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The Impact of Ex Ante Regulations and Ex Post Interventions on Bank Lending and Solvency
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Abstract

In this paper, I examine the impact of direct equity injections and troubled asset purchases on bank lending and solvency and analyze how ex ante tighter caps on leverage affect ex post decisions between both interventions. Extending the model of Bachmann (2018) by adding the government as a liquidity supplier, illiquid banks can either sell troubled assets at fire sale prices to collateralized financed liquid banks or to the government. If illiquid banks are forced to sell all troubled assets in order to meet premature withdrawals and the government is left with excess liquidity compared to direct equity injections, they can use these funds to bid up prices. Higher prices reduce future returns on buying illiquid assets and motivate liquid banks’ incentive to lend by crowding out their speculative motive for liquidity hoarding. As a result, troubled asset purchases weakly dominate direct equity injections in terms of lending and solvency, directly amplified by a drop in collateral liquidity. Additionally, regulating illiquid banks ex ante by tighter caps on leverage affects the government’s decisions about ex post interventions to effectively stabilize lending and solvency conditions, as the self-reinforcing downward spiral between fire sale prices and collateral liquidity is mitigated. Hence, I find that there exists an inherent nexus between ex ante regulations and ex post interventions.

Keywords: Fire Sales; Collateralized Financing; Troubled Asset Purchases; Capital Injections; Leverage Requirements; Lending; Solvency

JEL Classification: G01; G12; G21; G28

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1 Introduction

The changing nature of the financial system raises the question whether policy measures introduced to avoid traditional bank runs in the tradition of Diamond and Dybvig (1983) are still appropriate to intervene in current financial crises. Liquidity risks arising from maturity mismatches between short-term deposits and long-term investment can no longer be seen as the main source of bank vulnerability in modern financial crises. Financial instability is also driven by the growing application of increasingly complex financial instruments creating an amplifying nexus between market and funding liquidity potentially culminating in sudden freezes of short-term debt markets. Financial institutions were heavily leveraged by using long-term assets as collateral which exposes them to funding risks and insolvency as drastically seen in the recent financial crises (Brunnermeier and Pedersen (2009)). Significant contractions of bank lending activities, fire sales of various types of assets and dramatic reductions in bank leverage sharply aggravate the recession cycles in many countries around the world\(^1\). These phenomena highlight the importance of an appropriate implementation of policy interventions to effectively stabilize the financial system in economic and financial downturns on the one hand, and constitute the beginning of an intense debate on the well-suited timing of policy interventions on the other hand. Focusing on ex post interventions as preferably proposed by the "Greenspan doctrine", policymakers explored a variety of measures whereby especially equity injections and troubled asset purchases were implemented in the aftermath of the collapse of Lehman Brothers in September 14, 2008 (Psalida et al. (2009))\(^2\). The devastating consequences of the financial crisis have shown, however, that intervening ex post can create counter-productive effects which are, among others, mainly driven by excessive risk-taking ex ante (Jeanne and Korinek (2016)). This forces authorities to consider ex ante regulations more seriously by focusing on the procyclical nature of systemic risk (see, e.g., Brunnermeier et al. (2012) or Shin (2010)) and their impact on decisions about ex post interventions. Such ex ante requirements on higher capital buffers with a countercyclical component as well as a framework for ex ante liquidity regulations have since been implemented in the Basel III framework (Benoit et al. (2017)). Knowing the different implications of ex ante regulations defined in the Basel III framework and ex post interventions as proposed by the "Greenspan doctrine", the interdependence between both measures necessarily has to be considered as well.

In this paper, I build upon the model of Bachmann (2018) to examine the effects of ex post capital injections and troubled asset purchases on bank lending and solvency by introducing the government as an additional liquidity provider for illiquid banks. In a similar vein, the model integrates ex ante tighter leverage caps and shows how such requirements will affect ex post decisions between direct

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\(^{1}\)There is a growing literature that finds empirical evidence for all these striking events. To emphasize only a few, Ivashina and Scharfstein (2010) document that the lending volume of the fourth quarter of 2008 was 79 percent lower compared to the peak of the credit boom in the second quarter of 2007. Adrian and Shin (2010) provide evidence that fair value accounting leverage is strongly procyclical, that is, increasing in booms and decreasing in busts. de Haan and van den End (2013) and more recently Chu (2016) show very interesting results on asset fire sales.

\(^{2}\)For further information regarding the "Greenspan doctrine" see, e.g. Blinder and Reis (2005).
capital injections and troubled asset purchases ranked by the impact on bank lending activities and solvency improvements. As liquidity providers are endogenously exposed to collateral funding constraints, this paper moreover allows for analysing the overall effects on the timing nexus between ex ante tighter requirements and decisions about ex post interventions if short-term debt markets increasingly become less liquid.

The three period discrete time model in this paper assumes three different types of agents henceforth called seller bank, buyer bank and government. The key event happens at the intermediate date 1 at which the continuum of all seller banks are commonly hit by the same liquidity shock characterised by premature withdrawals. As prior providers of liquidity, buyer banks have an incentive to purchase illiquid assets at either date, as long as current fire sale prices are low enough to attract all their disposable cash holdings conditional on future returns in case of resurrection. The reason why assets have to be sold at fire sale prices in the secondary market is determined by the fact that natural buyers in the primary market are likely to be restricted by the same liquidity shock as well (Shleifer and Vishny (1992) and Allen and Gale (1994)). As future returns of such illiquid assets can potentially be extraordinarily high, buyer banks try to reduce the volume of term lending to industrials in order to meet their speculative motive for liquidity hoarding. In a similar vein, seller banks will refuse to sell more illiquid assets than necessary to meet premature liquidity needs. Such speculative motives for illiquidity seeking arise from the debt-overhang implication introduced by Myers (1977) as a special type of the risk-shifting motive of Jensen and Meckling (1976) through the reluctance to sell illiquid assets for safe cash. In addition to that, buyer banks’ long-term investment portfolio of illiquid assets and term lending is partly financed by collateralized borrowing in secured short-term debt markets. As the collateral potentially has to be liquidated in the secondary market conditional on the liquidity shock as well, buyer banks’ funding liquidity directly shrinks and amplifies the downward pressure on seller banks’ asset liquidity indirectly. Such collateral runs endogenously induce self-reinforcing feedback loops that potentially culminate in a complete evaporation of market and funding liquidity.

Up to now it was assumed that buyer banks’ amount of initially provided cash is sufficiently high to meet seller banks’ premature liquidity needs. But what happens if buyer banks’ available cash used for illiquid asset trading falls short of seller banks’ premature withdrawals? In such cases, the government will balance this excess demand by providing extra liquidity via direct capital injections or troubled asset purchases. The core model-condition which distinguishes the effectiveness between both interventions regarding bank lending and solvency is determined by the fire sale price for troubled assets in equilibrium. Thereby, the equilibrium condition is reached if all troubled assets are exhausted after trading with both buying agents conditional on just meeting seller banks’ premature withdrawals. This implies that the price for troubled asset purchases is the same for buyer banks and the government resulting in equally effective interventions. If, however, seller banks...

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3Please note that independent of the implemented characterisation of agency problems via risk-shifting, I will deliberately neglect the direct implications arising thereof by focusing on the indirect impacts triggered by fire sale prices only. Examining such mechanisms in more detail is left for further research.
are forced to demand additional liquidity to pay off premature withdrawals and the government is endowed with excess liquidity applied to the equilibrium condition, they are willing to pay higher prices for troubled assets than buyer banks as long as the invested amount of excess liquidity is not exhausted\(^4\). Higher prices reduce the future return for buying troubled assets resulting in higher volumes of term lending\(^5\). As buyer banks’ cash holdings are reduced by the government, I will refer to this as the crowding out effect of buyer banks’ speculative motive for liquidity hoarding. Moreover, seller banks’ solvency is improving as long as the relative impact on fire sale prices prevails over the relative impact on seller banks’ equity. Thus, the first main result of this paper points out that troubled asset purchases weakly dominate direct capital injections regarding buyer banks’ lending in general and seller banks’ solvency conditional on the previously mentioned condition.

The second finding is based on the endogenous amplification between market and funding liquidity. Liquidity shocks trigger a self-reinforcing downward funding liquidity spiral affecting fire sale prices and debt capacity interdependently. Being aware of these effects, secured creditors will hedge themselves against future losses by an exogenous decrease of the debt capacity depressing fire sale prices further more. Such higher exogenous exposure to collateral runs forces the government to intervene by providing more liquidity to satisfy the equilibrium condition on the one hand, and to meet seller banks’ solvency condition on the other hand. Hence, the range where troubled asset purchases strictly dominate direct capital injections regarding lending and solvency widens. This is induced by a lower crowding out effect of buyer banks’ speculative motive for liquidity hoarding.

