IS THERE AN EQUILIBRIUM RATE OF UNEMPLOYMENT IN THE LONG RUN?

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by

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Abstract

Distinguishing between profit led and growth led demand regimes, we analyze the conditions of existence and stability of long run equilibrium of unemployment. The model we employ has at its center the relation between growth and distribution. Growth can be either wage led or profit led. Distribution itself is a function of the unemployment rate, with higher unemployment leading to a higher profit share. We use Okun's Law to close the model, making the change of the rate of unemployment a function of growth. The interesting result of our analysis is that in profit led demand regime the short run and long run equilibrium are stable. However, if the demand regime is wage led, the same conditions that guarantee stability of the goods market equilibrium in the short run render impossible the existence of a long run equilibrium rate of unemployment, and vice versa. Thus, if Kalecki's proposition that higher wages lead to higher growth is true, there will be no equilibrium rate of unemployment in the long run that serves as an anchor for the economic system.

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Keywords

growth theory, unemployment, Keynesian economics

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Introduction

Most of the literature on European unemployment is built on the premises that there has been an increase in the equilibrium rate of unemployment, the NAIRU. The debate then is about what the cause of this increase is: overgenerous welfare states (Siebert 1997, Krugman 1994), increases in the real interest rate (Phelps and Fitoussi 1988), capital shortage (Rowthorn 1995, 1999, Bean and Dreze 1990) or some interaction between these (Blanchard 1999, Bean 1994). The present paper does not take for granted that there is such a thing as an equilibrium rate of unemployment in the long run, but seeks to establish under what conditions it exists.

The model we employ has at its center the relation between growth and distribution. Growth can be either wage led or profit led. Distribution itself is a function of the unemployment rate, with higher unemployment leading to a higher profit share. We use Okun's Law to close the model, making the change in the rate of unemployment a function of growth.

In this paper we define the long run with respect to the rate of unemployment. In the short run, last period's rate of unemployment is given, whereas the long run requires this period's and last period's unemployment rate to be identical.

Thus, with a given level of past unemployment, we get a short run equilibrium for growth and distribution that implies a certain change in unemployment. This gives the model a certain Keynesian flavor: in the short run the goods market dominates the labor market. The question this paper seeks to answer is, under what conditions this direction of causality is inverted in the long run, in other words: when will there be a rate of unemployment that the economy gravitates to in the long run.

The model

“One of the most active areas of cross-country research has been investigating the consequences of inequality for growth. Somewhat unusually for the growth literature, studies have tended to concur in finding a negative effect of high inequality on subsequent growth.”

Temple 1999, 146

Let us assume a demand function $g$

$$g_t = a_0 + a_1\pi_t,$$  \hspace{1cm} (1)

where $g$ and $\pi$ are growth and profit share respectively, with their subscripts denoting time. The sign of $a_1$ can be either positive or negative, giving rise to a profit led or wage led regime respectively. $a_0$ captures the effects of all other variables influencing growth.

We regard the wage led case as more interesting. Equation (1) can be derived from a variety of theories, but a negative correlation between growth and inequality is a stylized fact in growth theory, although there is no agreement why. We want to briefly mention two. First, the
political economy of growth argument is that high inequality leads to political instability which decreases investment (Alesina and Perotti 1996) or blocks the appropriate use of productive resources (Rodrik 1998). Second, a much older line of thought, Kaleckian macroeconomics posits a flexible capacity utilization in the long run because of monopolistic competition (Kalecki 1969, Steindl 1952). Since workers have a higher consumption propensity than capitalists, an increase in the wage share will increase growth (See Dutt 1984 and Rowthorn 1982 for modern formulations and Marglin and Bhaduri 1990 for a careful treatment of the underlying assumptions of the investment function). Note that if we follow the Kaleckian line of reasoning (1) is an IS-curve.

