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OPTIMAL TAXATION OF GAMBLING AND LOTTO

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OPTIMAL TAXATION OF GAMBLING AND LOTTO

by

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Abstract

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JEL Codes
D 81, H 71

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Abstract

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1 Introduction

In the United States nearly 60% of all adults played lottery at least once in a year (*Clotfelter and Cook, 1990).* In Austria the share is slightly lower at about 53 % (*Casino Austria AG, 2001*). Lotteries generate a small, but visible part of the state revenues (between 0.5% and 3%) and are an important source of finance for public purposes.

An ubiquitous phenomenon like lottery gambling deserves to be taken serious. Recent empirical studies show remarkable regularities in lottery buyer's behavior (*Beenstock and Haitovsky, 2001; Gulley and Scott,1995; Clotfelter and Cook, 1993; Shapira and Venezia, 1992; Walker, 1998, 2001).* Such observations require a consistent explanation in terms of rational economic behavior (*Fennema and Wakker, 1996; Forrest and Simmons, 2002).*

The analysis in this paper is based on an intertemporal, state dependent expected utility model, which will be outlined in the following section. The model will be used to analyze the demand for different gambles. Lotto is studied in some detail. Monopoly solutions are found and compared for various types of bets. Standard methods of estimating empirically the demand for lotto are criticized.

2 The NREU-Model

The 'Normal-Randomness-Expected-Utility' (NREU-)model (*Walther, 2003*) integrates the idea of elation and disappointment introduced by *Bell (1985)* and *Gul (1991)* into an expected utility framework. The basic idea of NREU is as follows. At the moment of decision the subject anticipates that resolution of uncertainty induces not only permanent wealth effects but also temporary and fading emotional shocks, referred to as 'elation' and 'disappointment'.[1] Emotional states of mind evolving over time are evaluated according to a preference function - more elation (less disappointment) is preferred to less elation (more disappointment). Expected flows of instantaneous utility from wealth and from emotions are assumed to be additively separable. Time preference is essential for trading off temporary net utility gains (or losses) from emotional effects against permanent utility of wealth consequences.

Emotional reactions are anticipated to arise if and only if the flow of utility of wealth, $u(w)$, happens to fall outside the barriers of $|u(s) + \sigma^-, u(s) + \sigma^+|$, where s is the security equivalent wealth and σ^+, σ^- are expected standard deviations of utility (see below for a formal definition). This assumption is justified by the observation that subjects get used to a certain degree of randomness in every day life. Only 'exceptional' ex post realizations will induce emotional reactions ('such an incredible luck!').[2]

[1] That emotions are indeed temporary and fading was shown in early psychological studies by *Brickman et al. (1968)* and recently confirmed (*Gilbert et al., 1998*).

[2] *Caplin* and *Leary* (2001) presented a different approach to integrate anticipated emotional psychological states like 'anxiety' or 'suspense'. The paper does not suggest, however, any clearcut specification, how anxiety and/or suspense are related to specific characteristics of different prospects.

Let $u(w)$ be the instantaneous flow of (well-behaved) utility from wealth ($u'(w) > 0, u''(w) < 0, u'(0) = +\infty, u'(+\infty) = 0$). A binary lottery ticket, $L = (p,\, w_1, w_2)$, is offered, where $w_1 < w_2$ realizes with probability p.[3] The moment of decision and the moment of resolution of uncertainty are both set equal to be zero. Let θ be the exponential rate of decay of the flow of utility from elation, alternatively the speed of adjustment towards a neutral state of 'emotional equilibrium'. The instantaneous flow of utility from elation, u_e, is defined as

$$u_e = e^{-\theta t}\alpha[u(w_2) - (u(s) + \sigma^+)] \tag{1}$$

$$\sigma^+ = (1-p)(u(w_2) - u(s)) \tag{2}$$

where α reflects the 'impact' effect of an emotional shock. Similarly, the flow of utility from disappointment, u_d, can be defined

$$u_d = e^{-\rho t}\beta[u(w_2) - (u(s) + \sigma^-)] \tag{3}$$

$$\sigma^- = p(u(w_1) - u(s)) \tag{4}$$

where β reflects the 'impact' effect and ρ the speed of adjustment. Substituting (2) into (1) and (4) into (3), we get for the expected utility of wealth and emotional states,

$$\begin{aligned} E(U) = \int_0^{+\infty} & e^{-\delta t}[pu(w_1) + (1-p)u(w_2) + \\ & (1-p)e^{-\theta t}\alpha p(u(w_2) - u(s)) + \\ & pe^{-\rho t}\beta(1-p)(u(w_1) - u(s))]dt \end{aligned} \tag{5}$$

where δ is the rate of time preference. Expected utility must be equal to the discounted flow of utility of the security equivalent wealth:

$$E(U) = U(s) = \int_0^{+\infty} e^{-\delta t}u(s)dt = \frac{u(s)}{\delta} \tag{6}$$

Setting the r.h.s. of (6) equal to the r.h.s. of (5), multiplying both sides of the equation by δ and integrating the r.h.s yields the following:

$$\begin{aligned} u(s) = {} & pu(w_1) + (1-p)u(w_2) + \\ & (1-p)p\frac{\delta\alpha}{\delta+\theta}(u(w_2) - u(s)) + p(1-p)\frac{\delta\beta}{\delta+\rho}(u(w_1) - (u(s)) \end{aligned} \tag{7}$$

[3] For different methods of generalization to multivalued and continuous prospects see Walther (2003 a). A simple procedure is the following one: (1) Expected utility of wealth of the multivalued prospects are evaluated. (2) For each multivalued prospect an equivalent binary prospect is defined, whose outcomes are the best and the worst of the multivalued prospect. (3) Binary prospects are evaluated via NREU and each prospect is ranked in accordance to the NREU values.

For the purpose of convenience only, let us define an elation parameter γ and a disappointment parameter μ in order to eliminate the clumsy fractions in (7).[4]

$$\gamma = \frac{\delta\alpha}{\delta + \theta} \tag{8}$$

$$\mu = \frac{\delta\beta}{\delta + \rho} \tag{9}$$

Solving equation (7) for $u(s)$ gives

$$u(s) = q(p)u(w_1) + (1 - q(p))u(w_2) \tag{10}$$

where

$$q(p) = p\frac{1 + (1-p)\mu}{1 + (1-p)p(\gamma + \mu)} \tag{11}$$

$q(p)$ is a mapping of p onto the unit interval. Additivity and monotonicity are fulfilled.

$$q'(p) = \frac{1 + \gamma p^2 + (1-p)^2 \mu}{(1 + p(1-p)(\mu + \gamma))^2} > 0 \tag{12}$$

Similar to RDEU (*Quiggin, J., 1993, Prelec, D., 1998*), S-shaped transformation of probability weights appears. Fig. 1 illustrates the S-shaped distortion of probability weights for the case of $\mu > \gamma$, caused by longer lasting disappointment ($\rho < \theta$) for two different rates of time preference. For $q(p) = p$ three solutions can be found ($p_0 = 0$, $p_1 = \frac{\mu}{\mu+\gamma}, p_2 = 1$). Emotional distortions are neutralized (or absent) in those cases.[5] Relative distortions are maximized where lines D and E are tangents to the weighting function. The ratio $\frac{\mu}{\mu+\gamma}$ will be called 'relative disappointment aversion'.

Small probabilities to win (on the r.h.s. of R) are systematically overvalued ('over-confidence'). The intuition behind this phenomenon is simple. Anticipated elation (disappointment) will be strong, if one gets a gain (a loss) against all odds. Those effects provide the basic foundation for the NREU-theory of gambling.

In the interval ($\frac{\mu}{\mu+\gamma} < p < 1$), the weighted probability of loss, $q(p)$, is distorted below p. Let us call this interval the 'elation interval'. A closer look at the behavior of $q'(p)$ shows that $\mu > \gamma$ is sufficient, but not necessary, for

$$q''(p) > 0 \tag{13}$$

[4] Additional parameters always have a smack of ad hoc-reasoning. Such criticism should be countered by the following arguments:
(1) Subjects surely differ with respect to their 'emotional calmness' and their time preference.
(2) Subjects surely value emotional reactions to suprising events. Wealth effects of transactions are sometimes less important than their emotional connotations. (Why do we wrap presents? Why don't we write out cheques?)

[5] In a carefully designed experiment *Anderhub et al. (2001)* linked higher rates of time preference to higher degrees of risk-aversion. While this will be true for symmetric bets in the NREU context, the same subjects may exhibit lower risk aversion for positively skewed bets.

