  which depend on estimated parameters) to the first two moments of the data and the Jacobian  $J_2$  of the mapping from the vector of estimated parameters to the

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discouraged-worker-effects are neglected in this paper.

<sup>54</sup>HP-filtered Euro area output is assumed to represent  $y_t^f := \frac{Y_t^f}{A_t}$ . The HP-filtering is done as it is assumed that only the short-term-fluctuations of vacancies and foreign output are meaningful for the purpose of this paper.

<sup>55</sup>See for example Canova and Sala (2009), Iskrev (2010b) or Ratto and Iskrev (2011).

<sup>56</sup>These are in turn inspired by Iskrev (2010b). Furthermore note that they build on the population objective function and therefore no data is needed to perform these routines (see Iskrev, 2010a).

<sup>57</sup>As these also include all correlations with previous periods, the number of moments is significantly higher than the number of estimated parameters.

policy function;  $J_1$  can be computed by direct differentiation of the policy function and  $J_2$  is significantly smaller than the overall Jacobian.

As the procedures indicated that the autocorrelation of  $e_t^c$  and the standard deviation of  $e_t^c$  are hard to identify, I excluded this domestic consumption shock from the estimation. This means that there are as many shocks as observable variables (14). Due to further identification issues I set the autocorrelation of  $e_t^i$  to 0 (due to the specification of (2.86), shocks to  $e_t^i$  have a persistent impact on investment anyway) and calibrate the autocorrelations of the innovations to the import demand shock  $e_t^m$ , the export demand shock  $e_t^{mf}$ , the risk premium  $e_t^{rp}$  and the foreign interest rate  $R_t$  (in all cases there are also other estimated parameters influencing the persistence of the concerned observable variables and there is a strong correlation with the respective standard deviations).

The application of these routines also indicated potential identification problems with other parameters, for example with  $\sigma_m$  and  $\sigma_{mf}$  ( $\sigma_m$  shows up together with  $\phi_m$  in equations (2.91) and (2.92)), the situation with  $\sigma_{mf}$  and  $\phi_{mf}$  is very similar); therefore I decided to give the production functions in equations (2.21) and (2.32) a Cobb-Douglas form by setting  $\sigma_m \rightarrow \infty$  and  $\sigma_{mf} \rightarrow \infty$ . The steady state share of fixed costs in production ( $\frac{\Phi}{y}$ ) is difficult to identify as well, therefore it is set to 0. Table 2.8 also gives an overview over all other deep parameters which have been calibrated. While the simulations indicated for all models that  $J_2$  has full rank in all draws, in several specifications with right-to-manage  $J$  did not have full rank in about 1% of the respective draws from the prior distributions (suggesting that  $J_1$  did not have full rank in these few cases). Figure 2.3 shows the ‘identification strength’ and the ‘sensitivity component’ (at the prior mean) of all estimated parameters for the best-performing model (for the ranking see table 2.1); both are clearly positive for all estimated parameters.<sup>58</sup> The ‘identification strength’ is described by Ratto and Iskrev (2011)[page 13] as ‘sort of a priori t-test’ for the respective estimated parameters and the ‘sensitivity component’ indicates by how much moments of observable variables change with the respective parameter (for the computation of these indicators see section 4.2 in Ratto and Iskrev, 2011).

Note that revisions to quarterly Austrian macroeconomic data have been substantial over the last years. Fluctuations of several macroeconomic aggregates were significantly lower in old releases (because of changes in seasonal adjustment); this especially affects the growth rates of (real) private consumption and the GDP deflator, where volatilities were lower by about 80% respectively 60% (at very unreasonable levels) in old data releases. Running the same estimation procedures with one of these older releases can lead to severe problems when estimating some of the worse (in terms of identification and/or marginal likelihood) models. Furthermore the ordering of marginal likelihoods would be affected; for example, models with rigid real wages would do relatively better when using older data releases.

### 2.5.3 Calibration and prior distributions

As certain steady state ratios are of crucial importance for labor market dynamics and therefore for overall model fit (see for example Costain and Reiter, 2008), I ensure that all relevant steady state ratios are the same in all specifications (for given parameter estimates). Therefore I assume that – ‘by coincidence’ – in the steady state the FOC for working hours of right-to-manage AND efficient bargaining hold at the same time; meaning that  $\frac{\zeta_L H^{\sigma_L}}{\lambda} = p_L \zeta_x \gamma H^{\gamma-1} = w$  (for details see appendix

<sup>58</sup>The patterns for the other models on top of the list in table 2.1 are relatively similar.

2.B). This restriction is also one of the reasons why I included flow fixed costs for labor service firms in this model: When  $\gamma$  is significantly smaller than 1,  $p_L \zeta_x \gamma H^\gamma - wH = (1 - \gamma)p_L \zeta_x \gamma H^\gamma$  becomes very large and – if there were no fixed costs – so would the value of posting a vacancy. This would imply an implausibly large share of hiring costs in GDP and would significantly dampen the response of employment to changes in  $P_{L,t}$ . The assumptions made to ensure that the ‘relevant ratios’ do not change in case of endogenous job destruction and do change as little as possible in case of contemporaneous hiring are described in appendix 2.B.

Table 2.8 gives an overview over all calibrated parameters. The growth rate of the technology process  $\mu^a$ , the unemployment rate, and the shares of consumption, investment and exports (it is assumed that the trade balance is zero in steady state) in GDP are chosen to match average ratios respectively average growth rates in the raw data. The other values for domestic (non-labor-market) structural parameters are mainly taken from Fenz et al. (2012).  $\sigma_L$  is chosen to be rather high and the ‘steady state minimum wage’ (= sum of the flow value of being unemployed and the disutility of work) is assumed to make up 70% of the actual steady state wage. This specification would imply – when using contemporaneous hiring and exogenous job destruction (which will be the case in the best specification; see section 2.5.4) and taking prior means for estimated parameters – an employers’ bargaining power  $\nu$  of around 0.65.

The calibrated foreign (Euro area) parameters are taken from the estimates of Christoffel et al. (2008) and Fenz et al. (2012). All other parameters or relevant steady state ratios are estimated (see table 2.9) or implied by estimated and calibrated parameters (see appendix 2.B).

As the model is estimated using Bayesian methods, I need to choose appropriate priors for all estimated parameters. The priors are the same over all specifications. However, several parameters do not show up in all specifications ( $\tilde{g}$ ,  $\zeta_{wr}$ ,  $\zeta_w$ ), which is of importance as the applied Bayesian model selection procedure assigns penalties for the number of estimated parameters (see for example Del Negro and Schorfheide, 2011).

All priors are given in table 2.9. As a rule, I assume that shock variances follow an inverted Gamma distribution and that all parameters which are restricted to lie between 0 and 1 (mainly the shock autocorrelations) follow a Beta distribution. Other non-negative parameters which might be close to 0 (but which could theoretically be also larger than 1) are assumed to follow a Gamma distribution as the alternative of a restricted normal distribution would introduce a kink close to 0. The other parameters are assumed to be normally distributed.

For choosing the priors of the shock standard deviations I apply the following approach: All domestic shock standard deviations are given a prior mean of 1, except the shocks for the demand for investment goods, export goods and imports (reflecting the relatively high volatility of these aggregates) and of working hours (due to measurement errors) and government consumption (as the residual of equation (2.99) also accounts for statistical discrepancies and changes in inventories). The foreign shocks are assigned lower priors, reflecting that not all demand and supply shocks in the Euro area are symmetric such that they will partly cancel out in the aggregate.

## 2.5.4 Estimation results

Table 2.1 shows the Laplace approximation<sup>59</sup> of marginal likelihoods of all 44 different specifications. It shows that the variation with contemporaneous hiring, exogenous job destruction, rigid

<sup>59</sup>Note that the properties of the maximizers (e.g. the Hessian matrix being close to singular or even negative definite) were relatively poor for some models at the bottom of this table (which can be interpreted as a sign for misspecification of these specific models).

nominal wages, convex vacancy posting costs and efficient bargaining has the highest marginal likelihood. Furthermore, the following patterns can be observed:

- The best specifications all have Calvo wages, which indicates an important role for wage rigidities. Though their papers refer to US data and use different models (for example, they do not have endogenous working hours, which means that the interpretation of the wage rate is different), this result is in line with Riggi and Tancioni (2010), who claim that nominal wage rigidities are doing better than real wage rigidities, and with Gertler et al. (2008), according to whom specifications with nominally rigid wages tend to perform better than the ones with flexible wages. Furthermore, note that according to firm-level data analyzed by Babecky et al. (2010), automatic indexation of wages to inflation is rare in Austria, but at the same time Holden and Wulfsberg (2009) find evidence for significant real wage rigidities in Austria, too.
- The best specifications with exogenous destruction all have efficient bargaining. As already mentioned in section 2.3.2, efficient bargaining tends to lead to a lower volatility of average working hours (relative to employment in persons); in the data the volatility of working hours is 'only' about 70% larger than the one of the employment rate.
- Out of the specifications with right-to-manage, models with Nash Bargaining on wages do significantly better than the ones with a simple sharing rule for match surpluses.
- Typically, specifications with convex vacancy posting costs are doing better than their counterparts with linear ones in case of efficient bargaining, but worse in case of right-to-manage determination of working hours.
- All specifications at the top of table 2.1 have contemporaneous hiring. However, there is an interaction effect with the determination of working hours. When using efficient bargaining, there seems to be an important role for such a timely response of employment in persons to changes in economic conditions. However, when using right-to-manage, specifications with lagged hiring tend to do relatively better. Note that the data on employment  $N_t$  and vacancies  $V_t$  both refer to averages over quarters. So it is postulated (implicitly) in the best specifications that vacancies are filled immediately after they are posted (i.e. after zero days instead of 3 months in case of lagged hiring) and not only within the same quarter.
- The best specifications with endogenous destruction do significantly worse than their counterparts with exogenous destruction. This is not so surprising in case of contemporaneous hiring as this characteristic also decreases the role of  $e^\theta$  in the one-step-ahead-forecast error of (un)employment (without incurring the penalty on having to estimate the additional parameter  $\tilde{g}$ ). Interestingly, however, endogenous destruction also performs relatively worse in case of lagged hiring.

When assigning the same prior model probabilities to all 44 different specifications, the ratio of posterior probabilities  $P$  of models  $x$  and  $y$  can be computed from the log-likelihoods  $l$  as follows:  $\frac{P(y)}{P(x)} = \frac{\exp(l(y))}{\exp(l(x))} = \exp(l(y) - l(x))$  (see for example Hoeting et al., 1999). This means that the posterior probability of the best specification is more than  $\exp(11)$  higher than the second best one, which can be interpreted as decisive evidence against the 43 models with lower marginal likelihoods. Given how far marginal likelihoods are apart, assigning different prior



**Table 2.1:** Log-likelihoods for different model specifications

Model Specification					Log-likelihood	Model
Hiring	Job dest.	Vacancies	Hours	Wages	(Laplace approx.)	probability
Contemp.	Exogenous	Convex	Eff. barg.	Calvo	-1224.3	1.0000
Contemp.	Exogenous	Linear	Eff. barg.	Calvo	-1235.5	0.0000
Contemp.	Exogenous	Convex	Eff. barg.	Real rig.	-1249.9	0.0000
Contemp.	Endogenous	Convex	Eff. barg.	Real rig.	-1260.1	0.0000
Contemp.	Exogenous	Linear	Eff. barg.	Real rig.	-1262.1	0.0000
Lagged	Exogenous	Convex	Eff. barg.	Calvo	-1264.9	0.0000
Contemp.	Endogenous	Linear	Eff. barg.	Real rig.	-1270.0	0.0000
Lagged	Exogenous	Linear	Eff. barg.	Calvo	-1293.3	0.0000
Lagged	Exogenous	Convex	Eff. barg.	Real rig.	-1297.9	0.0000
Lagged	Endogenous	Convex	Eff. barg.	Real rig.	-1302.3	0.0000
Lagged	Exogenous	Linear	Eff. barg.	Real rig.	-1322.3	0.0000
Lagged	Endogenous	Linear	Eff. barg.	Real rig.	-1324.3	0.0000
Contemp.	Endogenous	Convex	Eff. barg.	Flexible	-1324.6	0.0000
Lagged	Exogenous	Linear	Right to man.	Calvo/Nash B.	-1334.0	0.0000
Lagged	Exogenous	Linear	Right to man.	Real rig./Nash B.	-1337.1	0.0000
Lagged	Exogenous	Linear	Right to man.	Flexible/Nash B.	-1337.5	0.0000
Lagged	Exogenous	Convex	Right to man.	Flexible/Nash B.	-1342.1	0.0000
Lagged	Exogenous	Convex	Right to man.	Real rig./Nash B.	-1344.4	0.0000
Lagged	Exogenous	Convex	Right to man.	Calvo/Nash B.	-1346.8	0.0000
Lagged	Exogenous	Linear	Right to man.	Real rig./Sharing	-1355.1	0.0000
Contemp.	Exogenous	Linear	Right to man.	Flexible/Nash B.	-1362.9	0.0000
Contemp.	Exogenous	Linear	Right to man.	Real rig./Nash B.	-1365.3	0.0000
Lagged	Exogenous	Convex	Right to man.	Real rig./Sharing	-1367.0	0.0000
Contemp.	Exogenous	Linear	Right to man.	Calvo/Nash B.	-1369.9	0.0000
Contemp.	Endogenous	Linear	Eff. barg.	Flexible	-1379.9	0.0000
Contemp.	Exogenous	Linear	Right to man.	Real rig./Sharing	-1384.3	0.0000
Contemp.	Exogenous	Convex	Right to man.	Flexible/Nash B.	-1388.5	0.0000
Contemp.	Exogenous	Convex	Right to man.	Real rig./Nash B.	-1391.9	0.0000
Lagged	Exogenous	Linear	Right to man.	Flexible/Sharing	-1395.6	0.0000
Contemp.	Exogenous	Convex	Right to man.	Calvo/Nash B.	-1398.4	0.0000
Contemp.	Exogenous	Linear	Right to man.	Calvo/Sharing	-1402.6	0.0000
Contemp.	Exogenous	Convex	Right to man.	Real rig./Sharing	-1420.8	0.0000
Lagged	Exogenous	Linear	Eff. barg.	Flexible	-1424.6	0.0000
Lagged	Endogenous	Linear	Eff. barg.	Flexible	-1425.3	0.0000
Lagged	Exogenous	Convex	Right to man.	Flexible/Sharing	-1438.7	0.0000
Contemp.	Exogenous	Linear	Right to man.	Flexible/Sharing	-1444.1	0.0000
Contemp.	Exogenous	Convex	Right to man.	Calvo/Sharing	-1455.7	0.0000
Contemp.	Exogenous	Linear	Eff. barg.	Flexible	-1457.4	0.0000
Lagged	Exogenous	Convex	Eff. barg.	Flexible	-1490.7	0.0000
Lagged	Endogenous	Convex	Eff. barg.	Flexible	-1491.2	0.0000
Contemp.	Exogenous	Convex	Right to man.	Flexible/Sharing	-1513.0	0.0000
Contemp.	Exogenous	Convex	Eff. barg.	Flexible	-1569.4	0.0000
Lagged	Exogenous	Linear	Right to man.	Calvo/Sharing	n.a.	0.0000
Lagged	Exogenous	Convex	Right to man.	Calvo/Sharing	n.a.	0.0000

model probabilities would not make a big difference. Note in this context that the difference between the second- and the third-best specification is even larger than between the first two specifications and that the top two models are very similar (the only difference being convexity of vacancy posting costs). The large difference in marginal likelihoods also indicates that some models can be seen as severe miss-specifications.

The preference of the selection procedure of rigid nominal (and also real) wages over flexible wages is partly due to the fixing of the steady state 'minimum wage'  $b + \frac{\xi_L H^{1+\sigma_L}}{(1+\sigma_L)\lambda}$ , as rigid nominal (or real) wages could be somehow approximated using flexible wages by assigning an even higher value to this expression and reducing the bargaining power of workers at the same time. It would, however, make a significant difference to the reaction of labor market variables to permanent changes in shock processes (the only unit root shock process in my model is  $\mu_t^a$ , which cannot permanently influence the unemployment rate due to the assumed indexation of hiring costs and unemployment benefits) and policy variables (not discussed in this paper; see for example Costain and Reiter, 2008). Note that the estimation procedure used the additional degree of freedom granted by the Calvo parameter to assign a much lower (steady state) bargaining power for employees, namely 0.15 instead of the 0.65 implied by the means of the prior distributions (via higher values for  $\gamma$  and  $q_U$ ).

Table 2.9 shows mean and standard deviation of the prior distributions and the mean and several quantiles of the posterior distribution of the estimated parameters of this best-performing model. The results for the posterior distribution are based on 500,000 Metropolis-Hastings-draws, where the first 50% are discarded. Note that in many cases the posterior distribution differs substantially from the prior, which is, however, not a sufficient condition for identification.

Tables 2.10, 2.11 and 2.12 show a comparison of empirical moments of the observable variables with the theoretical moments generated by the estimated policy function and the standard deviations of shock innovations. Table 2.10 shows that standard deviations and (first-order) autocorrelations are relatively similar in the data and the model. However, they tend to be underestimated in the model simulations, which is most severe in case of the growth in real consumption, real investment and the GDP deflator. The simulated standard deviations tend to be a bit lower than in the data, too; the relative difference is largest for working hours, the two domestic price variables and growth in (domestic) GDP. Tables 2.11 and 2.12 show that cross-correlations between observable variables are relatively similar, too. Larger deviations can be observed for the correlation of real wage growth with domestic macro variables and for the correlation of unemployment and vacancies with working hours, foreign output and interest rates.

## 2.6 Explaining fluctuations in Austrian unemployment

Over the last decade there have been plenty of papers on the (in)ability of models with search unemployment to generate certain stylized labor market facts. After a short overview over the literature I will apply the best model of the comparison exercise from the previous section to contribute to this discussion.

### 2.6.1 A short overview over the literature

In his seminal contribution, Shimer (2005) claims that models of search unemployment where productivity shocks are the only source of economic fluctuations are incapable of accounting for the high relative volatility of labor market tightness (and unemployment) compared to labor productivity;<sup>60</sup> in simulations of his calibrated model the relative volatility of labor market tightness is below 10% of the one in the data. He conducts his analysis in a simple model with

<sup>60</sup>For an alternative discussion of this problem see Costain and Reiter (2008).

exogenous job destruction, linear hiring costs, flexible wages (with a medium bargaining power for workers), fixed working hours and without any other frictions in the economy.<sup>61</sup> The intuition for Shimer's result is as follows: An increase in labor productivity raises – when hiring costs and the flow value of being unemployed are fixed<sup>62</sup> – the value of a future match for entrepreneurs; it therefore leads to an increase in hiring and in labor market tightness. However, it also means – in the absence of wage rigidities – a significant increase of real wages via the worker's share in the increased revenue and via the increase in the value of being unemployed (due to the higher labor market tightness). These two channels significantly dampen the increase in the value of a match to the entrepreneur and thereby also the incentive to post more vacancies.

Among others, Mortensen and Nagypal (2007), Hagedorn and Manovskii (2008), Christoffel and Kuester (2009) and Pissarides (2009) discuss possible alternative approaches to get a higher relative volatility in models where labor productivity is the only source of aggregate fluctuations. Mortensen and Nagypal (2007) argue that several different deviations from the specification of Shimer (2005) can lead to a substantially higher relative volatility of the labor market tightness, for example introducing on-the-job-search. Hagedorn and Manovskii (2008) get to the desired results by raising both the bargaining power of firms and the outside option of workers to values close to 1. However, their calibration would imply that slight changes in unemployment benefits would lead to implausibly high changes in the unemployment rate (see Costain and Reiter, 2008). Christoffel and Kuester (2009) increase the relative volatility of unemployment in a model with endogenous working hours by using flow fixed costs of maintaining a match (like the ones introduced in equation (2.42))<sup>63</sup> and argue that specifications with right-to-manage-determination of working hours are also able to yield the desired level of the derivative of the unemployment rate with regard to unemployment benefits. Pissarides (2009) argues (referring to the empirical evidence discussed at the end of section 2.3.2) that wage rigidities should not be used to solve this volatility puzzle (like in Hagedorn and Manovskii, 2008) and adds a fixed matching component to the proportional vacancy cost of hiring.

Another possibility to increase the relative volatility of labor market tightness would be to find other sources of labor market fluctuations which do not have such a strong influence on labor productivity. One can argue that trying this is already mandated by the fact that the (absolute value of the) correlations of labor market tightness and the unemployment rate with labor productivity are rather low (as shown in table 2.3).<sup>64</sup> This could be done – still consistent with the basic philosophy of RBC models – via productivity shocks which do not have a strong immediate influence on labor productivity. For example, Faccini and Ortigueira (2010) show that investment-specific technology shocks can help increase the relative volatility of the unemployment rate compared to labor productivity.

One could also try to incorporate different real and monetary shocks on the demand side in a New Keynesian DSGE setting. However, relatively few of the contributions in this field try to explain short-term fluctuations of unemployment, the main focus seems to be on the implications

<sup>61</sup>His analysis is in continuous time, so no distinction between lagged hiring and contemporaneous hiring can be made. Furthermore, there are no analogues to the stages 2, 3 and 4 in my model.

<sup>62</sup>Note again that – to ensure stationarity of the unemployment rate – it is assumed in my model that the flow value of being unemployed  $b$ , fixed costs  $\Phi_L$  and hiring costs  $c_{V,t}$  are indexed to the unit-root labor productivity process  $A_t$  (but not to the stationary total factor productivity process  $e_t^A$ ).

<sup>63</sup>They previously employed such fixed costs in a New Keynesian DSGE model (see Christoffel and Kuester, 2008).

<sup>64</sup>Note that only in RBC models unemployment necessarily reacts negatively on a positive shock to labor productivity. In this model with significant nominal and real frictions, unemployment initially increases after a positive shock to labor productivity (see also figure 2.6).

of search unemployment for fluctuations of inflation and output in the context of monetary policy shocks. Notable exceptions to that rule are the calibrated models of Sveen and Weinke (2008) and Christoffel and Kuester (2008), who claim that demand and monetary policy shocks can contribute to explaining US unemployment fluctuations when choosing certain labor market specifications.

