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PIGOU'S MACROECONOMICS OF UNEMPLOYMENT (1933).
A SIMPLE MODEL*

HANSJÖRG KLAUSINGER

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Address of the Author:
Hansjörg Klausinger
Institute of Economic Theory and Policy
Vienna University of Economics and Business Administration
Augasse 2-6
A-1090 Vienna
Austria

1. Introduction

Recently there has been a renaissance of interest in the macroeconomics of Pigou. Once considered, thanks to Keynes's classification, as "the classical economist", Pigou was shown to have favoured moderately expansionist policies in the early 1930s rather similar to those advocated by Keynes himself. Furthermore, Keynes's "inconsistency thesis", according to which although Pigou's practical policy recommendations demonstrated wise judgement they could not be deduced from his theoretical account of unemployment, has come under attack. Thus emphasis has shifted to re-analysing Pigou's "Theory of Unemployment" (1933). Yet, to my knowledge, up to now there does not exist a formal model of Pigovian macroeconomics. In the following such a model is reconstructed in order to reexamine the logical consistency of Pigou's verbal results. Thereby special emphasis is laid on the evaluation of the employment effects of "public works" or similar policies towards unemployment, thus reconsidering Keynes's inconsistency thesis, too.

The next section at first states the model of a real economy. Then monetary factors are introduced, distinguishing between two types of monetary regimes and between rigid real and rigid money wages. For each of these different versions of the model comparative-static results are derived and then confronted with textual evidence from Pigou's TU. In the third section some aspects of Pigou's theory are critically commented upon in light of the formal model. A short conclusion follows.

2. Pigou's (1933) Macroeconomic Model of Unemployment

Pigou's mode of presenting his theory of unemployment is partially mathematical - some elements and relations between them are stated in formal terms - yet there is neither a consistent representation of the model as a whole nor are the results formally derived. Nevertheless the analysis is of that neoclassical kind, which even when stated in verbal terms lends itself easily to a formal description, and indeed in his contributions to the debate of Keynes's General Theory

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1 Hutchison (1976, ch. 6) has revised the conventional wisdom about Pigou's policy proposals, cf. also from a more Keynesian perspective Clarke (1988, ch. 8) and Dimand (1988, ch. 3). On Pigou's theory of unemployment see e.g. Collard (1981) and Solow (1980), and for a refutation of the inconsistency thesis Aslanbeigui (1992).

2 In the following "The Theory of Unemployment" (1933) is cited as TU.
Pigou (e.g. 1937, 1943) provided a formal model of his macroeconomics. Therefore the attempt is legitimate to capture the relevant relations of Pigou's macroeconomic analysis in a simple model. It should be noted that by concentrating on the macroeconomic aspect of the analysis a rather large part of Pigou's work is disregarded.  

The structure of the model is as follows: The model consists of a real and a monetary subsystem. The real subsystem is composed of three markets, for wage-goods, for non-wage-goods, and for labour, respectively. The monetary subsystem is analysed within a quantity theoretic framework. Within this model different versions are examined: After studying a real economy model, two kinds of monetary regimes (or "systems") are introduced, i.e. a "standard" and a "non-standard monetary system", as defined by Pigou. Finally, the analysis is carried over from the assumption of rigid (or fixed) real wages to rigid money wages.

2.1 The Real Economy Model

Pigou's real economy model (i.e. the real subsystem of the full model) consists of the following equations (for a glossary of symbols see the appendix):

\[
\begin{align*}
(1) \quad c_w &= vz + a = f(x), \quad f' > 0, f'' < 0, \\
(2) \quad f'(x) &= v, \\
(3) \quad c_n &= h(q) + b = g(y), \quad h' < 0, g' > 0, g'' < 0, \\
(4) \quad g'(y) &= rv, \\
(5) \quad Pq/P_n &= i, \\
(6) \quad v &= v_0.
\end{align*}
\]

Equations (1) and (3) are the equilibrium conditions of the market for wage-goods and for non-wage-goods, respectively, (2) and (4) are the profit maximising conditions for employment in these sectors. In particular, the demand for wage-goods, as specified in (1), conforms to a "classical" consumption function, where wage-earners spend their whole income \(vz\) on wage-goods; the demand for wage-goods by non-wage-earners is exogenous: \(a = a_0\). This is compatible with

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3 Some microeconomic aspects of TU are reviewed by Solow (1980).
Pigou's analysis: The "classical" hypothesis is entertained throughout the book (e.g. explicitly TU, 155); furthermore he considers non-wage-earners' demand for wage-goods to be "only a small proportion", "presumably inelastic" and therefore "highly stable" (TU, 146f.), so that the formulation in (1) constitutes a case covered by Pigou's theory. The condition (2) is explicitly stated by Pigou: "... the quantity of labour demanded ... is such that the value in terms of wage-goods of its marginal net product ... approximates to that rate of wage" (TU, 41). Non-wage-goods are on the whole identified by Pigou with investment goods and therefore demand depends in a familiar way on the rate of return of investment and the rate of interest, respectively. This is represented in (3) where the demand for investment $h$ determines the rate of return $q$. The exogenous component $b = b_0$ represents government demand for non-wage-goods as well as the effects on private demand by waves of optimism (or pessimism). Equation (5) stipulates the condition that rates of return for investment in real and financial capital must be equal. As the yield of investment is taken to be "real" in terms of a composite good consisting of wage- and non-wage-goods, it will be positively affected by an increase of the relative price of wage-goods.\(^4\) These characteristics of the investment goods market are only implicit in Pigou's analysis but typical for contemporary (monetary) business cycle theory, and it is well established that they are also part of Pigou's overall approach.\(^5\) There remains the labour market. Here again recent research has confirmed that Pigou did not assume full employment (as Keynes [1936, app. to ch. 19] maintains in his critique) but examined a case of fixed real wages, as in (6), where the real wage is defined by Pigou as the wage in terms of wage-goods. Employment is then determined by labour demand from (2) and (4).\(^6\)

The real (part of the) model is made up of the five equations (1) - (5) which determine the five endogenous variables $x, y, r, i$ and $q$. The real wage $v$ and the two demand variables $a$ and $b$ are exogenous. Besides the labour market and the market for wage-goods and for non-wage-goods there is a capital market (or market for loanable funds) implicit in the model. Therefore because of

\(^4\) Some calculation proves that the qualitative results do not change much when the expected yield is specified otherwise, e.g. weighed by a different price index.

\(^5\) Pigou dealt with monetary aspects more thoroughly in Pigou (1929); cf. also Aslanbeigui (1992, 423ff.).

\(^6\) Keynes's critique of Pigou's labour market analysis has been refuted by Collard (1981) and Aslanbeigui (1992, 418ff.).
Walras' law variations in the exogenous demands, \( \alpha \) or \( \beta \), imply equal variations in the (excess) supply of loanable funds, i.e. an increase in \( \alpha \) means a decrease in planned saving by non-wage-earners and similarly for \( \beta \).