The final result sheds new light on the interdependent impact of ex ante tighter leverage caps on the decision about ex post policy interventions. A lower amount of seller banks’ ex ante permitted leverage reduces the exposure to liquidity shocks and mitigates endogenous self-reinforcing spirals between seller bank’s asset and buyer bank’s collateral liquidity. The drop in troubled asset fire sale prices is less pronounced leading to a boost in term lending and a lower amount of cash holdings. The government expenditures necessary to meet seller banks’ solvency can be reduced and the decision on whether policy intervention is more effective will be affected as the range where troubled asset purchases strictly dominate direct capital injections shrinks. Summarizing, I find that there is an inherent nexus between ex ante regulations and ex post interventions providing new insights which basically confirm that tighter ex ante leverage caps mitigate ex post deteriorations in lending and solvency.

Despite the foundational papers by Bachmann (2018) and Diamond and Rajan (2011), this paper is closely related to a broad strand of theoretical literature that focuses on ex post interventions. Based on a similar debt overhang implication, Philippon and Schnabl (2013) show, among others, that banks will refuse to lend because they expect that other banks will do the same. In my model, however, lending is reduced because of the speculative motive for liquidity hoarding

\(^4\)You may think of it as a public purchase using a tender procedure where the highest bidder receives the surcharge for a given tranche of assets. Thereby, the government is willing to bid above the equilibrium fire sale price, as long as they are still endowed with excess liquidity applied to the equilibrium condition.

\(^5\)See, e.g. Puddu and Wälchli (2015) or more recently Rodnyansky and Darmouni (2017) for empirical evidence regarding the effects of troubled asset purchases on bank lending behaviour in the recent financial crisis.
indirectly induced by seller banks risk-shifting motive. This implies that the source of the debt overhang implication arises at the demand instead of the supply side of liquidity as implemented in Philippon and Schnabl (2013). Moreover, the authors focus on asymmetric informational problems induced by the debt overhang implication whereas this paper tries to emphasize the amplification between fire sales and collateral liquidity stemming indirectly from risk-shifting motives. Wilson (2012) compares troubled asset purchases and direct capital injections and finds that common stock capital infusions are weakly the most effective intervention to improve lending incentives. Considering the unpredictable evolution of asset prices, the author demonstrates that illiquid banks have no common incentive to sell their troubled assets voluntarily for its fair value. In my model, however, the government is willing to pay prices above their current fire sale valuation by using the same amount as they would have invested by direct equity injections. This implies that the tax-payers' burden is equally affected in both interventions. He and Krishnamurthy (2013) study the dynamics of risk premia during crises in asset markets if financial institutions face an equity capital constraint. By comparing direct capital injections, troubled asset purchases and reductions in borrowers costs, the authors find the former intervention to be the most effective one. Despite the fact that my model is static in time, probably the most important difference concerns the implementation of the model-constraint(s). As long as endogenous funding constraints are imposed only, I find that troubled asset purchases weakly dominate direct equity injections. This effect diminishes in favour of direct equity injections if (ex ante) equity constraints will be considered additionally which is in line with the main finding in He and Krishnamurthy (2013). By investigating different bailout strategies for banks that suffer from a debt overhang problem but have access to private informations on asset prices, Bhattacharya and Nyborg (2013) derive an equivalence result for direct equity injections and troubled asset purchases if future investment opportunities are reduced in relation to assets sold. Another very interesting, methodically yet clearly different, contribution is proposed by Kühl (2014). Using a New Keynesian dynamic stochastic general equilibrium model, he finds direct equity injections to be the most effective intervention in terms of a welfare perspective. Obviously, the analyses in my model are predominantly positive by emphasizing effects on the real economy regarding bank lending activities and solvency improvements only.

Further closely related papers that implement a similar analytical model-framework by examining different ex post interventions, however, are proposed by Diamond and Rajan (2005) and Acharya and Yorulmazer (2008). The former distinguish between pure liquidity infusion and pure recapitalization as ex post interventions and highlight that both can be counterproductive as bank liquidity and insolvency are mutually reinforcing. They find that pure liquidity infusion is most benign in terms of its spillovers and, at least, saves the most solvent banks. In Acharya and Yorulmazer (2008), illiquid banks are forced to sell assets at fire sale prices that will be purchased by liquid banks. To avoid inefficient allocations potentially triggered by liquidity evaporation, ex post interventions can be optimal if the fire sale mechanism is sufficiently pronounced which leads to higher incentives to buy from outside participants.

6A more detailed analytical tractability regarding asymmetric informations is left for further research.
Based on the implementation of the Basel III framework in the aftermath of the financial crisis, there is a growing literature at the intersection between micro- and macroeconomics focusing on ex ante policy regulations. Chari and Kehoe (2016) pointed out that optimal ex ante leverage constraints implement sustainable efficient outcomes without using any ex post interventions. Farhi and Tirole (2012) examine the implication of regulations in a model with coordination problems and show, among others, that ex ante caps on leverage provide an optimal regulation tool. Their central argument is that private leverage choices are expectationally linked to ex post interventions. Bianchi and Mendoza (2015) and more recently Gale et al. (2017) underpin that ex ante leverage constraints will lead to efficient outcomes ex post, whereby the former particularly propose ex ante taxations on debt in order to prevent excessive leverage when crises become more likely. Focusing on the interplay between capital and liquidity regulations, Kara and Ozsoy (2016) argue that capital requirements mitigate fire sales and improve financial stability which, in turn, reduces the incentive to hold liquidity explained by a lower benefit of liquidity. This is very similar to the results presented in this paper, as the speculative motive for liquidity hoarding is reduced by introducing tighter caps on leverage explained by lower expected future returns on buying troubled assets. Even though we share the same fire sale intuition, the market incompleteness in Kara and Ozsoy (2016) arises by a limited-commitment problem instead of the assumption of a limited set of natural buyers as proposed in this paper.

As there is very little work that analytically examines the interdependence between ex ante regulations and ex post interventions, however, this paper aims to fill this gap using a very different model approach to that proposed in Jeanne and Korinek (2016) by providing a theoretical framework that allows to investigate how ex ante tighter leverage caps will affect ex post decisions between using either direct capital injections or troubled asset purchases in order to efficiently boost bank lending and restore their solvency conditions. On the other hand, the endogenous implementation of the self-reinforcing interplay between market and funding liquidity allows to explain the impact of higher exposures to collateral runs on ex post decisions about policy interventions. Hence, this paper intends to provide new insights for policymakers as well as financial authorities to better prevent or mitigate the economically harmful repercussions of modern financial crises.

The rest of this paper is organized as follows. Section 2 introduces the basic concepts of the baseline model before extending it for policy analyses in Section 3. Following this, Section 4 analyses the overall impacts of ex ante prudential regulations and ex post policy interventions on bank lending and solvency by deriving the main results and presenting numerical examples. Section 5 concludes. All proofs are left to the Appendix.

2 The Baseline Model

The analytical framework in Bachmann (2018) consists of two types of financial institutions called seller and buyer banks, where each of them has a continuum of identical agents normalized to one. The key event of the three period discrete time model happens at the intermediate date 1, where
a fraction of seller banks’ depositors withdraw prematurely. Note that the occurrence of liquidity shocks are modeled as sunspot phenomena and are hence exogenous to the model – any information based reasons that can potentially trigger premature withdrawals are deliberately neglected. In order to meet this liquidity demand, seller banks can sell the necessary amount of illiquid assets to buyer banks, recall loans by liquidating borrowers’ projects or choose an optimal combination of both interventions. Buyer banks provide liquidity by purchasing seller banks’ illiquid assets at either date paying with cash. Buyer banks’ main source of cash is determined by collateralized borrowing in secured debt markets. Assets are used as collateral in order to finance long-term lending to industrials or illiquid asset purchases from seller banks depending on the expected return of both investments. In case of financial stress, seller banks are forced to sell illiquid assets for prices below fundamentals in order to attract buyer banks’ cash. Fire sale prices for illiquid assets increases the returns that are potentially available in the future, resulting in higher cash reserves on behalf of buyer banks. Such speculative motives for liquidity hoarding force the bank to reduce the amount of term lending to industrials to have more cash for buying illiquid assets. Simultaneously, however, creditors in secured debt markets ask for higher margins on collateral lending in order to hedge against potential collateral liquidations at fires sale prices in the secondary market. This amplifies the reduction of term lending conditional on the speculative motive of liquidity hoarding as less cash is generated by collateralized borrowing. On the other hand, seller banks will refuse to sell more illiquid assets than necessary to meet premature liquidity demand – a speculative motive of illiquidity seeking as seller banks gamble for resurrection in case of fires sales by expecting higher future relative returns if the bank survives conditional on the liquidity shock.

One of the key concepts that determines previously explained mechanisms is based on the expected return on illiquid assets and is expressed as

\[
\frac{1}{P_0} = \frac{q}{P_1} + (1 - q),
\]

(2.1)

where the LHIS represents the date-0 return on illiquid asset purchases and the RHS shows the expected return on buying those at date 1 conditional on the date-1 fire sale price \( P_1 \) and the probability of the date-1 liquidity shock \( q \). As illiquid assets trade at a discount to face value\(^7\) \((P_1 < 1)\) conditional on the date-1 liquidity shock at which a fraction \( f \) withdraw their deposits \( D \) prematurely, the return on illiquid asset purchases obviously increases for both dates 0 and 1.