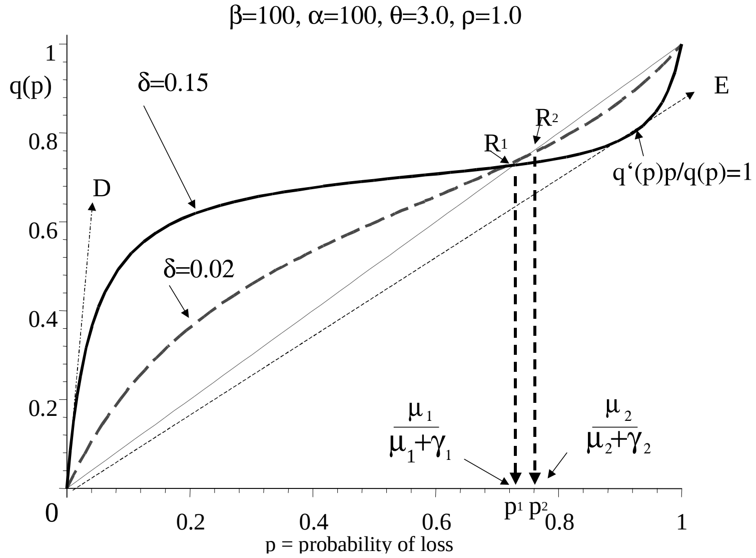

Figure 1: Emotional distortions and probability weighting

over the elation interval.[6]

Lower rate of time preference reduces distortions by making the monotonically rising function $q(p)$ less S-shaped. The point of intersection, R may shifts upward (as in fig.1 from $R_1 \rightarrow R_2$), if the relative weight of longer lasting disappointment increases due to $\rho < \theta$. Becoming a 'cool' decision-maker also lowers distortions.

Take a look at fig.2, reproduced from a careful study done by *Fehr-Duda and al.* (2004, p.29, fig.2-5). The picture confirms results of earlier experimental results (*Gonzalez and Wu*, 1999; *Bleichrodt and Pinter*, 2000) of significant gender specific differences with respect to probability weighting $w(p)$, where p is the probability to win.

In the context of NREU, the observed pattern supports the view that the utility flow from emotional reactions seems to be ranked somewhat higher by women (dotted $w(p)$-function) than by men (solid $w(p)$-function). Furthermore, women seem to be characterized by higher relative disappointment aversion. Those effects are stronger in contextual experiments, where 'vivid' images and examples strengthen emotional reactions. Emotional sensitivity matters.

[6] A short proof can be found in the Appendix (1).

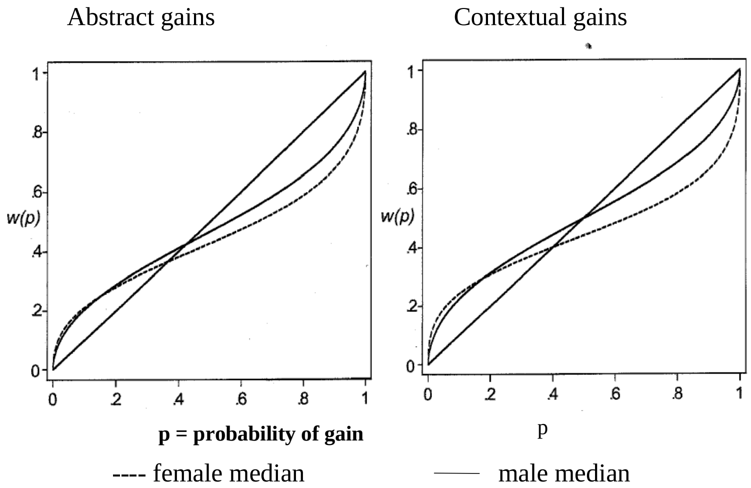

Figure 2: Gender specific probability weighting w(p)

3 Economics of Gambling

The most significant differences between various types of gambles is the method of introducing the house advantage and of defining the choice set for the gambler. Let us take a look at a simple gamble, which will serve as a state of reference. The subject maximizes

$$E(u(l,p)) = q(p)u(w_1) + (1 - q(p))u(w_2) \tag{14}$$

subject to (15), (16). Wealth in status ante is z, the choice variables are the wager, $0 \leq l \leq z$, and the 'riskiness' of the gamble (= probability of loss $0 \leq p \leq 1$). The distribution quota is $0 \leq k \leq 1$, w_1 and w_2 designate wealth in the worst and best state, respectively.

$$w_1 = z - l \tag{15}$$

$$w_2 = z - l + \frac{1}{1-p}kl \tag{16}$$

Winning the prize implies that the subject gets some multiple of the net wager kl. Let us call $1/(1-p)$ the 'multiplier'. A gamble, where the gambler can choose the multiplier, is called a *'flex-prize gamble'*. This type of gamble is different from a pure *'fix-prize gamble'*, where a fixed prize is offered, or genuine 'lotto gambles', where, from the gambler's point of view, $w_2 - z$ depends upon the expected sum of wagers put into the pot by all participating gamblers.

In the flex-prize gamble (15), (16) the expected profit of the lottery agency $\widehat{\pi}$ is equal to

$$\widehat{\pi} = pl - (1-p)(-l + \frac{1}{1-p}kl) = (1-k)l = tl \tag{17}$$

and does only depend on the tax rate ($t = 1 - k$) and the wager l. Let us call such bets pure *'wager tax bets'*, which will be analyzed in depth below.

Alternative to 'wager-tax bets' and similar to roulette-type gambles,[7] the probability of loss might be slightly increased by introducing one (or more) random draws assigned to the bank holder. Such bets are called '*bank-holder bets*'. We will focus on wager-tax bets exclusively.

3.1 The flex-prize/wager-tax bet

Let us study the pure flex-prize/wager-tax bet presented above. The subject maximizes (14) subject to (15), (16) by choice of (l^*, p^*). The first order conditions for an interior optimum within the elation interval ($l^* > 0, \mu/(\mu+\gamma) < p^* < 1$) are

$$\frac{\partial E(u)}{\partial l} = (1 - q(p))(\frac{k}{1-p} - 1)u'(w_2) - q\,(p)\,u'(w_1) = 0 \tag{18}$$

$$\frac{\partial E(u)}{\partial p} = (1 - q\,(p))u'\,(w_2)\,\frac{lk}{(1-p)^2} - q'\,(p)\,(u\,(w_2) - u\,(w_1)) = 0 \tag{19}$$

Equation (18) and (19) are called the 'wager' and the 'riskiness' (or risk-taking) condition, respectively. Both conditions imply the following one, which will be useful for later comparisons

$$\frac{l}{1-p} = \frac{q'\,(p)\,(u\,(w_2) - u\,(w_1))}{q(p)u'(w_1) + (1 - q(p))u'(w_2)} \tag{20}$$

A calibrated example is presented.

3.1.1 The best flex-prize/wager-tax bet

A fair bet (i.e. $k = 1$) is offered to a subject with wealth ($z = 10$) and preference structure $\mu = 25$, $\gamma = 4$, $u(w) = \ln(w)$. Fig.3 shows the wager condition, the risk-taking condition and the indifference curves in the (l, p) space for this type of bet.

Point A represents the bliss point (l_B^*, p_B^*) of a best bet (BB) equilibrium; the wager condition ($\partial E(u)/\partial l = 0$) and the riskiness condition ($\partial E(u)/\partial p = 0$) are fulfilled simultaneously. Note that the indifference curves are 'circles' in the (l, p) space. Note also that the optimum is placed to the r.h.s. of the maximum of the wager curve.

The wager condition (18) reveals that the expected marginal utility in state w_2 (state w_1) is increased (reduced) relative to an expected utility of wealth maximizer within the elation interval, because of $q(p) < p$. The acceptance of unfair bets becomes possible in spite of $u''(w) < 0$. For an attractive bet to exist, $\partial E(u)/\partial l > 0$ must be fulfilled for $w_1 = w_2$. This requires

$$\widehat{k} > \frac{1-p}{1-q\,(p)} \tag{21}$$

[7] For the nature and development of casino gambles c.f. *Eadington*, 1999.