Now the "reduced forms" for the real economy model can be derived.\(^7\) An equilibrium where for the sake of simplicity: \( \bar{\alpha} = \bar{\beta} = 0 \), is the point of departure. The model has a recursive structure, so that \( x, y, r \) and \( i \) can be determined in this order. First, totally differentiating (2): \( f''dx = dv \), which implies:

\[
\begin{align*}
(R1) \quad x &= x^R(v); \quad x^R_v = 1/f'' < 0, \\
where for sake of brevity reduced form partial derivatives are denoted by subscripts, e.g. \\
\frac{\partial x^R}{\partial v} &= x^R_v. \quad Then totally differentiating (1): \quad dc_w = v(dx + dy) + zdv + da = f'dx = vdx, \quad and substituting from (2) leads to: \quad vdy = -zdv - da, \quad and therefore:
\end{align*}
\]

\[
(R2) \quad y = y^R(v, a), \quad y^R_v = -z/v < 0, \quad y^R_a = -1/v < 0.
\]

Again totally differentiating (4) and substituting from (R2) leads to:

\[
(R3) \quad r = r^R(v, a), \quad r^R_v = -(rv + g''z)/v^2 = ?, \quad r^R_a = -g''/v^2 > 0.
\]

After some manipulation a sensible condition for determining the sign of \( r^R_v \) can be found. Note that the (structural form) elasticity of employment in the non-wage-goods sector \( y \) with regard to its product wage \( rv \) is given by \( \epsilon_{rv} = \frac{\partial y}{\partial (rv)} y = \frac{rv}{g''y} < 0 \); substituted into (R3) this renders the condition:

\[
(A1) \quad r^R_v = -\frac{r}{v} \left( \frac{1}{\epsilon_{rv} y} + 1 \right) \geq 0 \iff -\epsilon_{rv} \leq \frac{z}{y}.
\]

Condition (A1) is not implausible because of \( z/y > 1 \) (Pigou's [TU, 92] estimate for the ratio \( x/z = 3/4 \), so that \( z/y = 4 \)). In the following the qualitative results will always be given for the general case first, and then (in parentheses) for the case when condition (A1) holds.\(^8\)

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7 The solutions of the different versions of the model are distinguished by superscripts, e.g. "R" for the real economy model; see the appendix for a glossary.

8 The economic intuition behind (A1) is as follows: Starting from equation (4), if \( v \) increases, it is known from (R2) that \( y \) will fall and therefore \( g' \) rise. The reaction of \( r \) depends on which of these effects dominates. If the demand for labour in the non-wage-goods sector is not "too elastic" with regard to its product wage, then (A1) will be fulfilled and \( r \) will rise.
Next the solution for \( i \). First solving (5) for \( q \) leads to: 
\[
 dq = di - (1 - \lambda) q dr ,
\]
where for simplicity now and in the following: \( \hat{r} = 1 \); then substituting into (3) and again making use of (R2) gives:
\[
\begin{align*}
 i &= i^R(v, a, b), \\
 i^R_v &= (1 - \lambda) i^R - (z/h') = ? (> 0), \\
 i^R_a &= (1 - \lambda) i^R_a - (1/h') > 0, \\
 i^R_e &= -1/h' > 0.
\end{align*}
\]

At last the results for total employment, \( z = x + y \), follow trivially from (R1) and (R2):
\[
\begin{align*}
 z^R_v &= x^R_v + y^R_v < 0, \\
 z^R_a &= y^R_a < 0, \\
 z^R_e &= 0.
\end{align*}
\]

For convenience the qualitative results of the real economy model are summarised in Table 1.9

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<th>Table 1: The Real Economy</th>
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These results shall now be interpreted and confronted with textual evidence from TU.

The question of the "elasticity of demand in terms of wage-goods for labour as a whole" (TU, 73ff. and 88ff.) is central to Pigou's analysis: In the wage-goods sector a lower real wage directly increases employment and output. While the increase in demand induced by additional employment in the wage-goods sector just equals the increased output of wage-goods: 
\[
v dx - f'dx = 0,
\]
because of the lower real wage the demand for wage-goods per hour employed must fall: 
\[
z dv < 0.
\]
So there is a surplus of production over demand for wage-goods left which makes an increased employment in the non-wage-goods sector possible: 
\[
v dy > 0,
\]
so that \( y^R_v = -z/v \), as above. Combining the above results for \( x \) and \( y \) and defining the real wage elasticities of employment accordingly gives for \( e_{x,v} \):
\[
e_{x,v} = z^R_v + y^R_v \frac{v}{z} = e_{x,x} \frac{x}{z} + e_{x,y} \frac{y}{z} = e_{x,v} \frac{x}{z} - 1,
\]

9 In the following tables dependent variables are arranged in rows and independent variables in columns so that the respective fields show the signs of the comparative-static effects. These signs are valid for the general case; only when they are indeterminate in the general case, the signs will be given (in parentheses) for the case of condition (A1).
where \( e_{x,v} = \frac{1}{f''} \frac{v}{x} \), \( e_{y,v} = \frac{y}{v} \). This is exactly the value of the elasticity that Pigou derives for the special case where, as in the model above, "non-wage-earners have an absolutely rigid desire for wage-goods" (TU, 92). This value is the upper limit (numerically) for this elasticity. If non-wage-earners' demand for wage-goods were more elastic, e.g. by making it depend on profits, then the elasticity would be (numerically) lower (cf. TU, 92f).

Another question which Pigou attempts to answer is that of the interdependence between employment in the two sectors. The above model implies that, firstly, an exogenous increase in the demand for wage-goods \((da > 0)\) will decrease total employment, and secondly, that an exogenous increase in the demand for non-wage-goods \((db > 0)\), for instance "road-making" as part of a "public works" programme, will leave total employment unchanged. Pigou's statements fully comply with these results. First, for the sake of argument taking an employment effect from such "road making" in the non-wage-goods sector for granted,\(^{10}\) Pigou denies (for the real economy model) the possibility of "secondary employment" in the wage-goods sector (i.e. a multiplier effect of the Kahn-Keynes-type): "This argument, in the present connection, is invalid. When the real - not the money - rate of wages ruling in the wage-good industries is given, the quantity of labour demanded in these industries is determined ..." (TU, 75, emphasis in the original; cf. also 143f.). Secondly, even the primary employment effect in the non-wage-goods sector from public works is questionable. As for a given real wage employment in the wage-goods sector is fixed, such a primary effect presupposes that wage-goods can be transferred to the non-wage-goods sector from other uses (TU, 144ff.). Pigou mentions "the personal consumption of wage-goods by non-wage-earners, ... the storage of wage-goods, ... the purchase by non-wage-earners of non-wage-goods ... from abroad" (TU 146) as possible sources, none of which he considers as important. In the simple model none of these sources is available, so that the primary effect on employment is nil. Finally, the real economy model implies that an exogenous shift of demand from wage-goods to non-wage-goods \((da = -db < 0)\) increases total employment. Taking into account the different spending habits of wage-earners and non-wage-earners this provides the foundation

\(^{10}\) Obviously it is not true that "Professor Pigou accepts ... apparently the possibility of increased primary employment" (Keynes, 1936, 277).
for Pigou's strange (if only hypothetical) suggestion to promote employment by transfers of income from the former to the latter.\textsuperscript{11} However, it is crucial, even if difficult, to realise that these results are predicated upon the specific real economy model used, a model from which Pigou attempts to proceed in the course of his book to other more realistic ones.