## 2.6.2 The ability of my model to generate stylized facts of Austria

Table 2.2 compares the standard deviations of measures for labor productivity with the one of labor market variables<sup>65</sup> and shows relatively similar patterns for the Austrian data used in this paper and for the US data used by Shimer (2005). While the absolute standard deviations are not comparable due to different construction and filtering of the data,<sup>66</sup> in both cases the standard deviation of output per worker is substantially lower than the one of the labor market tightness (the difference in the US being larger). Table 2.3 shows that the correlations of labor market variables with measures for productivity are similar to the US case, too. The correlation between unemployment and vacancies (not shown in table 2.3) is relatively close to -1 in both cases; it is -0.70 in my data and -0.89 in the US data of Shimer (2005).

**Table 2.2:** Observed standard deviations and autocorrelations of labor market and productivity variables

		Austria		US (Shimer, 2005)	
		St. dev.	Autocorr.	St. dev.	Autocorr.
GDP per worker (unit root)	$\frac{Y}{N}$	1.2	0.78	-	-
GDP per hour (unit root)	$\frac{Y}{NH}$	1.0	0.76	-	-
GDP per worker (stationary)	$\frac{y}{N}$	1.9	0.84	2.0	0.88
GDP per hour (stationary)	$\frac{y}{NH}$	1.4	0.84	-	-
Unemployment	$U$	8.7	0.87	19.0	0.94
Vacancies	$V$	14.6	0.92	20.2	0.94
Labor market tightness	$\frac{V}{U}$	21.6	0.92	38.2	0.94

**Table 2.3:** Observed correlations of labor market variables with measures for productivity

		AT $\frac{Y}{N}$ (unit root)	AT $\frac{Y}{NH}$ (unit root)	AT $\frac{y}{N}$ (stationary)	AT $\frac{y}{NH}$ (stationary)	US $\frac{y}{N}$ (stationary)
Unemployment	$U$	-0.12	-0.22	-0.19	-0.31	-0.41
Vacancies	$V$	0.51	0.63	0.25	0.36	0.36
Labor market tightness	$\frac{V}{U}$	0.40	0.52	0.25	0.37	0.40

As table 2.12 shows, the simulated correlation of  $U$  and  $V$  is around -0.85 (which is rounded to -0.9 in the table) compared to -0.7 in the data (table 2.11). A comparison of tables 2.2 and

<sup>65</sup>Note that – as also indicated in tables 2.2, 2.3, 2.4 and 2.5 – labor market tightness refers is defined as the ratio of vacancies over unemployment in the context of this section. This stands in contrast to the definition given by equation (2.66), which is used in the best-performing model.

<sup>66</sup>Shimer (2005) looks at output and vacancies in the non-farm business sector only and does not divide output, unemployment and vacancies by the labor force. Furthermore, he uses an HP filter with the smoothing parameter  $\lambda = 10^5$  to detrend the quarterly data for labor market and productivity variables. As the subtraction of a quadratic trend does not remove a potential unit root from the output series (in contrast to the HP filter; the filtering of data in this paper is described in subsection 2.5.1), I also report moments of  $y_t := \frac{Y_t}{A_t}$ . This has the (non-negligible) disadvantage that an estimated data series (namely  $A_t$ ) is needed to calculate 'observed' moments.



**Table 2.7:** Forecast error variance decomposition for labor market tightness  $\frac{V}{U}$ 

Shock process		Forecasting horizon						
		1	2	4	8	16	40	100
Productivity (temporary)	$\epsilon^a$	0.244	0.224	0.194	0.135	0.110	0.117	0.115
Productivity (permanent)	$\epsilon^{\mu a}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Government consumption	$\epsilon^G$	0.033	0.021	0.012	0.008	0.006	0.005	0.005
Investment	$\epsilon^i$	0.016	0.012	0.009	0.005	0.005	0.004	0.004
Labor supply	$\epsilon^L$	0.064	0.058	0.051	0.037	0.028	0.025	0.025
Domestic inflation	$\epsilon^{\lambda_p}$	0.048	0.093	0.219	0.456	0.570	0.591	0.595
Matching efficiency	$\epsilon^\theta$	0.066	0.054	0.041	0.028	0.021	0.018	0.018
Import demand	$\epsilon^m$	0.082	0.089	0.084	0.062	0.047	0.043	0.043
Export demand	$\epsilon^{mf}$	0.261	0.268	0.243	0.172	0.128	0.118	0.118
Risk premium	$\epsilon^{rp}$	0.171	0.162	0.129	0.080	0.068	0.062	0.062
Import prices	$\epsilon^{\pi m}$	0.004	0.002	0.001	0.001	0.001	0.001	0.001
World inflation	$\epsilon^{\pi f}$	0.002	0.002	0.004	0.006	0.006	0.005	0.006
World interest rate	$\epsilon^R$	0.011	0.012	0.011	0.008	0.006	0.006	0.006
World demand	$\epsilon^{yf}$	0.000	0.000	0.000	0.001	0.003	0.003	0.003
Sum		1.000	1.000	1.000	1.000	1.000	1.000	1.000

Tables 2.6 and 2.7 show a forecast error variance decomposition<sup>67</sup> for unemployment and labor market tightness. It shows that the relative role of productivity shocks in explaining labor market variables is smaller than the one of (both domestic and foreign) demand shocks: Taken together, the risk premium shock<sup>68</sup> and the export demand shock<sup>69</sup> contribute significantly more to fluctuations of  $U$  and  $\frac{V}{U}$  than the productivity shocks. The role of the temporary productivity shock is not negligible, though. The contribution of the unit-root-productivity shock is very close to zero. This does not come as a big surprise as it is assumed that hiring costs and unemployment benefits are indexed to  $A_t$ .<sup>70</sup> When looking at longer forecasting horizons, the shock to the mark-up-process is the most important shock from the supply side.

Figures 2.4 to 2.10 in the appendix show the impulse responses of vacancies, unemployment, labor market tightness, output per worker and output per hour to the shocks with the largest contributions (where the dotted lines refer to the second variable on the respective sub-figures). Figure 2.6 shows that an increase in aggregate productivity pushes output per worker (or per hour) and unemployment into the same direction – a feature which is not atypical in New Keynesian models with rigid prices; it takes about 10 quarters after an innovation to  $e_t^a$  until the deviations of labor market tightness and unemployment from their steady state have the respective 'RBC-sign'. The demand shocks  $e_t^m$ ,  $e_t^{mf}$  and  $e_t^{rp}$  (figures 2.4, 2.5 and 2.10) push them into different directions and can help to produce the positive correlation between productivity

<sup>67</sup>This decomposition links the impulse responses of a certain variable to all possible shocks with the standard variances of these shocks (the textbook of Luetkepohl, 2006, provides an explanation of this concept on pages 63-66.). Tables 2.13, 2.14 and 2.15 in the appendix also show decompositions for working hours, output per worker and output per hour.

<sup>68</sup>The risk premium shock  $e_t^{rp}$  should be interpreted as domestic demand shock as an increase in  $e_t^{rp}$  leads to a decrease of both private consumption and investment for given Euro area interest rates. Note that the domestic interest rate  $R_t^f \bar{\phi}_{rp}$  ( $nfa_t, e_t^{rp}$ ) is treated as unobservable variable in this paper.

<sup>69</sup>The export demand shock  $e_t^{mf}$  captures developments in exports which cannot be directly related to movements in Euro area output and relative prices. The strong contribution from the import demand shock  $e_t^m$  should be related to the fact that exports are much more volatile than consumption and investment and that at the same time the import share of exports is much higher than that of private and government consumption.

<sup>70</sup>This was necessary to ensure that unemployment is stationary as a transformation would have been more difficult than with most other macroeconomic aggregates (like GDP, investment, ...). The alternative to this specification would have been to apply an HP-filter to the other domestic macroeconomic quantities. This would remove the unit root and would make  $A_t$  irrelevant.

measures and labor market tightness (table 2.3). It should also be noted that the changes in the two productivity measures (output per worker, output per hour) are driven by temporary productivity shocks to a very small extent (see tables 2.14 and 2.15).<sup>71</sup>

A further interesting aspect of the forecast error variance decomposition is that the contribution of the residual in the law of motion of (un-)employment, the matching efficiency process  $e_t^\theta$ , is 'only' about 50% for short horizons. These results stand in stark contrast to Lubik (2009), in whose smaller search unemployment model for the US economy a similar shock explains practically the whole variation in US unemployment, and to Konya and Krause (2009), where shocks to matching efficiency and to the outside option (equivalent to  $b$  in this paper) explain almost all of its fluctuations. Working hours are exogenous in both papers, so there the number of workers is the only available margin for adjusting employment after demand or supply shocks. Figure 2.9 shows that  $e_t^\theta$  pushes vacancies and unemployment into the same direction, which is at odds with the empirical evidence (see above). Higher matching efficiency increases the probability of finding a job (and therefore decreases unemployment) but at the same time less vacancy posting is necessary for entrepreneurs to get the desired number of new workers.<sup>72</sup> The role of  $e_t^\theta$  in explaining fluctuations in  $\frac{V}{U}$  is negligible. This is not surprising, however, as the matching efficiency shock serves as a wedge between (un)employment and vacancies (see also section 2.3.1).

#### 2.6.4 Historical shock decompositions for 2007 to 2011

Another way to look at the relative contribution of different shocks to movements in Austrian labor market variables is to conduct a historical shock decomposition, which is another by-product of the Bayesian estimation (combining estimation residuals with the policy function). This decomposition shows which shocks (i.e. which parts of the vector  $\epsilon_t$ ) can be attributed to the observed changes in variables. Figures 2.1 and 2.2 show the results of such a decomposition for labor market tightness and unemployment from 2007 to 2011 (Fenz et al., 2012, make a similar exercise with Austrian GDP in this time period).<sup>73</sup> While the decline in Austrian GDP from mid-2008 to mid-2009 has been substantial, the increase in unemployment has been relatively low by international standards (similar to Germany; see for example Stiglbauer, 2010) and it has been accompanied by a decline in working hours per employee. If working hours had not been used as endogenous observable variable, the contribution of the matching efficiency shock would have likely been larger to be able to capture this 'too small' response of employment (in persons) to the decline in economic activity.

Similar to the forecast error variance decomposition, figures 2.1 and 2.2 indicate that there is only a very limited role for (domestic and/or foreign) productivity shocks<sup>74</sup>, while external and domestic demand shocks explain a large part of the fluctuations (in case of the change in unemployment the matching efficiency shock is most important). As these 2 figures try to explain

<sup>71</sup>Like before in section 2.6.2, the unit-root-productivity process  $A_t$  was deducted from  $\frac{Y_t}{N_t}$  and  $\frac{Y_t}{\int_0^{N_t} H_t}$ . This is necessary as  $A_t$  dominates all other shock processes for longer forecasting horizons, but at the same time it has no influence on unemployment and labor market tightness (see again tables 2.6 and 2.7).

<sup>72</sup>Note, however, that vacancies are also treated as observable variable in Lubik (2009) and Konya and Krause (2009) (only for the US).

<sup>73</sup>To make these figures more readable, the 14 shocks have been allocated to 7 different groups (as explained in table 2.16).

<sup>74</sup>Note again that the unit-root process  $A_t$  not only affects Austrian productivity, but also the one of the rest of the world and that hiring costs and unemployment benefits are indexed to  $A_t$ .

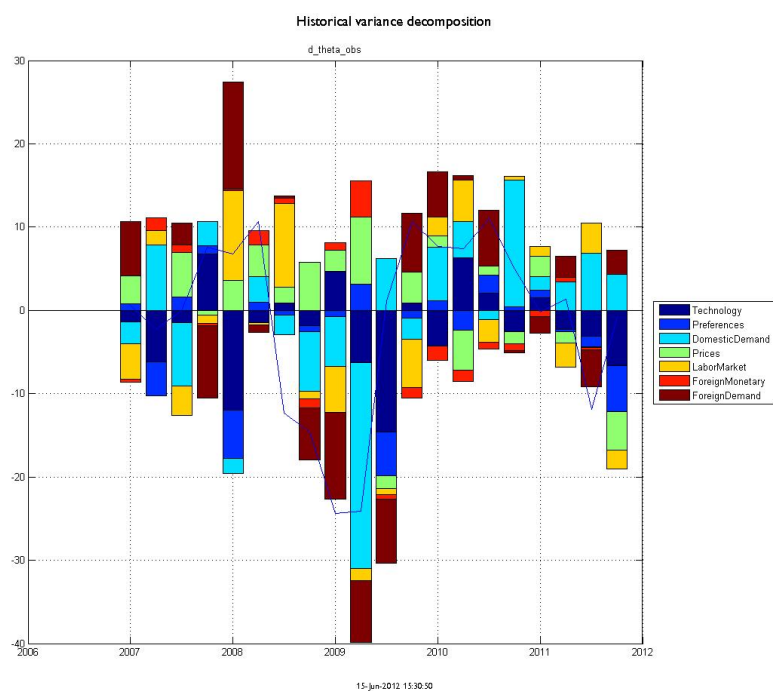


Figure 2.1: Historical shock decomposition for the change in labor market tightness

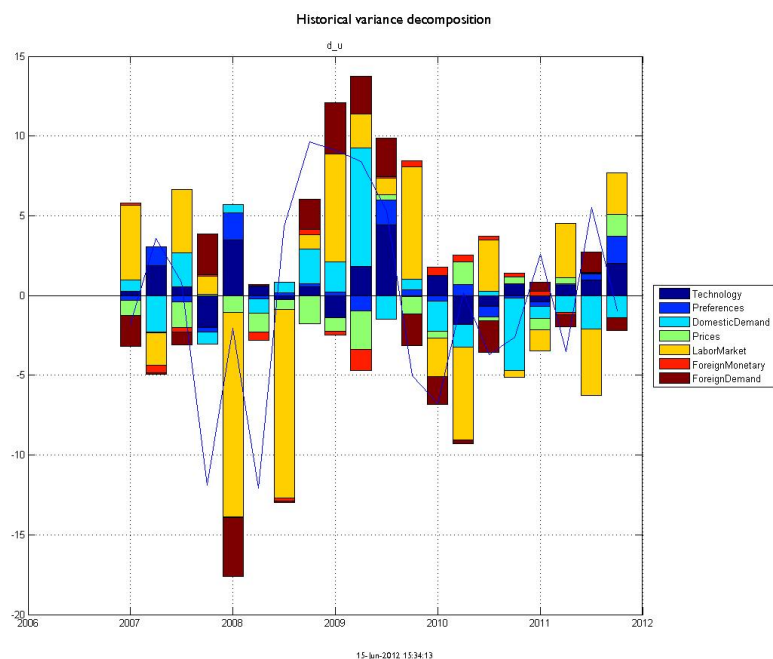


Figure 2.2: Historical shock decomposition for the change in the unemployment rate

the changes in variables (in contrast to tables 2.6 and 2.7, where deviations from steady state are analyzed), the contributions can be compared to the forecast error variance decomposition only for shorter horizons (where matching efficiency and demand shocks play a more significant



role) and the sum of the 'contributions' can substantially differ from the change in the variable in certain quarters (like in early 2008). Both figures indicate that in 2008/09 foreign demand contributed to the worsening of labor market variables relatively earlier than domestic demand. Due to the assumed Ricardian equivalence and the fact that government consumption shocks also include changes in inventories, not much can be said about the contribution of fiscal policy to changes in unemployment and vacancies in the context of this model.

## 2.7 Conclusions and possible extensions

Among the 44 compared specifications, the variation with contemporaneous hiring, exogenous job destruction, rigid nominal wages, convex vacancy posting costs and efficient bargaining has the highest marginal likelihood. Most importantly, the model comparison indicated a large role for wage rigidities and for a timely response of unemployment to changes in economic conditions. The best-performing model can reproduce relative volatilities of Austrian labor market variables relatively well, but the issue of too high simulated absolute volatilities deserves further investigation. Furthermore, shock decompositions indicate that fluctuations in Austrian labor market tightness and unemployment are driven more by demand shocks than by productivity shocks. And while the role of the residual (matching efficiency shock  $\varepsilon_t^{\theta}$ ) in the law of motion of employment is substantial, it is significantly smaller than in comparable contributions to the literature.

Possible extensions to my work include a systematic prior sensitivity analysis and changes in the overall setting which make more combinations with endogenous job destruction possible.

## Appendix

### 2.A Derivations of certain labor market equations

#### Hiring condition

Using that  $J_{V,i,t} = 0$  in equilibrium, (2.43) becomes:

$$\beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} J_{E,new,t+1} = \mathbb{E}_t \frac{A_t c_{V,t}}{q_V(\theta_t, e_{t+1}^\theta)}. \quad (2.140)$$

Putting that into (2.42) and considering that new matches get the average hourly wage yields:

$$J_{E,new,t} = P_{L,t} \zeta_x H_{new,t}^\gamma - A_t \Phi_L - W_t H_{new,t} + \mathbb{E}_t (1 - \delta) \frac{A_t c_{V,t}}{q_V(\theta_t, e_{t+1}^\theta)}. \quad (2.141)$$

Iterating (2.141) one period forward and plugging that into (2.140) yields (2.46).

#### Nash Bargaining

Maximizing  $J_{E,i,t}^v (J_{W,i,t} - J_{U,t})^{1-v}$  with respect to the wage rate yields:

$$v (J_{W,i,t} - J_{U,t}) \frac{\partial (J_{W,i,t} - J_{U,t})}{\partial W_{i,t}} + (1 - v) J_{E,i,t} \frac{\partial J_{E,i,t}}{\partial W_{i,t}} = 0.$$

In case of efficient bargaining ( $\frac{\partial (J_{W,i,t} - J_{U,t})}{\partial W_{i,t}} = H_{i,t}$  and  $\frac{\partial J_{E,i,t}}{\partial W_{i,t}} = -H_{i,t}$ ) this reduces to:

$$v (J_{W,i,t} - J_{U,t}) = (1 - v) J_{E,i,t}.$$

In case of right-to-manage the number of hours depends directly on the wage rate (rearranging (2.49) yields  $H_{i,t} = \left( \frac{P_{L,t} \zeta_x \gamma}{W_{i,t}} \right)^{\frac{1}{1-\gamma}}$ ), which makes things more complicated:

$$\begin{aligned} \frac{\partial (J_{W,i,t} - J_{U,t})}{\partial W_{i,t}} &= H_{i,t} + \frac{\partial H_{i,t}}{\partial W_{i,t}} (W_{i,t} - MRS_{i,t}) = \frac{H_{i,t}}{1-\gamma} \left( \frac{MRS_{i,t}}{W_{i,t}} - \gamma \right), \\ \frac{\partial J_{E,i,t}}{\partial W_{i,t}} &= \frac{\partial H_{i,t}}{\partial W_{i,t}} (P_{L,t} \zeta_x \gamma H_{i,t}^{\gamma-1} - W_{i,t}) - H_{i,t} = -H_{i,t}, \end{aligned}$$

where I made use of the fact that  $\frac{\partial H_{i,t}}{\partial W_{i,t}} = \frac{-1}{1-\gamma} \left( \frac{P_{L,t} \zeta_x \gamma}{W_{i,t}} \right)^{\frac{1}{1-\gamma}} \frac{1}{W_{i,t}} = \frac{-1}{1-\gamma} \frac{H_{i,t}}{W_{i,t}}$  and  $MRS_{i,t} = \frac{e_i^\zeta e_i^L \zeta_L H_{i,t}^{\sigma_L}}{\Lambda_t}$ . So the first order condition for the maximization problem from above is:

$$v \frac{\frac{MRS_{i,t}}{W_{i,t}} - \gamma}{1-\gamma} (J_{W,i,t} - J_{U,t}) = (1 - v) J_{E,i,t}. \quad (2.142)$$

**Nash bargaining in case of right-to-manage:** Subtracting equation (2.45) from (2.44) and rearranging leads to:

$$\begin{aligned} J_{W,i,t} - J_{U,t} - \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} (1 - \delta) (J_{W,i,t+1} - J_{U,t+1}) &= \\ = W_{i,t}^{nb} H_{i,t} - \left( A_t b + \frac{e_t^L e_t^c \zeta_L}{(1 + \sigma_L) \Lambda_t} H_{i,t}^{1+\sigma_L} \right) - \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} q_U(\theta_t, e_{t+1}^\theta) (J_{W,new,t+1} - J_{U,t+1}) \end{aligned} \quad (2.143)$$

Using (2.142), (2.143) can be rewritten to:

$$\begin{aligned} \frac{1 - \nu}{\nu} \frac{1 - \gamma}{\frac{MRS_{i,t}}{W_{i,t}^{nb}} - \gamma} J_{F,i,t} - \beta \frac{1 - \nu}{\nu} (1 - \delta) \mathbb{E}_t \frac{1 - \gamma}{\frac{MRS_{i,t+1}}{W_{i,t+1}^{nb}} - \gamma} \frac{\Lambda_{t+1}}{\Lambda_t} J_{F,i,t+1} &= \\ = W_{i,t}^{nb} H_{i,t} - \left( A_t b + \frac{e_t^L e_t^c \zeta_L}{(1 + \sigma_L) \Lambda_t} H_{i,t}^{1+\sigma_L} \right) - \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} q_U(\theta_t, e_{t+1}^\theta) (J_{W,new,t+1} - J_{U,t+1}) \end{aligned} \quad (2.144)$$