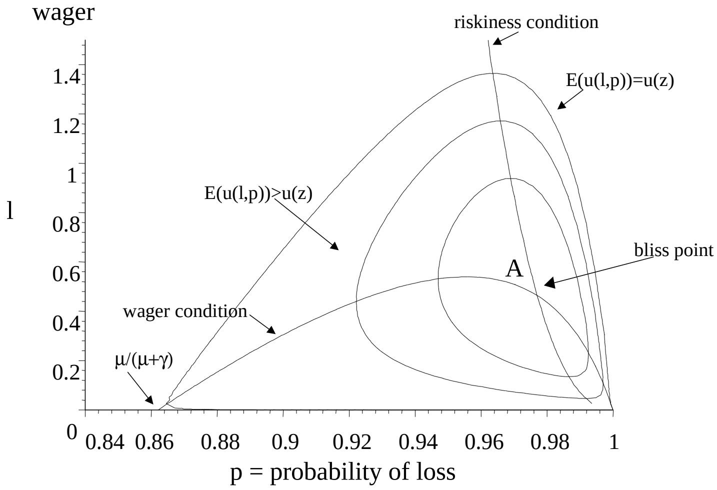

Figure 3: The best bet

Due to the u-shaped behavior of the r.h.s. of inequality (21), lower k reduces the probability range of acceptable bets. If an attractive bet $B(\widehat{k}, \mu/(\mu+\gamma) < \widehat{p} < 1)$ is offered, a utility maximizing wager $l^*(\widehat{p}, \widehat{k}) > 0$ is determined. Because of $u''(w) < 0$, $\partial E(u)/\partial l \partial l < 0$ is clearly fulfilled.

The risk-taking condition (19) can be interpreted as follows: The marginal benefit of increasing p is equal to the expected marginal utility of wealth w_2 times the marginal effect of the higher multiplier on wealth w_2. Note the peculiar shape of this expected marginal benefit (EMB), which may even be rising with p within the gambling interval due to the "multiplier" effect.[8] On the other hand, rising p also increases the distorted probability of loss $(q'(p) > 0; q''(p) > 0)$. Because $(u(w_2) - u(w_1))$ is also rising, the expected marginal loss of utility $(= q'(p)(u(w_2) - u(w_1)))$ clearly rises with higher p. It can be shown that the EMB(p) cuts the EML(p) from above within the gambling interval as long as any predetermined l is low enough for an attractive bet to exist. We may safely assume that a unique utility maximizing vector $(l^* > 0, \mu((\mu + \gamma) < p^* < 1)$ exists for relative degrees of risk aversion $R > 0$.[9]

The model allows to make specific behavioral predictions.

(1) Subjects with a lower rate of time preference are less inclined to gamble. If $\theta = \rho$, lower δ lowers μ and γ proportionally. Wager and risk-level both are falling. The gambling interval remains unchanged (fig.4).[10] The propensity to

[8] For relative risk aversion $0 < R < 2$ the 'multiplier effect' dominates and the expected marginal benefit (EMB) is rising monotonically as $p \to 1^-$. If $R > 2$ the $u''(w) < 0$ effect dominates and and EMB approaches zero as $p \to 1^-$.

[9] Detailed information is presented in the Appendix (2).

[10] If the utility flow of elation is anticipated to decay at a faster rate (e.g. $\theta > \rho$), lowering the rate of time preference increases the weight of relative disappointment aversion $(= \frac{\mu}{\mu+\gamma})$. The gambling interval shrinks, the wager decreases and risk-taking increases.

save and to gamble should be negatively correlated.

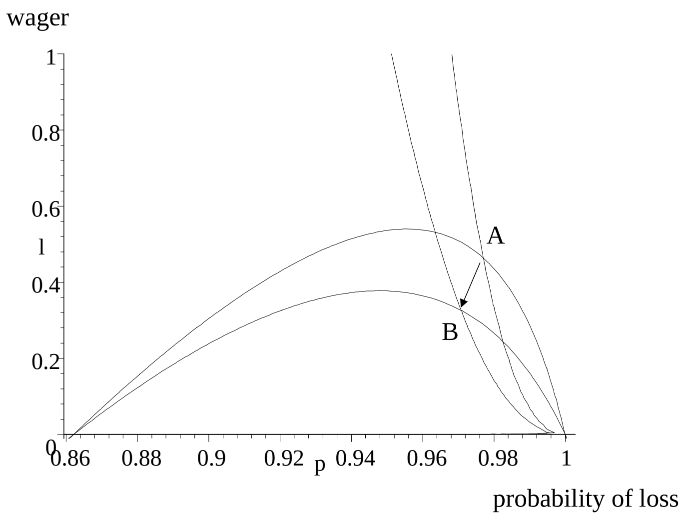

Figure 4: Lower rate of time preference

(2) If relative risk-aversion is constant the optimal wager rises in proportion to wealth and risk-taking remains constant.[11]

3.1.2 The monopolistic flex-prize/wager-tax bet

A pure monopoly exists. Let us first eliminate variables w_1, w_2 by use of (15), (16). The decision problem simplifies to maximizing expected net revenues, $\widehat{\pi} = l(1-k)$, subject to the conditions (18), (19) by choice of (l, p, k). While analytical solutions are cumbersome due to the non-linear probability weighting, numerical solutions for the monopolistic distribution quota, k^*, (as seen in fig.5) can be easily found.

An example for the given preference assumptions is presented in fig.5. Equilibrium shifts from A to B. Note that the subject lowers the wager and increases the level of risk-taking in response to lower distribution quota ($k^* = 0.584\,28$).[12] The acceptance region gets smaller and the preferred multiplier rises.

3.2 The fix-prize/wager-tax gamble

A **lottery agency** offers the following gamble to the representative gambler. A finite number of lottery tickets are issued, each with a unique number. The

[11] Let $u(w) = w^{1-R}/(1-R)$ and $l^* = \alpha^* z$. It is easy to show that $u'(w_2)/u'(w_1)$ is independent of z. Similarly, $u'(w_2)/(u(w_2) - u(w_1))$ is independent of z. For constant (α^*, p^*) wager and risk-taking conditions can be fulfilled for all values of $z > 0$.

[12] This result is fairly general, albeit for different degrees of relative risk aversion, R, upward (leftward) shifts of the wager (risk-taking) conditions over subintervals of the gambling interval are possible.Wealth and substitution effects have to be distinguished. For lower (higher) R the wager (risk-taking) level reacts more elastic.

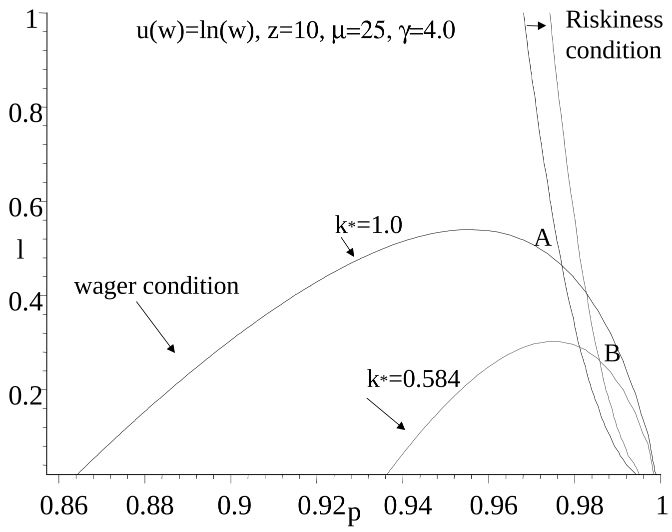

Figure 5: Monopoly in the flex-prize/wager tax bet

agency fixes the *price* of a single ticket ($= r$) and the *basic probability to win* ($= \sigma$) associated with each ticket. Let n be the demand for tickets by subject i (for notational simplicity the index i is suppressed). Risk-taking and wager are linked by the following linear relationships:[13]

$$p = 1 - n\sigma \tag{22}$$

$$l = nr \tag{23}$$

For the sake of simplicity, n is treated as a continuous variable. Note that by offering such restrictions, the flex-prize gamble from above is transformed into a 'fix-prize' gamble, where the prize is equal to $g_0 = kr/\sigma$:

$$w_2 = z - nr + k\frac{nr}{n\sigma} = z - rn + k\frac{r}{\sigma} \tag{24}$$

The subject i determines the optimal demand for tickets n_i^* by maximizing (14) subject to (15), (16), (22) and (23). Alternatively, the subject maximizes utility(14) by deciding upon l subject to (15), (16) and (25).

$$p = 1 - l\frac{\sigma}{r} \tag{25}$$

[13]This gamble is similar to the "Scratch Cards" or to the "Klassenlotterie", long ago the most important gamble of the Austrian state lottery.