Summarising the results from an exegetic point of view, the (real economy) model has performed well in reproducing Pigou's results.

2.2. The Monetary Subsystem

Monetary factors are introduced by Pigou at a rather late stage.\textsuperscript{12} The monetary subsystem of the model implicit in TU can be represented by the following two rather trivial equations:

\begin{align}
Y &= P_x (f + g/r), \\
Y &= MV(i), \quad V' > 0.
\end{align}

Equation (7) defines money income, (8) specifies the quantity theoretic framework, here for convenience formulated in terms of the equation of exchange. However, there is no explicit reference in TU to an interest sensitive income velocity of money, and it is therefore not certain if the simple model does not here attribute more coherence to Pigou's analysis than it really contains.\textsuperscript{13} The description of the money market has to be supplemented by assumptions on monetary policy or, as Pigou puts it, on the "monetary system". Pigou distinguishes between "a certain imaginary monetary system", which he calls "the standard system", and "systems of the general type of those that actually rule in the modern world" (TU, 187), i.e. the "non-standard" or "actual system".

\textsuperscript{11} "All gratuitous payments to poor people and all social services, in so far as they are financed at the expense of the richer non-wage-earning classes ... of necessity reduce pro tanto the quantity of labour demanded at a given real wage-rate." (TU, 155f.) Conversely total labour demanded will be increased by "contracting pensions and unemployment pay and remitting equivalent taxation on the well-to-do" (TU, 156).

\textsuperscript{12} Monetary factors are analysed in TU in Part II, ch. X (100-106) dealing with the elasticity of labour demand with regard to the money wage; in Part IV (185-243) on the assumption of rigid real wages, and in Part V, ch. IX (293-297) where at last rigid money wages are introduced.

\textsuperscript{13} The relation between Cambridge and Keynesian monetary thought is discussed by Patinkin (1982, ch. 6) and Laidler (1991, 60ff. and 86n.9), who do not find evidence for a systematic analysis of the interest rate as a determinant of velocity in e.g. Marshall and Pigou before 1930. However, an interest sensitive velocity is explicitly formulated in Pigou (1937, 409f.) when commenting on Keynes's \textit{General Theory}.  

The standard monetary system is defined "as one so constructed that, for all sorts of movements in the real demand function for labour or in real rates of wages ..., the aggregate money income is increased or diminished by precisely the difference made to the number of workpeople (or other factors of production) at work multiplied by the original rate of money wages" (TU, 205f.) With a standard system the price level should be approximately stable when technical efficiency is constant, and with technical progress it should fall roughly in proportion to the increase in productivity (cf. TU, 206ff.). Furthermore, employment tends to be more stable in the face of disturbances than with non-standard systems (cf. TU, 210). These claims will be examined below, when considering the full model.

According to the definition above the standard system can be specified as a reaction function targeting money income:

(9S) \[ Y = Y^S(z), \quad Y^S = W > 0. \]

From actual (non-standard) monetary systems the most simple one is chosen where the quantity of money is held constant:

(9N) \[ M = M_0. \]

Collecting equations and variables, the standard system consists of the three equations (7), (8) and (9S) with the three endogenous variables \( P_w, Y \) and \( M \), whereas in the non-standard system \( M \) is exogenous by (9N) so that two equations and two variables remain. Both types of monetary systems can now be solved for quasi-reduced forms with the variables of the real subsystem taken as exogenous.

Turning first to the standard system, the solution for \( P_w \) is obtained by setting (7) equal to (9S), again putting \( \bar{r} = 1 \):

\[ dY = P_w \left( f' dx + g' dy - g d\bar{r} \right) + \left( YdP_w / P_w \right) = W(dx + dy), \]

and after cancelling terms:

(S1) \[ P_w = P_w^S(\bar{r}), \quad P_w^S = \left( P_w g / Y \right) P_w = \lambda P_w > 0. \]

---

14 Similar rules of a secularly falling price level were favoured, as Robbins (1934, 20) puts it, by "the majority of economists of repute ... [i.e.] Marshall, Edgeworth, Taussig, Hawtrey, Robertson, Pigou".
The solution (S2) for $Y$ is, of course, given by (9S) and need not be repeated here. At last from the quantity equation (8) the solution for $M$ as an endogenous variable follows:

\[(S3) \quad M = M^S(x, y, i); \quad M^S_x = M^S_y = W/V > 0, \quad M^S_i = -e_{v,i} M / i < 0,\]

where $e_{v,i} = V'/V > 0$.

For the non-standard monetary system, the solution for $P_w$ follows from setting (7) equal to (8):

\[(N1) \quad P_w = P^N_w(x, y, r, i, M); \quad P^N_w_x = W - WP_w / Y < 0, \quad P^N_w_r = (P_w g / Y) P_w = \lambda P_w > 0, \quad P^N_w_i = Y = e_{v,i} P_w / i > 0, \quad P^N_w_M = P_w / M > 0.\]

And finally for $Y$ from (8):

\[(N2) \quad Y = Y^N(i, M); \quad Y^N_x = MV' = e_{v,x} Y / i > 0, \quad Y^N_M = Y / M > 0.\]

2.3 Rigid Real Wages with a Standard and Non-Standard Monetary System

For a monetary economy the real and the monetary subsystem must be combined to obtain the solutions for the full model. In this subsection the assumption of fixed real wages (6) is retained and its consequences are examined for both types of monetary system. Looking at the results above, it is recognised that there is still a recursive relation between real and monetary variables: The real variables are determined in the real subsystem independent of monetary factors whereas the ultimate exogenous variables affect the monetary subsystem directly and indirectly (i.e. through the real subsystem). Therefore the results for the real variables: $x, y, r, i$ and $z$, are identical with those of the real economy model, i.e. are given by (R1) - (R5) as in Table 1. Then the results for the monetary variables are obtained by substituting (R1) - (R4) into (S1) - (S3) and into (N1) and (N2) respectively.\(^{15}\)

The qualitative results for the standard monetary system are given in Table 2 and for the non-standard one in Table 3.\(^{16}\)

---

\(^{15}\) In this model a divergence between the proper and the actual rate of interest (in the standard and non-standard system respectively) is therefore impossible as long as real wages are fixed.

\(^{16}\) In the following only the qualitative results are reported in the text. For the calculation of full results see the appendix.
As predicted by Pigou, in the standard system the price level is stabilised in the face of disturbances by skilfully controlling monetary policy according to the rules laid out in (9S). Evidently positive shocks to the demand for wage-goods or for non-wage-goods have to be counteracted by monetary restriction.\textsuperscript{17}

While the quantity of money is endogenous in the standard system, it is exogenously fixed in the non-standard-system. Therefore the price level can no longer be kept stable in the face of the above mentioned demand shocks; furthermore there is now the additional possibility of monetary shocks. Thus in contrast to the standard system, the non-standard one is prone to the (potentially destabilising) effects of changes in the general price level (as analysed in TU, Pt. IV, chs. IX-XII).