The LHS of (2.144) can be rearranged by using (2.42):

$$\begin{aligned} \frac{1 - \nu}{\nu} \frac{1 - \gamma}{\frac{MRS_{i,t}}{W_{i,t}^{nb}} - \gamma} J_{F,i,t} - \beta \frac{1 - \nu}{\nu} (1 - \delta) \mathbb{E}_t \frac{1 - \gamma}{\frac{MRS_{i,t+1}}{W_{i,t+1}^{nb}} - \gamma} \frac{\Lambda_{t+1}}{\Lambda_t} J_{F,i,t+1} &= \\ = \frac{(1 - \nu)(1 - \gamma)}{\nu \left( \frac{MRS_{i,t}}{W_{i,t}^{nb}} - \gamma \right)} \left( J_{F,i,t} - \beta (1 - \delta) \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} J_{F,i,t+1} \right) & \\ + \beta \frac{1 - \nu}{\nu} (1 - \delta) \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} J_{F,i,t+1} \left( \frac{1 - \gamma}{\frac{MRS_{i,t}}{W_{i,t}^{nb}} - \gamma} - \frac{1 - \gamma}{\frac{MRS_{i,t+1}}{W_{i,t+1}^{nb}} - \gamma} \right) & \\ = \frac{1 - \nu}{\nu} \frac{1 - \gamma}{\frac{MRS_{i,t}}{W_{i,t}^{nb}} - \gamma} (P_{L,t} H_{i,t}^\gamma - A_t \Phi_L - W_{i,t}^{nb} H_{i,t}) & \\ + \beta \frac{1 - \nu}{\nu} (1 - \delta) \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} J_{F,i,t+1} \left( \frac{1 - \gamma}{\frac{MRS_{i,t}}{W_{i,t}^{nb}} - \gamma} - \frac{1 - \gamma}{\frac{MRS_{i,t+1}}{W_{i,t+1}^{nb}} - \gamma} \right) & \end{aligned}$$

Using this, (2.144) can be rewritten to:

$$\begin{aligned} \left( 1 + \frac{1 - \nu}{\nu} \frac{1 - \gamma}{\frac{MRS_{i,t}}{W_{i,t}^{nb}} - \gamma} \right) W_{i,t}^{nb} H_{i,t} &= \frac{1 - \nu}{\nu} \frac{1 - \gamma}{\frac{MRS_{i,t}}{W_{i,t}^{nb}} - \gamma} (P_{L,t} H_{i,t}^\gamma - A_t \Phi_L) + A_t b + \frac{e_t^L e_t^c \zeta_L}{(1 + \sigma_L) \Lambda_t} H_{i,t}^{1+\sigma_L} \\ &+ \beta \frac{1 - \nu}{\nu} (1 - \delta) \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} J_{F,i,t+1} \left( \frac{1 - \gamma}{\frac{MRS_{i,t}}{W_{i,t}^{nb}} - \gamma} - \frac{1 - \gamma}{\frac{MRS_{i,t+1}}{W_{i,t+1}^{nb}} - \gamma} \right) \\ &+ \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} q_U(\theta_t, e_{t+1}^\theta) (J_{W,new,t+1} - J_{U,t+1}). \end{aligned}$$

**Nash bargaining in case of efficient bargaining:** By Nash bargaining (efficient bargaining case)/surplus sharing (right-to-manage) it holds that in firm  $i$  for all  $t$ :

$$J_{W,i,t} - J_{U,t} = \frac{1-\nu}{\nu} J_{F,i,t}. \quad (2.145)$$

So (2.143) can be rearranged to:

$$\begin{aligned} & \frac{1-\nu}{\nu} \left( J_{F,i,t} - \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} (1-\delta) J_{F,i,t+1} \right) = \\ & = W_{i,t}^{nb} H_{i,t} - \left( A_t b + \frac{e_t^L e_t^c \zeta_L}{(1+\sigma_L) \Lambda_t} H_{i,t}^{1+\sigma_L} \right) - \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} q_U(\theta_t, e_{t+1}^\theta) (J_{W,new,t+1} - J_{U,t+1}). \end{aligned}$$

The LHS of the latter equation can be simplified by using (2.42):

$$\begin{aligned} & \frac{1-\nu}{\nu} (P_{L,t} H_{i,t}^\gamma - A_t \Phi_L - W_{i,t}^{nb} H_{i,t}) = \\ & = W_{i,t}^{nb} H_{i,t} - \left( A_t b + \frac{e_t^L e_t^c \zeta_L}{(1+\sigma_L) \Lambda_t} H_{i,t}^{1+\sigma_L} \right) - \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} q_U(\theta_t, e_{t+1}^\theta) (J_{W,new,t+1} - J_{U,t+1}). \end{aligned}$$

Solving for  $W_{i,t}^{nb}$  finally yields (2.50).

### Endogenous job destruction

**New equation for wages:** Subtracting equation (2.60) from (2.59) yields:

$$\begin{aligned} J_{W,i,t} - J_{U,t} &= W_{i,t} H_{i,t} - \frac{e_t^L e_t^c \zeta_L}{(1+\sigma_L) \Lambda_t} H_{i,t}^{1+\sigma_L} - A_t b - A_t \zeta_b b_{i,t} \\ &+ \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left( (1-\delta) G(\tilde{b}_{t+1}) (J_{W,i,t+1} - J_{U,t+1}) - q_U(\theta_t, e_{t+1}^\theta) G(\tilde{b}_{t+1}) (J_{W,new,t+1} - J_{U,t+1}) \right). \end{aligned}$$

Using that

$$\begin{aligned} & J_{W,i,t} - J_{U,t} - \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} (1-\delta) G(\tilde{b}_{t+1}) (J_{W,i,t+1} - J_{U,t+1}) = \\ & = \frac{1-\nu}{\nu} \left( J_{F,i,t} - \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} (1-\delta) G(\tilde{b}_{t+1}) J_{F,i,t+1} \right) \\ & = \frac{1-\nu}{\nu} (P_{L,t} \zeta_x H_t^\gamma - W_{i,t} H_t) \end{aligned}$$

and solving for  $W_{i,t}^{nb} H_t$  leads to (2.62).

**New equation for hiring:** Using that  $J_{V,t} = 0$ , (2.58) can be expressed as follows:

$$\frac{A_t c_{V,t}}{q_V(\theta_t, e_{t+1}^\theta)} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} G(\tilde{b}_{t+1}) J_{F,new,t+1}. \quad (2.146)$$

Putting this relation into (2.57) leads to

$$J_{F,new,t} = P_{L,t} \zeta_x H_t^\gamma - A_t \Phi_L - W_t H_t + \mathbb{E}_t (1-\delta) \frac{A_t c_{V,t}}{q_V(\theta_t, e_{t+1}^\theta)}; \quad (2.147)$$

iterating one period forward and plugging back into (2.146) yields (2.61).

## 2.B Further information on calibration and estimation strategy

### Implied parameters and ratios in all settings

- The share  $\mu$  of domestic goods for the production of final goods is given by  $\mu = \frac{1}{1+x_y}$ .
- The steady state return on capital is equal to the (net) interest rate on bonds plus a compensation for depreciation of the real capital stock:  $r_k = \frac{\mu^a}{\beta} - 1 + \tau$ .
- The mark-up in steady state can be computed by using that, due to the specification of production and price setting, capital must get a share of  $\alpha \frac{1+\frac{\Phi}{y}}{1+\lambda_p}$  of overall GDP as remuneration:  $\lambda_p = \frac{\alpha(1+\frac{\Phi}{y})}{r_k k_y} - 1$  where  $k_y := \frac{k}{y} = \frac{i_y}{1-\frac{1-\tau}{\mu^a}}$ .
- The steady state revenue of labor service firms  $p_L \zeta_x H^\gamma$  can be derived using that labor services must get a share of  $(1-\alpha) \frac{1+\frac{\Phi}{y}}{1+\lambda_p}$  of overall GDP as remuneration. So  $p_L \zeta_x H^\gamma = \frac{(1-\alpha)(y+\Phi)}{N(1+\lambda_p)}$ .
- As said before, I assume that 'by coincidence' in steady state both optimality conditions for working hours (efficient bargaining and right-to-manage) hold: This implies  $wH = \gamma p_L \zeta_x H^\gamma = \gamma \frac{(1-\alpha)(y+\Phi)}{N(1+\lambda_p)}$  and  $\frac{\zeta_L H^{1+\sigma_L}}{(1+\sigma_L)\lambda} = \gamma \frac{p_L \zeta_x H^\gamma}{1+\sigma_L} = \frac{\gamma}{1+\sigma_L} \frac{(1-\alpha)(y+\Phi)}{N(1+\lambda_p)}$ .

### Implied parameters and ratios in standard setting

- As unemployment is constant in the steady state, job creation needs to be equal job destruction. This leads to:  $\delta = \frac{U}{1-U} q_U$ .
- The flow value of being unemployed  $b$  is implied by the calibration that  $b + \frac{\zeta_L H^{1+\sigma_L}}{(1+\sigma_L)\lambda} = 0.7wH$ .
- The difference between the values of being employed and unemployed in steady state is given by:  $j_{wu} = \frac{wH - (b + \frac{\zeta_L H^{1+\sigma_L}}{\lambda})}{1 - \beta(1 - \delta - q_U)}$ .
- The flow fixed cost of labor service firms  $\Phi_L$  can be derived using that in steady state the hiring condition is  $\frac{c_V}{q_V} = \beta \left( p_L \zeta_x H^\gamma - \Phi_L - wH + (1 - \delta) \frac{c_V}{q_V} \right)$ . Solving for  $\Phi_L$  yields  $\Phi_L = p_L \zeta_x H^\gamma - wH + \frac{1 - \beta(1 - \delta)}{\beta} \frac{c_V}{q_V}$ .
- The firm's bargaining power in wage determination can be derived by using the wage equation in steady state:  $wH = (1 - \nu)(p_L \zeta_x H^\gamma - \Phi_L) + (1 - \nu)c_V \theta + \nu \left( b + \frac{H^{1+\sigma_L}}{(1+\sigma_L)\lambda} \right)$ ,<sup>75</sup> which leads to  $\nu = \frac{p_L \zeta_x H^\gamma - \Phi_L + c_V \theta - wH}{p_L \zeta_x H^\gamma - \Phi_L + c_V \theta - \left( b + \frac{H^{1+\sigma_L}}{(1+\sigma_L)\lambda} \right)}$ .

<sup>75</sup>Evaluating (2.50) at the steady state, one gets  $wH = (1 - \nu)(p_L \zeta_x H^\gamma - \Phi_L) + \nu \left( b + \frac{\zeta_L}{(1+\sigma_L)\lambda} H^{1+\sigma_L} \right) + \nu \beta q_U j_{wu}$ . Using that in the steady state there is Nash bargaining sharing in all matches, one can rearrange  $\nu \beta q_U j_{wu} = \nu \beta q_U \frac{1-\nu}{\nu} j_f = (1 - \nu) \beta q_U \frac{c_V}{\beta q_V} = (1 - \nu) c_V \theta$ , where  $j_f$  is the steady state value of  $\frac{J_{E, new, t}}{A_t}$ .

### Rescaling of parameters and estimation strategy in case of endogenous job destruction

When implementing endogenous job destruction, I assume that overall job destruction and overall job creation in steady state are like in the case of exogenous destruction.<sup>76</sup>

- For deriving new values for  $q_U$  and  $q_V$ , one only needs to bear in mind that a share  $\tilde{G}$  of new matches is destroyed before they actually start to work. So  $q_{U,end} = \frac{q_{U,exo}}{\tilde{G}}$  and  $q_{V,end} = \frac{q_{V,exo}}{\tilde{G}}$ .
- Overall job destruction is given by  $\delta N$  in case of exogenous destruction and  $(1 - \tilde{G}(1 - \delta))N$  in case of endogenous destruction. So I need that  $\delta_{exo} = 1 - \tilde{G}(1 - \delta_{end})$  which implies  $\delta_{end} = 1 - \frac{1 - \delta_{exo}}{\tilde{G}}$ .

The only additional variable which will be estimated is  $\tilde{g}$ . The other variables are calibrated and/or computed as follows:

- It is assumed that 75% of job destruction is endogenous in steady state (so  $\frac{\tilde{G}}{\delta_{exo}} = 0.75$ ).
- It is assumed that  $b_{end} = 0.6b_{exo}$ , which implies that  $\int_0^{\tilde{b}} bg(b)db = 0.4\tilde{G}b_{exo}$ .
- Let  $\tilde{w}\tilde{H}$  denote the wage in a match which is at the margin of being destroyed. In the steady state it holds that  $0 = p_L\zeta_x H^\gamma - \Phi_L - \tilde{w}\tilde{H} + (1 - \delta)\frac{c_V}{q_V(\theta)}$ , where  $\tilde{w}\tilde{H} = wH + \nu(1 - \xi_{wr})\zeta_b\tilde{b} - \nu(1 - \xi_{wr})\zeta_b\frac{\int_0^{\tilde{b}} bg(b)db}{\tilde{G}}$ . Solving for  $\tilde{b}$  leads to  $\tilde{b} = \frac{p_L\zeta_x H^\gamma - \Phi_L - wH + (1 - \delta)\frac{c_V}{q_V}}{\nu(1 - \xi_{wr})\zeta_b} + \frac{\int bg(b)db}{\tilde{G}}$ , where  $\Phi_L = p_L\zeta_x H^\gamma - wH + \frac{1 - \beta(1 - \delta)\tilde{G}}{\beta}\frac{c_V}{q_V}$ .
- Furthermore it is assumed that  $\zeta_b = 1$ .
- To ensure that the difference between the value of being employed and of being unemployed ( $j_{wu}$ ) is the same in the steady state, it has to hold that  $b_{end} = b_{exo} - \frac{\int_0^{\tilde{b}} bg(b)db}{\tilde{G}}$ .

### Rescaling of parameters in case of contemporaneous hiring

The implementation strategy for contemporaneous hiring is similar to the one of endogenous job destruction: I assume that observable<sup>77</sup> job destruction and observable job creation in steady state are the same as with lagged hiring.<sup>78</sup> The rescaling in case of exogenous destruction is as follows:

- The laws of motion of employment in steady state are  $N = (1 - \delta_{lag,exo})N + q_U(1 - N)$  in case of lagged hiring and  $N = (1 - \delta_{con,exo})N + q_U(1 - (1 - \delta_{con,exo})N)$  in case of contemporaneous hiring.
- Job finding and vacancy filling rates  $q_V$  and  $q_U$  are assumed to be like in the case of lagged hiring.
- Solving the law of motion for employment for the job destruction rate yields  $\delta_{lag,exo} = \frac{q_U(1 - N)}{N}$  and  $\delta_{con,exo} = \frac{q_U(1 - N)}{(1 - q_U)N}$ . So it has to hold that  $\delta_{con,exo} = \frac{\delta_{lag,exo}}{1 - q_U}$ .

The rescaling in case of endogenous destruction is as follows:

<sup>76</sup>I use the subscript *end* for the case of endogenous job destruction and *exo* for exogenous job destruction.

<sup>77</sup>Both 'observable job creation' and 'observable job destruction' exclude the case of workers whose job was destroyed in the same period in which they found another job and started working again.

<sup>78</sup>I use the subscript *lag* for the case of lagged hiring and *con* for contemporaneous hiring.

- The laws of motion of employment in steady state are  $N = (1 - \delta_{lag,exo})N + q_U(1 - N)$  in case of lagged hiring (and exogenous destruction) and  $N = \tilde{G}((1 - \delta_{con,end})N + q_U(1 - (1 - \delta_{con,end})N))$  in case of contemporaneous hiring (with endogenous destruction).
- Job finding and vacancy filling rates  $q_V$  and  $q_U$  are assumed to be like in the case of lagged hiring ( $q_{V,end} = \frac{q_{V,exo}}{\tilde{G}}$  and  $q_{U,end} = \frac{q_{U,exo}}{\tilde{G}}$ ).
- Solving the laws of motion for employment for the job destruction rate yields  $\delta_{lag} = \frac{q_U(1-N)}{N}$  and  $\delta_{con,end} = \frac{\tilde{G}q_U(1-N) - (1-\tilde{G})N}{\tilde{G}(1-q_U)N}$ .<sup>79</sup> So it has to hold that  $\delta_{con,end} = \frac{\delta_{lag,exo} - (1-\tilde{G})}{\tilde{G} - q_{U,exo}}$ .<sup>80</sup>

Furthermore note that the following steady state values change slightly:

- Due to the different hiring condition ( $\frac{c_V}{q_V} = \tilde{G} \left( p_L \zeta_x H^\gamma - \Phi_L - wH + \beta(1 - \delta) \frac{c_V}{q_V} \right)$ ), it now holds that  $\Phi_L = p_L \zeta_x H^\gamma - wH + \frac{1 - \beta(1 - \delta)\tilde{G}}{\tilde{G}} \frac{c_V}{q_V}$ , where  $\tilde{G} = 1$  in case of exogenous job destruction.
- As the steady state expression for the wage is now  $wH = (1 - \nu)(p_L \zeta_x H^\gamma - \Phi_L) + (1 - \nu)\beta(1 - \delta)c_V\theta + \nu \left( b + \frac{H^{1+\sigma_L}}{(1+\sigma_L)\lambda} \right)$ ,<sup>81</sup> the expression for the firm's bargaining power in wage determination changes to  $\nu = \frac{p_L \zeta_x H^\gamma - \Phi_L + \beta(1 - \delta)\tilde{G}c_V\theta - wH}{p_L \zeta_x H^\gamma - \Phi_L + \beta(1 - \delta)\tilde{G}c_V\theta - \left( b + \frac{H^{1+\sigma_L}}{(1+\sigma_L)\lambda} \right)}$ , where  $\tilde{G} = 1$  in case of exogenous job destruction.
- In case of endogenous job destruction, the value of  $\tilde{b}$  also changes slightly. The term  $(1 - \delta) \frac{c_V}{q_V(\theta)}$  in the job destruction equation is multiplied by  $\beta$  in case of contemporaneous hiring, so  $\tilde{b} = \frac{p_L \zeta_x H^\gamma - \Phi_L - wH + \beta(1 - \delta) \frac{c_V}{q_V}}{\nu(1 - \xi_{wr})\zeta_b} + \frac{\int b g(b) db}{\tilde{G}}$ .

<sup>79</sup>  $N = \tilde{G}((1 - \delta)N + q_U(1 - (1 - \delta)N)) = \tilde{G}((1 - \delta)N - q_U(1 - \delta)N) + \tilde{G}q_U = \tilde{G}(1 - \delta)(1 - q_U)N + \tilde{G}q_U$ . So  $1 - \delta = \frac{N - \tilde{G}q_U}{\tilde{G}(1 - q_U)N}$ . This leads to  $\delta = 1 - \frac{N - \tilde{G}q_U}{\tilde{G}(1 - q_U)N} = \frac{\tilde{G}(1 - q_U)N - N + \tilde{G}q_U}{\tilde{G}(1 - q_U)N} = \frac{\tilde{G}q_U(1 - N) - (1 - \tilde{G})N}{\tilde{G}(1 - q_U)N}$ .

<sup>80</sup>  $\delta_{con,end} = \frac{\tilde{G}q_{U,end}(1 - N) - (1 - \tilde{G})N}{\tilde{G}(1 - q_{U,end})N} = \frac{q_{U,exo}U - (1 - \tilde{G})N}{\tilde{G}(1 - q_{U,end})N} = \frac{q_{U,exo}U}{N} \frac{1}{\tilde{G}(1 - q_{U,end})} - \frac{(1 - \tilde{G})N}{\tilde{G}(1 - q_{U,end})N} = \frac{\delta_{lag,exo}}{\tilde{G} - q_{U,end}\tilde{G}} - \frac{1 - \tilde{G}}{\tilde{G} - q_{U,end}\tilde{G}}$ .

<sup>81</sup> Evaluating the wage equation at the steady state yields  $wH = (1 - \nu)(p_L \zeta_x H^\gamma - \Phi_L) + \nu \left( b + \frac{c_L}{(1 + \sigma_L)\lambda} H^{1 + \sigma_L} \right) + \nu\beta(1 - \delta)\tilde{G}q_U j_{wu}$ , where  $\nu\beta(1 - \delta)\tilde{G}q_U(1 - \delta)j_{wu} = \nu\beta(1 - \delta)\tilde{G}q_U \frac{1 - \nu}{\nu} j_f = (1 - \nu)\beta(1 - \delta)\tilde{G}q_U \frac{c_V}{q_V} = \beta(1 - \delta)\tilde{G}(1 - \nu)c_V\theta$ .