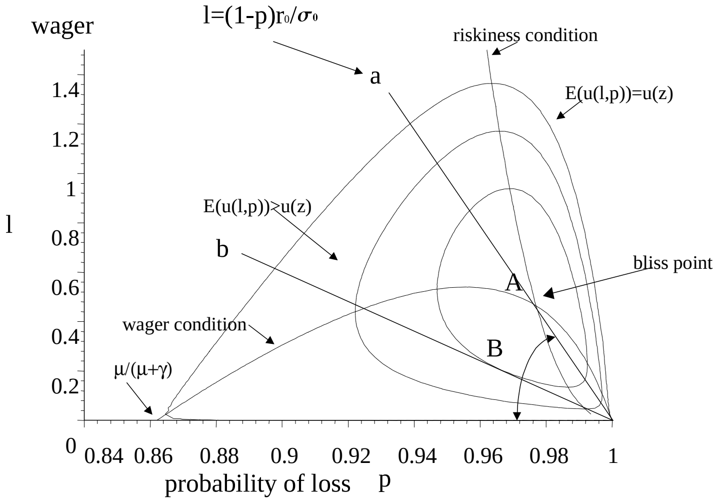

Figure 6: Fix-prize bets

Substituting (25) into (14), the first order optimum condition for this fix prize bet can be determined:

$$\frac{\partial E(u)}{\partial l} = q'(p)\left(\frac{\sigma}{r}\right)(u(w_2) - u(w_1)) \tag{26}$$

$$-(q(p)u'(w_1) + (1 - q(p))u'(w_2)) = 0 \tag{27}$$

$$\Longrightarrow \frac{r}{\sigma} = \frac{q'(p)(u(w_2) - u(w_1))}{q(p)u'(w_1) + (1 - q(p))u'(w_2)} \tag{28}$$

By fixing r/σ, the choice set gets linearly restricted (e.g. lines a or b in fig. 6). Variations of r/σ generate a reaction path of all points, where the rays are tangent to the indifference curves.

3.2.1 The best fix-prize/wager tax bet

Comparing (20) and (28) it becomes clear that the best bet A can be transformed into a fix-prize bet, by setting $r/\sigma = l_B^*/(1 - p_B^*)$ (line a).

If preferences are heterogenous, a broad variety of linear 'menus' can be offered, where each subject chooses a different fix-prize bet. 'Scratch-cards' may be an example for such gambles.

3.2.2 The monopolistic fix-prize/wager-tax bet

A monopolistic agency maximizes expected tax revenues ($\widehat{\pi} = nr(1-k) = l(1-k)$). Let us map the decision problem again into the (l, p) space by eliminating w_1, w_2 using (15), (16). By choice of (l, p, k) the agency maximizes

$$\begin{aligned} \widehat{\pi} &= l(1-k) \text{ s.t.} & (29) \\ \frac{l}{1-p} &= \frac{q'(p)(u(w_2)-u(w_1))}{q(p)u'(w_1)+(1-q(p))u'(w_2)} & (30) \end{aligned}$$

After (l^*, p^*, k^*) have been determined, the ratio of the price per ticket, r, relative to the basic probability to win has to be determined according to

$$\frac{rn}{\sigma n} = \frac{l^*}{1-p^*}$$

The question remains, how to determine the absolute value of r and σ together with n^*.We will return to that issue in the next section.

While analytical solutions are complex due to the non-linear weighting function, numerical solutions can be easily calculated for particular cases. Fig.7 illustrates for the same preferences as above Cournot's point (= C) in a fix-prize gamble. Note that the movement to point C can be separated into two steps:

- The distribution quota, k^*, falls (to $k^* = 0.591\,58$) (shifting a hypothetical flex-prize equilibrium from point A to point B).
- The price per unit of probability to win is raised above the point B level.

The linear ('ticket') restriction line, a, represents an additional instrument to raise monopolistic 'exploitation' above the level of a flex-prize bet. Relative to the flex-prize monopoly, k^* and l^* rise slightly (k^* somewhat less than l^*) and skewness increases. The indifference circle around B shows that, for identical house advantage, the subject prefers a flex-prize bet at point B with lower risk, lower wager and lower prize.

Holding k at the level of k^* constant, an implicit reaction path $l^* = l(k^*, r/\sigma)$ to c.p. variations of r/σ can be derived from (30).

Because $l^* = n^* r$, the partial elasticity of the demand for tickets, n, with respect to the price of a single ticket, r, must be just equal to one at point C, as can be seen by looking at the slope of the reaction path. To the left (right), it must be smaller (larger) than one. Furthermore, in the profit maximum, $\partial(n^* r(1-k))/\partial k = 0$. This will be the case, if

$$\frac{\partial n^*}{\partial k}\frac{k}{n^*} = \frac{k}{1-k} \tag{31}$$

Note that there does exist a possibility of increasing monopoly profits even further by offering a non-linear price/prize schedule. The gambler is ready to trade off higher probability to win and higher wager along the flex-prize indifference circle tangent to the price line at point C. Iso-expected profit lines, $\widehat{\pi} = (1-k^*)l$, are horizontal lines in the (l, p) space at the given k^*. The agency might increase expected profits by combining a fix- and a flex-prize bet menu.

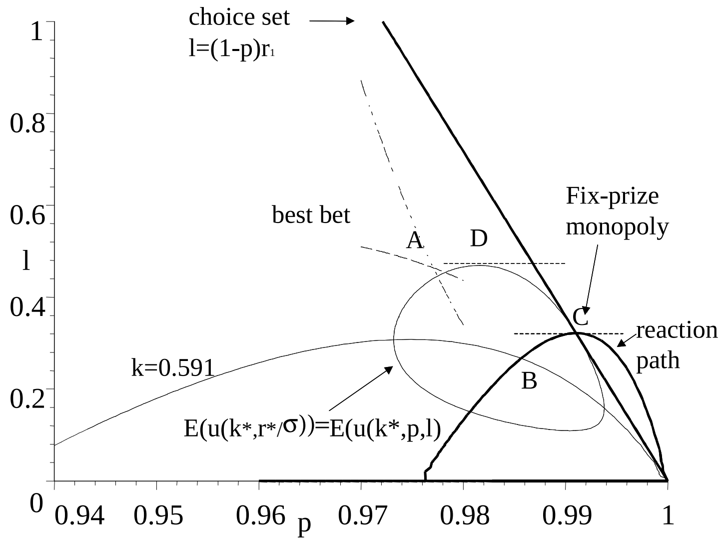

Figure 7: Fix-prize monopoly

3.2.3 The 'splitting' problem

In the fix-prize gamble, the wager l and the probability of loss p were linked together by the 'ticket constraints' (22) and (23). If r doubles and σ doubles, the demand for the tickets n^* falls by 50 %. The subject values only (l^*, p^*), and is not interested in the number of tickets as such.

The assumption that only r/σ matters is far from innocuous. NREU-gamblers will enjoy 'splitting' the probability to win. Anticipated elation during the resolution process can be 'stretched' and increased up to a certain point by checking several tickets step by step rather than a single one.

A simple example illustrates this argument. Assume that the subject has an initial wealth of $z = 10$ and agrees to accept a fair bet ($k = 1$) with a probability of loss $p = 0.95$ and a wager of $l = 0.5$. In the worst case the wealth is $w_1 = z - l = 9.5$, in the best case $w_2 = 10 - 0.5 + (1 - 0.95)^{-1} \times 0.5 = 19.5$. Again, let $\mu = 25.0$, $\gamma = 4.0$, $u(w) = \ln(w)$. Then for a single drawing,

$$E(u) = q(p)u(w_1) + (1 - q(p))u(w_2) = 2.324 > u(z) = 2.302 \qquad (32)$$

Now, the subject gets two tickets offered instead of one, each at a price of 0.25 \$. Total supply of tickets, T_s, is also doubled (and σ reduced), so that aggregate (p, l) remain the same, if the subject buys two tickets. The winning number is drawn and compared with the first and second ticket successively. This is tantamount to participating in two successive lotteries with identical potential gains and losses as in the single ticket lottery. Probability of loss $\overline{p}$ is higher in each step according to

$$\overline{p} = 1 - \frac{1 - p}{n} = 1 - \frac{0.05}{2} = 0.975 \qquad (33)$$

If the time lag between checking the first and the second ticket is very short, we may ignore differences of discounted values of anticipated utility altogether and get

$$E(u) = 2(q(\overline{p})u(w_1) + (1 - q(\overline{p}))u(w_2)) = 2 \times 2.303 = 4.606 > u(z) = 2.302 \tag{34}$$

The subject is better off by 'splitting'.