### 2.4 Rigid Money Wages with a Standard and Non-Standard Monetary System

In most parts of the TU Pigou examines the economy under the assumption of fixed real wages. Only near the end of the book (on page 293 of 313 pages) Pigou refers to the fact that those "factors of inertia, which, in an economy where wage-rates were always contracted for in kind, would tend to keep real wages stable in the face of changing demand, in a money economy tend to keep money wages stable" (TU, 294). Although it is in such an economy with rigid money wages that the distinction between monetary systems becomes most relevant, Pigou restricts the analysis of it to a single chapter of less than five pages, concluding with the thesis that with rigid

\textsuperscript{17} As the interest rate is still determined within the real subsystem, the quantity of money, and not the rate of interest, is the relevant monetary instrument. Cf. below.
money wages "the volume of employment is substantially more variable than it would be" with rigid real wages (TU, 296). Therefore it is the task of this model to conclude this unfinished business and to look more carefully than Pigou did whether the substitution of rigid money for rigid real wages makes a qualitative difference to the results already derived.

As a first consequence of money wage rigidity, equation (6) of the real subsystem must be replaced by two new equations (6a) and (6b), which define the real wage and state the exogeneity of the money wage:

\[(6a) \quad v = \frac{W}{P_w},\]

\[(6b) \quad W = W_0.\]

By this modification the model loses its recursive structure. To find the solution it is now necessary to work back from (6a, b) to (R1) - (R4) and (S1) - (S3) or (N1) and (N2).

The qualitative results for the standard monetary system are again summarised in Table 4:

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<th>$a$</th>
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<tbody>
<tr>
<td>$P_w$</td>
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<td>+</td>
<td>0</td>
</tr>
<tr>
<td>$v$</td>
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<tr>
<td>$x$</td>
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The main results turn out as expected. The real wage $v$ responds positively to changes in the money wage $W$ and negatively - for the money wage given - to increases in the demand for wage goods $a$. Therefore employment in the wage-goods sector $x$ is (no longer independent but) positively related to demand - and employment in the non-wage-goods sector $y$ is negatively related.

Most relevant are, of course, the effects on total employment $z$, which are analysed in detail below:
\[ z_{W}^{MS} = x_{W}^{MS} + y_{W}^{MS} = z_{v}^{R} \frac{v}{W} < 0, \]
\[ z_{a}^{MS} = x_{a}^{MS} + y_{a}^{MS} = -\frac{1}{v}\Delta(1 - \lambda - \lambda \frac{g''}{f''}) = -\frac{1}{v}\Delta(1 - \lambda - \lambda \frac{e_{x,v}}{e_{y,rv}}) = ?, \]
\[ z_{b}^{MS} = 0. \]

where \( \Delta = 1 + \lambda w_{c}^{R} = (1 - \lambda) - \lambda (g''z/v) > 0 \) \((\Delta \geq 1 \text{ if } A1)\).

These results shall be confronted with those of Pigou. First, by looking at the employment effects of money wage variations, the elasticities with regard to real and money wages can be compared for the standard system. Obviously
\[ e_{W}^{MS} = \frac{z_{W}^{R}}{z} \frac{W}{W} = e_{v}^{R} \frac{1}{\Delta}, \]
and therefore: \( |e_{W}^{R}| \geq |e_{W}^{MS}| \Leftrightarrow \Delta \geq 1 \Leftrightarrow \lambda_{c}^{R} \geq 0; \) i.e. under assumption \((A1)\) the elasticity with regard to money wages within the standard system will be smaller than that of real wages, except in the case of \( \lambda_{c}^{R} = 0 \) when they will be equal. Now look at Pigou's result \((TU, 102f).\) He makes the relation between these two elasticities depend on the monetary policy target function: \( Y = \psi(z) \)
(in my symbols), where obviously for the standard system: \( \psi' = W. \) Inserting in Pigou's formula\(^{18}\) one arrives at \( e_{W}^{R} = e_{W}^{MS} \). So Pigou would conclude that in the standard system money wage variations have the same effects as real wage variations in the real economy. Yet it follows from the model that this is only true when (as Pigou implicitly does) induced effects on \( r \) are ignored.

The second result concerns shifts in the demand for wage-goods. With rigid money wages the sign of this effect is now indeterminate. As a point of reference it can be asked what the result will be when production conditions in both sectors are symmetric, i.e. when the product wage elasticities of employment are identical in both sectors and the respective shares in money income equal those in employment, i.e.:

\[ e_{x,v} = e_{y,rv}, \quad \frac{x}{y} = \frac{P_{w}f}{P_{w}g} = \frac{1 - \lambda}{\lambda}. \]

\(^{18}\) In effect the formula (in TU, 103) reads as follows: \( \frac{1}{E_{m}} = \frac{1}{E_{r}} - \frac{xF'(x)}{F(x)} + \frac{y\psi'(x)}{\psi(x)}, \) where \( E_{m} \) and \( E_{r} \) refer to the money and real wage elasticities, \( x \) is total employment (i.e. \( z \) in terms of the model) and \( F(x) \) is defined as "the value, in terms of wage-goods of the aggregate real output" \((TU, 102), \) i.e. in terms of the simple model:
\[ Y/P_{w} = f(x) + g(y)/r, \] and accordingly \( F' = v. \) Evidently Pigou's formulation makes only sense when \( r \) is held constant. Under this condition after inserting \( \psi' = W \) the two last terms cancel and the elasticities become equal.
In this case the demand effect vanishes: 
\[ z_{a}^{MS} = -\frac{1}{\nu\Lambda} \left(1 - \lambda - \frac{1 - \lambda}{\lambda}\right) = 0. \]

Again it must be noted that Pigou did not realise (or at least he did not clarify it for the reader) that the rather strange result of \( z_{a}^{r} < 0 \) for the real economy cannot be carried over to the monetary economy even with a standard system.

Finally considering the monetary variables, just as with rigid real wages the standard monetary system by suitable reactions of monetary policy stabilises the price level against demand shifts when money wages are rigid and even against variations in the money wage.

These results are now contrasted with those of the non-standard monetary system, which are summarised in Table 5.19

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As before, the reaction of total employment is most interesting:

\[ z_{a}^{MEV} = z_{a}^{r} \frac{v}{W\Lambda} < 0, \quad z_{a}^{MEV} = z_{a}^{r} v_{a}^{MEV} + y_{a}^{r} = -\frac{1}{\nu\Lambda} \left(1 - \mu - \frac{e_{xv}}{e_{yv}} \frac{x}{y} + \frac{v^{2}e_{yv}}{f_{hi}^{y}}\right) = ?, \]

where

\[ z_{b}^{MEV} = z_{b}^{r} v_{b}^{MEV} > 0, \quad z_{M}^{MEV} = z_{M}^{r} v_{M}^{MEV} > 0; \]

and \( \Lambda = 1 + v \left(\frac{W}{Y} z_{a}^{r} + \mu v_{r} - e_{yv} \frac{z}{f_{hi}^{y}}\right) = 1 + \mu v_{r} + \ldots = ?, \) and \( \mu = \lambda + \left(1 - \lambda\right) e_{yv} > \lambda > 0. \) In \( \Lambda \) only the term, \( \mu v_{r}^{r} \), is indeterminate; the remaining terms are positive. Therefore (A1) is sufficient for \( \Lambda > 1 > 0.20 \)

---

19 In Table 5 the condition \( \Lambda > 0 \) is assumed for the general case, which is weaker than (A1).
The sign of the effect of shifts in the demand for wage-goods is again indeterminate. However, when assuming symmetry (A2) as above, the result becomes definite:

$$z_{a}^{\lambda v} = \frac{1}{\nu \Lambda} \left( \frac{\lambda - \mu}{\lambda} - \frac{v^{2} e_{v,i}}{f' h' i} \right) = e_{v,i} \left( \frac{1- \lambda}{\lambda} + \frac{v^{2}}{f' h' i} \right) > 0.$$ 

Therefore, in the non-standard system the effect is plausibly positive. Furthermore, the effect of shifts in the demand for non-wage-goods is positive, too, so that there are definitely primary (and secondary) employment effects of e.g. deficit-financed public works. Again it must be noted that Pigou (at least) did not emphasise that the transition from the real economy model to one of rigid money wages with a standard and then with a non-standard system in the end may reverse the sign of those effects, so that the policy prescriptions valid for the real economy cannot be maintained.