Distributional equilibrium, a result of wage bargaining (see e.g. Sen and Dutt 1993) is given by

\[ \pi_t = d_0 + d_2 u_t, \]

where \( u \) is the unemployment rate, and thus \( \Delta \pi = d_2 \Delta u \). By (2) we capture the disciplinary effect unemployment has on wages, thus \( d_2 > 0 \).

The change in the unemployment rate is given by

\[ \Delta u_t = n - g_t, \]

where \( n \) is the growth of the labor force, assumed to be constant. (3) is a version of Okun's Law without technical progress. In the language of Harrod, \( g \) is the warranted rate of growth and \( n \) the natural rate of growth.

Equations (1) – (3) summarize the model. The goods market equilibrium determines the change in the rate of unemployment, rather than the level of unemployment. Therefore the full effects of unemployment on the goods market will only be felt in the long run.

\[ u_t = u_{t-1} + \Delta u_t \]

\[ \pi_t = \pi_{t-1} + \Delta \pi_t = d_0 + d_2 u_{t-1} + d_2 \Delta u_t \]

In the short run, i.e. with a given rate of last year's rate of unemployment, the profit function becomes:

\[ \pi_t = d_0 + d_2 u_{t-1} + d_2 (n - g_t) \]

**Short run equilibrium**

Solving (1) and (4) we get

\[ g_t = a_0 + a_1 [d_0 + d_2 u_{t-1} + d_2 (n - g)] \]

\[ g_t (1 + a_1 d_2) = a_0 + a_1 (d_0 + d_2 u_{t-1} + d_2 n) \]
\[ g_{**} = \frac{a_0 + a_1 (d_0 + d_2 u_{t-1} + d_2 n)}{1 + a_i d_2} \]  

and

\[ \pi_{**} = \frac{a_0}{1 + a_i d_2} + \left( 1 + \frac{a_1}{1 + a_i d_2} \right) (d_0 + d_2 u_{t-1} + d_2 n) \]

Equations (5) and (6) give the short run equilibrium of the goods market. Note that both of them depend on last year's unemployment rate.

**Stability in the short run**

We assume adjustment behavior of the following form

\[
\begin{align*}
\dot{g} &= \lambda_g (g(\pi) - g) \\
\dot{\pi} &= \lambda_\pi (\pi(g) - \pi)
\end{align*}
\]

where \( g(\pi) \) and \( \pi(g) \) are equations (1) and (4) respectively and we assume both \( \lambda_g \) and \( \lambda_\pi \) to be positive. We then get the Jacobian:

\[
J = \begin{bmatrix}
-\lambda_g & \lambda_g a_1 \\
-\lambda_\pi d_2 & -\lambda_\pi 
\end{bmatrix}
\]

For the equilibrium to be stable the \( \text{tr}(J) < 0 \) and \( \det(J) > 0 \).

\[
\begin{align*}
\text{tr}(J) &= -\lambda_g - \lambda_\pi < 0 \\
\det(J) &= \lambda_g \lambda_\pi (1 + a_i d_2)
\end{align*}
\]

Signing these two expressions is straightforward for \( \text{tr}(J) \), which is negative, but not for \( \det(J) \). Here we have to distinguish two cases, the profit led regime and the wage led regime.

I. profit led regime: \( a_i > 0 \)
then the equilibrium is always stable since \( 1 + a_i d_2 > 0 \) holds.

II. wage led regime: \( a_i < 0 \)
Now the equilibrium will be stable if

\[ 1 + a_i d_2 > 0 \]

holds, or: \( a_i < \frac{-1}{d_2} \)
Since \( a_i = \frac{\partial g}{\partial \pi} \bigg|_{\pi_0} \) and \( -1 = \frac{\partial g}{\partial \pi} \bigg|_{\pi_*} \), the stability condition requires that the \( \pi \)-curve is steeper than the IS-curve. Graphically, the \( g^* \)-curve has to be flatter than the \( \pi \)-curve (see phase diagram). Intuitively, a flat \( g \)-curve means that growth does not react strongly on income distribution and a steep \( \pi \)-curve means that distribution does not react strongly on unemployment.