Of course, no gambler enjoys to spend weeks comparing thousands of numbers. Marginal benefits of splitting decline as the probability to win continues to fall. As more and more tickets are added discounting and boredom becomes relevant. Lottery agencies can easily account for the desire to 'split' by reducing the basic probability to win per ticket and increasing the supply of tickets above the number of gamblers $T_s > m_0$.

In the following part, we will more or less ignore the 'splitting' aspect of choosing n and focus on the instrumental value of n via (22) and (23). Whenever necessary, however, we will qualify our results by returning to the 'splitting' issue.

3.3 The 'lotto' gamble

3.3.1 The basic model

The lottery agency fixes a certain supply of tickets, T_s. A unique number is assigned to each ticket. Each ticket is offered at the same price r and guarantees the same basic chance to win, $\sigma = 1/T_s$. $\widehat{m}$ subjects are expected to participate in the gamble. For the moment only, let us assume that $\widehat{m}$ is equal to the exogenous size of the market m_0.

Let T_s be the supply of tickets, which is set equal to $T_s \geq \widehat{m}$.

$$\widehat{m} = m_0 \tag{35}$$

$$T_s \geq \widehat{m} \tag{36}$$

$$\sigma = (T_s)^{-1} \tag{37}$$

$$l_i = n_i r \tag{38}$$

The subject has to decide upon the number of tickets, n_i. For the purpose of comparing solution with the former gambles, let us map the problem into the (l, p)-space. The expected sum of wagers from subjects $j \neq i$ is given at $W_{-i} = \widehat{l}_{j \neq i}(m_0 - 1)$, where $\widehat{l}_{j \neq i}$ is the expected average wager set by subjects $j \neq i$. kW_{-i} is assumed to be large enough to make the bet attractive.

By choice of l_i the subject i maximizes

$$\begin{aligned} E(u_i) &= q(p_i)u(w_1) + (1 - q(p_i))u(w_2) \qquad \text{s.t.} && (39)\\ w_1 &= z - l_i && (40)\\ w_2 &= z - l_i + kl_i + k\widehat{l}_{j \neq i}(m_0 - 1)) && (41)\\ p_i &= 1 - \frac{l_i}{r}\sigma && (42) \end{aligned}$$

The first order condition is given as (43)

$$\frac{r}{\sigma} = \frac{q'(p_i)(u(w_2) - u(w_1))}{q(p_i)u'(w_1) + (1-k)(1-q(p_i))u'(w_2)} \tag{43}$$

In the Nash-equilibrium the expected average wager, $\widehat{l}_{j\neq i}$, of subjects $j \neq i$ is equal to the actual average wager, $\bar{l}_{j\neq i}$, and equal to the optimal wager l_i^* of the representative subject.

$$\widehat{l}_{j\neq i} = \bar{l}_{j\neq i} = l_i^* \tag{44}$$

The last condition together with (43) determines (l_i^*, p_i^*) and the *individual demand* for tickets in the Nash-equilibrium $(n_i^* = l_i^*/r)$.[14] 00

Equation (45) defines the market clearing condition. Market clearing implies that a single gambler necessarily wins the prize. The price of a single ticket, r^*, is determined by the market clearing condition (45).

$$T_D = \frac{l_i}{r} m_0 = T_s \tag{45}$$

3.3.2 The best of all lotto bets

A benevolent agency intends to organize the best lotto gamble (BLG) for the representative gambler. Setting $k = 1$, the critical issue is to determine the optimal scale of the gamble m_0^*. This can be done by maximizing indirect utility (as a function of (l^*, p^*, m_0)) subject to (43) and $m_0 = (1-p)^{-1}$. Fig. 8 illustrates the results calculated on basis of our familiar example.

After substituting the solution m_0^* into (43), the *willingness to pay* curve can be plotted (43). It tells us, which amount of money the representative subject is willing to spend for a certain probability to win in a community of size m_0^*, if a Nash-equilibrium is realized. The slope of the choice set is determined by the market clearing condition, requiring $r/\sigma = l^* m_0^*$. The choice set line relates the demand for tickets (= probability to win) and the payment necessary to buy the tickets. Two possible Nash-equilibria exist, point L and point D, differing with regard to their welfare and stability characteristics.

The BLG (situated at point L) departs slightly from the best flexible bet A. The wager is somewhat higher and the scale and the multiplier somewhat smaller. Utility is also somewhat lower than in the best flex-prize bet A. Comparing the first order conditions (43) and (27) it becomes clear, why the solutions differ. Substituting the solution vector for the bliss point A (l_B^*, p_B^*) into (43), subtracting (43) from (27) gives

$$(1 - q(p_B^*))u'(w_2^*)k > 0 \tag{46}$$

Obviously, both conditions cannot be fulfilled at (l_B^*, p_B^*) simultaneously. Intuitively, in the fix prize gamble $\partial w_2/\partial l_i = -1$, in the lotto gamble $\partial w_2/\partial l_i =$

[14] The second order condition with respect to the number of tickets is presented in the Appendix.

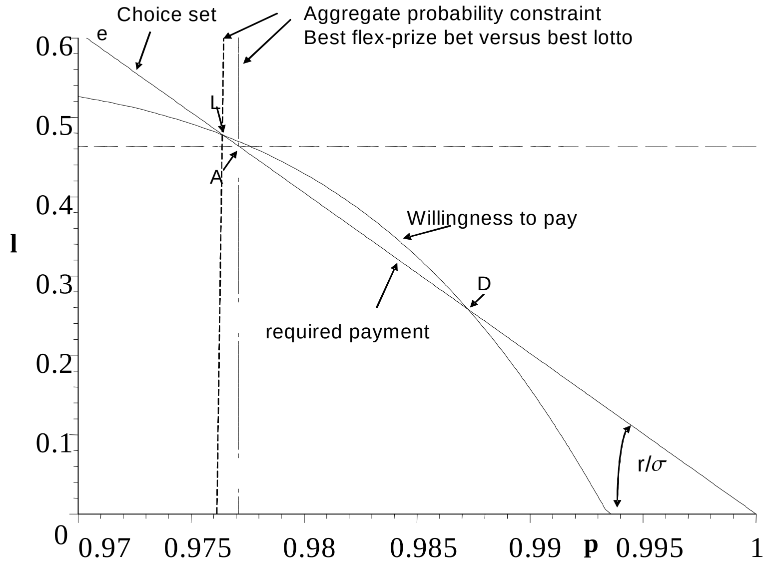

Figure 8: Best flex-prize bet versus best lotto bet

$-(1-k)$: the subject gets back part of its own wager in the case of winning. This induces a small incentive to set the wager somewhat higher in the lotto bet than in the pure fix-prize bet. While this effect is small prima facie, it gets amplified by second and third round effects.

Equilibrium L is a stable one. The reasons are that (a) $\partial E(u)/\partial l > 0$ to the r.h.s. (and < 0 to the l.h.s.) and (b) to increase (reduce) the wager, more (less) tickets must be bought at the price r along the choice set e.

Note that there does exist a second unstable Nash-equilibrium at point D violating the market clearing condition (excess supply of tickets exists).

The 'stability' issue will be discussed in some detail in the context of monopoly below.

3.3.3 The monopolistic lotto bet

A monopolistic agency exists, which tries to maximize expected profits (= expected tax revenues). Costs are assumed to be zero. One has to be careful to distinguish three possible scenarios: (1) The agency maximizes profits on basis of an exogenously given scale determined by historical incidence. (2) The agency acts like a 'meta agency', separating regions of optimal size to maximize average profit per capita in each region. (3) Or the agency maximizes aggregate profits by simultaneously determining

(a) the optimal monopolistic scale m_0^* of the gamble ("state-' or 'nation-wide', 'EU-wide'?), determining also the basic probability to win $\sigma^* = (m_0^*)^{-1}$

(b) the optimal redistribution share k^*

(c) the optimal price per ticket r^*.

Let us start with the first scenario.