Moreover, comparing the respective results proves that generally the effects of the real disturbances, $a$ and $b$, are smaller in the standard than in the non-standard system - which on the other hand means that in the latter case demand management (e.g. public works) is more effective.

Again assuming (A1) so that $\Lambda > \Lambda$, it turns out that the employment effect of money wage variations is greater in the standard than in the non-standard system.

Next the behaviour of the interest rate in the standard and in the non-standard system is examined.

The reactions as prescribed by the management of the standard system Pigou calls those of the "proper rate" (TU, 211). In the non-standard system the reactions of the actual interest rate diverge from those of the "proper rate". Because of these divergences the interest rate does not fully adjust to real disturbances (such as variations in $a$ or $b$) in the non-standard system, so that real disturbances are intensified and monetary ones are introduced (cf. TU, 210). The divergences can be calculated as follows:

20 Another sufficient condition for $\Lambda > 0$ is: (A3) $e_{v,i} < 1$. Then: $1 - \mu = \left( 1 - \lambda \right) \left( 1 - e_{v,i} \right) > 0$, and therefore:

$$1 + \mu v_R = \left( 1 - \mu \right) - \mu \frac{g'' \pi}{\nu} > 0.$$
\[ i_{w}^{\text{MS}} - i_{w}^{\text{MV}} = i_{w}^{R} \frac{1}{P_{w}} \left( \frac{1}{\Delta} - \frac{1}{\Lambda} \right) = \gamma (> 0), \]
\[ i_{a}^{\text{MS}} - i_{a}^{\text{MV}} = i_{a}^{R} \left( v_{a}^{\text{MS}} - v_{a}^{\text{MV}} \right) = i_{a}^{R} \left( \frac{1 - \lambda}{\Lambda} \right) \left( \frac{P_{w}}{Y} - \frac{e_{V_{d}}}{h' i} - \frac{e_{V_{d} G''}}{v'} \right) = \gamma (> 0), \]
\[ i_{b}^{\text{MS}} - i_{b}^{\text{MV}} = -i_{b}^{R} \frac{\varepsilon_{V_{d}}}{h' i} = ? (> 0). \]

Therefore when (A1) holds, then \( i_{w}^{R} > 0, \Lambda > \Delta \geq 1 \), and Pigou's conjectures are confirmed. Note also that, contrary to the non-standard system, in the standard system the rate of interest (and the other real variables) cannot be affected by monetary disturbances.

At last, by holding the quantity of money constant the price level is affected by disturbances in the non-standard system, whereas as before in the standard system the price level is kept stable by suitably adjusting monetary policy.

3. Comments and Criticism

Drawing on insights from the formal model, Pigou's approach is now critically evaluated with a view to its relation to what was later on described as classical and Keynesian macroeconomics.

3.1 Pigou's Model: A Case of "Barter Illusion"?

Pigou starts his analysis of unemployment by defending his decision to take a real economy model as point of departure: Although these phenomena can be studied "either from the money end or ... from the real end [, the] two studies, if made complete and carried through correctly, must necessarily come to the same thing" (TU, v). Nevertheless most critics have noticed, that the structure of the TU is ill-suited to make the reader realise that it is meant to apply to a monetary economy: The monetary system is introduced after 185 pages devoted to the real economy, the relevance of rigid money wages is mentioned only in two short chapters, and, most disturbingly, differences of crucial results between the real and the money economy are neglected.

The formal model has pointed at some of these neglected differences: First, under plausible assumptions a variation in demand for non-wage-goods has no effect on employment if either real wages are rigid or the monetary system is a standard one, whereas the effect is positive with rigid
money wages and a non-standard system. Second, the sign of the effect of a demand shift towards wage-goods is negative in the case of rigid real wages; it is (under conditions of symmetry) zero with rigid money wages and a standard system, and positive with a non-standard system. In such a case when the sign of the effect is reversed, it does not make good sense to start the analysis from the real end and give policy prescriptions predicated on that model, without even hinting at the fact that these will change drastically when applied to a monetary economy. With rigid money wages even a standard system cannot reproduce the results of the real economy in Pigou's analysis: They are similar inasmuch as money wage variations in the former model have the same comparative-static effects as real wage variations in the latter. Yet, the reactions to demand shifts are qualitatively different. As reconstructed by the simple model, the results of the real economy are therefore equivalent to those of a monetary economy when real wages are fixed, but qualitatively different when money wages are fixed irrespective of the monetary system.

So the criticism that Pigou's analysis suffers from "barter illusion"\(^\text{21}\) is more justified than e.g. conceded by Aslanbeigui (1992, 424f.), as Pigou did not completely carry through his analysis to a monetary economy as promised. However, it is true, that the results of the model, if taken as a reconstruction of "what Pigou should have said" on unemployment in a monetary economy, are not inconsistent with his activist policy proposals during the 1930s.

3.2 Some Strange Features of Pigou (1933)

Some other strange features, if not weaknesses, of Pigou's analysis shall be mentioned:

(1) A crucial cause for the strangeness, even to contemporaries, of some results of the model stems from Pigou's rigid distinction of wage- and non-wage-goods and from using a "classical" consumption function.\(^\text{22}\) From these assumptions derives the importance that Pigou attributes to a kind of "wages fund" that is uniquely determined by the ruling real wage. This in turn leads to the counterintuitive effects of "public works" and "transfers" in the real economy model.

\(^21\) Aslanbeigui (1992, 424) attributes this term to Dillard (1988).

\(^22\) The assumption of different saving behaviour of workers and capitalists was not uncommon in the contemporary business cycle literature, cf. Keynes's "widow's cruse" (Keynes, 1930a, 125) or Robertson (1934).
A feature of the formal model which Pigou neglected throughout his verbal analysis is the potential relevance of effects on the relative price of wage-goods \( r \) in terms of the model. The possibility of relative price effects is, to my knowledge, never mentioned in TU. When, for instance, Pigou traces the effects of a demand shift from wage- to non-wage-goods in an economy with rigid real wages and a standard monetary system, he correctly derives the (positive) effect on total employment. Yet then he concludes that: "the rate of interest, in terms of wage-goods - and everything else of relevance - remains unaltered ... money wage rates and prices remaining the same as before ..." (TU, 204f.), a statement which is crucially incomplete, as the formal model tells that the rate of interest and the relative price of wage-goods must change. Furthermore, neither is the sign of such relative price effects determinate nor, as assumption (A1) of the model makes clear, is it irrelevant.