Buyer banks will reduce their lending volume \( I(1/P_0) \) at date 0 in anticipation of future fire sales conditional on (2.1), which increases their disposable amount of cash defined as

\[
\theta + (1 - H(P_0)) C - I \left( \frac{1}{P_0} \right),
\]

(2.2)

where \( C \) is the assets’ net worth at date 2 and \( \theta \) shows the amount of cash endowment at date 0.

\(^7\)Note that illiquid assets trade at a discount to face value if and only if the buyer bank’s amount of disposable cash falls short of premature liquidity needs \( fD \).
Moreover it is assumed that the change in the volume of term lending to industrials is diminishing over date-1 fire sale prices $P_1$. Hence, $I'(1/P_0) > 0$ and $I''(1/P_0) < 0$ conditional on (2.1). Finally, in order to model the impact of date-1 fire sale prices on collateral haircuts, it is important to incorporate the risk of collateral illiquidity triggered by a potential sale in the secondary market for prices below fundamentals. As the risk of future fire sales increases conditional on the liquidity shock, the current debt capacity endogenously shrinks as creditors hedge themselves against future losses by a precautionary rise in current haircuts. Thus, the haircut function $H(P_0)$ will be

$$
\left(1 - \frac{1}{P_1^q} (1 - q)\right)^{1-\delta}
$$

conditional on (2.1), where $\delta \in [0,1]$ defines the speed of convergence over the interval of asset liquidity $P_1$. Haircuts approach one if $1 - \delta$ is close to zero even for very high realisations of $P_1$, that is, if collateral illiquidity becomes highly probable conditional on seller banks’ future liquidity shocks, the creditor will increase the haircut on collateral lending towards one even for currently very liquid assets. Considering such worst case scenarios, the overall amplifying downward pressures on fire sale prices triggered by increased exposures to collateral runs can therefore potentially end in funding freezes. Summarizing, I find that haircuts are increasing if common liquidity shocks are more likely, the exposure to collateral runs is increasing or date-1 fire sale prices are falling. Thus, the haircut function (2.3) is decreasing in $P_1$ and increasing in $q$ and $\delta$.

The reason why assets are illiquid or trade below fundamentals was not explicitly derived up to now. As long as the cash received from selling assets at full face value to buyer banks exceeds the premature demand for liquidity at date 1, there is no immediate need to sell at lower prices. If, however, the cash received from selling to buyer banks at full face value falls short of premature withdrawals, the seller bank is forced to sell the necessary amount of currently illiquid assets at fire sale prices in order to meet the date-1 excess demand for liquidity. In addition to the fraction $\beta$ of illiquid assets described so far, seller banks’ asset endowment is composed of the fraction $1 - \beta$ of loans employed at date 0 which can be recalled at date 1 by liquidating borrowers’ projects. This creates an additional source of liquidity for seller banks to meet premature liquidity needs. By combining both sources, seller bank’s date-1 liquidity constraint will be

$$
(1 - \beta) \frac{Z}{2} (1 - P_1^2) + \left[ \theta + (1 - H(P_0))C - I\left(\frac{1}{P_0}\right) \right] = fD, \quad (2.4)
$$

where the first term represents the cash generated by the liquidation of borrowers’ projects assuming uniformly distributed liquidation values of seller bank’s loan portfolio. The last term in square brackets is the amount of cash received from selling the necessary amount of illiquid assets for fire sale prices to buyer banks. As the date-2 expected return on illiquid assets is the same for selling at either dates 0 or date 1 conditional on (2.1), I will assume that illiquid asset trading is realised at date 1.
Despite this, the crucial condition for the liquidity constraint (2.4) determining the fire sale price \( P_1 \), is seller bank’s solvency conditional on the date-1 liquidity shock. As assets are assumed to be traded at date-1 only, seller bank’s date-2 equity will be

\[
E^2_2 = \left\{ \left[ \beta Z - \frac{\theta + (1 - H(P_0))C - I\left(\frac{1}{P_0}\right)}{P_1} \right] + (1 - \beta)P_1Z - (1 - f)D \right\}
\]  

(2.5)

where the first term in square brackets is the illiquid asset endowment left after trading with buyer banks, the second term represents the endowment left after loan liquidation and the last term is the amount of deposits remaining at date 2. Thus, in order to specify seller bank’s solvency condition conditional on the liquidity shock, (2.5) will be discounted to date-1 using the fire sale price \( P_1 \).

The intuition leading to (2.6) is explained by ask and bid prices for illiquid asset trading. As long as the price the buyer bank is willing to pay exceeds the price the seller bank is willing to offer, seller bank’s equity is positive and hence the solvency condition (2.6) is satisfied.

Assumption 1 in Bachmann (2018) explicitly specifies that some illiquid assets are assumed to remain on seller banks’ balance sheets after trading with buyer banks. In the following, I will deliberately admit for situations where no assets are left after trading with the buyer bank and the subsequently introduced government agent in order to examine, among others, the direct impact on price deviations and their indirect effects on term lending and seller bank’s solvency conditions. This will constitute one of the core conditions that have to be considered in the following analyses.

3 Modeling Policy Measures

In this section, I will extend the baseline model described in Section 2 by introducing the government as an additional agent in order to implement ex ante policy regulations, ex post policy interventions and their intertemporal interdependence. The real economy impact of subsequently introduced interventions and regulations is assumed to be established by examining their effects on bank lending and solvency. Thereby it is important to note that both policy measures are modeled affecting seller banks as the date-1 liquidity shock deteriorates buyer banks’ funding but does not trigger immediate liquidity needs.

3.1 Ex Post Interventions

According to Song (2011), I assume that the government is endowed with a fixed amount \( gZ \) and has to decide whether to intervene at date 0 or at date 1 by using the full amount either for direct

\footnote{Please note that date-0 price impacts on trading are implicitly considered even if illiquid asset trading is explicitly modeled at date 1 only. Moreover, it is assumed that seller banks are solvent in the long run, that is, \( Z > D \).}
capital injection (Intervention (A)) or for buying a given amount of illiquid assets (Intervention (B)).

Given the government decides to intervene ex post conditional on the date-1 liquidity shock, they will acquire a stake \( gZ \) of seller bank’s date-2 equity and inject an amount \( \alpha Z \) of date-1 liquidity. Hence, the seller bank’s date-1 liquidity and solvency conditions for interventions (A) and (B) are respectively modified as

\[
\alpha Z + (1 - \beta) \frac{Z}{2} (1 - P_{1A}^2) + \left[ \theta + (1 - H(P_{0A})) C - I \left( \frac{1}{P_{0A}} \right) \right] = f D, \tag{3.1}
\]

\[
\alpha Z + (1 - \beta) \frac{Z}{2} (1 - L_{1}^2) + \left[ \theta + (1 - H(P_{0B})) C - I \left( \frac{1}{P_{0B}} \right) \right] = f D, \tag{3.2}
\]

\[
P_{1A} \beta Z - \left[ \theta + (1 - H(P_{0A})) C - I \left( \frac{1}{P_{0A}} \right) \right] + (1 - \beta) P_{1A}^2 Z - (1 - f) D P_{1A} - g Z P_{1A} \geq 0 \tag{3.3}
\]

and

\[
\left\{ P_{1B} \beta (\beta - g) - \left[ \theta + (1 - H(P_{0B})) C - I \left( \frac{1}{P_{0B}} \right) \right] \right\} + (1 - \beta) P_{1B} L_{1} Z - (1 - f) D P_{1B} \geq 0 \tag{3.4}
\]

where seller bank’s assets are decomposed by \( \alpha Z + \beta Z + (1 - \beta) Z = Z + g Z \). The first term of (3.1) shows the amount of liquidity provided by the government, the second term represents the cash generated from recalled loans and the last term in square brackets is the amount of cash received from selling a given amount of illiquid assets to buyer banks. Focusing on (3.4), the first term in curly brackets represents seller bank’s illiquid asset endowment left after trading with buyer banks and the government, the second term is the endowment left after loan liquidation and the last term shows the amount of deposits remaining. Please note that the LHS of conditions (3.3) and (3.4) is nothing but seller bank’s date-2 equity discounted to date 1 by the fire sale price conditional on date-1 illiquid asset trading only. Now, as it is assumed that the government intervenes with a fixed amount \( g Z \) at date 1 conditional on the liquidity shock, seller bank’s date-2 equity and date-1 liquidity endowments are equally affected due to \( \alpha Z = g Z \). This relationship between seller bank’s solvency and liquidity conditions is independent of the intervention and is determined only by the date-1 fire sale discount \( P_{1} \).