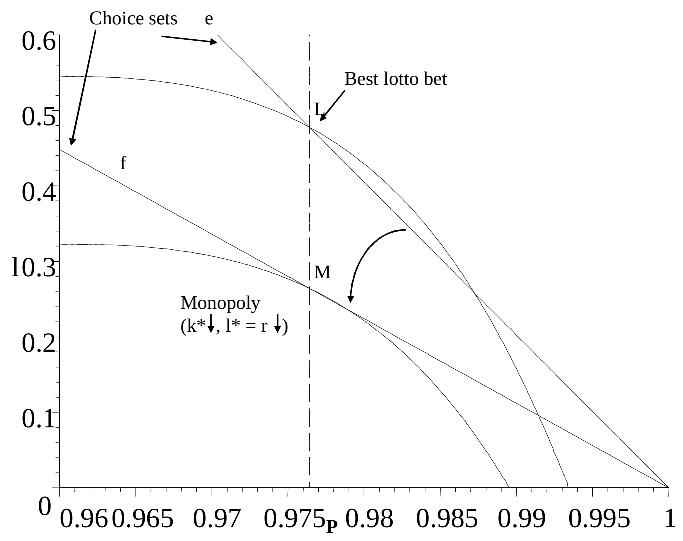

Figure 9: Shifting from the best lotto bet to monopoly

The fix-scale lotto Let us assume that the scale of the gamble is exogenously given at the BLG-level ($T_s = m_0 = \sigma_0^{-1} = (1-p_0)^{-1}$).

The monopolistic agency maximizes - by choice of (k^*, l^*) - expected total profits

$$\widehat{\pi} = l(1-k)m_0 \tag{47}$$

subject to

$$\begin{aligned} \frac{l}{1-p_0} &= \frac{q'(p_0)(u(w_2)-u(w_1))}{q(p_0)u'(w_1)+(1-q(p_0))u'(w_2)(1-k)} && (48) \\ w_1 &= z-l && (49) \\ w_2 &= z-l+kl+kl(m_0-1) && (50) \end{aligned}$$

Market clearing implies that the representative gambler buys at the price $r^* = l^*$ just one ticket and $\sigma_0 = 1-p_0$. It is illuminating to study our familiar example ($u(w) = \ln(w), z = 10.0, \mu = 25, \gamma = 4$).

Switching from the the BLB (L) to monopoly (M) at the present scale has the following consequences (fig. 9). The distribution quota, k^*, and the long-run equilibrium ticket price, r^*, both will be lower. Note, that the Nash-equilibrium shifts towards the abyss of downward instability at point M, because the choice set, line f, becomes tangent to the willingness to pay-curve.

The variable-scale lotto Table 1 illustrates the consequences of varying the scale of the gamble. The bold - second - line represents a gamble of the same size as the best of all lotto bets (point M in fig. 9).

Varying the scale of lotto

m_0	k^*	l^*=r^*	$\widehat{\pi}$	$E(u^*)$-$u(z)$	$\widehat{\pi}/m_0$	r^*/σ	w_2^*-z
31.82	0.839	0.172	0.88	0.0139	0.028	5.48	4.43
42.42	**0.751**	**0.264**	**2.79**	**0.0164**	**0.066**	**11.20**	**8.15**
84.84	0.619	0.340	10.98	0.0111	0.129	28.84	17.51
169.68	0.556	0.276	20.83	0.0046	0.123	46.90	25.80
339.37	0.526	0.179	28.86	0.0015	0.085	60.85	31.80
678.73	0.511	0.103	34.11	0.0004	0.043	69.77	35.56

Table 1

Several conclusions can be drawn:

- The smallest lottery in the first line of table 1 is inferior to the second line - expected profits *and* the utility gain from gambling both are lower than in the second line. For very 'small' monopolies cooperation is useful and can improve even the gamblers welfare.
- Utility for the gambler is still relatively highest at the BLB-scale and decreases for larger communities.
- The larger the lotto community, the lower will be the distribution quota k.
- With increasing scale the optimal wager, l^*, $(= r^*)$ rises first and falls afterwards.
- The price per unit of probability to win will be higher, the larger the lotto community.
- The potential prize will rise more than proportionally first, and then less than proportionally.
- Profits per gambler, $\widehat{\pi}/m_0 = l(1-k)$, first increase and then decrease. A 'meta' lotto agency, controlling the largest community in the last line of table 1, could split the market into sub-regions to increase aggregate profits[15].
- Aggregate monopoly profits,$\widehat{\pi} = m_0 l^*(1-k^*)$, rise (at a decreasing rate) with the size of the lotto community. This conclusion depends, however, critically on the absence of transaction costs and/or competing alternatives (like splitting the market!). Even small transaction costs restrict the potential size of the lotto because $E(u^*) > u(z)$ might be violated. Furthermore, gamblers might switch to smaller lotto gambles in other communities, if the gamble becomes too large and unfair.

[15] Splitting may also be done by introducing additional drawings per week, e.g. two instead of one, as it was done in Austria 1996. None of the basic parameters σ,k,r was adjusted simultaneously, albeit they should have been adjusted upwards. Nevertheless overall revenues rose. Rising frequency of jackpots was observered (i.e. excess supply of tickets). After a certain period, k was increased.

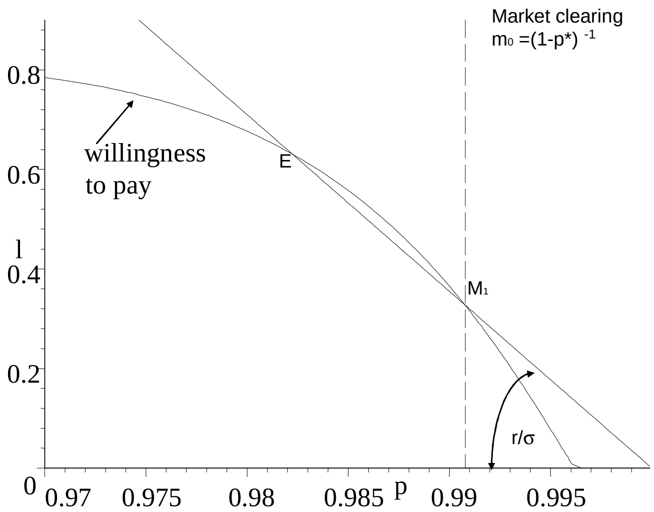

Figure 10: The lotto monopoly of optimal 'regional' size

Stability of Lotto It is impossible to discuss the question of stability without making specific assumptions about expectations. Two cases have to be distinguished:

(1) If (a) the representative subject expects

$$\widehat{W}_{j\neq i,t} = kl_{j\neq i,t-1} \times (m_0 - 1)$$

(expectations are endogenous in the short-run) and (b) if excess demand is satisfied by issuing more tickets, any random deviation of demand for tickets to the l.h.s. of M_0 triggers an expansion process towards equilibrium E.

This case is illustrated in fig. 10 for a monopoly designed to maximize profits per capita. Note that this gamble is more skewed to the right (i.e. larger) than the BLB. The 'willingness to pay' curve for the given scale is steeper than the choice set line at M_0 implying instability. If the equilibrium E realizes, it will sooner or later end like a 'bubble', because the market clearing condition is violated. Subjects will recognize that σ is 'diluted' at E.

(2) If expectations are exogenous in the short-run (bold willingness to pay in fig. 11), random deviations of the aggregate demand for tickets do not change the expected sum of wagers by subjects $j \neq i$ in the future ($\widehat{W}_{j\neq i,t} = E(W_{j\neq i})$ = equilibrium demand realized at M_0). As can be seen in fig. 11 'exogenous' expectations guarantee stability. Lotto gets transformed to a quasi-fix-prize-bet.

One obvious possibility to deal with the risk of downward instability at point M is to introduce minimum prize guarantees.

Jackpots If the undistributed prize of the last period, R, is added to the prize of the present period,

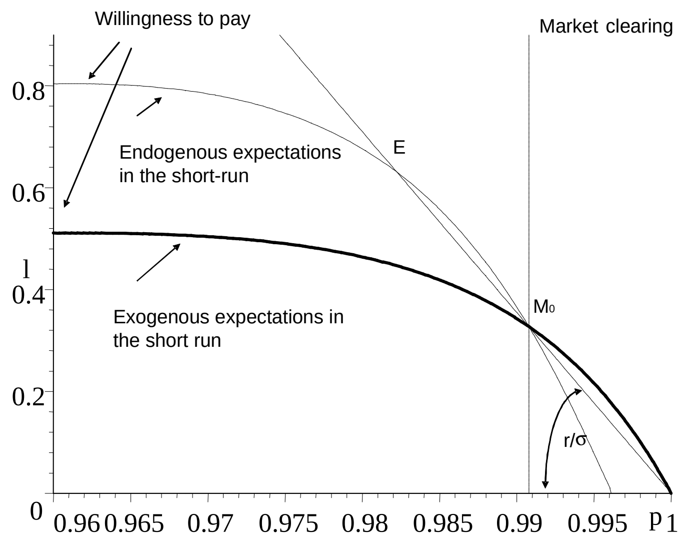

Figure 11: Short-run stability and instability

$$w_2 = z - (1-k)l + R + k\widehat{l}_{j\neq i}(m_0 - 1) \tag{51}$$

This lowers $u'(w_2)$ and increases $(u(w_2) - u(w_1))$. Condition (48) implies that willingness to pay must rise increasing the demand for tickets. Jackpots can be seen as a 'signalling device'. The representative subject surely recognizes that jackpots raises the willingness to pay for *all* participating gamblers. Common knowledge implies that expectations will be of the 'elastic' type if a jackpot is offered.