**Unemployment in the short run**

Having a stable equilibrium on the goods market does not imply a stable unemployment rate

\[
\Delta u = n - g^*
\]

\[
\Delta u = n - a_0 + a_1 (d_0 + d_2 u_{t-1} + d_2 n)
\]

\[
\Delta u = \frac{a_0 + a_1 (d_0 + d_2 u_{t-1})}{1 + a_1 d_2} + \left( \frac{1}{1 + a_1 d_2} \right) n
\]

\[
\Delta u = \frac{n - a_0 - a_1 d_0}{1 + a_1 d_2} - \left( \frac{a_1 d_2}{1 + a_1 d_2} \right) u_{t-1}
\]

\[
u_t - u_{t-1} = \frac{n - a_0 - a_1}{1 + a_1 d_2} - \left( \frac{a_1 d_2}{1 + a_1 d_2} \right) u_{t-1}
\]

\[
u_t = \frac{n - a_0 - a_1 d_0}{1 + a_1 d_2} + \left( 1 - \frac{a_1 d_2}{1 + a_1 d_2} \right) u_{t-1}
\]

Equation (9) is the short run rate of unemployment.

\[
u_t^{**} = \frac{n - a_0 - a_1 d_0}{1 + a_1 d_2} + \frac{1}{1 + a_1 d_2} u_{t-1}
\]

**The long run: existence and stability**

In the long run, the equilibrium rate of unemployment is:

\[
u^{***} = \frac{n - a_0 - a_1 d_0}{a_1 d_2}
\]

the corresponding long run equilibrium values of \( \pi \) and \( g \) are:
\[
\pi *** = \frac{n - a_0}{a_1}
\]

\[
g *** = a_0 + a_1 \left[ \frac{n - a_0}{a_1} \right]
\]

\[
g *** = n
\]

(12) is unsurprising, since only the equality of warranted and natural rate of growth will allow for a stable rate of unemployment, i.e. for an equilibrium rate of unemployment.

Two questions arise, first whether the equilibrium values are positive, i.e. whether there is an economically meaningful solution, and second whether they are stable. First, a positive equilibrium rate of unemployment exists if

\[
\frac{n - a_0}{a_1} > d_0
\]

holds. Moreover for \(\pi\) to be positive,

\[
\frac{n - a_0}{a_1} > 0
\]

Second, for stability it turns out that the term

\[
\frac{1}{1 + a_id_2}
\]

is crucial. For convergence towards a stable equilibrium point the absolute value of (14) has to be less than unity (a detailed discussion of difference equation (9) can be found in the appendix). We will have to distinguish three cases.

First, the profit led regime: \(a_i > 0\)
Note that a profit led regime requires \(n > a_0\), in words: natural growth has to exceed autonomous growth, for a positive profit share to exist.
In the profit led regime, the denominator of (14) is positive and greater than unity, hence the overall expression will be positive and less than unity. Thus the unemployment rate will converge to the equilibrium value.

Second, a stable (short run) wage led regime \(a_i < 0\) and \(1 + a_id_2 > 0\) (see case II above and equation (8) holds).
A positive equilibrium rate of unemployment exists if \(\frac{n - a_0}{a_1} > d_0\) (i.e. 13 holds). The LHS has negative denominator, thus it will be negative unless autonomous accumulation exceeds the natural rate of growth. (The RHS is always positive, therefore a negative LHS is sufficient for no positive equilibrium rate of unemployment rate to exist.) A positive rate of unemployment
can thus exist, only if autonomous accumulation exceeds the growth of the labor force.

However, even if it existed, this equilibrium would not be stable. Since the denominator of (14) has to be positive, but $a_1$ is negative, the numerator has to be less than unity and the overall expression (14) will be greater than unity. Thus the unemployment rate will not converge, but explode.

Note that, while the economy is moving along a series of stable short term equilibria, there exists no stable long run equilibrium, if an equilibrium exists at all.