The relation between prices for illiquid asset trading and their indirect impact on buyer bank’s disposable amount of cash turned out to be yet somewhat more complex but highly important to understand the core results of this paper presented in Section 4. To compare the different effects between direct capital injections and illiquid asset purchases, the government uses the same amount \( g \) for both measures. As long as not all illiquid assets have to be sold to meet premature withdrawals conditional on the liquidity shock after trading with the buyer bank and the government, prices for illiquid assets are equal for both interventions. To put it simply, illiquid assets trade at a discount to face value conditional on the date-1 liquidity shock resulting in higher returns on buying such illiquid assets. Based on the speculative motive for liquidity hoarding, buyer banks employ all their cash holdings for illiquid asset trading. If, however, seller bank’s liquidity shortfall cannot be
balanced with illiquid asset sales to buyer banks only, an additional source of liquidity is needed. The government provides this extra liquidity by buying out the necessary amount of illiquid assets to meet seller bank’s premature withdrawals. If not all illiquid assets are exhausted after trading with the government, there is no reason for price deviations of illiquid assets between intervention (A) and (B). Moreover, as the amount of buyer bank’s lending to industrials (sometimes briefly called bank lending as well) and the haircut depend on fire sale prices only, buyer bank’s disposable amount of cash remains unchanged comparing both interventions as well. Subsequently, I will refer to this as case (I).

However, what are the effects on prices in case (II) when no illiquid assets are left after trading with both buying agents and seller banks fail to meet their premature withdrawals? To answer this question, comparing the different impact on seller bank’s asset endowment triggered by the respective intervention in more detail is necessary. As already mentioned above, the amount of government expenditures $g$ is the same for both interventions for comparability reasons. Consider now the case that the liquidity injection $\alpha Z$ to seller banks using direct capital injection would lead to a higher demand for illiquid assets than seller banks can supply if the same amount $gZ$ would have been used to intervene via illiquid asset purchases otherwise, that is,

$$Z(\beta - g) < \frac{\theta + (1 - H(P_{0A}))C - I \left( \frac{1}{P_{0A}} \right)}{P_{1A}}. \quad (3.5)$$

Conditional on seller bank’s anticipation of such an excess demand for illiquid assets, the government is willing to pay higher prices compared to buyer banks in order to balance (3.5) until the equilibrium condition is reached. Thereby, the equilibrium condition is satisfied if

$$Z(\beta - g) = \frac{\theta + (1 - H(P_{0B}))C - I \left( \frac{1}{P_{0B}} \right)}{P_{1B}}. \quad (3.6)$$

the amount of illiquid assets left after trading with the government equals the maximum amount that can be purchased by buyer banks using intervention (B). Moreover note that government’s willingness to pay higher prices is explained by the fact that the difference in $g$ between intervention (A) and (B) applied to the equilibrium condition (3.6), gives the opportunity to bid for the same amount $Z(\beta - g)$ with higher prices. By doing so, the government has a twofold impact on buyer banks: Lower expected returns on buying illiquid assets increase the amount of bank lending and creditors will adjust to lower future risks on collateral illiquidity by reducing the haircut. According to the speculative motive for liquidity hoarding in case of fire sales, buyer banks try to reduce their current cash holdings in anticipation of lower future returns on illiquid assets purchases as lending becomes more attractive.

**Assumption 1** In order to satisfy the speculative motive for liquidity hoarding even in such cases, the impact on bank lending is assumed to prevail the impact on haircuts in reaction to higher future prices for illiquid assets purchases.
This implies that government’s willingness to pay higher prices for illiquid assets purchases decreases buyer banks’ incentive to hold cash as the return on buying these assets in the future is lower compared to government’s direct capital injections regarding case (II). Thus, I will refer to this as crowding out of buyer bank’s speculative motive for liquidity hoarding.

**Lemma 1** Conditional on the date-1 liquidity shock we find that

(i) \( L_1 = P_{1A} = P_{1B} \) if the seller bank is endowed with some illiquid assets after trading and

(ii) \( L_1 < P_{1A} < P_{1B} \) if no illiquid assets are remaining on the seller bank’s balance sheet after trading.

**Proof.** See Appendix A. ■

The intuition behind the different effects on illiquid asset prices in (i) and (ii) were already discussed in detail. Focusing on the not yet explained impact on the loan liquidation price \( L_1 \), the change in buyer bank’s disposable amount of cash for both interventions has to be considered. As explained above, buyer bank’s cash holdings are equally affected regarding intervention (A) and (B) in case (I) and hence prices for asset trading do not diverge from loan liquidation prices in order to meet premature withdrawals conditional on (3.1). In case (II), however, buyer bank’s disposable amount of cash is lower for intervention (B) than for (A) as the return on buying illiquid assets is higher for intervention (A) than for (B). To balance this liquidity shortfall, seller banks are forced to recall more loans by reducing the loan liquidation price below the price for illiquid assets if the government intervenes by illiquid asset purchases conditional on (3.2).

### 3.2 Ex Ante Regulations

As introduced in Section 2 in a preliminary way, the key feature for a potential market freeze on the demand side of liquidity is triggered by a self-reinforcing need in selling illiquid assets at fire sale prices to meet premature withdrawals conditional on the date-1 liquidity shock. If this liquidity demand cannot be served with asset sales only, the seller bank is forced to recall more loans additionally. In order to counter against such an ex post deleveraging cycle and to prevent the market from potential failures, I will examine the impact of ex ante tighter leverage ratio requirements on ex post decisions of government interventions regarding bank lending and solvency. Subsequently, seller bank’s ex ante leverage ratio \( L_R \) will be defined as \( D/(Z - D) \) the ratio between debt and equity at date 0 conditional on the non-occurrence of the liquidity shock. This implies that seller bank’s debt and asset endowments are the same at both dates 0 and 2, as ”ex ante” describes the basic model conditions before the overall effects triggered by the liquidity shock are considered. This is implemented by the fact that assets trade at full face value in the absence of the shock. Based on this relation, tighter caps on seller bank’s ex ante leverage ratios can be modeled affecting either the asset or the liability side of their balance sheet. Focusing on the asset side, requiring to keep (more) better valued assets in relation to the amount of outstanding debt reduces
the maximum amount of permitted leverage. Hence, an ex ante tighter requirement on leverage reduces seller bank’s leverage ratio as $L_R$ is decreasing in $Z$.

Despite this immediate impact, the model should, moreover, be able to track the indirect dynamic impacts on leverage changes conditional on the liquidity shock. To meet this requirement, seller bank’s date-1 leverage ratio will be defined as

$$
(1 - f)D + P_1 \beta Z - \left[ \theta + (1 - H(P_0)) C - I \left( \frac{1}{P_0} \right) \right] + (1 - \beta) P_1^2 Z - (1 - f)D,
$$

(3.7)

where the numerator shows the amount of debt remaining after date-1 withdrawals whereas the denominator represents seller bank’s equity conditional on the assumption that illiquid asset trading takes place at date 1 only.

### 3.3 Sequence of Events

The timeline of the baseline model according to Bachmann (2018) will be adjusted by introducing the government as a new agent. All else being unchanged, the government intervenes at date 1 conditional on the liquidity shock by providing an additional source for seller banks’ liquidity in order to meet premature withdrawals. In anticipation of date-1 fire sales, the decision on whether policy intervention is used will be based on the impact on bank lending and seller bank’s solvency. On the other hand, the government can affect the financial system by ex ante regulations in order to protect banks from failing and credit markets from collapsing conditional on the date-1 liquidity shock ex post. Figure 1 illustrates the timeline of the model.

**Figure 1:** Sequence of Events
4 Analyses and Results

In this section, I will emphasize the reasons why government interventions and regulations are set up before analysing the key mechanisms of direct capital injections and illiquid assets purchases as well as their impact on buyer banks’ lending and seller banks’ solvency. Following this, I will introduce the purpose of ex ante regulations by focusing on tighter requirements on seller bank’s leverage and their impact on ex post decisions between the two distinguished policy interventions.

Based on liquidity driven premature withdrawals, the bank is forced to sell the necessary amount of assets at fire sale prices and/or to recall loans by liquidating borrowers’ projects prematurely. As the expected return on buying illiquid assets increases conditional on date-1 fire sales, buyer banks cut term lending to industrials in order to meet their speculative motive for liquidity hoarding. Simultaneously, however, buyer bank’s amount of disposable cash shrinks as the creditor protects himself from collateral illiquidity by increasing the amount of haircuts. This, in turn, forces buyer banks to drop lending to industrials further more, resulting in a higher need for selling illiquid assets to funding-liquidity restricted buyer banks. This self-reinforcing process between endogenous fire sales and collateral runs deteriorates term lending and makes seller bank’s insolvency more likely. In the worst case, seller banks will slip into insolvency and credit as well as collateral markets become illiquid. To counter such seemingly unrelated feedback loops, governments should feel compelled to regulate and intervene using reasonable and effective instruments to boost bank lending and restore solvency conditions.