Finally, it is not clear whether Pigou introduces the interest rate into the model as part of the real or of the monetary sector. In the simple model above the rate of return of investment is specified in real terms so that the condition of equal rates of return (5) belongs to the real sector. As long as real wages are fixed, the interest rate is therefore determined in the real sector which is prior (and dichotomised with respect) to the monetary one. Only when money wages are fixed, monetary factors will influence the rate of interest. It may be doubted if this reconstruction corresponds to Pigou's verbal analysis where he speaks of possible divergences of the interest rate from its proper value even in an economy with fixed real wages - a result impossible to reproduce in the model. Yet, Pigou's result can only be arrived at when the expected yield were defined in monetary terms, so that an absolute price level (and a definite quantity of money) would be needed to solve for the rate of interest. But this means plainly that it could not be defined in the real economy model, where it is dealt with by Pigou.

### 3.3 Pigou's Macroeconomics and Keynesian Policy Proposals

Pigou's book on unemployment, strange as this might have been in 1933, is primarily theoretical and statements about policy can be found only scattered throughout the text. Nevertheless, con-
sidering Keynes's inconsistency thesis it is legitimate to ask about the context of such prescrip-
tions and whether they follow from the formal model.

It is evident from what was said above that Keynesian policy conclusions cannot follow from the
real economy model, and Keynes was obviously correct when pointing at that inconsistency. Yet
it is possible to discover a consistent justification of such policies when looking at Pigou's inter-
pretation of the monetary system. The standard monetary system as conceived by Pigou is one
that (especially with fixed money wages) stabilises total employment and the price level. Yet, to
establish such a standard system in the face of cyclical disturbances, it is necessary to skilfully
control the interest rate, i.e. to set it at the value of the "proper rate". Maintenance of the standard
system is therefore not an exercise in laissez-faire but to the contrary a difficult task of monetary
policy. In fact, Pigou believes that it is not always "practicable" to maintain the standard system,
as "the actual rate of bank interest cannot fall below nil ... [whereas] the proper money rate ... may
... be a negative rate" (TU, 212f.), so that unaided bank rate policy must fail. In this case "a policy
of public works" is needed which "pushes up the proper rate" (TU, 213).23 (Of course, Pigou
cannot thereby be said to have anticipated Keynes's "liquidity trap" as a similar argument is
familiar from the "Treatise on Money" [Keynes, 1930b, 325ff.].) This means that as long as the
standard system can be maintained there is no need for fiscal policy, which moreover is (as the
model demonstrates) ineffective. Yet, when the economy cannot be stabilised within the standard
system, then fiscal policy is necessary and effective.

There is another and related argument for fiscal policy in TU. If the monetary system is non-stand-
dard, disturbances will cause a change in the general price level. Yet, "when prices have risen or
fallen ... that fact generates among business men an expectation that they will rise or fall further"
(TU, 241), thereby causing a fall or rise of the real rate of interest. A "cumulative and progressive
[process] of great importance" possibly sets in which would justify fiscal policy action: "... a small
injection of money into the income-expenditure circuit in bad times in connection with skilfully
chosen public works ... might lead to a progressive and far-reaching improvement in the
employment situation." (TU, 243) The effects analysed by Pigou are rather similar to those at

23 Patinkin (1976, 132n.; 1982, 168) has repeatedly pointed to this passage of Pigou's TU.
which Keynes (1936, ch. 19) points as possible causes for the ineffectiveness of money wage reductions in times of depression; however, that is a conclusion not explicitly drawn by Pigou.

Summarising, there can be found two kinds of arguments in TU for justifying activist fiscal policy: First, fiscal policy might be necessary to maintain the standard monetary system and a stable price level when monetary policy cannot fulfil this task alone. Second, if the standard system fails, price deflation might degenerate into a cumulative downward process that again only fiscal policy is able to stop.24

3.4 Rigid Money Wages and Unemployment

As can be seen from the model and from Pigou's analysis, rigid money wages in the face of disturbances to aggregate demand may represent a cause of unemployment as well as a justification for expansionist policy. In principle, Pigou does not consider either the wage rate or the state of demand as the sole cause of unemployment so that consequently neither wage reduction nor expansion of demand is the only possible remedy (cf. TU, 253). Rather the relevant factor is whether current wage rates form part of a "wage policy" conceived as a "permanent plan" that typically aims at some desired level of the real wage (ib.). From this point of view Pigou's models can be classified into those with rigid money wages as applying to the short run (relevant for the business cycle) and those with real wages determined by wage policy as applying to the long run.25

Whereas with "perfectly free competition ... and labour perfectly mobile ... there will always be a strong tendency for wage-rates to be so related to demand that everybody is employed" (TU, 252), "the goal at which wage policy aims is ... a wage-rate substantially higher than the rate which ... would yield nil unemployment" (TU, 253). Therefore with wage policy given, long-run government policies can have no effect on unemployment (cf. TU, 248).

Nevertheless, Pigou's long-run ineffectiveness thesis does not restrict activist government policy to the depression phase of the business cycle; in his view even the heavy unemployment in Great

24 A similar argument refers to the danger of "secondary deflation" as a justification for "pump priming", cf. Röpke (1933) and Robertson (1931, 61).

25 Of course, this presupposes that workers - presumably by changing the money wage - can also control the real wage, an assumption that Keynes (1936, ch. 2) vehemently contested.
Britain during the 1920s would have constituted an example for "a short-period malady, needing treatment only for a few difficult years" (TU, 250) - besides, a useful hint at the duration ("years") of the short-period.

In the context of the distinction between short and long run this means that rigid money wages might constitute a justification for activist policy in two respects: First, policy can be used as a remedy against the effects of temporary disturbances and against the danger that these effects might generate cumulative processes. And second, as can be concluded from Pigou's statement concerning the 1920s, policy could be used as a substitute for money wage reductions if the adjustments brought about by expansionist policy were not in conflict with wage policy - a qualification which, as many "classical economists" criticised, is crucially absent from Keynesian analysis. Anyway, it must be noticed that for Pigou money wage rigidity, due to some meta-economic reasons,\(^{26}\) did constitute a possible justification for activist policy.

4. Conclusion

Looked at anew in the light of this simple model, Pigou's macroeconomics of unemployment despite some strange features and deficiencies must be judged as providing a logically coherent framework of analysis not inconsistent with his practical policy recommendations in the 1930s. In effect, in his attempts at a synthesis of short run, i.e. business cycle, and long run analysis and by integrating the phenomenon of rigid money wages Pigou's analysis seems more similar to "Keynesian" than to "classical" macroeconomics as these were to be reconstructed by the so-called neoclassical synthesis. Perhaps the gap between Pigovian and Keynesian economics is smaller than that to the Economics of Keynes. But this is another story.