## 2.C Further tables and figures

Table 2.8: Calibrated parameters

Parameter		Value
<b>Shock autocorrelations</b>		
Investment	$\rho_i$	0
Import demand	$\rho_m$	0.75
Export demand	$\rho_{mf}$	0.75
Risk premium	$\rho_{rp}$	0.75
<b>Model-specific deep parameters</b>		
Indexation parameter for wage setting	$\gamma_w$	0.5
Convexity of vacancy costs	$\psi_V$	1
<b>Other domestic deep parameters</b>		
Discount factor	$\beta$	$0.995\mu^a$
Share of capital	$\alpha$	0.31
Depreciation rate	$\tau$	0.025
Share of fixed cost in production	$\frac{\Phi}{y}$	0
Degree of habit formation	$\kappa$	0.4
Disutility of hours worked	$\sigma_L$	10
Indexation parameter for price setting	$\gamma_p$	1
Risk premium coefficient	$\phi_{rp}$	0.01
Hiring costs*labor market tightness	$c_V\theta$	0.3
Parameter in final goods CES function	$\sigma_m$	$\rightarrow \infty$
<b>Foreign deep parameters</b>		
Degree of habit formation	$\kappa_f$	0.566
Indexation parameter for price setting	$\gamma_{pf}$	0.424
Inflation coefficient in Taylor rule	$\psi_f$	1.9
Persistence interest rate	$\rho_r$	0.855
Foreign inverse elasticity of labor supply	$\sigma_l^f$	5.42
Parameter in final goods CES function	$\sigma_{mf}$	$\rightarrow \infty$
<b>Steady state values</b>		
Growth of permanent technology shock	$\mu^a - 1$	0.003
Share of consumption in GDP	$c_y$	0.5512
Share of investment in GDP	$i_y$	0.2214
Share of exports in GDP	$x_y$	0.4855
Unemployment rate	$u$	0.0521
Threat point of worker	$\frac{b + \frac{\xi_f H^{1+\sigma_L}}{(1+\sigma_L)\lambda}}{wH}$	0.7



Table 2.9: Estimation results for structural parameters

Parameter		Type	Prior		Posterior		
			Mean	SE	Mean	5%	95%
<b>Shock autocorrelations</b>							
Productivity (temporary)	$\rho_a$	beta	0.750	0.150	0.921	0.861	0.985
Productivity (permanent)	$\rho_{\mu_a}$	beta	0.750	0.150	0.332	0.229	0.442
Government consumption	$\rho_G$	beta	0.750	0.150	0.651	0.492	0.818
Labor supply	$\rho_L$	beta	0.750	0.150	0.913	0.851	0.982
Matching efficiency	$\rho_\theta$	beta	0.750	0.150	0.754	0.631	0.883
Import prices	$\rho_{\pi_m}$	beta	0.750	0.150	0.886	0.802	0.980
World demand	$\rho_{yf}$	beta	0.750	0.150	0.742	0.662	0.827
<b>Variances of shock innovations</b>							
Productivity (temporary)	$\epsilon^a$	invg	1.000	Inf	0.984	0.813	1.148
Productivity (permanent)	$\epsilon^{\mu_a}$	invg	1.000	Inf	0.550	0.470	0.625
Government consumption	$\epsilon^G$	invg	4.000	Inf	3.506	2.994	4.003
Investment	$\epsilon^i$	invg	4.000	Inf	2.581	2.214	2.947
Labor supply	$\epsilon^L$	invg	4.000	Inf	6.164	5.231	7.059
Domestic inflation	$\epsilon^{\lambda_p}$	invg	1.000	Inf	0.232	0.197	0.266
Matching efficiency	$\epsilon^\theta$	invg	1.000	Inf	0.521	0.367	0.667
Import demand	$\epsilon^m$	invg	4.000	Inf	1.028	0.882	1.180
Export demand	$\epsilon^{mf}$	invg	4.000	Inf	3.366	2.624	4.078
Risk premium	$\epsilon^{rp}$	invg	1.000	Inf	0.641	0.546	0.735
Import prices	$\epsilon^{\pi_m}$	invg	1.000	Inf	0.961	0.814	1.100
World inflation	$\epsilon^{\pi^f}$	invg	0.500	Inf	0.152	0.129	0.174
World interest rate	$\epsilon^R$	invg	0.500	Inf	0.101	0.087	0.116
World demand	$\epsilon^{yf}$	invg	0.500	Inf	1.828	1.520	2.131
<b>Other structural parameters in all models</b>							
Foreign trade adjustment costs	$\phi_{mf}$	gamma	0.400	0.200	0.254	0.108	0.392
Domestic trade adjustment costs	$\phi_m$	gamma	0.400	0.200	0.040	0.013	0.066
Calvo parameter prices	$\zeta_p$	beta	0.750	0.150	0.952	0.927	0.978
Job finding prob.	$q\bar{U}$	beta	0.750	0.150	0.886	0.860	0.913
Matching function elasticity	$\eta$	beta	0.500	0.150	0.056	0.034	0.076
Production elasticity wrt hours	$\gamma$	beta	0.600	0.150	0.888	0.809	0.972
Investment adjustment costs	$\chi$	gamma	0.250	0.100	0.191	0.098	0.282
Parameter of capital utilisation function	$\psi$	norm	4.000	1.500	4.266	2.183	6.180
Output coefficient in Taylor rule	$\psi_{yf}$	gamma	0.300	0.150	0.291	0.189	0.388
Foreing Calvo parameter prices	$\tilde{\zeta}_p$	beta	0.750	0.150	0.939	0.918	0.959
<b>Model-specific structural parameters</b>							
Pdf at threshold (end. dest.)	$\tilde{g}$	invg	0.005	0.075	-	-	-
Real wage rigidity	$\zeta_{wr}$	beta	0.500	0.100	-	-	-
Calvo parameter wages	$\zeta_w$	beta	0.750	0.150	0.976	0.957	0.996

**Table 2.10:** Standard deviations and autocorrelations of observable variables in the data and the model

		St. dev. Data	St. dev. Model	Autocorr. Data	Autocorr. Model
GDP growth	$\Delta Y$	0.83	1.76	0.20	0.36
Consumption growth	$\Delta C$	0.97	1.68	-0.34	0.30
Investment growth	$\Delta I$	2.41	3.61	-0.15	0.49
Export growth	$\Delta X$	2.47	2.54	0.31	0.22
Import growth	$\Delta M$	2.49	2.14	0.13	0.07
Wage growth	$\Delta W$	0.67	0.90	0.34	0.13
Working hours	$H$	0.83	2.02	0.85	0.95
Unemployment	$U$	8.69	13.39	0.87	0.91
Vacancies	$V$	14.61	26.27	0.92	0.94
GDP deflator	$\Pi^d$	0.26	0.77	-0.11	0.85
Consumption deflator	$\Pi$	0.37	0.70	0.02	0.43
Foreign output	$y^f$	1.26	1.03	0.90	0.83
Foreign inflation	$\Pi^f$	0.20	0.23	0.22	0.42
Foreign interest rate	$R^f$	0.26	0.27	0.92	0.89

**Table 2.11:** Cross-correlations of observable variables in the data

		$\Delta Y$	$\Delta C$	$\Delta I$	$\Delta X$	$\Delta M$	$\Delta W$	$H$	$U$	$V$	$\Pi^d$	$\Pi$	$y^f$	$\Pi^f$	$R^f$
GDP growth	$\Delta Y$	1.0	0.2	0.4	0.7	0.5	-0.3	0.1	0.2	0.1	-0.1	0.2	0.1	0.1	-0.2
Consumption growth	$\Delta C$	0.2	1.0	0.1	0.1	0.3	0.0	0.0	0.0	0.0	0.2	-0.2	-0.1	0.0	-0.1
Investment growth	$\Delta I$	0.4	0.1	1.0	0.2	0.4	-0.3	0.1	0.1	0.1	-0.2	0.2	0.1	0.0	-0.1
Export growth	$\Delta X$	0.7	0.1	0.2	1.0	0.7	-0.4	0.3	0.1	0.0	0.0	0.4	0.1	0.1	-0.3
Import growth	$\Delta M$	0.5	0.3	0.4	0.7	1.0	-0.4	0.2	0.1	0.1	0.1	0.3	0.1	0.0	-0.3
Wage growth	$\Delta W$	-0.3	0.0	-0.3	-0.4	-0.4	1.0	-0.1	-0.1	0.0	-0.1	-0.6	0.0	0.0	0.1
Working hours	$H$	0.1	0.0	0.1	0.3	0.2	-0.1	1.0	0.1	0.0	0.1	0.1	0.2	0.2	-0.1
Unemployment	$U$	0.2	0.0	0.1	0.1	0.1	-0.1	0.1	1.0	-0.7	-0.1	0.0	-0.7	-0.2	-0.6
Vacancies	$V$	0.1	0.0	0.1	0.0	0.1	0.0	0.0	-0.7	1.0	0.1	0.1	0.8	0.2	0.6
GDP deflator	$\Pi^d$	-0.1	0.2	-0.2	0.0	0.1	-0.1	0.1	-0.1	0.1	1.0	0.2	0.2	0.1	0.1
Consumption deflator	$\Pi$	0.2	-0.2	0.2	0.4	0.3	-0.6	0.1	0.0	0.1	0.2	1.0	0.2	0.1	0.0
Foreign output	$y^f$	0.1	-0.1	0.1	0.1	0.1	0.0	0.2	-0.7	0.8	0.2	0.2	1.0	0.4	0.8
Foreign inflation	$\Pi^f$	0.1	0.0	0.0	0.1	0.0	0.0	0.2	-0.2	0.2	0.1	0.1	0.4	1.0	0.4
Foreign interest rate	$R^f$	-0.2	-0.1	-0.1	-0.3	-0.3	0.1	-0.1	-0.6	0.6	0.1	0.0	0.8	0.4	1.0

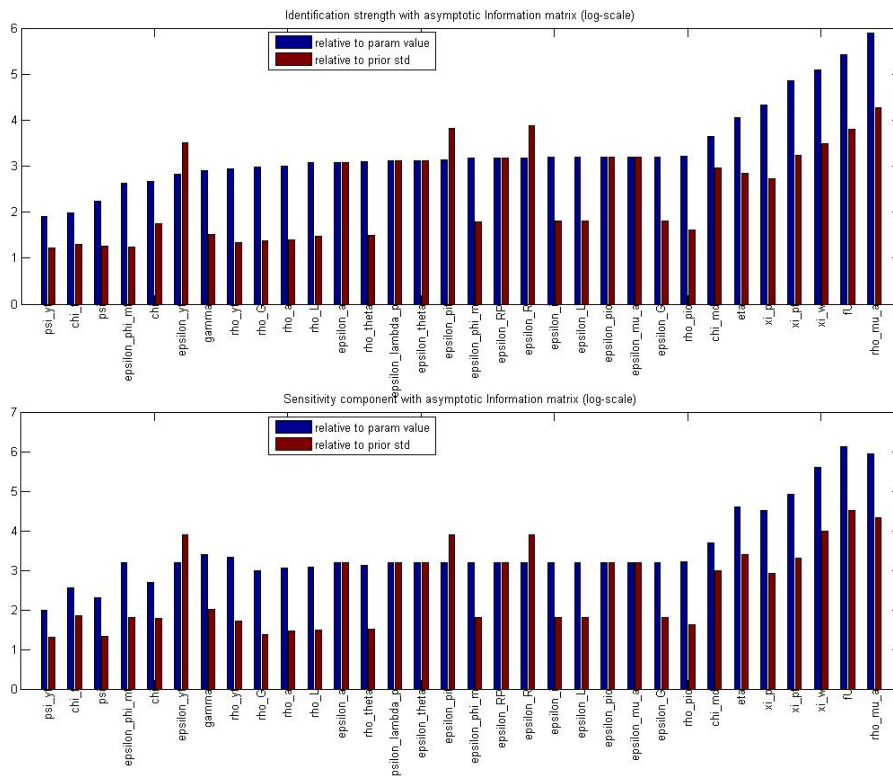
**Table 2.12:** Cross-correlations of observable variables in the model

		$\Delta Y$	$\Delta C$	$\Delta I$	$\Delta X$	$\Delta M$	$\Delta W$	$H$	$U$	$V$	$\Pi^d$	$\Pi$	$y^f$	$\Pi^f$	$R^f$
GDP growth	$\Delta Y$	1.0	0.7	0.6	0.7	0.3	0.5	0.1	-0.2	0.2	-0.5	-0.4	0.0	0.0	0.0
Consumption growth	$\Delta C$	0.7	1.0	0.4	0.3	0.3	0.4	0.0	-0.1	0.2	-0.2	-0.3	-0.1	-0.1	0.0
Investment growth	$\Delta I$	0.6	0.4	1.0	0.2	0.3	0.3	0.1	-0.3	0.3	-0.5	-0.4	-0.1	0.0	0.0
Export growth	$\Delta X$	0.7	0.3	0.2	1.0	0.4	0.3	0.0	-0.1	0.1	-0.2	-0.2	0.0	0.0	-0.1
Import growth	$\Delta M$	0.3	0.3	0.3	0.4	1.0	0.0	0.1	-0.1	0.1	0.0	0.2	-0.1	-0.1	0.0
Wage growth	$\Delta W$	0.5	0.4	0.3	0.3	0.0	1.0	0.1	-0.2	0.3	-0.3	-0.6	-0.1	-0.1	0.0
Working hours	$H$	0.1	0.0	0.1	0.0	0.1	0.1	1.0	-0.5	0.6	0.1	0.1	0.0	0.0	0.0
Unemployment	$U$	-0.2	-0.1	-0.3	-0.1	-0.1	-0.2	-0.5	1.0	-0.9	0.0	0.0	0.0	0.0	0.0
Vacancies	$V$	0.2	0.2	0.3	0.1	0.1	0.3	0.6	-0.9	1.0	0.0	0.0	0.0	0.0	0.0
GDP deflator	$\Pi^d$	-0.5	-0.2	-0.5	-0.2	0.0	-0.3	0.1	0.0	0.0	1.0	0.7	0.0	0.0	0.0
Consumption deflator	$\Pi$	-0.4	-0.3	-0.4	-0.2	0.2	-0.6	0.1	0.0	0.0	0.7	1.0	0.0	0.1	0.1
Foreign output	$y^f$	0.0	-0.1	-0.1	0.0	-0.1	-0.1	0.0	0.0	0.0	0.0	0.0	1.0	0.3	0.3
Foreign inflation	$\Pi^f$	0.0	-0.1	0.0	0.0	-0.1	-0.1	0.0	0.0	0.0	0.0	0.1	0.3	1.0	0.3
Foreign interest rate	$R^f$	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.3	1.0



**Table 2.15:** Forecast error variance decomposition for output per working hour  $\frac{y}{NH}$  (stationary)

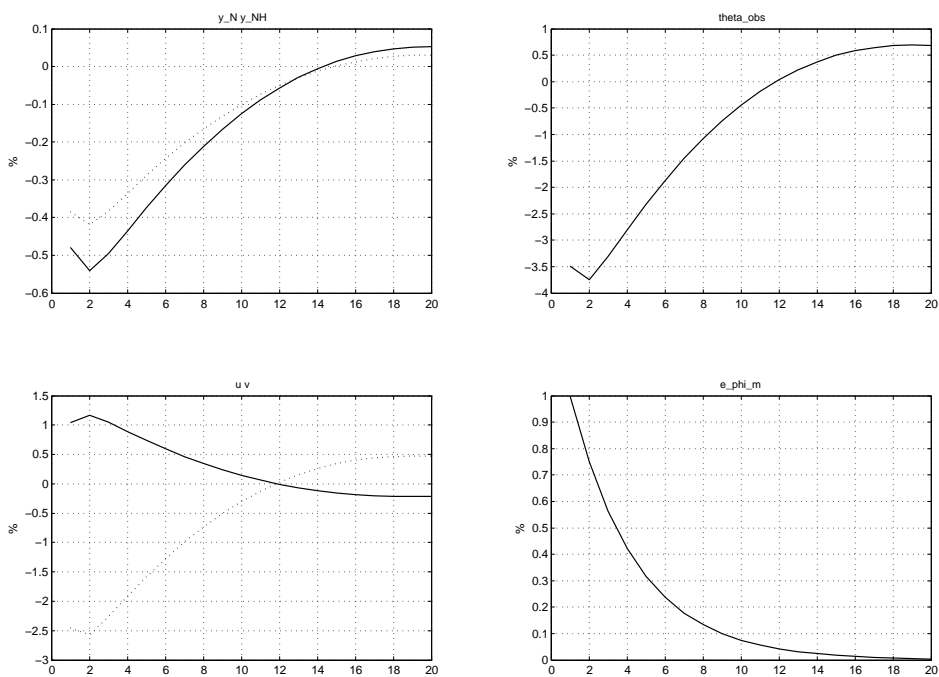
Shock process		Forecasting horizon						
		1	2	4	8	16	40	100
Productivity (temporary)	$\epsilon^a$	0.092	0.084	0.085	0.104	0.180	0.276	0.279
Productivity (permanent)	$\epsilon^{\mu a}$	0.003	0.004	0.003	0.002	0.001	0.002	0.002
Government consumption	$\epsilon^G$	0.077	0.048	0.025	0.012	0.007	0.006	0.006
Investment	$\epsilon^i$	0.026	0.020	0.014	0.007	0.004	0.004	0.003
Labor supply	$\epsilon^L$	0.140	0.120	0.098	0.064	0.036	0.031	0.030
Domestic inflation	$\epsilon^{\lambda p}$	0.019	0.061	0.197	0.457	0.568	0.507	0.504
Matching efficiency	$\epsilon^\theta$	0.017	0.013	0.009	0.004	0.002	0.002	0.002
Import demand	$\epsilon^m$	0.120	0.123	0.109	0.071	0.040	0.034	0.034
Export demand	$\epsilon^{mf}$	0.287	0.312	0.283	0.179	0.100	0.085	0.085
Risk premium	$\epsilon^{rp}$	0.204	0.202	0.163	0.088	0.051	0.044	0.044
Import prices	$\epsilon^{\pi m}$	0.003	0.001	0.001	0.001	0.001	0.001	0.002
World inflation	$\epsilon^{\pi f}$	0.000	0.001	0.002	0.004	0.003	0.003	0.003
World interest rate	$\epsilon^R$	0.011	0.012	0.012	0.008	0.004	0.004	0.004
World demand	$\epsilon^{yf}$	0.000	0.000	0.000	0.000	0.001	0.001	0.001
Sum		1.000	1.000	1.000	1.000	1.000	1.000	1.000



**Figure 2.3:** Identification strength at the prior mean for the best-performing model

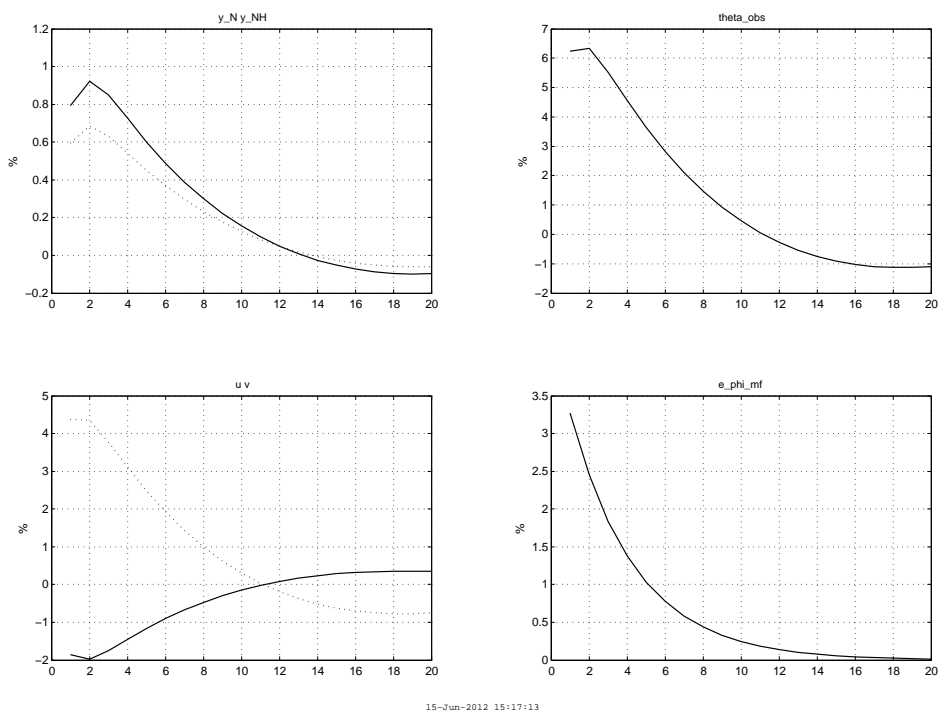
**Table 2.16:** Shock groups for historical decomposition

Group name	Included shocks
Technology	Productivity (temporary) $\epsilon^a$ , Productivity (permanent) $\epsilon^{\mu^a}$
Preferences	Labor supply $\epsilon^L$
Domestic demand	Government consumption $\epsilon^G$ , Investment $\epsilon^i$ , Risk premium $\epsilon^{rp}$
Prices	Domestic inflation $\epsilon^{\lambda_p}$ , World inflation $\epsilon^{\pi^f}$ , Import prices $\epsilon^{\pi^m}$
Labor market	Matching efficiency $\epsilon^\theta$
Foreign monetary	World interest rate $\epsilon^R$
Foreign demand	World demand $\epsilon^{y^f}$ , Export demand $\epsilon^{\phi_{mf}}$ , Import demand $\epsilon^{\phi_m}$

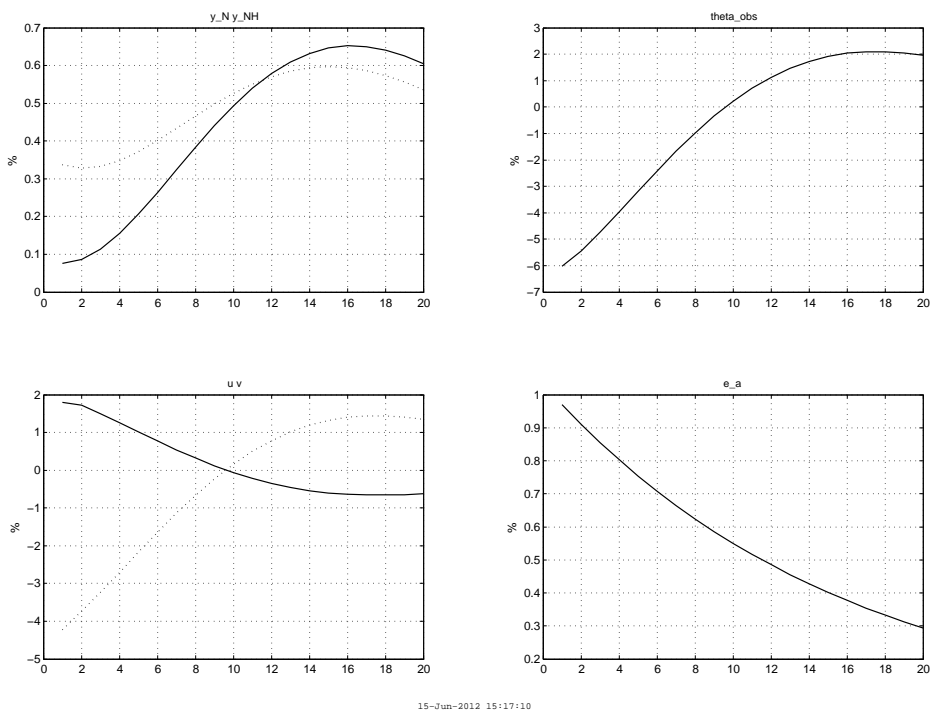


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**Figure 2.4:** Impulse responses to import demand shock