In fig.12 the resulting effects are illustrated.

M_0 represents the long-run equilibrium due to 'inelastic' prize expectations. A jackpot is introduced. The willingness to pay curve shifts upwards and becomes steeper due to the 'expectation effect'. If the representative subject ignores 'dilution' of probability to win completely, 'overshooting' towards the 'bubble' point A happens. Even if the representative subject anticipates that 'dilution' will happen ex post, some 'overshooting' in response to shifts of the willingness-to-pay' curve is inevitable due to a 'prisoners dilemma': If, in equilibrium B, 1000 tickets could be sold, adding a single ticket to the basket of 10 tickets held by any subject increases the probability to win by 0.0009 89 percent points. This is nearly as high as under conditions of excess supply, where the probability rises by $\Delta n\sigma = 0.001$ If, however, all 100 subjects do the same, ex post probability to win cannot rise. Adding R to the prize will have different effects, when it is recognized by gamblers as a 'jackpot' or as a permanent shift. In the latter case, subjects anticipate the decline of the basic probability to win. Holding the price per ticket r constant, the choice set line turns upwards, as the perceived $\widehat{\sigma}$ falls, until point B realizes.

Overshooting implies that empirical estimates of the elasticity of l with

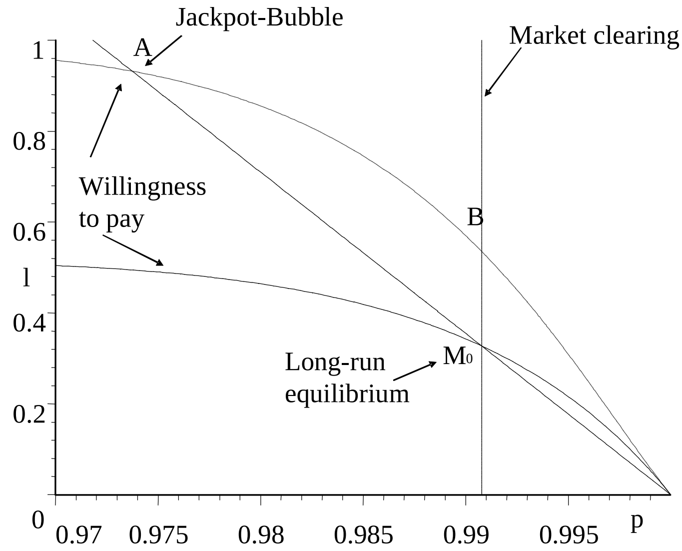

Figure 12: The jackpot bubble

respect to k using the extreme variability of demand induced by roll-overs are biased upwards. A further argument can be brought forward in support of this conjecture. With higher frequency of jackpots intertemporal substitution becomes attractive. Rational gamblers shift spending from regular drawings to jackpot-drawings. This reduces not only the probability of multiple jackpots, but also the basis for future roll-overs - a strategy, which might become 'self defeating'. An early signal for such developments might be a rising variance of demand between rollover drawings and regular drawings.

'Overshooting' induced by jackpots can be a 'Bonanza' for lottery agencies: the additional profits at point A are higher than the amount of profits foregone by offering the jackpot. Jackpots are an instrument to exploit the 'bubble' equilibria created by 'overshooting'.

3.4 Comparing the different gambles

Table 2 shows, at a glance, the simulated results for the different gambles analyzed so far.

Comparing different gambles
$(u(w) = \ln(w), z = 10.0, \mu = 25.0, \gamma = 4.0)$

Bet	t=1-k	l	p	E(u)	l(1-k)	l/(1-p)
Fair flex-prize bet	0	0.46237	0.97710	2. 3327	0	20.233
Flex-prize monopoly	0.4157	0.26235	0.98583	2. 3132	0.1091	18.516
Fix-prize monopoly	0.4084	0.32210	0.99102	2.3109	0.1316	35.864
Best lotto bet	0	0.47738	0.97643	2.3327	0	20.251
Lotto monopoly	0.4086	0.32729	0.99078	2.3111	0.1337	35.498

Table 2

Results indicate that the lotto monopoly is superior with regard to expected profits per capita to other wager-tax bets. Fix-prize and lotto monopoly are more skewed towards higher risk-levels than flex-prize bets. In spite of a higher tax rate, from the gambler's point of view the lotto monopoly seems to be slightly superior relative to the fix-prize monopoly. Intuitively speaking, because part of the wager set by subject i is redistributed in case of winning, the 'distortion' towards 'skewness' present in the fix-prize bet is slightly offset.

3.5 The regressive nature of the lotto tax

With constant relative risk-aversion, the long-run equilibrium wager rises in proportion to wealth.

A different picture develops, if any *non-representative* subject becomes richer or poorer relative to the representative subject. Let us assume that the expected amount of money put into the pot by 'other' subjects $W_{j \neq i} = \widehat{l}_{j \neq i} m_0 = \widehat{n}_{j \neq i} r m_0$ is given. Let us eliminate l_i and p_i from the utility function by substituting $l_i = n_i r$ and $p_i = 1 - n(m_0)^{-1}$. The first order condition $\partial E(u)/\partial n_i = 0$ can be used to draw an Engel-curve for the individual demand for tickets in fig. 13. The model predicts poor (but not extremely poor) individuals to spend in relation to wealth more for lottery tickets than richer ones. The share of spending relative to wealth is maximized at point A in fig 13.

A proportional tax on wagers in a lotto gamble will therefore be a regressive one.

This is different from flex-prize gambles, where subjects choose their wager in proportion to wealth, if relative risk aversion is constant. Taxing those gambles will be less regressive or might even be progressive depending upon the relative degree of risk-aversion.[16]

A further factor pointing to the regressive nature of lotto taxes exists. In the NREU-model the propensity to accept unfair bets depends upon time preference. Some types of poorness might be linked to high rates of time preference (lack of planning ability, lack of investment in human capital). Naturally, these subjects will be taxed more heavily.

3.6 The 'expected value' approach

The presented model might be criticized for being to specific, using 'arbitrary' elation and disappointment parameters, utility of wealth functions and so on. Nevertheless it is at least (or tries at least to be) a consistent model of rational gamblers behavior, rooted in (intertemporal) state dependent expected utility theory. Although in many ways explorative, it supports the view that a meaningful taxonomy of different gambling activities on basis of the assumptions of rational behavior is possible. And it may sharpen our understanding of the problems ahead for empirical work.

[16] Scott and Garen (1994) have shown that the demand for lotto has an $\cap$ shape relationship with income and that unemployed individuals are more likely to play lotto - a finding compatible with the presented model.

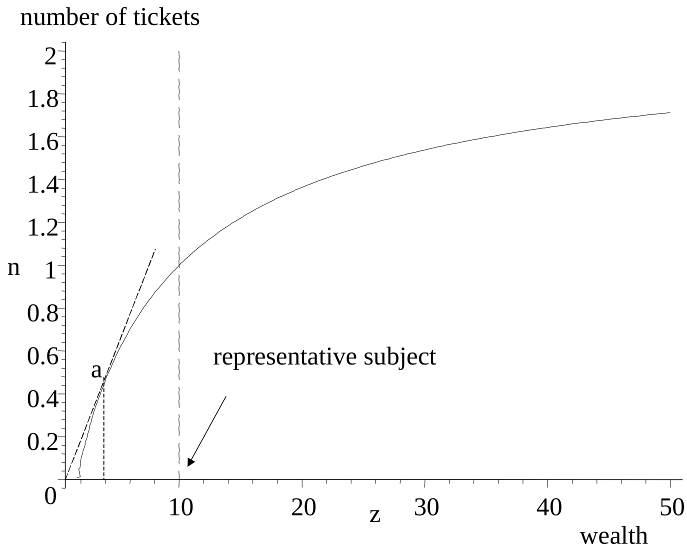

Figure 13: The Engel-curve for lotto tickets

A large part of the empirical literature (Sprowls (1970), Scoggins (1955), Cook and Clotfelter (1993), Walker (1998), Farrell (1997) uses the 'expected value', V, of a lottery ticket as an indicator of it's attractiveness and $(1-V)$ as the 'price' variable. Following Walker (1998), this expected value, V, is defined as the probability that *at least one ticket wins the prize* times the return per ticket. If R is the rollover

$$V(R,k,\sigma;n) = (1-(1-\sigma)^n)\frac{R+krn}{rn} \tag{52}$$

The probability that at least one ticket wins rises at a decreasing rate with n. The average return falls with rising n or is constant, depending on R being positive or zero. In the latter case the 'price' is obviously equal to $1-k$ and independent of the price per ticket, r. It is postulated that aggregate demand for tickets rises with expected return.

$$n = f(V) \qquad f'(V) > 0 \tag{53}$$

Equilibrium demand for tickets is determined by the point of intersection between these two curves, assumed to be located in the falling part of the $V-$curve. Theoretical foundations in terms of optimizing behavior are completely absent.