\(^{26}\) Keynes's policy prescriptions drawn from the "Treatise on Money" were also based on the argument of money wage rigidity due to meta-economic (social or political) forces, cf. Clarke (1988, 166f.).
Glossary of Symbols

\[\nu = \frac{W}{P_w}\] real wage (in terms of wage-goods)

\[W\] money wage

\[P_w\] price of wage-goods (w)

\[P_n\] price of non-wage-goods (n)

\[r = \frac{P_w}{P_n}\] relative price of wage-goods

\[c_w\] demand for wage-goods

\[a\] non-wage-earners' (autonomous) demand for wage-goods

\[c_n\] demand for non-wage-goods

\[b\] (autonomous) demand for non-wage-goods

\[h\] interest sensitive demand for non-wage-goods

\[f\] production (function) for wage-goods

\[g\] production (function) for non-wage-goods

\[x\] employment in the wage-goods sector

\[y\] employment in the non-wage-goods sector

\[z = x + y\] total employment

\[q\] expected yield of investment

\[i\] rate of interest

\[Y = P_w f + P_n g\] money income

\[P = (1 - \lambda)P_w + \lambda P_n\] price level

\[\lambda = \frac{P_n g}{Y}\] share of non-wage-goods in money income

\[M\] quantity of money

\[V\] income velocity of money

Subscripts:

\[e_{x,v}\] elasticity of x with regard to v

\[\partial s/\partial j = s_j\] partial derivative of s with regard to j

Superscripts are used for distinguishing solutions of different models:

\[R\] real economy model

\[S\] monetary model, standard system

\[N\] monetary model, non-standard system

\[RS\] full model with rigid real wages and standard system

\[RN\] full model with rigid real wages and non-standard system

\[MS\] full model with rigid money wages and standard system

\[MN\] full model with rigid money wages and non-standard system
References


Appendix: Calculation of Full Results

Derivation of (R4):

From (5) with $\bar{r} = 1$ follows: $dq = di - (1 - \lambda)qdr$; substituting into (3):

$$dc_n = h' dq + db = h'[di - (1 - \lambda)qdr] + db = h'di - h'(1 - \lambda)q(r_v dv + r_a da) + db =$$

$$= g'dy = rv\left(-\frac{z}{v} dv - \frac{1}{v} da\right) = -zd\nu - da$$

$$h'di = h'(1 - \lambda)q(r_v dv + r_a da) - zd\nu - da - db$$

Which easily leads to (R4).

Derivation of (RS1) - (RS4):

The full model with the *standard monetary system* is solved by substituting (R1) - (R4) into (S1) - (S3). This gives:

\begin{align*}
\text{(RS1)} & \quad Y = Y^S[v^R(v), y^R(v, a)] = Y^{RS}(v, a), \quad \frac{\partial Y^{RS}}{\partial v} = Wz_v^R < 0, \quad \frac{\partial Y^{RS}}{\partial a} = -P_w < 0. \\
\text{(RS2)} & \quad P_w = P_w^S[r^R(v, a)] = P_w^{RS}(v, a), \quad \frac{\partial P_w^{RS}}{\partial v} = \lambda P_w r_v^R = ? (\geq 0), \quad \frac{\partial P_w^{RS}}{\partial a} = \lambda P_w r_a^R > 0. \\
\text{(RS3)} & \quad M = M^S[x^R(v), y^R(v, a), i^R(v, a, b)] = M^{RS}(v, a, b); \\
& \quad \frac{\partial M^{RS}}{\partial v} = \frac{W}{V} z_v^R - e_{v,i} M \left[(1 - \lambda)r_v^R + \frac{v}{h'i} y_v^R\right] = ? (< 0), \\
& \quad \frac{\partial M^{RS}}{\partial a} = \frac{W}{V} y_a^R - e_{v,i} M \left[(1 - \lambda)r_a^R + \frac{v}{h'i} y_a^R\right] < 0, \quad \frac{\partial M^{RS}}{\partial b} = e_{v,i} M \frac{M}{h'i} < 0.
\end{align*}

Solving for the price level $P$: $dP = (1 - \lambda)dP_w + \lambda dP_v = dP_w - \lambda P_w dr$, so that:

\begin{align*}
\text{(RS4)} & \quad \frac{\partial P^{RS}}{\partial j} = \frac{\partial P_w^{RS}}{\partial j} - \lambda P_w \frac{\partial r^R}{\partial j} = 0, \quad j = v, a; \quad \frac{\partial P^{RS}}{\partial b} = 0.
\end{align*}

Derivation of (RN1) and (RN2):

By the same procedure as above the solutions for the *non-standard monetary system* can be found, now substituting (R1) - (R4) into (N1) and (N2), so that:
\[ Y = Y^R(v, a, b, M) = Y^R(v, a, b, M), \quad \frac{\partial Y^R}{\partial v} = M V^R_a > 0 \]

\[ \frac{\partial Y^R}{\partial a} = M V^R_a > 0, \quad \frac{\partial Y^R}{\partial b} = M V^R_b > 0, \quad \frac{\partial Y^R}{\partial M} = \frac{Y}{M} > 0. \]

\[ P_w = P^N_w(x^R(v), y^R(v, a), r^R(v, a), i^R(v, a, b), M) = P^R(v, a, b, M); \]

\[ \frac{\partial P^R}{\partial v} = P_w\left(-\frac{W P^R_y}{Y} r^R_v + \mu r^R_v + v \frac{\partial v^R}{\partial v'} \right) > 0, \quad \frac{\partial P^R}{\partial b} = -v^R_v h' > 0, \quad \frac{\partial P^R}{\partial M} = \frac{P_w}{M}. \]

where \( \mu = \lambda + (1 - \lambda) v^R_v > \lambda > 0. \)

**Derivation of (MS1) - (MS10):**

The modified model no longer possesses a recursive structure. To find the solution it is now necessary to work back from (6a, b) to (R1) - (R4) and (S1) - (S3). At first, (6a, b) is substituted into (S1):

\[ (MS1) \quad P_w = P^S_w\left(r^R\left[v^M(W, P_w), a\right]\right) = P^M_S(W, a), \]

which yields:

\[ dP_w = \lambda P_w\left[r^R_w\left(dW - v dP_w\right) + r^R_s da\right], \]

so that

\[ (0 \leq) \frac{\partial P^M_S}{\partial W} = \frac{\Delta - 1}{v \Delta} = \frac{1}{v} \left(1 - \frac{1}{\Delta}\right) < \frac{1}{v} = \frac{P_w}{W}, \quad \frac{\partial P^M_S}{\partial a} = \frac{AP_w r^R_s}{\Delta} > 0, \]

where \( \Delta = 1 + \lambda v^R_v = (1 - \lambda) - \lambda \frac{\partial v^R}{v} > 0. \) Evidently (A1) implies \( \Delta \geq 1. \)

Furthermore, substituting (MS1) into \( d\nu = \left(dW - v dP_w\right)/P_w, \) gives:

\[ \nu = v^M\left[W, P^M_S(W, a)\right] = v^M_S(W, a); \]

\[ \frac{\partial v^M_S}{\partial W} = \frac{1}{P_w} \left(1 - \nu \frac{\Delta - 1}{v \Delta}\right) = \frac{1}{P_w \Delta}\left(\Delta - \Delta + 1\right) = \frac{\nu}{\Delta}, \]

\[ \frac{\partial v^M_S}{\partial a} = -\frac{\lambda P_w r^R_s}{\Delta^2} = \frac{\nu}{\Delta} r^R_s. \]
\( x = x^R \left[ v^{MS} (W, a) \right] = x^{MS} (W, a), \quad \frac{\partial x^{MS}}{\partial W} = x^R \frac{\partial v^{MS}}{\partial W} < 0, \quad \frac{\partial x^{MS}}{\partial a} = x^R \frac{\partial v^{MS}}{\partial a} > 0. \)