**Figure 2.5:** Impulse responses to export demand shock



**Figure 2.6:** Impulse responses to (temporary) productivity shock

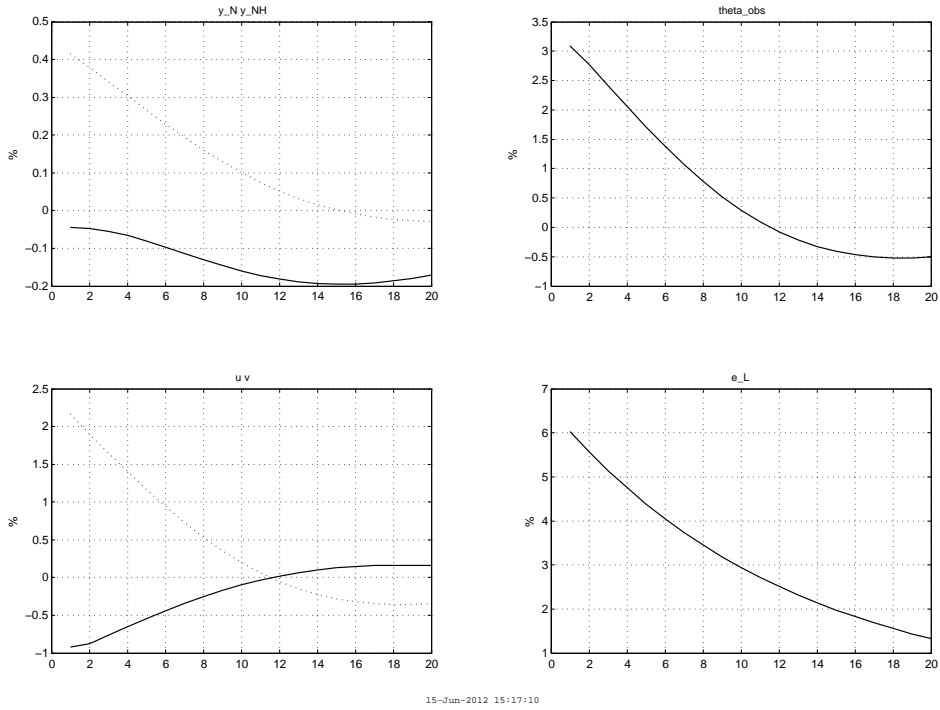


Figure 2.7: Impulse responses to disutility of work shock

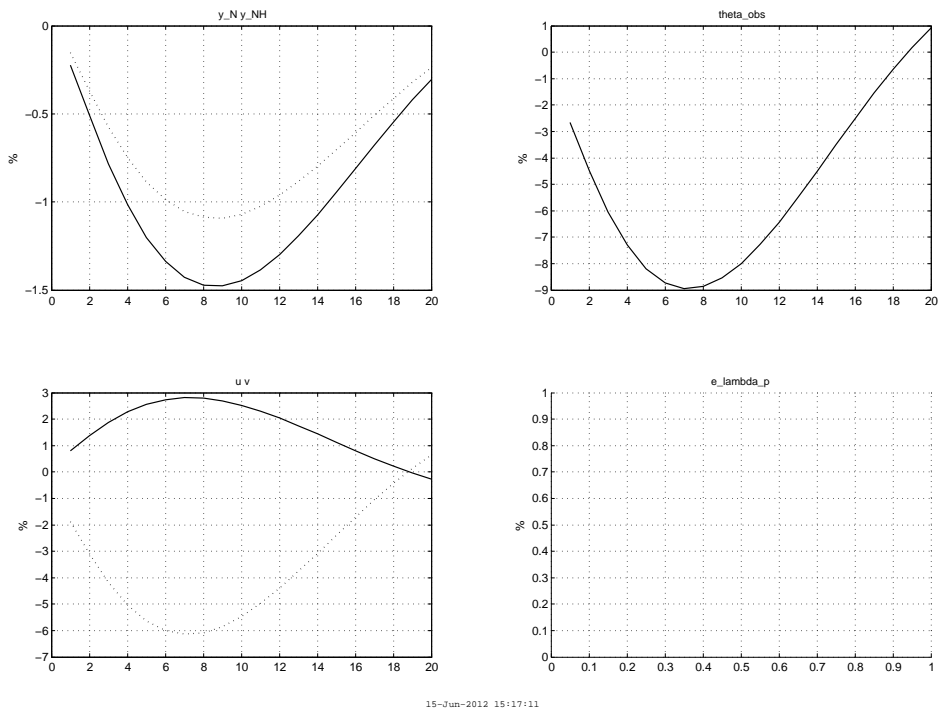


Figure 2.8: Impulse responses to price mark-up shock

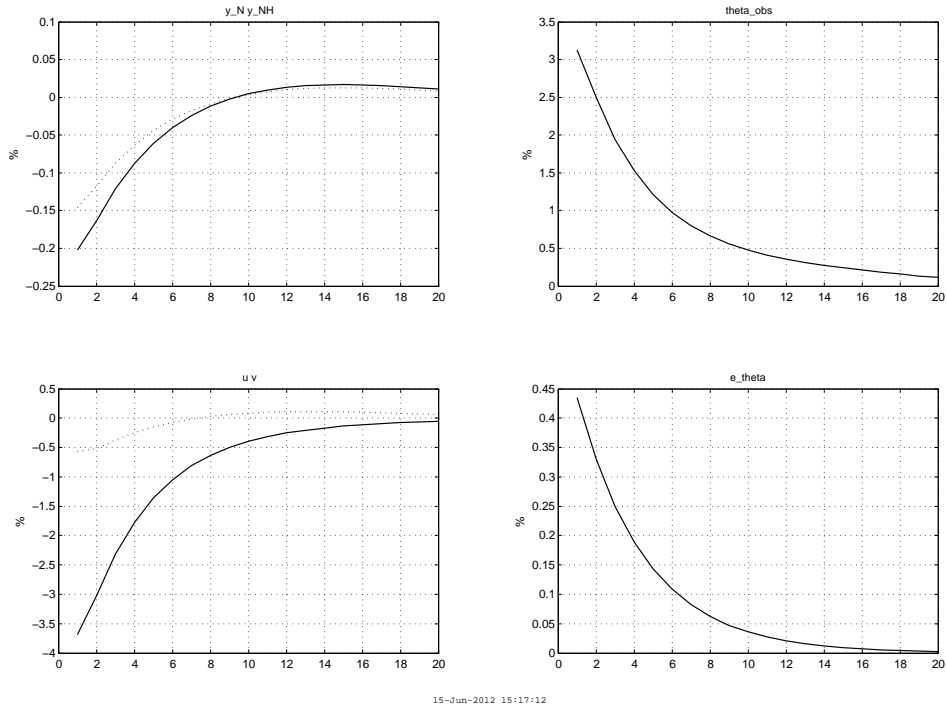


Figure 2.9: Impulse responses to matching efficiency shock

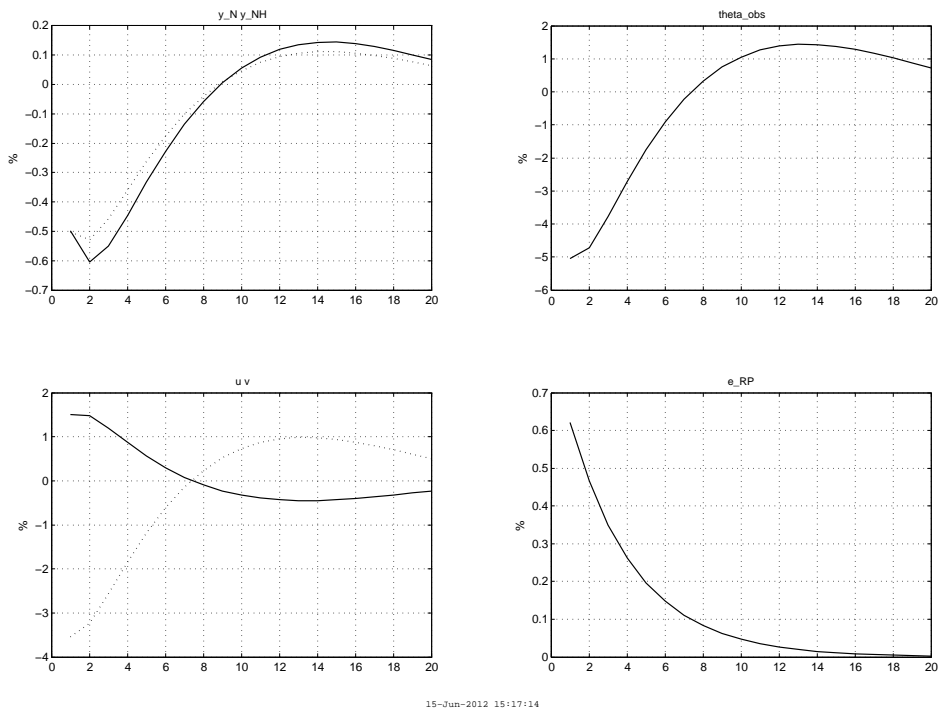


Figure 2.10: Impulse responses to risk premium shock



## Chapter 3

# International Fragmentation, Unskilled Unemployment and the Role of Nontradable Goods

Lukas Reiss<sup>1</sup>

JEL codes: F16, E24, F11

### Abstract

In this paper I analyze the effects of international outsourcing on the employment of unskilled labor in a Heckscher-Ohlin-World (2 countries with different relative endowments: unskilled-labor-abundant South and skill-abundant North). Relocation of parts of the value added chain in manufacturing (due to a relative increase in Southern productivity) makes Northern production more skill-intensive. This causes a shift of part of the unskilled labor force to the non-trading sectors which are assumed to be highly intensive in this factor. Sectoral unemployment rates of the low skilled are increasing, the overall effect on unskilled unemployment depends heavily on the characteristics of the non-trading sectors. Furthermore, under certain circumstances this increase in outsourcing can also be accompanied by an increase in the relative price of labor-intensive goods and the relative employment in labor-intensive sectors.

### 3.1 Introduction

From the 1970s to the mid-2000s, Western Europe and the US experienced relative increases in unemployment rates of low skilled<sup>2</sup> and the decrease of their relative wages, where in the US the decrease in relative wages was more pronounced while in Western Europe the increase in the relative unemployment rate was stronger.<sup>3</sup> In the same time period trade has been liberalized

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<sup>1</sup>Lukas.Reiss@oeb.at. Many thanks to Christian Haefke, Ingrid Kubin and Michael Rauscher for helpful comments. Remaining mistakes are all mine.

<sup>2</sup>The use of the words 'skilled' and 'unskilled' in this paper is not meant to indicate that people without a university (or high school) degree are less able than people who have a diploma; it is just following a convention in the literature which refers to worse prospects at the job market.

<sup>3</sup>See for example chapter 13 in Borjas (2005), chapter 10 in Cahuc and Zylberberg (2004) and Pierrard and Sneessens (2008).

and world trade has substantially grown. Is there any relationship, namely a differential response of these two regions' labor markets? And what may cause them to respond differently?

This naturally raises two questions:

1. Which factors contributed to the losses of unskilled labor in Western Europe and the US?
2. Which factors can be made responsible for the differential response of (Continental) Western Europe and the US?

### 3.1.1 The importance of international outsourcing

Concerning the first question, the different (but partially interconnected) possible reasons which are discussed most prominently in the literature are: migration, institutional changes, international trade and (skill-)biased technological progress (see for example Cahuc and Zylberberg, 2004, chapter 10). The latter is typically argued to be the by far most important contributor, which is also claimed in several empirical studies. However, a comparatively smaller role of international trade is usually acknowledged too.<sup>4</sup>

The potential distributional impact of international trade is an old issue in economics, which – probably most importantly (in the context of physical capital and labor) – is for example analyzed in the famous Heckscher-Ohlin-model.<sup>5</sup>

Over the last two decades there have also been several contributions analyzing the impact of international trade on the (un)employment of skilled and unskilled workers: These include Krugman (1995), Davis (1998), Sener (2003), Moore and Ranjan (2005), Egger and Kreickemeier (2008), Keuschnigg and Ribi (2009), Mitra and Ranjan (2009) and Helpman et al. (2010).

Most of the above mentioned papers do not fully account for trade in intermediate goods. Trade in intermediate goods is relevant in both theoretical and empirical terms. According to OECD (2007), imports in intermediate goods already make up around 10% of GDP in most Western European countries (and upward trending), even Germany – Europe's largest economy – has a share of 10%. In the larger and less open US economy it is 3%.<sup>6</sup> The empirical evidence on the effects of increased outsourcing is mixed; for example OECD (2007) find negative effects of offshoring and import penetration on demand for unskilled labor.

Taking into account of the existence of outsourcing is very important when comparing the theoretical predictions of trade models with data: It can be taken as a reply to the common argument (for example made by Moore and Ranjan, 2005) that international trade cannot contribute too much to the relative decrease in unskilled wages and employment as relative prices of skilled goods and relative employment in skill-intensive sectors did not increase too much over the last decades, contrary to predictions of the Heckscher-Ohlin-model for the effects of opening up trade with labor-abundant countries. The answer of authors like Feenstra and Hanson (2001)<sup>7</sup> is that one has

<sup>4</sup>There is also the reasoning that both factors are closely interrelated as for example globalization might enforce (skill-biased) technological progress (see again Cahuc and Zylberberg, 2004, chapter 10).

<sup>5</sup>Among others, chapter 2 in Feenstra (2004) provides a theoretical discussion of this model. An overview of the empirical literature of the 1980s and 1990s of the impact of international trade on the relative wage of unskilled workers can for example be found in Feenstra and Hanson (2001).

<sup>6</sup>These numbers are actually underestimating the extent of international outsourcing as also parts of trade in final goods can be seen as a part of that. For example firms could only keep headquarter services in-house and subcontract all manufacturing of final goods (see for example Feenstra and Hanson (1996) who include that aspect into their empirical definition of outsourcing).

<sup>7</sup>This paper provides a much longer discussion on these arguments. In addition it contains a very detailed empirical analysis of possible effects of outsourcing on the share of low skilled in the overall wage bill (with mixed results).

to look at relative employment shares and prices of different fragments/subsectors (where the fit with the theoretical implications is much better) instead of more aggregated sectors.

Another important aspect of international outsourcing is the scale effect – namely that the relative productivity of outsourcing sectors increases, which may even lead – depending on whether there are demand-side restrictions – to an overall increase of employment in this sector (compared to the pre-outsourcing situation). These effects are empirically very hard to distinguish from the ones of skill-biased-technological change. The possibility of an increase in employment in a sector which increases its extent of outsourcing is emphasized in the theoretical work of Mitra and Ranjan (2010)<sup>8</sup> and the empirical paper of Amiti and Wei (2005). However, both contributions do not account for skill and it is unlikely that the skill composition of a sector remains the same after an increase in outsourcing. In a model with heterogeneous skill (without unemployment), Grossman and Rossi-Hansberg (2008) show that – under certain assumptions – these productivity effects can be so large that the factor affected by outsourcing can even benefit from it.

### 3.1.2 The importance of non-trading sectors

Concerning the second question, most of the literature refers to factors like relatively higher unemployment benefits, employment protection and union power in Continental Europe (see for example chapter 10 in Cahuc and Zylberberg (2004), Pierrard and Sneessens (2008) and Mortensen and Pissarides (1999)). Within the above cited literature on unskilled unemployment and trade, Egger and Kreckemeier (2008) and Mitra and Ranjan (2009) also discuss the role of fair wage constraints. All 4 factors are thought to lead to a relatively smaller reaction of wages to a negative shock, making a stronger reaction of employment ‘necessary’.<sup>9</sup> In the following, I will argue that the characteristics of non-trading sectors might be an alternative reason for this differential response.

Though trade in services is becoming more important in both the popular media (e.g. call-centers and programmers in India) and in empirics (service offshoring has strongly increased over the last years – however, from a very low starting point – see OECD, 2007), there are still many kinds of services which can be considered as non-tradable – especially consumer services.

One interesting aspect in the context of outsourcing is that consumer services are presumably relatively intensive in unskilled labor compared to manufacturing. When the share of unskilled labor used in manufacturing decreases after an increase in international fragmentation, there will be a shift of part of the unskilled labor force from manufacturing to the non-trading sector. I will argue that their ‘ability to absorb labor’ can differ significantly across industrial economies. This may be one of the causes of the differential response to an increase in international trade mentioned in the first paragraph.

The above mentioned papers on unskilled unemployment and trade do either not incorporate non-trading sectors, or only skilled workers are employed in them. In the model of Mitra and Ranjan (2010), the non-trading sector employs the same kind of workers whose jobs might be outsourced in manufacturing, but labor is homogeneous (my model will build on their contribution).

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<sup>8</sup>Actually this theoretical paper does not only analyze the effects of outsourcing, it also incorporates a sector producing non-tradable goods.

<sup>9</sup>It should be noted, however, that the effects of employment protection on the response of unemployment to a negative shock can be ambiguous (see for example Mortensen and Pissarides, 1999).

### 3.1.3 Overview over the rest of the paper

In the following, I will – building on the contributions of Feenstra and Hanson (1996) and Mitra and Ranjan (2010) – set up a model to analyze the effects of an increase in international outsourcing from rich (abundant in skilled labor) countries to poorer (abundant in unskilled labor) ones on employment of unskilled labor. Thereby I account for the role of the characteristics of sectors producing non-tradable goods (as possible cause for a differential response). The rest of the paper is organized as follows: The model setting will be explained in section 3.2, followed by an analysis of the effects of an increase in outsourcing in this model (sections 3.3 and 3.4). Section 3.5 compares my results with the existing literature, and section 3.6 concludes and talks about possible extensions.

## 3.2 Model setting

The economy in my model is characterized by

- 2 factors of production: skilled labor  $H$  and unskilled labor  $L$  (skill is exogenous),
- 2 goods: non-tradable good  $Z$  which is produced using only unskilled labor ('consumer services'<sup>10</sup>) and a tradable composite manufacturing good  $Y$  which is produced out of a continuum of intermediate goods  $X(i)$  using both skilled and unskilled labor, and
- 2 regions: North  $N$  is relatively skill-abundant and South  $S$  is unskilled-labor-abundant, and the latter has also a lower total factor productivity in manufacturing.

In both regions there is a continuum of workers who provide one unit of labor each and who are not mobile across regions (they cannot migrate from North to South or vice versa). Skilled workers are always employed, they can switch from one firm to the other without any frictions<sup>11</sup> and get paid their marginal revenue product which has to be equal in all subsectors of the respective region. Unskilled workers are subject to search-unemployment. There is no on-the-job-search. The unemployed can search either in consumer services (indexed by  $z$ ) or in manufacturing (index  $x$ ) and have to be indifferent between these two options.

The identical utility function of all workers (and entrepreneurs) is

$$Utility(C(t)) = \int_0^{\infty} C(t)e^{-rt}dt, \quad (3.1)$$

where

$$C(t) = Y(t)^{\mu}Z(t)^{1-\mu}. \quad (3.2)$$

This specification implies that there is no consumption smoothing over time (as lifetime utility is linear in consumption) and that in each period a share  $\mu$  of aggregate income goes into manufacturing goods and the rest into consumer services. In the following I drop the time index and only look at the steady state.

<sup>10</sup>Business services can be thought of as being a part of the composite manufacturing good  $Y$ .

<sup>11</sup>One could think of them being employed by competitive temp agencies.

### 3.2.1 Manufacturing sector

Following the model of Feenstra and Hanson (1996),<sup>12</sup> I assume that the final manufacturing good  $Y$  is assembled out of a continuum of intermediate goods  $X(i)$  with a Cobb-Douglas production function:

$$\ln Y = \int_0^1 \alpha(i) \ln X(i) di, \quad (3.3)$$

where  $\int_0^1 \alpha(i) di = 1$ . Intermediate goods are produced via

$$X(i) = A_{x,l} L(i)^{\gamma(i)} H(i)^{1-\gamma(i)}, \quad (3.4)$$

where  $A_{x,l}$  denotes the country-specific total factor productivity in manufacturing ( $l = N, S$ , throughout section 3.2 region indices will be dropped) and the elasticity  $\gamma(i) \in (0, 1)$  is decreasing in  $i$ . The latter means that – for given wage rates – the skill intensity of production will be increasing in  $i$ .

In each  $i$ , there are infinitely many firms producing  $X(i, k)$  units each ( $\int X(i, k) dk = X(i)$ ). They have to pay a fee of  $c_{V,x}$  for posting vacancies and employ only one unit of unskilled labor ( $L(i, k) = 1 \forall i, k$ ). So it holds that:

$$X(i, k) = A_x H(i, k)^{1-\gamma(i)}. \quad (3.5)$$

Unskilled workers are subject to search-unemployment. If they apply in the manufacturing sector, they do not know in which subsector  $i$  and firm  $k$  they will work after being matched. The number of matches is determined by a linear homogeneous matching function  $Match(V_x, U_x)$  where  $U_x$  denotes the mass of unskilled unemployed in sector manufacturing and  $V_x$  is the mass of vacancies. Let  $\theta_x := \frac{V_x}{U_x}$  denote the labor market tightness in sector  $x$ . The probability of filling a vacancy is given by  $q_V(\theta_x) := \frac{Match(V_x, U_x)}{V_x} = Match(1, \frac{U_x}{V_x}) = Match(1, \frac{1}{\theta_x})$ , which is decreasing in  $\theta_x$ , and the job finding rate is  $q_U(\theta_x) := \frac{Match(V_x, U_x)}{U_x} = Match(\theta_x, 1) = Match(1, \frac{1}{\theta_x})\theta_x = q_V(\theta_x)\theta_x$ , which is increasing in  $\theta_x$ . When unemployed, workers receive a flow of  $b$  (value of home production plus unemployment benefits, the latter being financed by lump-sum taxes).

Let  $J_F$ ,  $J_W$ ,  $J_U$  and  $J_V$  denote the values of a match to the employer, of a match to the unskilled employee, of being unemployed and of posting a vacancy, respectively. The asset equations for firm  $k$  in subsector  $i$  are:<sup>13</sup>

$$rJ_{F,x}(i, k) = A_x H(i, k)^{1-\gamma(i)} P_x(i) - QH(i, k) - W_x(i) - \delta(J_{F,x}(i, k) - J_{V,x}(i, k)), \quad (3.6)$$

$$rJ_{W,x}(i, k) = W_x(i) - \delta(J_{W,x}(i, k) - J_{U,x}), \quad (3.7)$$

$$rJ_{U,x} = b + q_U(\theta_x) (EJ_{W,x} - J_{U,x}), \quad (3.8)$$

$$rJ_{V,x}(i, k) = -c_{V,x} + q_V(\theta_x) (J_{F,x}(i, k) - J_{V,x}(i, k)), \quad (3.9)$$

where  $r$  is the exogenous discount rate,  $\delta$  is the exogenous job destruction rate,  $Q$  is the wage rate of a skilled worker,  $W_x(i)$  are wages of the unskilled, and  $P_x(i)$  is the price of one unit of  $X(i)$ .

<sup>12</sup>However, they use a Leontief production function for aggregating the two types of labor.