4.1 Ex Post Interventions

The government intervenes at date 1 and has to decide between direct capital injections and buying troubled illiquid assets using the same amount of expenditure $g$ to restore seller bank’s balance sheets. In both cases, the government takes a stake worth $g$ of seller bank’s equity which pays off at date 2. As introduced in Section 3.1, there exists the equilibrium condition (3.6) at which both interventions (A) and (B) are equivalent regarding all examined model parameters.

Let us start with case (I) in which not all of seller banks’ illiquid assets are exhausted after trading with both buying agents conditional on the date-1 liquidity shock. Additional liquidity provided by the government reduces seller banks’ liquidity pressure to meet premature withdrawals and enables them to sell less illiquid assets at fire sale prices to buyer banks. This limits the downward price spiral and reduces the expected return on buying illiquid assets cheaply in the future. The lower attractiveness for liquidity hoarding increases the incentive to invest in term lending to industrials. Simultaneously, seller bank’s date-1 insolvency becomes more unlikely as the price drop for illiquid assets is mitigated by government intervention. Considering that the impact on date-1 fire sale prices is the same for both interventions in case (I) conditional on Lemma 1, it follows that term lending is boosted by the same amount independent of whether the government injects the necessary amount of liquidity by illiquid asset purchases or by direct capital injections (see Figure 2). By using the same intuition it can be established that the improvement regarding
seller bank’s solvency condition is equally affected by both government interventions.

The more interesting case (II) arises if all illiquid assets are exhausted after trading conditional on the liquidity shock. As already explained in detail in Section 3.1, the government is willing to pay higher prices for illiquid assets than buyer banks as long as the former is endowed with excess liquidity compared to the equilibrium condition (3.6). This, moreover, implies that there necessarily must be an amount of government expenditure \( g \) left compared to direct capital injections conditional on (3.5). Hence, the government has to decide whether to use this remainder to bid for higher prices and crowd out buyer banks’ current incentive to buy such illiquid assets, or they forfeit this task and risk seller bank’s insolvency as well as potential credit and collateral freezes. I rule out such conditions by assuming that the full amount of government expenditure must be used in both interventions for comparability reasons. Moreover, in such cases it would never be reasonable to use higher expenditure for direct capital injections than for illiquid asset purchases.

Following this and Lemma 1 it should be obvious that buyer bank’s term lending to industrials is strictly higher using government expenditure for illiquid asset purchases compared to direct capital injections if no illiquid assets are left after trading conditional on the liquidity shock. However, the lending gap triggered by these different price impacts between both interventions is diminishing in the amount of government expenditure as shown in Figure 2, which is based on a lower price sensitivity of the change in lending volumes as asset liquidity increases. This implies that the impact of a reduced drop in fire sale prices on term lending is higher for less liquid assets than for highly liquid ones as long as the equilibrium condition (3.6) is not satisfied.

**Proposition 2**

(i) If the seller bank is endowed with some illiquid assets conditional on the date-1 liquidity shock, direct capital injections and illiquid asset purchases are equivalent regarding buyer bank’s lending and seller bank’s solvency.

(ii) If all illiquid assets are exhausted conditional on the date-1 liquidity shock, illiquid asset purchases weakly dominate direct capital injections regarding buyer bank’s lending. The lending gap between both interventions is diminishing in the amount of government expenditure.

(iii) If all illiquid assets are exhausted conditional on the date-1 liquidity shock, illiquid asset purchases weakly dominate direct capital injections regarding seller bank’s solvency as long as the direct relative impact on prices prevails over the indirect relative impact on seller bank’s equity.

**Proof.** See Appendix A. ■

Turning to (iii) we find that independent of the intervention used, the seller bank’s solvency condition (2.6) is increasing on the asset side and decreasing on the liability side in terms of the first order partial derivatives with respect to the date-1 fire sale price. Thus, the seller bank’s solvency condition improves as long as the price impact on illiquid assets prevails over the impact on the amount of outstanding debt. Combining this effect with Lemma 1, we can conclude that the improvement of the solvency condition due to intervention (B) weakly dominates intervention (A) as long as the relative increase in asset prices is higher than their relative overall impact on seller bank’s equity as depicted in Figure 2.
Figure 2: Buyer bank’s lending and seller bank’s solvency for different exposures to collateral runs by focusing on the representation of changes in the likelihood switching between case (I) and (II), where DCI and IAP denote direct capital injections and illiquid asset purchases, respectively. Thereby, the following parameter values are used: $Z = 1$, $D = 0.72$, $b = 0.5$, $t = 0.4$, $C = 0.86$, $f = 0.75$ and $q = 0.7$.

But what are the impacts on lending and solvency if the exposure to collateral runs increases? The following proposition summarizes the formal results answering this question.

**Proposition 3** A higher exposure to collateral runs
(i) expands the parameter range for case (II),
(ii) deteriorates buyer bank’s lending and seller bank’s solvency for both interventions in both cases (I) and (II),
(iii) and forces the government to increase expenditure conditional on seller bank’s solvency needs.

**Proof.** See Appendix A.

First of all, a higher exposure to collateral runs is modeled affecting the haircut on collateralized borrowing as already explained in Section 2. An increased parameter $\delta$ highlights higher expected risks of collateral illiquidity resulting in a lower amount of buyer banks’ available cash holdings as the debt capacity shrinks endogenously. Thus, buyer banks will reduce the volume of term lending in order to satisfy their speculative motive for liquidity hoarding culminating in a greater drop in liquidity for seller banks. This is obviously true for both cases (I) and (II) as the reduced incentive for liquidity holding results from lower expected returns on buying illiquid assets which triggers the same directional effects independent of the amount of assets left after trading (see Figure 3). Despite this, the government is forced to intervene with higher expenditure in order to balance seller
bank’s liquidity shortfall which, in turn, expands the parameter range for case (II) furthermore (see Figure 2). The intuition behind this mechanism is determined by the fact that an increase in the exposure to collateral runs increases the necessary amount of government expenditure to satisfy the equilibrium condition (3.6). This implies that the parameter range where illiquid asset purchases are strictly dominating over direct capital injections widens conditional on higher exposures to collateral runs (see Figure 2). Moreover, the lending gap between both interventions is increasing for fixed government expenditures as asset illiquidity is more likely (see Figure 3). In a similar vein, seller bank’s solvency condition deteriorates by the amplified price drop conditional on higher exposures to collateral runs. In reaction to this, the government is forced to intervene with higher expenditure to restore seller bank’s solvency as shown in Figure 3.

4.2 Interdependence of Ex Ante Regulations and Ex Post Interventions

Despite the effects on buyer bank’s lending and seller bank’s solvency triggered by the government’s decision between illiquid asset purchases and direct capital injections previously explained in Section 4.1, I will analyse how these ex post decisions are affected by regulating seller bank’s leverage ex ante. Thus, the key question to be answered in this section is as follows: How does an ex ante tighter constraint on seller bank’s leverage ratio affect the government’s ex post decision between illiquid asset purchases and direct capital injections by targeting a boost of buyer bank’s lending and an improvement of seller bank’s solvency? The following proposition summarizes the formal results answering this question.
Proposition 4 A reduced amount of permitted leverage (i) expands the parameter range for case (I), (ii) increases lending in case (I) but reduces lending in case (II), (iii) improves seller bank’s solvency in case (I) and improves seller bank’s solvency if the impact on loan liquidation prices prevails over the impact on illiquid asset prices in case (II).

Proof. See Appendix A. ■

As already mentioned in Section 3.2, tighter leverage requirements are implemented on the asset side of seller bank’s balance sheet by a forced increase in the assets’ net worth $Z$. This enables seller banks to reduce the necessary amount of illiquid asset sales and to recall less loans by liquidating fewer borrowers’ projects in order to meet premature withdrawals conditional on the forced assets’ net worth improvement. Hence, as seller banks’ need in ex post deleveraging is reduced conditional on tighter ex ante leverage caps, the likelihood that not all illiquid assets have to be sold to cover premature liquidity demand increases, that is, the parameter range for case (I) expands (see Figure 4). The key intuition driving these impacts is determined by the fact that the self-reinforcement between asset fire sales and funding liquidity is mitigated by introducing ex ante constraints on seller bank’s leverage ratio. The reduced necessity of ex post deleveraging reduces the drop in date-1 fire sale prices affecting the creditor to allow for lower haircuts as the risk of collateral illiquidity shrinks. The lower expected future return on illiquid assets increases buyer bank’s incentive to invest in term lending to industrials based on the crowding out effect of their speculative motive for liquidity hoarding. Hence, from case (I) immediately follows that buyer bank’s volume of term lending is expanded by the same amount for both government interventions conditional on Lemma 1 (see Figure 5).