Third, an unstable (short run) wage led regime $a_1 < 0$ and $1 + a_1 d_2 < 0$ (equation 8 does not hold)

Whether this case has a stable long run equilibrium depends on whether $|1 + a_1 d_2| > 1$. If so, there, again, is no stable equilibrium value. If not, there exists a value of $u$ around which the unemployment rate may oscillate. However, since the goods market equilibrium is unstable to begin with, it is unlikely that the equilibrium level is ever reached.

**Conclusion**

We seeked to establish under what conditions a long run equilibrium rate of unemployment exists. Profit led and wage led regime exhibited very different properties.

A profit led demand regime is stable on the goods market in the short run and exhibits a unique equilibrium rate of unemployment towards which, the actual growth rate will converge.

A wage led regime can exhibit a stable short run equilibrium, but these do not lead to a long term equilibrium. If the goods market is stable in the short run, unemployment, and inversely growth, will explode in the long run.

An unstable wage led regime, i.e. a wage led regime that exhibits an unstable goods market in the short run, may have an equilibrium value towards which it gravitates. However the movement towards this equilibrium could be reached only by way of oscillation, which, given the unstable goods market, makes it unlikely that the equilibrium value is ever reached.

Thus, while there is an equilibrium rate of unemployment in a profit led regime in the long run, there is none of practical relevance in a wage led regime. The condition for short run stability at the same time prohibits stability in the long run. This finding may be interesting to Keynesians as well as Marxians. For Keynesians it will imply the need for permanent government intervention. To Marxians it may serve as an illustration of the inherent contradictions of capitalism.

The implications for the interpretation of the rising unemployment rates in Europe, are that there need not be an equilibrium rate of unemployment. If the economies are wage led (individual countries are likely to be profit led because of exports, but collectively they may well be wage led, see Bowles and Boyer 1995), rising unemployment rates cannot be interpreted as a rising NAIRU since no equilibrium rate of unemployment will exist in the long run. Interventions on the goods market, then, could not only increase employment but, if unemployment were decreased enough, could make use of a virtuous circle of self-generating employment increases.
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**Appendix**

The appendix provides a careful analysis of the difference equation (9).

\[
u_t = \frac{n-a_0-a_id_0}{1+a_id_2} + \frac{1}{1+a_id_2}u_{t-1}\]

**Complementary solution**

assume \(u_t = Ab^t\) and \(Ab^t - \frac{1}{1+a_id_2}Ab^{t-1} = 0\)

and solving for b we get:

\[
u_c = A\left(\frac{1}{1+a_id_2}\right)^t
\]

**Particular solution**

\[
u_p = \frac{n-a_0-a_id_0}{1+a_id_2} + \frac{1}{1+a_id_2}u_p
\]

\[
u_p\left(1-\frac{1}{1+a_id_2}\right) = \frac{n-a_0-a_id_0}{1+a_id_2}
\]

\[
u_p = \frac{n-a_0-a_id_0}{1+a_id_2}
\]

\[
u_p = \frac{n-a_0-a_id_0}{a_id_2}
\]

**General solution**

\[
u_t = u_c + u_p
\]

\[
u_t = A\left(\frac{1}{1+a_id_2}\right)^t + \frac{n-a_0-a_id_0}{a_id_2}
\]

where
\[ u_0 = A \left( \frac{1}{1 + a_1 d_2} \right)^0 + \frac{n - a_0 - a_1 d_0}{a_1 d_2} \]

\[ u_0 = A + \frac{n - a_0 - a_1 d_0}{a_1 d_2} \]

\[ A = u_0 - \frac{n - a_0 - a_1 d_0}{a_1 d_2} \]

\[ u_t = \left( u_0 - \frac{n - a_0 - a_1 d_0}{a_1 d_2} \right) \left( \frac{1}{1 + a_1 d_2} \right)^t + \frac{n - a_0 - a_1 d_0}{a_1 d_2} \]
Phasediagram