<sup>13</sup>The value of being unemployed  $J_{U,x}$  does not carry indices for subsector and firm. This is due to the assumption that workers applying in  $x$  do not know to which firm (and subsector) they might be matched.

Due to free entry the value of posting a vacancy has to be zero:

$$J_{V,x}(i, k) = 0. \quad (3.10)$$

Flow profits of firms and wages are determined via Nash Bargaining where workers are assumed to have bargaining power  $(1 - \nu)$ . Using this assumption and the zero-profit-condition (3.10),  $W_x(i)$  can be expressed in the following way (derivation in appendix 3.A):

$$W_x(i) = W_x = b + \frac{(1 - \nu)c_{V,x}}{\nu} \left( \theta_x + \frac{r + \delta}{q_V(\theta_x)} \right). \quad (3.11)$$

The wage is increasing in the flow value of being unemployed  $b$ , the bargaining power  $(1 - \nu)$ , the labor market tightness  $\theta_x$  and the costs of posting a vacancy  $c_{V,x}$ . The higher  $b$  and  $\theta_x$  (implying a higher job finding probability), the better the outside option for the worker; and the higher  $c_{V,x}$  and  $\theta_x$  (via a lower probability of filling a vacancy), the higher is the rent generated by the match (as the expected costs for the employer of forming a new match are higher).

Using free entry (3.10) in (3.6) and computing the first-order-conditions yields (derivation in appendix 3.A):

$$P_x(i) = \frac{1}{A_x \gamma(i)^{\gamma(i)} (1 - \gamma(i))^{1 - \gamma(i)}} \left( W_x(i) + (r + \delta) \frac{c_{V,x}}{q_V(\theta_x)} \right)^{\gamma(i)} Q^{1 - \gamma(i)}. \quad (3.12)$$

This is close to the typical expression for a cost function in the Cobb-Douglas-case (see for example Mas-Colell et al., 1995, chapter 5, example 5.C.1). The higher  $i$ , the lower is  $\gamma(i)$  and so the higher is the weight on the remuneration of high skilled labor and the lower the weight on the cost of unskilled labor. This cost of unskilled labor is larger than the wage rate as  $(r + \delta) \frac{c_{V,x}}{q_V(\theta_x)}$  goes to the entrepreneur (to make up for the costs of filling a vacancy).

### 3.2.2 Consumer services

Consumer services are produced using unskilled labor only. Every one-job-firm produces  $A_z$  units, so aggregate production of  $z$  is:<sup>14</sup>

$$Z = A_z(L_z - U_z). \quad (3.13)$$

The matching technology is the same as in the manufacturing sector, but that the cost of posting a vacancy  $c_{V,z}$  can differ from the one in manufacturing. The asset equations are given by:

$$rJ_{F,z} = A_z P_z - W_z - \delta(J_{F,z} - J_{V,z}), \quad (3.14)$$

$$rJ_{W,z} = W_z + \delta(J_{U,z} - J_{W,z}), \quad (3.15)$$

$$rJ_{U,z} = b + q_U(\theta_z)(J_{W,z} - J_{U,z}), \quad (3.16)$$

$$rJ_{V,z} = -c_{V,z} + q_V(\theta_z)J_{F,z} = 0. \quad (3.17)$$

<sup>14</sup>This setting is inspired by Mitra and Ranjan (2010) who incorporate the non-trading sector in a very similar way (in their case of 'perfect labor mobility').

Similar steps as in the previous section lead to

$$W_z = b + \frac{(1-\nu)c_{V,z}}{\nu} \left( \theta_z + \frac{r+\delta}{q_V(\theta_z)} \right), \quad \text{and} \quad (3.18)$$

$$P_z = \frac{1}{A_z} \left( W_z + (r+\delta) \frac{c_{V,z}}{q_V(\theta_z)} \right) = \frac{1}{A_z} \left( b + \frac{c_{V,z}}{\nu} \left( (1-\nu)\theta_z + \frac{r+\delta}{q_V(\theta_z)} \right) \right). \quad (3.19)$$

### 3.2.3 Labor market equilibrium

Unemployed unskilled workers have to be indifferent between searching in  $x$  or in  $z$ , which means that  $J_{U,x} = J_{U,z}$ . Combining equations (3.8), (3.16), (3.11) and (3.18), this is equivalent to (for the intermediate steps see appendix 3.A):

$$c_{V,x}\theta_x = c_{V,z}\theta_z. \quad (3.20)$$

Note that this equation implies an important trade-off: A relatively higher  $c_V$  means a relatively higher wage and a relatively lower job finding probability.<sup>15</sup>

All unskilled workers either belong to the labor force in  $x$  or to the one in  $z$ :

$$L = L_x + L_z. \quad (3.21)$$

In the steady state the outflow of unemployment (newly formed jobs) has to be equal to the inflow (destroyed jobs). So in sector  $x$ , it has to hold that  $q_U(\theta_x)U_x = \delta(L_x - U_x)$ . This means that the mass of unemployed in equilibrium is:

$$U_x = \frac{\delta}{\delta + q_U(\theta_x)} L_x, \quad (3.22)$$

which depends positively on the separation rate  $\delta$ , negatively on the job finding rate  $q_U(\theta_x)$  and positively on labor force  $L_x$  (as  $U_x$  denotes the mass of unemployed in  $x$  and not the unemployment rate). As the job destruction rate  $\delta$  is assumed to be constant, the level of the unemployment rate is only determined by the job finding probability. So when later a shift in the trade structure will lead to a change in the steady state unemployment rate, this will be via a change in job creation (and not in job destruction).

Making the same steps for sector  $z$  yields:

$$U_z = \frac{\delta}{\delta + q_U(\theta_z)} L_z. \quad (3.23)$$

Aggregate unemployment  $U = U_x + U_z$  is then equal to:

$$U = \delta \left( \frac{L_x}{\delta + q_U(\theta_x)} + \frac{L_z}{\delta + q_U(\theta_z)} \right). \quad (3.24)$$

To get to the overall unemployment rate of the unskilled, one has to divide (3.24) by  $L$ . Equation (3.24) shows that the sectoral composition of the labor force is crucial. If  $q_U(\theta_x) \neq q_U(\theta_z)$ , then the sectoral unemployment rates will differ and so a change in the relative size of the labor force

<sup>15</sup>Equations (3.11) and (3.18) indicate that  $\frac{\partial \ln(W-b)}{\partial \ln c_V} = 1$  for given  $\theta$  but  $0 < \frac{\partial \ln(W-b)}{\partial \ln \theta} < 1$  for given  $c_V$ . Furthermore, note again that the probability to find a job increases in  $\theta$ .

in  $x$  will – for given  $\theta_x$  and  $\theta_z$  – lead to a change of the overall unemployment rate (for given sectoral unemployment rates). This point will be crucial later in sections 3.3.3 and 3.3.4.

### 3.3 Effects of change in outsourcing structure

#### 3.3.1 Introducing international trade

As a next step I introduce trade into this model. I assume that all intermediate goods can be traded freely. So each good is produced in the region where unit cost is lower. Using this in (3.12) yields:

$$P_x(i) = \frac{1}{\gamma(i)\gamma^{(i)}(1-\gamma(i))^{1-\gamma(i)}} \min \left[ \frac{WC_{x,N}^{\gamma(i)} q_N^{1-\gamma(i)}}{A_{x,N}}, \frac{WC_{x,S}^{\gamma(i)} q_S^{1-\gamma(i)}}{A_{x,S}} \right], \quad (3.25)$$

where  $WC_x := W_x + (r + \delta) \frac{c_{V,x}}{q_V(\theta_x)} = b + \frac{c_{V,x}}{v} \left( (1-\nu)\theta_x + \frac{r+\delta}{q_V(\theta_x)} \right)$ .

As stated before, the South is assumed to be more abundant in unskilled labor ( $\frac{H_N}{L_N} > \frac{H_S}{L_S}$ ) and to have a lower total factor productivity ( $A_{x,S} < A_{x,N}$ ). If the endowments are different enough, it holds that  $\frac{WC_{x,S}}{A_{x,S}} < \frac{WC_{x,N}}{A_{x,N}}$  and  $\frac{Q_S}{A_{x,S}} > \frac{Q_N}{A_{x,N}}$  (this implies that  $\frac{Q_N WC_{x,S}}{Q_S WC_{x,N}} < 1$ ). This means that compared to total factor productivity, unskilled labor is relatively cheaper in the South and skilled labor relatively cheaper in the North. So there is a unique  $i^*$  for which:

$$\begin{aligned} \frac{WC_{x,N}^{\gamma(i^*)} W_{N,S}^{1-\gamma(i^*)}}{A_{x,N}} &= \frac{WC_{x,S}^{\gamma(i^*)} Q_S^{1-\gamma(i^*)}}{A_{x,S}}, \\ \frac{WC_{x,N}^{\gamma(i)} Q_N^{1-\gamma(i)}}{A_{x,N}} &< \frac{WC_{x,S}^{\gamma(i)} Q_S^{1-\gamma(i)}}{A_{x,S}} \quad \text{for } i > i^*, \text{ and} \\ \frac{WC_{x,N}^{\gamma(i)} Q_N^{1-\gamma(i)}}{A_{x,N}} &> \frac{WC_{x,S}^{\gamma(i)} Q_S^{1-\gamma(i)}}{A_{x,S}} \quad \text{for } i < i^*. \end{aligned} \quad (3.26)$$

So everything with  $i > i^*$  (the relatively skill-intensive intermediate goods) is produced in the North, the rest (the relatively unskilled-labor-intensive inputs) in the South (see figure 3.1).<sup>16</sup>  $i^*$  will from now on be called ‘outsourcing frontier’.

#### 3.3.2 Distribution of income

Given this frontier one can compute the relative shares of North and South and of  $H$  and  $L$  in world income. A share  $\int_{i^*}^1 \alpha(i) di$  of overall revenue from manufacturing will be generated in the North.

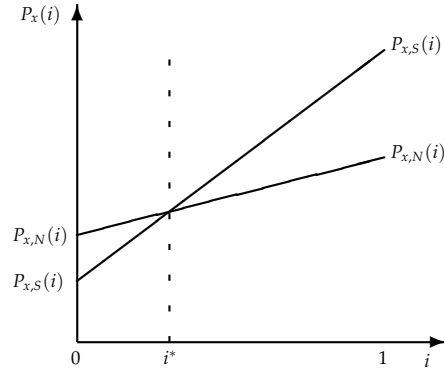
Northern income is chosen to be the numeraire. As all income of workers and entrepreneurs<sup>17</sup> is assumed to be spent on consumption, this means that:

$$Q_N H_N + WC_{x,N} (L_{x,N} - U_{x,N}) + P_{z,N} A_{z,N} (L_{z,N} - U_{z,N}) = 1.$$

<sup>16</sup>Note that the ordinate in figures 3.1 and 3.2 has a logarithmic scale. Furthermore,  $P_x(i)$  has been pre-multiplied by  $\gamma(i)\gamma^{(i)}(1-\gamma(i))^{1-\gamma(i)}$ . This has no effect on any results in this paper, but eases the visual presentability of the function  $P_x(i)$ .

<sup>17</sup>I assume that the vacancy posting cost  $c_V$  is paid to households; this ensures that GDP equals consumption in the Northern economy.





**Figure 3.1:** Prices as a function of  $i$  and the outsourcing frontier

From (3.2) it can be seen that a fraction  $\mu$  of income is spent on goods from sector  $x$  and the rest on services:

$$WC_{x,N}(L_{x,N} - U_{x,N}) + Q_N H_N = \mu, \quad (3.27)$$

$$P_{z,N} A_{z,N}(L_{z,N} - U_{z,N}) = 1 - \mu. \quad (3.28)$$

Using that (3.3) and (3.4) are Cobb-Douglas as well, it becomes clear that a share  $\frac{\int_{i^*}^1 (1-\gamma(i))\alpha(i)di}{\int_{i^*}^1 \alpha(i)di}$  of revenue from Northern manufacturing goes to skilled labor and the rest to unskilled workers and entrepreneurs:

$$WC_{x,N}(L_{x,N} - U_{x,N}) = \mu \frac{\int_{i^*}^1 \gamma(i)\alpha(i)di}{\int_{i^*}^1 \alpha(i)di}, \quad (3.29)$$

$$Q_N H_N = \mu \frac{\int_{i^*}^1 (1-\gamma(i))\alpha(i)di}{\int_{i^*}^1 \alpha(i)di}. \quad (3.30)$$

Note that the choice of numeraire is not irrelevant in this model as the flow value of being unemployed  $b$  and the vacancy posting costs ( $c_{V,x}$ ,  $c_{V,z}$ ) are fixed in terms of it. Northern income was chosen as it can be expected that the decrease in the share of the overall pie after outsourcing ( $di^* > 0$ ) is approximately offset by the growth of the whole pie as the shift will be assumed to be due to a productivity increase in the South. This choice has similar implications as a fair-wage-constraint, which is for example employed by Egger and Kreickemeier (2008), because also there a trade-induced increase in wage inequality goes hand in hand with an increase in unskilled unemployment (see section 3.5).

An intuitive alternative choice would have been to assume quasilinear preferences with the Northern service good as numeraire, but that would have caused severe problems. Using (3.18) in (3.19) and setting  $P_z = 1$ , one can see that such an assumption would pin down the labor market tightness in  $z$  – and by equation (3.20) also the one in  $x$ . So the choice of the numeraire remains an open issue.

### 3.3.3 Relocation of production and Northern unemployment

One possible reason for an increase in  $i^*$  can be a relative increase in Southern productivity  $A_{x,S}$ . Appendix 3.A shows that

$$\frac{di^*}{d \ln A_{x,S}} > 0. \quad (3.31)$$

A higher  $A_{x,S}$  means that intermediate goods with an index slightly above the 'old'  $i^*$  will now be cheaper to produce in the South (see figure 3.2).<sup>18</sup> This result stands in contrast to typical small-open-economy models of a Heckscher-Ohlin-type, where a factor-neutral productivity increase by 1% would lead to a 1% increase in all factor remunerations.

The increase in outsourcing from the North to the South leads to an even stronger specialization of the North in skill-intensive production as the outsourced parts are the least skill-intensive. Therefore the relative wage of skilled labor increases.<sup>19</sup>

To analyze the effects of such an increase in outsourcing on unskilled unemployment in the North, one has to derive the labor market equilibrium in the North for a given  $i^*$ . Using (3.22) in (3.29) yields:

$$\frac{q_U(\theta_{x,N}) \left( b + \frac{c_{V,x,N}}{v} \left( \frac{r+\delta}{q_V(\theta_{x,N})} + (1-\nu)\theta_{x,N} \right) \right)}{\delta + q_U(\theta_{x,N})} L_{x,N} = \mu \frac{\int_{i^*}^1 \gamma(i) \alpha(i) di}{\int_{i^*}^1 \alpha(i) di}. \quad (3.32)$$

Using (3.23) in (3.28) yields a similar expression for the service sector:

$$\frac{q_U(\theta_{z,N}) \left( b + \frac{c_{V,z,N}}{v} \left( \frac{r+\delta}{q_V(\theta_{z,N})} + (1-\nu)\theta_{z,N} \right) \right)}{\delta + q_U(\theta_{z,N})} L_{z,N} = 1 - \mu. \quad (3.33)$$

Rearranging the last expression and substituting out  $P_{z,N}$ ,  $L_{z,N}$  and  $\theta_{z,N}$  results in (derivation in appendix 3.A):

$$L_{x,N} = L_N - \frac{1 - \mu}{b + \frac{c_{V,z,N}}{v} \left( \frac{r+\delta}{q_V \left( \frac{c_{V,x,N} \theta_{x,N}}{c_{V,z,N}} \right)} + \frac{(1-\nu)c_{V,x,N} \theta_{x,N}}{c_{V,z,N}} \right)} \left( \frac{\delta}{q_U \left( \frac{c_{V,x,N} \theta_{x,N}}{c_{V,z,N}} \right)} + 1 \right). \quad (3.34)$$

The latter equation shows that  $L_{x,N}$  is increasing in  $\theta_{x,N}$  as  $q_V(\theta)$  is decreasing in  $\theta$  and  $q_U(\theta) = q_V(\theta)\theta$  is increasing in  $\theta$ . This implies that the LHS in equation (3.32) is increasing in  $\theta_{x,N}$ , while the RHS is decreasing in  $i^*$ .<sup>20</sup> So it holds that:

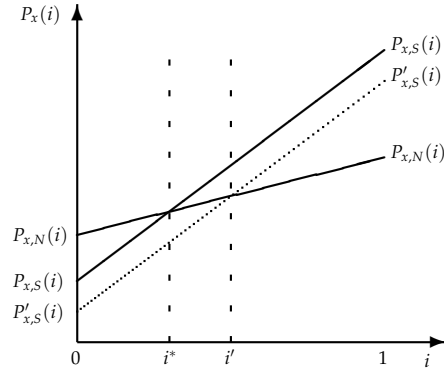
$$\frac{d\theta_{x,N}}{di^*} < 0. \quad (3.35)$$

Due to the increase in outsourcing, unskilled labor cannot be used as productively as before and less vacancies for them are posted in sector  $x$ . This leads to a decrease of the wage rate and of the unskilled labor force in  $x$ . By equation (3.21) the latter means an increase in the labor force  $L_z$  in the non-trading sector.

<sup>18</sup>Note that figure 3.2 is a bit simplifying as the increase in  $A_{x,S}$  induces changes in factor prices which would actually also change the slopes of the  $P_x(i)$ -functions.

<sup>19</sup>This can be seen from equation (3.30) where  $Q_N$  is an increasing function of  $i^*$ .

<sup>20</sup> $\frac{\partial}{\partial i^*} \left( \frac{\int_{i^*}^1 \gamma(i) \alpha(i) di}{\int_{i^*}^1 \alpha(i) di} \right) = \frac{-\gamma(i^*) \alpha(i^*) \left( \int_{i^*}^1 \alpha(i) di \right) + \alpha(i^*) \int_{i^*}^1 \gamma(i) \alpha(i) di}{\left( \int_{i^*}^1 \alpha(i) di \right)^2} = -\gamma(i^*) \alpha(i^*) \frac{\int_{i^*}^1 (\gamma(i^*) - \gamma(i)) \alpha(i) di}{\left( \int_{i^*}^1 \alpha(i) di \right)^2} < 0$  as  $\gamma(i)$  is decreasing in  $i$  by assumption and so  $\int_{i^*}^1 (\gamma(i^*) - \gamma(i)) \alpha(i) di > 0$ .



**Figure 3.2:** Relocation of production

However, to ensure that unemployed unskilled workers are still indifferent between searching in  $x$  and searching in  $z$ , the labor market tightness in  $z$  has to decrease too ( $\frac{d\theta_{z,N}}{di^*} < 0$ ; see equation (3.20)).<sup>21</sup> By equations (3.22) and (3.23) this means that both sectoral unemployment rates increase:

$$\frac{d\frac{U_{x,N}}{L_{x,N}}}{di^*} > 0 \quad \text{and} \quad \frac{d\frac{U_{z,N}}{L_{z,N}}}{di^*} > 0. \quad (3.36)$$

Note that the effect on overall unskilled unemployment  $U_N$  is ambiguous. If the hiring costs in  $z$  are sufficiently lower than in  $x$  and the unemployment rate in  $z$  is therefore lower too, then the shift of labor force from the high-unemployment sector  $x$  to the low-unemployment-sector  $z$  could lead to a decrease in the overall unemployment rate in spite of the increase in both sectoral unemployment rates.<sup>22</sup> Note again that in this case wages in  $z$  are lower than in  $x$  (see section 3.2.3).

The effect on the overall unemployment rate is also closely linked to the effect on wage inequality (between skilled and unskilled) in this model. As indicated before,  $W_x$  and  $W_z$  will decrease in terms of the numeraire and  $Q$  will increase. The only possible scenario in which this would not necessarily lead to an increase in the relative wage of skilled labor,<sup>23</sup> would be a case where  $c_{V,z} > c_{V,x}$  (the opposite from the previous paragraph): Then an increase in the labor force of  $z$  would mean that relatively more unskilled workers earn a wage in the high-unemployment-high-wage-sector. And this effect could – at least in principle – outweigh the impact of the relative increase of  $Q$  compared to  $W_x$  and  $W_z$ .

### 3.3.4 Possible differential response

The dependence of the labor market effects on the relative hiring costs in  $x$  and  $z$  introduces the possibility of a differential response of unemployment rates to outsourcing due to differences in the characteristics of the non-trading sectors.

<sup>21</sup> As  $\theta_z$  is falling, the price of good  $z$  is falling as well in terms of the numeraire (this can be seen from equation (3.19)).

<sup>22</sup> Mitra and Ranjan (2010) make a similar point (case of 'perfect labor mobility').

<sup>23</sup> The relative wage in this model is derived by comparing  $Q$  to the weighted average of  $W_x$  and  $W_z$ :  $\frac{Q}{\frac{W_x(L_x - U_x) + W_z(L_z - U_z)}{L_x + L_z - (U_x + U_z)}}$ .

Imagine two disconnected pairs which both consist of a Northern and a Southern model economy: Germany (outsourcing to countries in Central Eastern Europe) on the one hand and the US (outsourcing to Mexico and South Asia) on the other.<sup>24</sup> Even if unemployment benefits and matching technology as well as the hiring costs in manufacturing were the same in both countries, differences in the characteristics of the service sectors would lead to a differential response to an increase in outsourcing. For example, if hiring costs were relatively lower in the US service sector, this would lead to a smaller response of unskilled unemployment and a larger response of wage inequality. This could also be one of the explanations of the relatively higher share of services and construction in overall employment in the US and the UK. According to the OECD, in 2010 services and construction accounted for 'only' 79% of overall employment in the Euro area, but for 88% in the US and 87% in the UK (these numbers also include business services).

In the context of this paper, these hiring costs  $c_V$  can be thought of as a proxy for other sector-specific factors which drive wages and the cost of hiring in the same direction (like training costs).

Note that the analysis in this model abstracts from skilled unemployment and so an increase in unskilled unemployment is equivalent to an increase in overall unemployment. If one were also to account for skilled unemployment, the (likely negative) effect of an increase in outsourcing on the overall unemployment rate should be significantly dampened by the expected decrease in skilled unemployment.