The different impact between both government interventions on ex post term lending triggered by ex ante tighter leverage requirements in case (II), however, turned out to be somewhat more complex again. The basic intuition is similar to the concept already explained in Section 3.1 and Section 4.1. Considering the remainder between direct capital injections and illiquid asset purchases applied to the equilibrium condition (3.6), the government is willing to pay higher prices than buyer banks in order to provide the necessary amount of additional liquidity to seller banks. Moreover note that the higher the realisation of the remainder applied to the equilibrium condition the higher the government’s willingness to bid even more for illiquid asset purchases resulting in a higher crowding out effect of buyer bank’s disposable cash. This means that the ”distance” of the remainder applied to the equilibrium condition matters for the extent of the price impact on illiquid assets. To put it simple, the greater the interval characterizing case (II), the higher the price impact and hence the crowding out effect of buyer banks’ cash holdings. Thus, as an increase in the assets’ net worth forced by tighter leverage requirements reduces the extent of the remainder applied to the equilibrium condition, the government’s necessity and willingness to pay higher prices is mitigated. Lower future prices for illiquid assets raise buyer bank’s incentive to hold enough liquidity to buy such assets cheaply in the future in order to satisfy their speculative motive for liquidity hoarding.
Figure 4: Buyer bank’s lending and seller bank’s solvency for different ex ante leverage ratio requirements by focusing on the representation of changes in the likelihood switching between case (I) and (II), where DCI and IAP denote direct capital injections and illiquid asset purchases, respectively. Thereby, the following parameter values are used: $D = 0.72$, $b = 0.5$, $t = 0.4$, $C = 0.86$, $f = 0.65$, $q = 0.8$ and $\delta = 0.6$.

Simultaneously, buyer banks will reduce their term lending conditional on higher future returns on buying illiquid assets endogenously induced by mitigating the crowding out effect of buyer banks’ incentive for liquidity hoarding as shown in Figure 5.

Let us finally focus on ex post solvency issues triggered by ex ante tighter leverage ratio requirements. Based on the reduced necessity of seller banks’ ex post deleveraging considering case (I), the drop in date-1 fire sale prices is mitigated which obviously results in an improvement of seller banks’ solvency (see Figure 5). Turning to case (II), the different impact on illiquid asset prices and loan liquidation prices has to be considered\(^9\). On the one hand, the reduced government’s need to pay higher prices triggered by tighter leverage requirements would lead to a deterioration of seller banks’ solvency condition. On the other hand, as the higher returns on illiquid assets that are potentially available in the future increase buyer banks’ cash holdings used to trade with seller banks, fewer loans have to be recalled prematurely. This immediately implies higher liquidation prices conditional on uniformly distributed loan liquidation values ranging between full face value and zero. As long as this increase in loan liquidation prices prevails over the reduction in illiquid asset prices, seller bank’s solvency condition is finally improving (see Figure 5).

\(^9\)Note that in case (I), both impact are equal conditional on Lemma 1.
Figure 5: Buyer bank’s lending and seller bank’s solvency for different ex ante leverage ratio requirements by focusing on the representation of the impact on lending changes and solvency improvements, where DCI and IAP denote direct capital injections and illiquid asset purchases, respectively. Thereby, the following parameter values are used: $D = 0.72$, $b = 0.5$, $t = 0.4$, $C = 0.86$, $f = 0.65$, $q = 0.8$ and $\delta = 0.6$.

5 Concluding Remarks

The theoretical framework implemented in this paper is well suited to examine the impact of ex ante policy regulations and ex post policy interventions on bank lending and solvency in times of financial stress on the one hand, and enables to explain how governments ex post decisions will be affected by ex ante regulations on the other hand. External interventions are demanded by illiquid financial institutions which are commonly hit by the same liquidity shock triggered by premature withdrawals. Going beyond the baseline model of Bachmann (2018), not only liquid buyer banks but also the government will provide liquidity. By doing so, liquid buyers intervene by purchasing troubled assets only whereas the government has to decide whether to intervene by direct capital injections or by troubled asset purchases conditional on the effectiveness on boosting liquid (buyer) bank’s lending and restoring illiquid (seller) bank’s solvency.

The core condition driving the different model results is determined by seller bank’s asset endowment conditional on the liquidity shock. If there are some troubled assets left after trading, both interventions boost buyer bank’s lending and restore seller bank’s solvency equally. The explanation offered in this paper is based on fire sale prices of troubled assets. As long as some of those assets are left, there is no reason for different bid prices between liquid buyer banks and the government. If, however, seller banks cannot cover premature liquidity needs even by selling all their assets at fire sale prices and the government is left with excess liquidity compared to direct capital injections, they are willing to pay higher prices for troubled asset purchases than liquid buyer banks. This induces a crowding out of buyer banks’ incentive for liquidity hoarding as term lending becomes
more attractive. Simultaneously, seller bank’s solvency is improving as long as governments’ impact on fire sale prices prevails over their counter impact on seller bank’s equity.

As the baseline model characterizes modern financial crises by relating traditional bank runs to collateral based runs in order to explain the amplification between market and funding liquidity, we are able to examine the impact of higher exposures to collateral runs on decisions about ex post policy interventions additionally. A part of buyer bank’s funding is realised by collateralized borrowing in secured short-term debt markets depending on creditors’ expectations of future debt capacities. Higher risk of future collateral illiquidity makes creditors more sensitive to asset fire sales resulting in a lower amount of secured lending which forces illiquid banks to sell even more troubled assets at fire sale prices to meet premature withdrawals. This amplification of the self-reinforcement between fire sales and collateral illiquidity mitigates the crowding out effect of buyer banks’ speculative motive for liquidity hoarding as the future return on buying troubled assets increases steadily. This implies that situations in which illiquid banks exhaust all their troubled assets are more likely, buyer bank’s volume of lending is reduced and seller bank’s solvency is worsened conditional on higher exposures to collateral runs. To counter these effects, the government is forced to intervene.

Finally, the model enables to examine the intertemporal interdependence between ex ante policy regulations and ex post decisions on whether intervention will be more effective regarding buyer bank’s lending and seller bank’s solvency. The government regulates illiquid banks ex ante by a reduction of the maximum amount of permitted leverage in order to reduce the need for extensive deleveraging conditional on the liquidity shock ex ante. If some troubled assets are left after trading, I find that ex ante tighter leverage requirements boost buyer bank’s lending and improve seller bank’s solvency as the ex post drop in fire sale prices is mitigated. Both effects occur independently of the intervention mechanism. Focussing on the case where all troubled assets are exhausted after trading it becomes apparent that buyer banks’ lending shrinks as the government’s necessity to bid higher prices is less pronounced. This, in turn, lowers the crowding out effect of buyer banks’ incentive for liquidity hoarding resulting in reduced lending volumes.

Summarizing, I find that troubled asset purchases weakly dominate direct capital injections in terms of buyer bank’s lending which is even more pronounced in case of higher exposures to collateral runs. The same holds for illiquid bank’s solvency improvement considering the condition on the interplay between fire sale prices and seller bank’s equity. Moreover, tighter ex ante leverage requirements will affect ex post decisions about policy interventions conditional on an extended parameter range where direct capital injections and troubled asset purchases improve buyer bank’s lending and restore illiquid bank’s solvency equally effective. The model developed in this paper provides new insights for policymakers as well as financial authorities to implement commonly aligned regulatory strategies and policy interventions which should prevent or at least mitigate the economically harmful repercussions of modern financial crises.
Appendix A  Proofs

Proof of Lemma 1: Let us start with (i) and the slightly modified liquidity and solvency conditions

$$\begin{align*}
\alpha Z + (1 - \beta)\frac{Z}{2}(1 - P_{1A}^2) + \left[ \theta + (1 - H(P_0A(P_{1A})))C - I \left( \frac{1}{P_0A(P_{1A})} \right) \right] &= fD, \quad \text{(A.1)} \\
\alpha Z + (1 - \beta)\frac{Z}{2}(1 - L_1^2) + \left[ \theta + (1 - H(P_0B(P_{1B})))C - I \left( \frac{1}{P_0B(P_{1B})} \right) \right] &= fD, \quad \text{(A.2)}
\end{align*}$$

$$P_{1A}\beta Z - \left[ \theta + (1 - H(P_0A(P_{1A})))C - I \left( \frac{1}{P_0A(P_{1A})} \right) \right] + (1 - \beta)P_{1A}^2Z - (1 - f)DP_{1A} - gZP_{1A} \geq 0 \quad \text{(A.3)}$$

and

$$\left\{ P_{1B}Z(\beta - g) - \left[ \theta + (1 - H(P_0B(P_{1B})))C - I \left( \frac{1}{P_0B(P_{1B})} \right) \right] \right\} + (1 - \beta)P_{1B}L_1Z - (1 - f)DP_{1B} \geq 0 \quad \text{(A.4)}$$

for direct capital injection (A) and illiquid asset purchases (B) respectively. We find that

$$P_{1A} = P_{1B} \quad \text{(A.5)}$$

as long as

$$Z(\beta - g) > \frac{\theta + (1 - H(P_0A(P_{1A})))C - I \left( \frac{1}{P_0A(P_{1A})} \right)}{P_{1A}}$$

some illiquid assets are left after trading conditional on the date-1 liquidity shock. Based on (A.5),

$$\theta + (1 - H(P_0A(P_{1A})))C - I \left( \frac{1}{P_0A(P_{1A})} \right) = \theta + (1 - H(P_0B(P_{1B})))C - I \left( \frac{1}{P_0B(P_{1B})} \right)$$

buyer bank’s amount of disposable cash is the same for both interventions (A) and (B). Using this equality we find that

$$L_1 = P_{1A} = P_{1B} \quad \text{(A.6)}$$

conditional on (A.1), (A.2) and (A.5).