The presented approach might be used to criticize this simple model.

(1) Why should rational, utility maximizing subjects care about the probability 'that at least one ticket wins the prize' times 'the average expected return per ticket'? This might be relevant for someone, who buys all tickets, but this person would not only be bored, but also wasting money in an unfair gamble.

(2) The probability that at least one ticket wins the prize increases with the demand for tickets - the price is correspondingly lower. This is contradictory to

our model, where the 'dilution' effect, if recognized, is raising the implicit price r/σ per ticket. If probability illusion is present, the price remains constant.

(2) Variations in R are treated as equivalent to variations of k[17]. This ignores the 'overshooting' problem completely - the basis for the profitability of jackpots, as long as they remain rare events.

(3) The expected value model implies that the demand for tickets is completely independent of the price per ticket r, if $R = 0$. The NREU-model presented above clearly shows that setting r is far from tivial. It can be set too high c.p. and even destabilize the gamble.

3.7 The optimal design of lotto

Maximizing tax revenues from lotto is an ambitious task requiring the *simultaneous* determination of the size of the gamblers pool m (small lotteries might have to cooperate, large lotteries might have to split); the basic probability to win σ relative to the size of the gambler pool to satisfy preferences for 'splitting'; the distribution quota, k; the price of tickets, r; to determine the optimal degree of 'excess supply' to create profitable (i.e. not to many) jackpots. Serious recommendations based on structural data require first of all cross-sectional analysis to identify the best performing lotteries.

4 Summary

The paper develops a consistent framework for analyzing different types of gambles. Gambling is explained by using an intertemporal state dependent expected utility model with probability weighting. Expected utility of short-run and fading emotions is anticipated in addition to expected utility of wealth. Flex- and fix-prize/wager tax gambles are analyzed and compared with lotto. Monopolistic tax rates are compared for different gambles. Maximizing expected profits from lotto implies that the house advantage, the price per ticket and the scale of the gamble have to be adjusted simultaneously. It is shown that lotteries can take advantage from splitting (or increasing) the gamblers pool. Very large lotteries with low basic probabilities to win should have higher tax rates, lower prices per ticket and higher prizes. Gamblers preferences for 'splitting' tickets are explained. It is argued that jackpots induce temporary 'overshooting' of the demand for tickets in a lotto gamble. Jackpots are profitable only, as long as they remain to be rare events.

[17] Forrest et al. (2003) test the traditional approach of using the effective prize against a model in the spirit of Conlisk (1996) pointing to an additional, specific utility from gambling. When this is tested against the traditional model using data from the U.K. National Lottery, it is found that jackpot considerations exert an influence over and above that of variations in effective price.

Appendix

1. Let us assume $\mu \geq \gamma$. Then it can easily be shown that the nominator of

$$q'(p) = \frac{1 + \gamma p^2 + (1-p)^2 \mu}{(1 + p(1-p)(\mu + \gamma))^2} \tag{54}$$

must rise with rising p within the interval $\mu/(\mu+\gamma) < p \leq 1$. The denominator of $q'(p)$ on the other hand necessarily falls. Therefore $q''(p) > 0$ must be fulfilled within the elation interval.

2. The riskiness condition can also be written as

$$\frac{u'(w_2)}{\frac{u(w_2)-u(w_1)}{w_2-w_1}} = \frac{1-p}{1-q(p)} q'(p) = \frac{1-p}{1-q(p)} \frac{d(1-q(p))}{d(1-p)} \tag{55}$$

The elasticity of the weighted probability to win with respect to the true probability to win is balanced against the ratio of the marginal utility of wealth w_2 relative to the average increase of utility. Let us offer, in a first step, an attractive bet $B(\widehat{k}, p_0)$, where p_0 is selected from an ϵ^+-neighborhood of the lower border of the gambling interval. The wager condition determines the optimal wager $l_0 = l(p_0)$, which will be positive, but very small near the lower border. The l.h.s. of marginal to average utility in (55) will then be approximately equal to one. The r.h.s., however, will be smaller than one, because

$$\lim_{p \to \mu/(\mu+\gamma)^+} \frac{1-p}{1-q(p)} q'(p) = \frac{\mu+\gamma}{\mu\gamma + \mu + \gamma} < 1 \tag{56}$$

The risk-taking condition is therefore violated and $\partial E(u)/\partial p > 0$ at $l_0 = l(p_0)$. If p rises above p_0, it can be shown that the r.h.s. of (55) increases in an U-shaped (if $\gamma < \mu < 2\gamma$) or strictly convex manner (if $\mu \geq 2\gamma$) up to

$$\lim_{p \to 1^-} \frac{1-p}{1-q(p)} q'(p) = 1 \tag{57}$$

Let us assume constant relative risk aversion R. If $p \to 1^-$ and wealth $w_2 \to +\infty$ the l.h.s. declines monotonically and approaches

$$\lim_{p \to 1^-} \frac{u'(w_2)}{\frac{u(w_2)-u(w_1)}{w_2-w_1}} = \lim_{p \to 1^-} (1-R) \frac{1 - \frac{w_1}{w_2}}{1 - \left(\frac{w_1}{w_2}\right)^{1-R}} \tag{58}$$

$$= \left\{ \begin{array}{c} 1-R \\ 0 \end{array} \text{ if } \begin{array}{c} 0 < R < 1 \\ R \geq 1 \end{array} \right. \tag{59}$$

Obviously, an optimal $p_1 = p(l_0)$ exists. It can also be shown that increasing the given wager l_0 shifts the l.h.s. downwards, reducing the preferred risk-level.

For the case of a fair bet $(k = 1)$ and $u(w) = \ln(w)$ it is easy to show that any simultaneous solution for the flex-prize bet must be to the r.h.s. of the maximum of the wager curve. Solving explicitly for the w ager curve gives

$$\frac{l^*}{z} = 1 - \frac{q(p)}{p} \tag{60}$$

Therefore, the wager will be maximized at $\widehat{p}$, where

$$\frac{q'(\widehat{p})}{q(\widehat{p})}\widehat{p}=1 \tag{61}$$

The risk-taking condition (55)

$$\frac{u'(w_2)}{\frac{u(w_2)-u(w_1)}{w_2-w_1}}=\frac{(1-p)\,q(p)}{(1-q(p))p}\times\frac{q'(p)\,p}{q(p)} \tag{62}$$

can be fulfilled simultaneously with the wager condition, if and only if

$$\frac{u'(w_2)}{\frac{u(w_2)-u(w_1)}{w_2-w_1}}=\frac{u'(w_2)}{u'(w_1)}\times\frac{q'(p)\,p}{q(p)} \tag{63}$$

For this to be valid

$$\frac{q'(p)\,p}{q(p)}>1 \tag{64}$$

must be the case, which implies $p^*>\widehat{p}$. Implicitly, the subject trades off a lower wager versus higher risk.

3. The second order condition for the optimal number of tickets is given by

$$\begin{aligned}\frac{\partial v}{\partial n\partial n} &= -q''(p)\,\sigma^2\,(u(w_2)-u(w_1))+2r\sigma q'(p)\,(u'(w_1)-u'(w_2)\,(1-k)) \\ &\quad +q(p)\,u''(w_1)\,r^2+(1-q(p))\,u''(w_2)\,r^2\,(1-k)^2 \end{aligned} \tag{65}$$

The first term on the right side is surely negative ($q''(p)>0$ in the gambling interval) and so are the third and the forth terms (for $k<1$). Only the second term is positive. Because σ is very small in a typical lotto bet, we can safely assume that NREU is strictly concave in n.

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