### 3.4 International fragmentation versus trade in final goods

The setting employed so far serves very well for emphasizing the role of non-tradable goods and for deriving the implications of increased outsourcing for labor markets. However, while in section 3.1.1 I emphasize the importance of distinguishing between trade in final goods and trade in intermediate goods, there is nothing in my analysis which would change if I just relabelled the intermediate manufacturing goods as final goods (and interpret  $Y$  in equation (3.3) as an aggregator for deriving utility). The case is similar with several other papers discussing the impact of 'offshoring' or 'outsourcing' on unskilled unemployment (see section 3.5).

So to be able to compare changes in relative employment and relative prices of different sectors (as in Feenstra and Hanson, 2001), I need to split up the manufacturing process further into the following two stages:<sup>25</sup>

1. Different intermediate goods  $X(i, j)$  (with differing labor elasticities  $\gamma(i, j)$ ) are bundled to
2. different final goods  $X(i)$  (with labor elasticities  $\gamma(i) = \int_0^1 \gamma(i, j) dj$ ).

Production functions are given by:

$$\begin{aligned} \ln Y &= \int_0^1 \ln X(i) di \quad \text{and} \\ \ln X(i) &= \int_0^1 \ln X(i, j) dj. \\ X(i, j) &= A_x L(i, j)^{\gamma(i, j)} H(i, j)^{1-\gamma(i, j)}, \end{aligned}$$

<sup>24</sup>In reality the US and Germany obviously engage in trade with each other.

<sup>25</sup>To make things a bit easier, I set  $\alpha(i, j) = 1 \forall i, j$ .

where the labor elasticity  $\gamma(i, j)$  is increasing in  $j$  for given  $i$ . Similar to before I have:

$$P(i, j) = \frac{1}{\gamma(i)\gamma(i, j)(1 - \gamma(i, j))^{1-\gamma(i, j)}} \min \left[ \frac{WC_{x,N}^{\gamma(i, j)} Q_N^{1-\gamma(i, j)}}{A_{x,N}}, \frac{WC_{x,S}^{\gamma(i, j)} Q_S^{1-\gamma(i, j)}}{A_{x,S}} \right].$$

For each  $i$ , there will be a cut-off point  $j^*(i)$  for which everything with  $j(i) < j^*(i)$  ( $j(i) > j^*(i)$ ) is produced in the South (North). As productivity  $A_{x,l}$  is assumed to be the same across all sectors (within each country),  $\gamma(i, j^*(i)) =: \gamma^*$  is identical for all  $i$ .<sup>26</sup>

The effects of an increase in outsourcing on labor markets (possibly caused by a relative increase in Southern productivity) are qualitatively the same as before. But the enriched production structure allows me to look at relative prices and relative employment of different sectors before and after the overall increase in outsourcing.

Appendix 3.B shows that the effect of an increase in Southern productivity on the relative production (i.e. relative to overall production) of good  $\tilde{i}$  can be expressed as follows:

$$\begin{aligned} \frac{d \ln X(\tilde{i})}{d \ln A_{x,S}} - \frac{d \ln Y}{d \ln A_{x,S}} &= j^*(\tilde{i}) - \int_0^1 j^*(i) di \\ &+ \left( \int_0^{j^*(\tilde{i})} \gamma(\tilde{i}, j) dj - \int_0^1 \int_0^{j^*(i)} \gamma(i, j) dj di \right) \frac{d \ln N_{x,S}}{d \ln A_{x,S}} \\ &+ \left( \int_{j^*(\tilde{i})}^1 \gamma(\tilde{i}, j) dj - \int_0^1 \int_{j^*(i)}^1 \gamma(i, j) dj di \right) \frac{d \ln N_{x,N}}{d \ln A_{x,S}} \\ &+ \gamma^* \left( -\frac{N_{x,S}(\tilde{i})}{N_{x,S}} + \frac{N_{x,N}(\tilde{i})}{N_{x,N}} \right) \int_0^1 \frac{dj^*(i)}{d \ln A_{x,S}} di \\ &+ (1 - \gamma^*) \left( -\frac{H_S(\tilde{i})}{H_S} + \frac{H_N(\tilde{i})}{H_N} \right) \int_0^1 \frac{dj^*(i)}{d \ln A_{x,S}} di. \end{aligned} \quad (3.37)$$

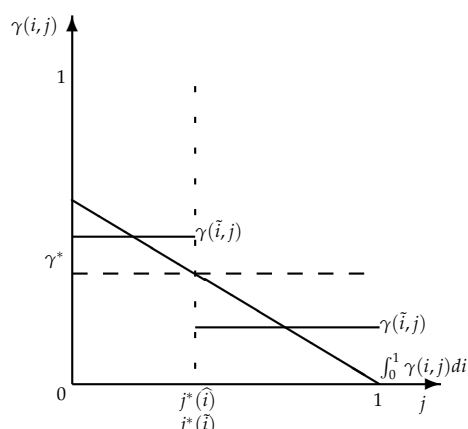
This long expression can be disentangled as follows:

1. The terms in the first line denote the gains from the productivity increase in the South (as intermediate goods which were already produced in the South before the increase in  $A_{x,S}$  can now be produced with less factor input).
2. The terms in the second and third line denote the effects of the change in the number of unskilled workers in both Southern and Northern manufacturing. This change is due to a shift to the service sector and higher unemployment, the number of unskilled workers in manufacturing decreases.
3. The terms in the last two lines denote the effects of increased outsourcing on the intra-manufacturing-sector-availability of labor (in the South  $N_{x,S}$  and  $H_S$  are distributed to a wider range of manufacturing tasks, while labor becomes less scarce in remaining Northern manufacturing).

One case where sector  $\tilde{i}$  is relatively labor-intensive (meaning that  $\int_0^1 \gamma(\tilde{i}, j) dj > \int_0^1 \int_0^1 \gamma(i, j) dj di$ ) and an increase in Southern productivity increases the relative price of  $\tilde{i}$  is illustrated in figure 3.3 (details can be found in appendix 3.B).<sup>27</sup> There the first two lines of (3.37) both cancel out, the

<sup>26</sup>Note, however, that some sectors might produce everything in one region (when either  $\gamma(i, j) > \gamma^* \forall j$  or  $\gamma(i, j) < \gamma^* \forall j$ ).

<sup>27</sup>Note that  $\tilde{i}$  is only very slightly more labor intensive than the average.



**Figure 3.3:** Outsourcing frontier for different manufacturing sectors

item in the third line is negative and the sum of the fourth and the fifth line is smaller than zero too (see again appendix 3.B).

As  $P(i)X(i)$  is identical over all goods, this relative decrease of  $X(\tilde{i})$  means that the relative price of labor-intensive good  $\tilde{i}$  increases. Furthermore, while the other sectors relocate parts of their production to the South, sector  $\tilde{i}$  does not, which also means an increase in relative employment (compared to overall manufacturing) of this sector in the North. The results of section 3.3.3 also imply that – like with skill-biased technological change – there is an increase in the skill-intensity in all manufacturing sectors of the economy (due to the shift of unskilled labor into unemployment and into the service sector).

This means that in this model – and in contrast to typical models analyzing trade in final goods only – an increase in outsourcing to labor-abundant countries cannot only lead to an increase in unskilled unemployment, but can also be accompanied by an increase in the relative price of labor-intensive goods and the relative employment in labor-intensive sectors.

### 3.5 Comparison to other results in the literature

The result that an increase in trade or opening up trade increases unemployment and/or wage inequality is not new in the literature. For example, it is also the case in Krugman (1995), Davis (1998), Sener (2003), Moore and Ranjan (2005), Egger and Kreickemeier (2008)<sup>28</sup>, Keuschnigg and Ribi (2009), Mitra and Ranjan (2009) and Helpman et al. (2010).

The analysis of Sener (2003) and Helpman et al. (2010) is made in the context of opening up/liberalizing North-North-trade (trade of countries which are similar in terms of factor endowments and productivity). So the channels leading to their results are completely different to my work. In Sener (2003) trade liberalization increases the gains to innovation and makes therefore working in R&D (= high skilled jobs) more attractive, while at the same time increasing turnover and job destruction in the manufacturing industry (= low skilled jobs). In Helpman et al. (2010), one crucial factor for the increase in unemployment and inequality is the expansion of the manufacturing sector following an increase in trade of its goods. This leads to a shift

<sup>28</sup>Egger and Kreickemeier (2008) also discuss cases where international fragmentation benefits unskilled labor.

of a part of the labor force from the low-income-low-unemployment numeraire sector to the high-wage-high-unemployment manufacturing sector.<sup>29</sup>

The contributions of Krugman (1995), Davis (1998), Moore and Ranjan (2005), Mitra and Ranjan (2009), Egger and Kreickemeier (2008) and Keuschnigg and Ribi (2009) all model North-South-trade in a Heckscher-Ohlin-like-setting. In the first four papers an exogenous (trade- or fragmentation-driven) increase of the relative price of the skill-intensive good is analyzed, while in Egger and Kreickemeier (2008) it becomes possible to split up previously integrated production structures and in Keuschnigg and Ribi (2009) there is a decrease in transport costs. In a model without unemployment this would typically lead to an increase of the relative wage of the skilled, but due to frictions (minimum wages in the former two, fair wages in the third and search frictions in the latter three) there is an increase in unemployment of the unskilled. The difference of my contribution to these six papers is two-fold:

First, the proposed reason for a differential response is different in my model. In Krugman (1995) and Davis (1998) the prevailing minimum wage is the reason for the different response of 'Europe' (Northern country with binding minimum real wage constraint) and 'the US' (Northern country with flexible labor markets) to opening up trade with a unskilled-labor-abundant country (leading to an increase in inequality in 'the US' and an increase in unemployment in 'Europe'),<sup>30,31</sup> while Egger and Kreickemeier (2008) and Mitra and Ranjan (2009) claim that stricter 'fair wage constraints' in Europe could lead to this result. Moore and Ranjan (2005) and Keuschnigg and Ribi (2009) do not explicitly talk about a differential response of different Northern economies to 'globalization' but mention that the higher unemployment benefits, the higher the reaction of unemployment rates and the lower of wages. So in all of these papers the characteristics of the trading sectors (real wage rigidities and unemployment benefits) are the reason for the differential response. However, in my paper I argue that there can be a differential response of different Northern economies to the same shock even if the characteristics of the manufacturing sectors (including unemployment benefits) are the same. Namely, possible differences in the characteristics of the non-trading sectors could matter (see section 3.3.4).

Second, with the exception of Egger and Kreickemeier (2008), none of the above papers makes the crucial distinction between the skill-intensity of sectors/final goods versus the one of subsectors/intermediate goods. Krugman (1995) and Davis (1998) explicitly model trade in final goods only; Moore and Ranjan (2005), Keuschnigg and Ribi (2009) and Mitra and Ranjan (2009) model trade in intermediate goods (the latter two use the terms outsourcing respectively offshoring). However, in all three contributions these two intermediate goods (the skill-intensive and the labor-intensive one) are boundled into just one final good, so all of them could be easily rewritten into a model with two consumption goods, and so the above mentioned distinction cannot be made. However, this distinction – which I explicitly make in section 3.4 – is crucial, as an increase of the relative price of skill-intensive final goods (which is needed to have adverse effects for unskilled labor in Heckscher-Ohlin-type-models with trade in final goods) is often claimed to be

<sup>29</sup>The mechanism making workers indifferent between applying in these 2 sectors is somehow different from the one in my model, however: In Helpman et al. (2010), all workers in the numeraire sector receive the same wage regardless of their ability which is not observable to themselves. In manufacturing, there is a (costly) screening for abilities with a certain ability threshold below which workers are rejected (and therefore unemployed); the wage rate for the accepted workers is higher than in the numeraire sector.

<sup>30</sup>The first to analyze the effects of minimum wages in a Heckscher-Ohlin-world was Brecher (1974) in a more theoretical contribution.

<sup>31</sup>The first to analyze the effects of minimum wages in a Heckscher-Ohlin-world was Brecher (1974) in a more theoretical contribution.

at odds with empirical evidence (see section 3.1 and Feenstra and Hanson (2001)).

Only Egger and Kreickemeier (2008) make this distinction: In their model, there are three final goods ( $Y$ , skill-intensive  $X$ , labor-intensive  $Z$ ) and the good with the intermediate skill intensity ( $Y$ ) can be internationally fragmented. However, their production structure introduces an ambiguity in the distributional effects of international outsourcing, which does not exist in my model. Similar to the more general fragmentation model of Kohler (2003), in Egger and Kreickemeier (2008) international fragmentation benefits the factor which is used intensively in the fragmenting sector. So if (relatively labor-intensive) fragments are outsourced to Southern economies, the impact on wage premiums and unemployment rates depends on the relative skill intensity of the outsourcing sector. If it is relatively skill-intensive (i.e. production of  $Y$  and  $Z$  in the pre-fragmentation-equilibrium), the results are relatively similar to mine; if it is relatively labor-intensive (i.e. production of  $X$  and  $Y$  in the pre-fragmentation-equilibrium), unskilled labor gains from international fragmentation. In my model the relative gain of certain sectors from international outsourcing is reflected in an increase in their produced quantities but – thanks to the general equilibrium setting – leads at the same time to a decrease of the relative prices of their goods (which can be easily quantified due to the Cobb-Douglas-specification, see section 3.4). Due to the latter aspect, there can be no gain for unskilled labor after an increase in outsourcing to labor-abundant countries in my model (in contrast to the case of a small open economy in Egger and Kreickemeier, 2008).

The models of Matusz (1996), Hoon (2001), Mitra and Ranjan (2010) and Felbermayr et al. (2011) all have only one type of labor input. Furthermore, they all emphasize the efficiency gains from opening up or liberalizing trade while I analyze an increase in trade caused by higher competitiveness of a trading partner and stress the relative loss incurred by unskilled labor.

Given these settings, it comes at no surprise that in Matusz (1996) and Hoon (2001) opening up trade is unambiguously positive for employment and that in the other two contributions the effects tend to be positive with some ambiguities: In the perfect-labor-mobility case in Mitra and Ranjan (2010), sectoral unemployment rates decrease after outsourcing becomes possible, but the effect on overall employment can be negative. In Felbermayr et al. (2011) the effect on employment is positive in two trade liberalization scenarios and ambiguous in one.

With one exception, these four models are therefore not comparable with mine. The work of Mitra and Ranjan (2010) is similar to mine in terms of the modelling approach: The (homogeneous) unemployed workers have to be indifferent between searching in a sector where international outsourcing is possible and one where it is not. They get to a similar dependence of the overall labor market effects of outsourcing on the relative hiring costs in the two sectors, but the effect of international outsourcing on sectoral employment rates is positive (i.e. in the 'normal case' outsourcing reduces unemployment). Furthermore, in their contribution it is not explicitly mentioned to be a reason for a differential response of Northern economies to international outsourcing. They analyze the potential impact of labor mobility between the trading and the non-trading sector on the response to international outsourcing instead.

### 3.6 Conclusion and possible extensions

In this paper I have analyzed the effects of international outsourcing on the employment of unskilled labor in a Heckscher-Ohlin-World (2 countries with different relative endowments).



An increase in outsourcing from the skill-abundant North to the labor-abundant South makes Northern production more skill-intensive. This causes a shift of part of the unskilled labor force to the non-trading sectors and increases sectoral unemployment rates. The characteristics of this non-trading sector are then crucial to determine the overall effect of unskilled unemployment and can be a cause for a differential response of different Northern economies (like Germany and the US) to an increase in outsourcing. Furthermore, I show that in my setting – in contrast to typical models analyzing trade in final goods only – an increase in outsourcing to labor-abundant countries can also be accompanied by an increase in the relative price of labor-intensive goods and the relative employment in labor-intensive sectors in the North.

Possible extensions to this work would be to look at adjustment paths from one steady state to the other and to get away from the purely unit-cost-based location decision; the latter would be necessary to calibrate the model as now the effects of changes in Southern productivity on relative production would be very large.

## Appendix

### 3.A Derivations related to employment effects

#### Intermediate Steps for Equations (3.11) and (3.20)

Combining Nash bargaining and the zero-profit-condition (3.6) and (3.9) yields

$$\begin{aligned} J_{W,x}(i,k) - J_{U,x} &= \frac{(1-\nu)}{\nu} J_{E,x}(i,k) \\ &= \frac{(1-\nu)}{\nu} \frac{c_{V,x}}{q_V(\theta_x)}. \end{aligned} \quad (3.38)$$

This expression is independent of  $i$ . So  $J_{W,x}(i,k) = \mathbb{E}J_{W,x} = J_{W,x}$ . Subtracting (3.8) from (3.7), solving for  $W_x(i) = W_x$  and then plugging in from above leads to:

$$\begin{aligned} W_x &= b + (r + \delta + q_U(\theta_x))(J_{W,x} - J_{U,x}) \\ &= b + (r + \delta + q_U(\theta_x)) \frac{(1-\nu)}{\nu} \frac{c_{V,x}}{q_V(\theta_x)} \\ &= b + \frac{(1-\nu)c_{V,x}}{\nu} \left( \theta_x + \frac{r + \delta}{q_V(\theta_x)} \right). \end{aligned}$$

Plugging that into the expression of the value of being unemployed (3.8) yields

$$rJ_{U,x} = b + q_U(\theta_x) \frac{(1-\nu)}{\nu} \frac{c_{V,x}}{q_V(\theta_x)} = b + \frac{(1-\nu)}{\nu} c_{V,x} \theta_x. \quad (3.39)$$

#### Intermediate Steps for Equation (3.12)

Taking the first derivative of (3.6) with regard to  $H(i,k)$  yields:

$$(1 - \gamma(i))A_x H(i,k)^{-\gamma(i)} P_x(i) = Q.$$

Solving for  $H(i,k) = H(i)$  yields:

$$H(i,k) = \left( \frac{A_x(1 - \gamma(i))P_x(i)}{Q} \right)^{\frac{1}{\gamma(i)}}. \quad (3.40)$$

Using (3.10) and (3.6) to rewrite (3.9) leads to:

$$(r + \delta)J_{V,x}(i) = A_x H(i,k)^{1-\gamma(i)} P_x(i) - QH(i,k) - W_x(i) - (r + \delta) \frac{c_{V,x}}{q_V(\theta_x)} = 0. \quad (3.41)$$

Combining (3.40) and (3.41) and rearranging yields:

$$\begin{aligned} 0 &= A_x P_x(i) \left( \frac{A_x(1 - \gamma(i))P_x(i)}{Q} \right)^{\frac{1-\gamma(i)}{\gamma(i)}} - Q \left( \frac{A_x(1 - \gamma(i))P_x(i)}{Q} \right)^{\frac{1}{\gamma(i)}} - W_x(i) - (r + \delta) \frac{c_{V,x}}{q_V(\theta_x)} \\ &= A_x^{\frac{1}{\gamma(i)}} P_x(i)^{\frac{1}{\gamma(i)}} Q^{\frac{\gamma(i)-1}{\gamma(i)}} \left[ (1 - \gamma(i))^{\frac{1-\gamma(i)}{\gamma(i)}} - (1 - \gamma(i))^{\frac{1}{\gamma(i)}} \right] - W_x(i) - (r + \delta) \frac{c_{V,x}}{q_V(\theta_x)}. \end{aligned}$$

Finally one gets to (3.12) by solving for  $P_x(i)$ .

### Intermediate Steps for Equation (3.34)

Using (3.19) in (3.33) results in:

$$\frac{f(\theta_{z,N}) \theta_{z,N} \left( b + \frac{c_{V,z,N}}{v} \left( \frac{r+\delta}{f(\theta_{z,N})} + (1-v)\theta_{z,N} \right) \right)}{\delta + f(\theta_{z,N}) \theta_{z,N}} L_{z,N} = 1 - \mu.$$

Rewriting that expression using (3.20) and (3.21) yields

$$\frac{f\left(\frac{c_{V,x,N}\theta_{x,N}}{c_{V,z,N}}\right) \frac{c_{V,x,N}\theta_{x,N}}{c_{V,z,N}} \left( b + \frac{c_{V,z,N}}{v} \left( \frac{r+\delta}{f\left(\frac{c_{V,x,N}\theta_{x,N}}{c_{V,z,N}}\right)} + \frac{(1-v)c_{x,N}\theta_{x,N}}{c_{V,z,N}} \right) \right)}{\delta + f\left(\frac{c_{V,x,N}\theta_{x,N}}{c_{V,z,N}}\right) \frac{c_{V,x,N}\theta_{x,N}}{c_{V,z,N}}} (L_N - L_{x,N}) = 1 - \mu;$$

solving for  $L_{x,N}$  leads to (3.34).