Focusing on (ii), all seller banks’ illiquid assets are exhausted after trading using the same amount $g$ either for direct capital injection or for buying illiquid assets, that is,

$$Z(\beta - g) < \frac{\theta + (1 - H(P_0A(P_{1A})))C - I \left( \frac{1}{P_0A(P_{1A})} \right)}{P_{1A}} \quad \text{(A.7)}$$

where it must hold that

$$Z(\beta - g) = \frac{\theta + (1 - H(P_0B(P_{1B})))C - I \left( \frac{1}{P_0B(P_{1B})} \right)}{P_{1B}} \quad \text{(A.8)}$$

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Based on (A.7) and (A.8) we find that

\[ P_{1A} < P_{1B} \]  

(A.9)

the price for illiquid assets is lower in case of direct capital injections compared to illiquid asset purchases. Moreover,

\[ \theta + (1 - H(P_0A(P_{1A})))C - I \left( \frac{1}{P_{0A}(P_{1A})} \right) > \theta + (1 - H(P_0B(P_{1B})))C - I \left( \frac{1}{P_{0B}(P_{1B})} \right) \]  

(A.10)

buyer banks’ disposable amount of cash is lower if government expenditures are used for illiquid asset purchases than for direct capital injections conditional on (A.9),

\[ \frac{\partial H(P_0(P_1))}{\partial P_1} \left( \frac{1}{P_0(P_1)} \right) C = H'(P_0(P_1)) \frac{\partial P_0(P_1)}{\partial P_1} C < 0 \]  

(A.11)

the general first order condition for the haircut function \( H(P_0(P_1)) \),

\[ \frac{\partial I \left( \frac{1}{P_0(P_1)} \right)}{\partial P_1} = I' \left( \frac{1}{P_0(P_1)} \right) \frac{\partial P_0(P_1)}{\partial P_1} \frac{\partial P_0(P_1)}{\partial P_0(P_1)} C < 0 \]  

(A.12)

the general first order condition for the lending function \( I \left( \frac{1}{P_0(P_1)} \right) \) and Assumption 1. Hence, finally we can state

\[ L_1 < P_{1A} < P_{1B} \]  

(A.13)

conditional on (A.1), (A.2) and (A.10).■

**Proof of Proposition 2**: The first bullet (i) is an immediate result of Lemma 1. Credit supply and solvency are equally affected independent of which policy intervention is used. Thus,

\[ I \left( \frac{1}{P_{0A}(P_{1A})} \right) = I \left( \frac{1}{P_{0B}(P_{1B})} \right) \]  

(A.14)

and

\[ P_{1A}\beta Z - \left[ \theta + (1 - H(P_0A(P_{1A})))C - I \left( \frac{1}{P_{0A}(P_{1A})} \right) \right] + (1 - \beta)P_{1A}^2 Z - (1 - f)DP_{1A} - gZP_{1A} \]

\[ = \left\{ P_{1B}Z(\beta - g) - \left[ \theta + (1 - H(P_0B(P_{1B})))C - I \left( \frac{1}{P_{0B}(P_{1B})} \right) \right] \right\} + (1 - \beta)P_{1B}L_1 Z - (1 - f)DP_{1B}. \]  

(A.15)

■
Focusing on the first part of (ii) we find
\[ I\left(\frac{1}{P_{0B}(P_{1B})}\right) > I\left(\frac{1}{P_{0A}(P_{1A})}\right) \]  
(A.16)
that term lending to industrials is higher if the government intervenes through illiquid asset purchases than by direct capital injections conditional on (A.9) and (A.12).

For the second part of (ii) it is necessary to determine the impact on \( P_{1B} \) (and \( P_{1A} \)) conditional on a change in the amount of government expenditure \( g \) (and \( \alpha \)) first. Total differentiation of (A.8) (and (A.1)) yields
\[ \frac{dP_{1B}}{dg} = \frac{P_{1B}Z}{Z(\beta - g) + \frac{\partial H(P_{0B}(P_{1B}))}{\partial P_{1B}}C + \frac{\partial I\left(\frac{1}{P_{0B}(P_{1B})}\right)}{\partial P_{1B}}} > 0 \]  
(A.17)
and
\[ \frac{dP_{1A}}{d\alpha} = \frac{-Z}{-\frac{\partial H(P_{0A}(P_{1A}))}{\partial P_{1A}}C - \frac{\partial I\left(\frac{1}{P_{0A}(P_{1A})}\right)}{\partial P_{1A}}} > 0 \]  
(A.18)
conditional on (A.11), (A.12) and Assumption 1. Based on (A.9), (A.12), (A.17), (A.18) and
\[ \frac{\partial^2 I\left(\frac{1}{P_{0B}(P_{1B})}\right)}{\partial P_{1B}^2} < 0 \]  
(A.19)
the general second order condition for the lending function, the second part of (ii) immediately follows.

To show the statement in (iii) we have to match the solvency condition for intervention (A) against (B). By simplifying and rearranging (A.3) and (A.4) we can formulate
\[ P_{1A}\left[\beta Z + (1 - \beta)\frac{Z}{2}P_{1A} - (1 - f)D - gZ\right] \geq P_{1B}\left[\beta Z + (1 - \beta)L_1Z - (1 - f)D - gZ\right] - (1 - \beta)\frac{Z}{2}L_1^2. \]  
(A.20)
Condition (A.20) is satisfied if
\[ \frac{P_{1B}}{P_{1A}} > \frac{x + (1 - \beta)\frac{Z}{2}\left(P_{1A} + L_1^2\right)}{x + (1 - \beta)L_1Z} \]  
(A.21)
the direct relative impact on prices prevails over the indirect relative impact on equity, where \( x = \beta Z - (1 - f)D - gZ. \)

**Proof of Proposition 3:** Let us start with (i) and the slightly modified equilibrium condition
\[ Z(\beta - g) = \frac{\theta + (1 - H(P_{0B}(P_{1B}), \delta))C - I\left(\frac{1}{P_{0B}(P_{1B})}\right)}{P_{1B}}. \]  
(A.22)
Holding \( P_{0B}(P_{1B}) \) constant, totally differentiating (A.22) yields

\[
\frac{dg}{d\delta} = -\frac{\frac{\partial H(P_{0B}(P_{1B}),\delta)}{\partial \delta} C}{-ZP_{1B}} > 0 \tag{A.23}
\]

where

\[
\frac{\partial H(P_{0B}(P_{1B}),\delta)}{\partial \delta} = -(1 - P_{0B}(P_{1B}))^{1-\delta} \ln (1 - P_{0B}(P_{1B})) > 0 \tag{A.24}
\]

conditional on the haircut function (2.3) and \( 0 < 1 - P_{0B}(P_{1B}) < 1 \). Hence, the parameter range for case (II) expands as the new equilibrium condition refers to a higher \( g \).

To show part (ii) we start our analyses with case (II). Let (j) be the case with a lower exposure to collateral runs compared to (k), that is, \( \delta_{(j)} < \delta_{(k)} \). In order to derive the impact on buyer bank’s lending conditional on a change in the exposure to collateral runs, we need to know how \( P_{1B} \) is affected by \( \delta \). Calculating the total derivative of (A.22) w.r.t \( \delta \) we find

\[
\frac{dP_{1B}}{d\delta} = \frac{-\frac{\partial H(P_{0B}(P_{1B}),\delta)}{\partial \delta} C}{Z(\beta - g) + \frac{\partial H(P_{0B}(P_{1B}),\delta)}{\partial P_{1B}} C + \frac{\partial I(1P_{0B}(P_{1B}))}{\partial P_{1B}}} < 0 \tag{A.25}
\]

conditional on (A.11), (A.12), (A.24) and Assumption 1. This implies that

\[
\left| \frac{dP_{1B}}{d\delta} \right|_{(k)} > \left| \frac{dP_{1B}}{d\delta} \right|_{(j)} \tag{A.26}
\]

and hence

\[
I \left( \frac{1}{P_{1B}} \right)_{(k)} < I \left( \frac{1}{P_{1B}} \right)_{(j)} \tag{A.27}
\]

the first partial statement of (ii) follows conditional on (A.12). The same obviously holds for intervention (A) as

\[
\frac{dP_{1A}}{d\delta} = \frac{\frac{\partial H(P_{0A}(P_{1A}),\delta)}{\partial \delta} C}{-(1 - \beta)P_{1AZ} - \frac{\partial H(P_{0A}(P_{1A}),\delta)}{\partial P_{1A}} C - \frac{\partial I(1P_{0A}(P_{1A}))}{\partial P_{1A}}} < 0 \tag{A.28}
\]

the total derivative of (A.1) is negative conditional on (A.11), (A.12), (A.24) and Assumption 1. Thus,

\[
\left| \frac{dP_{1A}}{d\delta} \right|_{(k)} > \left| \frac{dP_{1A}}{d\delta} \right|_{(j)} \tag{A.29}
\]

and hence

\[
I \left( \frac{1}{P_{1A}} \right)_{(k)} < I \left( \frac{1}{P_{1A}} \right)_{(j)} \tag{A.30}
\]

the second partial statement of (ii) follows conditional on (A.12) as well. Turning to case (I) we obviously find the same impacts for both interventions as derived in conditions (A.29) and (A.30) conditional on (A.14). This means that buyer bank’s lending is equally reduced for both
interventions (A) and (B) conditional on a higher exposure to collateral runs. Finally, in order to
derive the overall impact on the seller bank’s solvency condition affected by a change in the exposure
to collateral runs, we have to examine the effect on the loan liquidation price \( L_1 \). Based on the
slightly modified case (II) liquidity condition

\[
\alpha Z + (1 - \beta) \frac{Z}{2}(1 - L_1^2) + \left[ \theta + (1 - H(P_{0B}(P_{1B}), \delta))C - I \left( \frac{1}{P_{0B}(P_{1B})} \right) \right] = fD
\]

we find

\[
\frac{dL_1}{d\delta} = \frac{\partial H(P_{0B}(P_{1B}), \delta)}{\partial \delta} C - (1 - \beta) Z L_1 < 0
\]

(A.31)

that an increase in the exposure to collateral runs depresses the loan liquidation price \( L_1 \) conditional
on (A.24). Following the same intuition as in deriving (A.26) or (A.29) we can equally state

\[
\left| \frac{dL_1}{d\delta} \right| (k) > \left| \frac{dL_1}{d\delta} \right| (j)
\]

(A.32)

Using the RHS of (A.20) to specify the solvency condition \( SC_B(P_{1B}, L_1) \) regarding intervention B
we find

\[
\frac{dSC_B(P_{1B}, L_1)}{d\delta} = [\beta Z + (1 - \beta)L_1Z - (1 - f)D - gZ] \frac{dP_{1B}}{d\delta} + [(1 - \beta)Z(P_{1B} - L_1)] \frac{dL_1}{d\delta} < 0
\]

(A.33)

that the overall impact on seller bank’s solvency is negative conditional on an increase in the
exposure to collateral runs, considering (A.13), (A.25), (A.31) and seller bank’s date-2 solvency.
Combing conditions (A.26) and (A.32) it follows

\[
\left| \frac{dSC_B(P_{1B}, \delta)}{d\delta} \right| (k) > \left| \frac{dSC_B(P_{1B}, \delta)}{d\delta} \right| (j)
\]

(A.34)

that the deteriorating impact on seller bank’s solvency condition is more pronounced the higher the
exposure to collateral runs. Focusing on intervention (A) we define seller bank’s solvency \( SC_A(P_{1A}) \)
by the LHS of (A.20) and find

\[
\frac{dSC_A(P_{1A})}{d\delta} = [\beta Z + (1 - \beta)P_{1AZ} - (1 - f)D - gZ] \frac{dP_{1A}}{d\delta} < 0
\]

(A.35)

the same qualitative result as in (A.33) conditional on (A.28) and seller bank’s date-2 solvency.
Using (A.29) obviously follows

\[
\left| \frac{dSC_A(P_{1A})}{d\delta} \right| (k) > \left| \frac{dSC_A(P_{1A})}{d\delta} \right| (j)
\]

(A.36)

that the deteriorating impact on seller bank’s solvency condition is more pronounced the higher the
exposure to collateral runs – equal to intervention (B). Turning to case (I) we immediately find the
same impacts as derived in (A.36) for both interventions (A) and (B) conditional on (A.15).

(iii) In order to improve seller bank’s solvency (affected by asset prices) conditional on higher exposures to collateral runs, the government is forced to increase their expenditures even more compared to cases when collateral runs are less likely. As the qualitative results are the same for both cases (I) and (II) as well as for both interventions (A) and (B), we chose case (II) and intervention (B) for simplicity. Let (j) again be the case with a lower exposure to collateral runs compared to (k), that is, \( \delta(j) < \delta(k) \). Combining (A.17) and (A.25) we find

\[
\frac{dP_{1B}}{dg} + \frac{dP_{1B}}{d\delta} < \frac{dP_{1B}}{dg}
\]

that the amount of government expenditure increases with higher exposure to collateral runs to counter price depressions. Hence,

\[
\left| \frac{dP_{1B}}{dg} \right|_{(k)} < \left| \frac{dP_{1B}}{dg} \right|_{(j)}
\]

conditional on (A.26) and (A.37) holding \( g \) constant.

Proof of Proposition 4: Let us start with (i) and

\[
Z(\beta - g) > \frac{fD - (1 - \beta)Z^2(1 - P_{1A}^2) - gZ}{P_{1A}}
\]

the necessary condition for case (I) conditional on (A.1) and seller bank’s asset endowment \( \alpha Z + \beta Z + (1 - \beta)Z = Z + gZ \). Bullet (i) immediately follows using (A.39), as the LHS is increasing and the RHS is decreasing in the amount of seller bank’s asset net worth \( Z \).

Focusing on the first part of (ii), we need to know the impact on \( P_{1A} \) triggered by an increase in the amount of assets’ net worth \( Z \) first. Total differentiation of (A.1) yields

\[
\frac{dP_{1A}}{dZ} = \frac{-\alpha - \frac{1}{2}(1 - \beta)(1 - P_{1A}^2)}{-(1 - \beta)P_{1A}Z - \frac{\partial H(P_{0A}(P_{1A}))}{\partial P_{1A}}C - \frac{\partial I(P_{0A}(P_{1A}))}{\partial P_{1A}}C - \frac{\partial I}{\partial P_{1A}}P_{1A}^2} > 0
\]

conditional on (A.11), (A.12) and Assumption 1. Hence, buyer bank’s lending volume increases considering (A.12). Using the same approach applied to the equilibrium condition (A.8) for the second part of (ii) we find

\[
\frac{dP_{1B}}{dZ} = \left[ -\frac{\partial H(P_{0B}(P_{1B}))}{\partial P_{1B}}C - \frac{\partial I(P_{0B}(P_{1B}))C - I(P_{0B}(P_{1B}))}{\partial P_{1B}} \right] P_{1B} \frac{P_{1B}^2(\beta - g)}{(\theta + (1 - H(P_{0B}(P_{1B})))C - I(P_{0B}(P_{1B})))} < 0
\]

that tighter leverage ratio requirements implemented by higher valued assets depresses date-1 fire sale prices conditional on (A.11), (A.12) and Assumption 1. Thus, buyer bank’s amount of lending decreases considering (A.12).
(iii) By defining seller bank’s solvency $SC_A(P_{1A})$ for case (I) using the LHS of condition (A.20) we find
\[
\frac{\partial SC_A(P_{1A})}{\partial P_{1A}} = \beta Z + (1 - \beta) P_{1A} Z - (1 - f) D - gZ > 0 \quad \text{(A.42)}
\]
that seller bank’s solvency condition is improving knowing that illiquid asset prices are increasing conditional on (A.40) as long as they are solvent in the long run. For case (II) it is necessary to examine how changes in date-1 fire sale prices $P_{1B}$ will affect loan liquidation prices $L_1$. Thus, total differentiation of (A.2) yields
\[
\frac{dL_1}{dP_{1B}} = \frac{\partial H(P_{0B}(P_{1B}))}{\partial P_{1B}} C + \frac{\partial I(1/P_{0B}(P_{1B}))}{\partial P_{1B}} - (1 - \beta) L_1 Z < 0, \quad \text{(A.43)}
\]
conditional on (A.11), (A.12) and Assumption 1. Using this and the RHS of (A.20) to define seller bank’s solvency $SC_B(P_{1B}, L_1)$ we find
\[
dSC_B(P_{1B}, L_1) = [\beta Z + (1 - \beta) L_1 Z - (1 - f) D - gZ] dP_{1B} + [(1 - \beta) Z(P_{1B} - L_1)] dL_1 \quad \text{(A.44)}
\]
which is positive as long as
\[
\left| \frac{\partial SC_B(P_{1B}, L_1)}{\partial L_1} dL_1 \right| > \left| \frac{\partial SC_B(P_{1B}, L_1)}{\partial P_{1B}} dP_{1B} \right|
\]
the impact on liquidation prices prevails over the impact on date-1 fire sale prices conditional on (A.41), (A.43), Lemma 1 and the date-2 solvency assumption. ■
References