### Intermediate Steps for Equation (3.31)

Using equations (3.29) - (3.30) (and the equivalent expressions for the South), (3.26) can be expressed as:

$$\frac{A_{x,S}}{A_{x,N}} = \left( \frac{L_{x,N} - U_{x,N}}{L_{x,S} - U_{x,S}} \right)^{\gamma(i^*)} \left( \frac{\int_0^{i^*} \gamma(i)\alpha(i)di}{\int_{i^*}^1 \gamma(i)\alpha(i)di} \right)^{\gamma(i^*)} \left( \frac{H_N}{H_S} \right)^{1-\gamma(i^*)} \left( \frac{\int_0^{i^*} (1-\gamma(i))\alpha(i)di}{\int_{i^*}^1 (1-\gamma(i))\alpha(i)di} \right)^{1-\gamma(i^*)}.$$

Taking logs on both sides and rearranging leads to:

$$\begin{aligned} 0 = \Omega(A_{x,S}, A_{x,N}, i^*) &:= -\ln \frac{A_{x,S}}{A_{x,N}} + \gamma(i^*) \ln \left( \frac{L_{x,N} - U_{x,N}}{L_{x,S} - U_{x,S}} \right) + (1 - \gamma(i^*)) \ln \left( \frac{H_N}{H_S} \right) \\ &+ \gamma(i^*) \ln \int_0^{i^*} \gamma(i)\alpha(i)di - \gamma(i^*) \ln \int_{i^*}^1 \gamma(i)\alpha(i)di \\ &+ (1 - \gamma(i^*)) \ln \int_0^{i^*} (1 - \gamma(i))\alpha(i)di \\ &- (1 - \gamma(i^*)) \ln \int_{i^*}^1 (1 - \gamma(i))\alpha(i)di. \end{aligned} \quad (3.42)$$

Applying the implicit function theorem, the effect of an unexpected increase in Southern productivity  $A_{x,S}$  on the outsourcing frontier  $i^*$  can be computed as follows:

$$\frac{di^*}{d \ln A_{x,S}} = -\frac{\frac{\partial \Omega}{\partial \ln A_{x,S}}}{\frac{\partial \Omega}{\partial i^*}}. \quad (3.43)$$

Taking derivatives of (3.42) yields

$$\frac{\partial \Omega}{\partial \ln A_{x,S}} = -1, \quad (3.44)$$

and

$$\begin{aligned} \frac{\partial \Omega}{\partial i^*} &= \frac{\partial \gamma(i^*)}{\partial i} \left( \ln \frac{(L_{x,N} - U_{x,N}) H_S}{(L_{x,S} - U_{x,S}) H_N} + \ln \frac{\int_0^{i^*} \gamma(i) \alpha(i) di \int_{i^*}^1 (1 - \gamma(i)) \alpha(i) di}{\int_{i^*}^1 \gamma(i) \alpha(i) di \int_0^{i^*} (1 - \gamma(i)) \alpha(i) di} \right) \\ &+ \gamma(i^*) \frac{\gamma(i^*) \alpha(i^*)}{\int_0^{i^*} \gamma(i) \alpha(i) di} + \gamma(i^*) \frac{\gamma(i^*) \alpha(i^*)}{\int_{i^*}^1 \gamma(i) \alpha(i) di} + (1 - \gamma(i^*)) \frac{(1 - \gamma(i^*)) \alpha(i^*)}{\int_0^{i^*} (1 - \gamma(i)) \alpha(i) di}. \end{aligned} \quad (3.45)$$

Combining equations (3.43), (3.44) and (3.45) results in:

$$\begin{aligned} \left( \frac{di^*}{d \ln A_{x,S}} \right)^{-1} &= \frac{\partial \Omega}{\partial i^*} = \frac{\partial \gamma(i^*)}{\partial i} \ln \frac{Q_N WC_{x,S}}{Q_S WC_{x,N}} + \gamma(i^*) \frac{\gamma(i^*) \alpha(i^*)}{\int_0^{i^*} \gamma(i) \alpha(i) di} + \gamma(i^*) \frac{\gamma(i^*) \alpha(i^*)}{\int_{i^*}^1 \gamma(i) \alpha(i) di} \\ &+ (1 - \gamma(i^*)) \frac{(1 - \gamma(i^*)) \alpha(i^*)}{\int_0^{i^*} (1 - \gamma(i)) \alpha(i) di} + (1 - \gamma(i^*)) \frac{(1 - \gamma(i^*)) \alpha(i^*)}{\int_{i^*}^1 (1 - \gamma(i)) \alpha(i) di} \\ &+ \gamma(i^*) \frac{dN_{x,N}}{N_{x,N}} \frac{di^*}{di^*} - \gamma(i^*) \frac{dN_{x,S}}{N_{x,S}} \frac{di^*}{di^*}, \end{aligned} \quad (3.46)$$

where I used equations (3.29) and (3.30) for rearranging terms. All but the last two terms in (3.46) are unambiguously positive (as  $\frac{\partial \gamma(i^*)}{\partial i} < 0$  and  $\frac{Q_N WC_{x,S}}{Q_S WC_{x,N}} < 1$ ). To keep things simple, I assume that labor markets in the South are competitive and not subject to a search friction. Then it holds that:

$$N_{x,S} = \frac{\mu \int_0^{i^*} \alpha(i) \gamma(i) di}{\int_0^{i^*} \alpha(i) di} N_S = \frac{\mu \int_0^{i^*} \alpha(i) \gamma(i) di}{\mu \int_0^{i^*} \alpha(i) \gamma(i) di + (1 - \mu) \int_0^{i^*} \alpha(i) di} N_S,$$

where  $N_S = L_S$  is exogenous. As the overall skill intensity in the Southern manufacturing sector increases with the rise in  $i^*$ , the share of the unskilled labor force working in the manufacturing sector decreases:

$$\begin{aligned} \frac{dN_{x,S}}{di^*} &= \frac{\mu \alpha(i^*) \gamma(i^*) \left( \mu \int_0^{i^*} \alpha(i) \gamma(i) di + (1 - \mu) \int_0^{i^*} \alpha(i) di \right)}{\left( \mu \int_0^{i^*} \alpha(i) \gamma(i) di + 1 - \mu \right)^2} N_S \\ &- \frac{(\mu \alpha(i^*) \gamma(i^*) + (1 - \mu) \alpha(i^*)) \mu \int_0^{i^*} \alpha(i) \gamma(i) di}{\left( \mu \int_0^{i^*} \alpha(i) \gamma(i) di + 1 - \mu \right)^2} N_S \\ &= \frac{\mu(1 - \mu) \alpha(i^*) \gamma(i^*) \int_0^{i^*} \alpha(i) di - (1 - \mu) \alpha(i^*) \mu \int_0^{i^*} \alpha(i) \gamma(i) di}{\left( \mu \int_0^{i^*} \gamma(i) di + 1 - \mu \right)^2} N_S \\ &= \frac{\mu(1 - \mu) \alpha(i^*) \int_0^{i^*} \alpha(i) di}{\left( \mu \int_0^{i^*} \gamma(i) di + 1 - \mu \right)^2} (\gamma(i^*) - 1) N_S < 0. \end{aligned} \quad (3.47)$$

So  $-\gamma(i^*) \frac{dN_{x,S}}{N_{x,S}} \frac{di^*}{di^*} > 0$ .

To get to the intended result of  $\frac{di^*}{d \ln A_{x,S}} > 0$ , the only thing left to show is that  $\gamma(i^*) \frac{dN_{x,N}}{N_{x,N}} \frac{di^*}{di^*}$  (which

is negative) is sufficiently small in absolute value. Rearranging equation (3.32) yields:

$$N_{x,N} = \frac{\mu \frac{\int_{i^*}^1 \gamma(i)\alpha(i)di}{\int_{i^*}^1 \alpha(i)di}}{b + \frac{c_{V,x}}{v} \left( (1-\nu)\theta_{x,N} + \frac{r+\delta}{q_V(\theta_{x,N})} \right)}. \quad (3.48)$$

$\frac{dN_{x,N}}{di^*}$  can be computed as follows:

$$\frac{dN_{x,N}}{di^*} = \frac{\partial N_{x,N}}{\partial i^*} + \frac{\partial N_{x,N}}{\partial \theta_{x,N}} \frac{d\theta_{x,N}}{di^*}. \quad (3.49)$$

It has already been shown in section 3.3.3 that  $\frac{d\theta_{x,N}}{di^*} < 0$ . Taking derivatives of (3.48) with regard to  $\theta_{x,N}$  and  $i^*$  leads to:

$$\begin{aligned} \frac{\partial N_{x,N}}{\partial \theta_{x,N}} &= \frac{-\mu \frac{\int_{i^*}^1 \gamma(i)\alpha(i)di}{\int_{i^*}^1 \alpha(i)di}}{\left( b + \frac{c_{V,x}}{v} \left( (1-\nu)\theta_{x,N} - \frac{r+\delta}{q_V(\theta_{x,N})} \right) \right)^2} \left( \frac{c_{V,x}}{v} \left( (1-\nu) - \frac{r+\delta}{(q_V(\theta_{x,N}))^2} \frac{\partial q_V}{\partial \theta_{x,N}} \right) \right) < 0, \text{ and} \\ \frac{\partial N_{x,N}}{\partial i^*} &= \frac{\mu \frac{-\gamma(i^*)\alpha(i^*) \int_{i^*}^1 \alpha(i)di + \alpha(i^*) \int_{i^*}^1 \gamma(i)\alpha(i)di}{\left( \int_{i^*}^1 \alpha(i)di \right)^2}}{b + \frac{c_{V,x}}{v} \left( (1-\nu)\theta_{x,N} + \frac{r+\delta}{q_V(\theta_{x,N})} \right)} = \frac{\mu \frac{\alpha(i^*) \int_{i^*}^1 \alpha(i)(\gamma(i) - \gamma(i^*))di}{\left( \int_{i^*}^1 \alpha(i)di \right)^2}}{b + \frac{c_{V,x}}{v} \left( (1-\nu)\theta_{x,N} + \frac{r+\delta}{q_V(\theta_{x,N})} \right)} \\ &= N_{x,N} \frac{\alpha(i^*) \int_{i^*}^1 \alpha(i)(\gamma(i) - \gamma(i^*))di}{\left( \int_{i^*}^1 \alpha(i)di \right) \left( \int_{i^*}^1 \alpha(i)\gamma(i)di \right)} < 0. \end{aligned}$$

As  $\frac{\partial N_{x,N}}{\partial \theta_{x,N}} < 0$  and  $\frac{d\theta_{x,N}}{di^*} < 0$ , it holds that:

$$\frac{dN_{x,N}}{di^*} > \frac{\partial N_{x,N}}{\partial i^*} = \frac{\alpha(i^*)N_{x,N}}{\int_{i^*}^1 \alpha(i)\gamma(i)di} \left( \frac{\int_{i^*}^1 \alpha(i)\gamma(i)di}{\int_{i^*}^1 \alpha(i)di} - \gamma(i^*) \right).$$

Using this expression and taking one of the summands of (3.46), one can finally show that

$$\begin{aligned} \gamma(i^*) \left( \frac{\frac{dN_{x,N}}{di^*}}{N_{x,N}} + \frac{\gamma(i^*)\alpha(i^*)}{\int_{i^*}^1 \gamma(i)\alpha(i)di} \right) &> \gamma(i^*) \left( \frac{\alpha(i^*)}{\int_{i^*}^1 \alpha(i)\gamma(i)di} \left( \frac{\int_{i^*}^1 \alpha(i)\gamma(i)di}{\int_{i^*}^1 \alpha(i)di} - \gamma(i^*) \right) + \frac{\gamma(i^*)\alpha(i^*)}{\int_{i^*}^1 \gamma(i)\alpha(i)di} \right) \\ &> \gamma(i^*) \left( -\frac{\alpha(i^*)}{\int_{i^*}^1 \alpha(i)\gamma(i)di} \gamma(i^*) + \frac{\gamma(i^*)\alpha(i^*)}{\int_{i^*}^1 \gamma(i)\alpha(i)di} \right) = 0, \end{aligned}$$

which implies that  $\left( \frac{di^*}{d \ln A_{x,S}} \right)^{-1} > 0$ .

### 3.B Derivations related to international fragmentation

This appendix shows that an increase in  $A_{x,S}$  and therefore in outsourcing to the labor-abundant South will not necessarily mean that the relative price of skill-intensive goods increases or that the relative employment in skill-intensive sectors increases. In the following I will use the words 'good' and 'sector' equivalently. Sector  $\tilde{i}$  will be labelled as labor-intensive if  $\int_0^1 \gamma(\tilde{i}, j) dj > \int_0^1 \gamma(i, j) dj di$ .

Production of good  $X(i)$  is given by:

$$\begin{aligned}
\ln X(i) &= \int_0^{j^*(i)} (\ln A_{x,S} + \gamma(i,j) \ln N_S(i,j) + (1 - \gamma(i,j)) \ln H_S(i,j)) dj \\
&\quad + \int_{j^*(i)}^1 (\ln A_{x,N} + \gamma(i,j) \ln N_N(i,j) + (1 - \gamma(i,j)) \ln H_N(i,j)) dj \\
&= \int_0^{j^*(i)} \left( \ln A_{x,S} + \gamma(i,j) \ln \frac{\gamma(i,j) N_{x,S}}{\int_0^1 \int_0^{j^*(i)} \gamma(i,j) dj di} + (1 - \gamma(i,j)) \ln \frac{(1 - \gamma(i,j)) H_S}{\int_0^1 \int_0^{j^*(i)} (1 - \gamma(i,j)) dj di} \right) dj \\
&\quad + \int_{j^*(i)}^1 \left( \ln A_{x,N} + \gamma(i,j) \ln \frac{\gamma(i,j) N_{x,N}}{\int_0^1 \int_{j^*(i)}^1 \gamma(i,j) dj di} + (1 - \gamma(i,j)) \ln \frac{(1 - \gamma(i,j)) H_N}{\int_0^1 \int_{j^*(i)}^1 (1 - \gamma(i,j)) dj di} \right) dj
\end{aligned} \tag{3.50}$$

where the last equality is due to the Cobb-Douglas-structure production function, which implies the following distribution of unskilled and skilled labor over the manufacturing subsectors in North and South:

$$H_S(i,j) = \frac{(1 - \gamma(i,j))}{\int_0^1 \int_0^{j^*(i)} (1 - \gamma(i,j)) dj di} H_S, \tag{3.51}$$

$$H_N(i,j) = \frac{(1 - \gamma(i,j))}{\int_0^1 \int_{j^*(i)}^1 (1 - \gamma(i,j)) dj di} H_N, \tag{3.52}$$

$$N_{x,S}(i,j) = \frac{\gamma(i,j)}{\int_0^1 \int_0^{j^*(i)} \gamma(i,j) dj di} N_{x,S}, \tag{3.53}$$

$$N_{x,N}(i,j) = \frac{\gamma(i,j)}{\int_0^1 \int_{j^*(i)}^1 \gamma(i,j) dj di} N_{x,N}. \tag{3.54}$$

Differentiating (3.50) with regard to  $\ln A_{x,S}$  yields:

$$\begin{aligned}
\frac{d \ln X(i)}{d \ln A_{x,S}} &= j^*(i) \\
&\quad + \left( \ln A_{x,S} + \gamma^* \ln \frac{\gamma^* N_{x,S}}{\int_0^1 \int_0^{j^*(i)} \gamma(i,j) dj di} + (1 - \gamma^*) \ln \frac{(1 - \gamma^*) H_S}{\int_0^1 \int_0^{j^*(i)} (1 - \gamma(i,j)) dj di} \right) \frac{dj^*(i)}{d \ln A_{x,S}} \\
&\quad - \left( \ln A_{x,N} + \gamma^* \ln \frac{\gamma^* N_{x,N}}{\int_0^1 \int_{j^*(i)}^1 \gamma(i,j) dj di} + (1 - \gamma^*) \ln \frac{(1 - \gamma^*) H_N}{\int_0^1 \int_{j^*(i)}^1 (1 - \gamma(i,j)) dj di} \right) \frac{dj^*(i)}{d \ln A_{x,S}} \\
&\quad - \int_0^{j^*(i)} \left( \gamma(i,j) \frac{\gamma^*}{\int_0^1 \int_0^{j^*(i)} \gamma(i,j) dj di} + (1 - \gamma(i,j)) \frac{1 - \gamma^*}{\int_0^1 \int_0^{j^*(i)} (1 - \gamma(i,j)) dj di} \right) dj \int_0^1 \frac{dj^*(i)}{d \ln A_{x,S}} di \\
&\quad + \int_{j^*(i)}^1 \left( \gamma(i,j) \frac{\gamma^*}{\int_0^1 \int_{j^*(i)}^1 \gamma(i,j) dj di} + (1 - \gamma(i,j)) \frac{1 - \gamma^*}{\int_0^1 \int_{j^*(i)}^1 (1 - \gamma(i,j)) dj di} \right) dj \int_0^1 \frac{dj^*(i)}{d \ln A_{x,S}} di \\
&\quad + \int_0^{j^*(i)} \gamma(i,j) dj \frac{d \ln N_{x,S}}{d \ln A_{x,S}} + \int_{j^*(i)}^1 \gamma(i,j) dj \frac{d \ln N_{x,N}}{d \ln A_{x,S}}.
\end{aligned} \tag{3.55}$$

The terms in the second and third line cancel out as  $P_S(i, j^*(i)) = P_N(i, j^*(i))$  and therefore  $X_S(i, j^*(i)) = X_N(i, j^*(i))$ . Furthermore  $\gamma^*$  can be taken out of the integrals. So (3.55) can be

simplified to

$$\begin{aligned} \frac{d \ln X(i)}{d \ln A_{x,S}} &= j^*(i) + \int_0^{j^*(i)} \gamma(i,j) dj \frac{d \ln N_{x,S}}{d \ln A_{x,S}} + \int_{j^*(i)}^1 \gamma(i,j) dj \frac{d \ln N_{x,N}}{d \ln A_{x,S}} \\ &+ \gamma^* \left( -\frac{\int_0^{j^*(i)} \gamma(i,j) dj}{\int_0^1 \int_0^{j^*(i)} \gamma(i,j) dj di} + \frac{\int_{j^*(i)}^1 \gamma(i,j) dj}{\int_0^1 \int_{j^*(i)}^1 \gamma(i,j) dj di} \right) \int_0^1 \frac{dj^*(i)}{d \ln A_{x,S}} di \\ &+ (1 - \gamma^*) \left( -\frac{\int_0^{j^*(i)} (1 - \gamma(i,j)) dj}{\int_0^1 \int_0^{j^*(i)} (1 - \gamma(i,j)) dj di} + \frac{\int_{j^*(i)}^1 (1 - \gamma(i,j)) dj}{\int_0^1 \int_{j^*(i)}^1 (1 - \gamma(i,j)) dj di} \right) \int_0^1 \frac{dj^*(i)}{d \ln A_{x,S}} di. \end{aligned}$$

Using equations (3.51) to (3.54) this can be rewritten to

$$\begin{aligned} \frac{d \ln X(i)}{d \ln A_{x,S}} &= j^*(i) + \int_0^{j^*(i)} \gamma(i,j) dj \frac{d \ln N_{x,S}}{d \ln A_{x,S}} + \int_{j^*(i)}^1 \gamma(i,j) dj \frac{d \ln N_{x,N}}{d \ln A_{x,S}} \\ &+ \gamma^* \left( -\frac{N_{x,S}(i)}{N_{x,S}} + \frac{N_{x,N}(i)}{N_{x,N}} \right) \int_0^1 \frac{dj^*(i)}{d \ln A_{x,S}} di \\ &+ (1 - \gamma^*) \left( -\frac{H_S(i)}{H_S} + \frac{H_N(i)}{H_N} \right) \int_0^1 \frac{dj^*(i)}{d \ln A_{x,S}} di. \end{aligned} \quad (3.56)$$

When comparing this derivative for sector  $\tilde{i}$  with the average over the whole manufacturing sector, one gets to:

$$\begin{aligned} \frac{d \ln X(\tilde{i})}{d \ln A_{x,S}} - \frac{d \ln Y}{d \ln A_{x,S}} &= j^*(\tilde{i}) - \int_0^1 j^*(i) di \\ &+ \left( \int_0^{j^*(\tilde{i})} \gamma(\tilde{i},j) dj - \int_0^1 \int_0^{j^*(i)} \gamma(i,j) dj di \right) \frac{d \ln N_{x,S}}{d \ln A_{x,S}} \\ &+ \left( \int_{j^*(\tilde{i})}^1 \gamma(\tilde{i},j) dj - \int_0^1 \int_{j^*(i)}^1 \gamma(i,j) dj di \right) \frac{d \ln N_{x,N}}{d \ln A_{x,S}} \\ &+ \gamma^* \left( -\frac{N_{x,S}(\tilde{i})}{N_{x,S}} + \frac{N_{x,N}(\tilde{i})}{N_{x,N}} \right) \int_0^1 \frac{dj^*(i)}{d \ln A_{x,S}} di \\ &+ (1 - \gamma^*) \left( -\frac{H_S(\tilde{i})}{H_S} + \frac{H_N(\tilde{i})}{H_N} \right) \int_0^1 \frac{dj^*(i)}{d \ln A_{x,S}} di. \end{aligned} \quad (3.57)$$

One numerical example where this expression becomes negative for a labor-intensive sector  $\tilde{i}$  (labor-intensive meaning  $\int_0^1 \gamma(\tilde{i},j) dj > \int_0^1 \int_0^1 \gamma(i,j) dj di$ ) would be the following (also illustrated in figure 3.3):

- $\gamma(\tilde{i},j) = 0.48$  for  $j \leq 0.4$  and  $\gamma(\tilde{i},j) = 0.185$  for  $j > 0.4$ ,
- $\int_0^1 \gamma(i,j) di = 0.6 - 0.6j$ ,
- $j^*(\tilde{i}) = \int_0^1 j^*(i) di = 0.4$  and  $\gamma^* = 0.36$  (this can be achieved by appropriately choosing other exogenous parameters like labor supply and total factor productivity).

Then it holds that:

- $j^*(\tilde{i}) - \int_0^1 j^*(i) di = 0$ ;

- $\int_0^{j^*(\tilde{i})} \gamma(\tilde{i}, j) dj - \int_0^1 \int_0^{j^*(i)} \gamma(i, j) dj di = 0$  as production in sector  $\tilde{i}$  is as labor intensive as the average in the South (note again that  $\frac{d \ln N_{x,N}}{d \ln A_{x,S}} < 0$ );
- $\int_{j^*(\tilde{i})}^1 \gamma(\tilde{i}, j) dj > \int_0^1 \int_{j^*(i)}^1 \gamma(i, j) dj di$  as production in sector  $\tilde{i}$  is relatively more labor intensive than the average in the North (note again that  $\frac{d \ln N_{x,N}}{d \ln A_{x,S}} < 0$ ); and
- $\gamma^* \left( -\frac{N_{x,S}(\tilde{i})}{N_{x,S}} + \frac{N_{x,N}(\tilde{i})}{N_{x,N}} \right) + (1 - \gamma^*) \left( -\frac{H_S(\tilde{i})}{H_S} + \frac{H_N(\tilde{i})}{H_N} \right) = -0.006 < 0$  as  $\frac{H_S(\tilde{i})}{H_S} = \frac{N_{x,S}(\tilde{i})}{N_{x,S}} = 1$  while  $\frac{N_{x,N}(\tilde{i})}{N_{x,N}}$  and  $\frac{H_N(\tilde{i})}{H_N}$  are very close to 1 and  $\gamma^*$  is significantly smaller than  $(1 - \gamma^*)$  (note again that  $\int_0^1 \frac{dj^*(i)}{d \ln A_{x,S}} di > 0$ ).



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