\( y = y^R \left[ v^{MS} (W, a), a \right] = y^{MS} (W, a), \quad \frac{\partial y^{MS}}{\partial W} = y^R \frac{\partial v^{MS}}{\partial W} < 0, \quad \frac{\partial y^{MS}}{\partial a} = y^R \frac{\partial v^{MS}}{\partial a} + y^R_a = -\frac{1 - \lambda}{\nu \Delta} < 0. \)

\( r = r^R \left[ v^{MS} (W, a), a \right] = r^{MS} (W, a), \quad \frac{\partial r^{MS}}{\partial W} = r^R \frac{\partial v^{MS}}{\partial W} = \begin{cases} 0, \\
(\geq 0), \\
(\geq 0). 
\end{cases} \)

\( \frac{\partial r^{MS}}{\partial a} = r^R \frac{\partial v^{MS}}{\partial a} + r^R_a = \frac{1}{\Delta} r_a > 0. \)

\( \frac{\partial x^{MS}}{\partial W} = \frac{\partial x^{MS}}{\partial a} + \frac{\partial y^{MS}}{\partial a} = z_v^R \frac{\nu}{W \Delta} < 0, \quad \frac{\partial x^{MS}}{\partial b} = 0, \)

\( \frac{\partial y^{MS}}{\partial a} = \frac{\partial x^{MS}}{\partial a} + \frac{\partial y^{MS}}{\partial a} = -\frac{1}{\nu \Delta} \left( 1 - \lambda - \frac{\nu}{f''} \right) - \frac{1}{\nu \Delta} \left( 1 - \lambda - \frac{\nu}{f''} \right) e_{x,v} x \)?

\( i = i^R \left[ v^{MS} (W, a), a, b \right] = i^{MS} (W, a, b), \quad \frac{\partial i^{MS}}{\partial W} = i^R \frac{\partial v^{MS}}{\partial W} = \begin{cases} 0, \\
(> 0). 
\end{cases} \)

\( \frac{\partial i^{MS}}{\partial a} = i^R \frac{\partial v^{MS}}{\partial a} + i^R_a = \frac{1}{\Delta} \left[ \frac{\lambda v^2}{h'} \left( r^R a y_a^R - r^R a y_v^R \right) + \left( 1 - \lambda \right) r_a^R + \frac{\nu}{h'} y_a^R \right] = \frac{1}{\Delta} \frac{\nu}{h'} \left( 1 + \frac{g''}{\nu} \right) > 0, \quad \frac{\partial i^{MS}}{\partial b} = i^R_b = -\frac{1}{h'} > 0. \)

\( P^{MS} = P_w - \lambda P_w^r, \quad \frac{\partial P^{MS}}{\partial j} = \frac{\partial P^w}{\partial j} - \lambda P_w \frac{\partial P^{MS}}{\partial j} = 0, \quad j = W, a, b. \)

\( Y = Y^S \left[ x^{MS} (W, a), y^{MS} (W, a) \right] = Y^{MS} (W, a), \quad \frac{\partial Y^{MS}}{\partial W} = W \frac{\partial x^{MS}}{\partial W} < 0, \quad \frac{\partial Y^{MS}}{\partial a} = W \frac{\partial x^{MS}}{\partial a} = ? (= 0, \text{if A2}). \)

\( M = M^S \left[ x^{MS} (W, a), y^{MS} (W, a), i^{MS} (W, a, b) \right] = M^{MS} (W, a, b), \quad \frac{\partial M^{MS}}{\partial W} = \lambda \frac{\partial x^{MS}}{\partial W} < 0, \quad \frac{\partial M^{MS}}{\partial a} = \lambda \frac{\partial x^{MS}}{\partial a} = ? (= 0, \text{if A2}). \)

\( \frac{\partial M^{MS}}{\partial b} = -e_{v,j} M \frac{\partial x^{MS}}{\partial b} < 0, \quad \frac{\partial M^{MS}}{\partial a} = \frac{W}{V} \frac{\partial x^{MS}}{\partial a} - e_{v,j} M \frac{\partial x^{MS}}{\partial a} = ? (= 0, \text{if A2}). \)

Derivation of (MN1) - (MN7):

The procedure is the same as above, starting with the solution for the price of wage-goods. At first, (6a, b) is substituted into (N1):
\[ P_w = P_w^N \left\{ x^M (W, P_w), y^R (v^M (, a), r^R (v^M (, a), i^R [v^M (, a, b)], M) \right\} = P_w^MN (W, a, b, M), \]

which yields:

\[
dP_w = -\frac{WP_w}{Y} \left( x_v^R + y_v^R \right) \frac{1}{P_w} \left( dW - v dP_w \right) + \frac{1}{P_w} \left( dW - v dP_w \right) + r_a^R \frac{1}{P_w} \left( dW - v dP_w \right) + r_b^R \frac{1}{P_w} \left( dW - v dP_w \right) + \lambda P_w \left[ r_a^R \frac{1}{P_w} \left( dW - v dP_w \right) + r_b^R \frac{1}{P_w} \left( dW - v dP_w \right) + \right] + e_{v,j} \frac{P_w}{i} \left[ i_a^R \frac{1}{P_w} \left( dW - v dP_w \right) + i_b^R \frac{1}{P_w} \left( dW - v dP_w \right) + \right] + P_w \frac{i}{M} dM
\]

\[
dP_w \left[ 1 + v \left( \frac{W}{Y} z_v^R + \lambda z_v^R + \frac{e_{v,j}}{i} \right) \right] = \frac{dP_w}{\Lambda} \left[ 1 + v \left( \frac{W}{Y} z_v^R + \lambda z_v^R + \right) \right] = \frac{dP_w}{\Lambda} \left[ 1 + v \left( \frac{W}{Y} z_v^R + \lambda z_v^R + \right) \right] = \Lambda dP_w = \frac{\Lambda - 1}{\Lambda} dW + \left( -\frac{WP_w}{Y} y_a^R + \lambda P_w r_a^R + e_{v,j} \frac{P_w}{i} \right) + e_{v,j} \frac{P_w}{i} dM
\]

where \( \Lambda = 1 + v \left( \frac{W}{Y} z_v^R + \mu v^R - e_{v,j} \right) = 1 + \mu v^R + \ldots \), and \( \mu = \lambda + (1 - \lambda) e_{v,j} > \lambda > 0 \). In

\( \Lambda \) the term, \( \mu v^R \), is indeterminate, all the remaining terms are positive. Therefore (A1) is

sufficient for \( \Lambda > 1 > 0 \).

Obviously another sufficient condition for \( \Lambda > 0 \) is: \( e_{v,j} < 1 \) (A3). For then:

\( 1 - \mu = (1 - \lambda)(1 - e_{v,j}) > 0 \), and therefore: \( 1 + \mu v^R = (1 - \mu) - \mu \frac{g''}{V} > 0 \).

Thus, assuming \( \Lambda > 0 \):

\[
\frac{\partial P_w^N}{\partial W} = -\frac{\Lambda - 1}{\Lambda} \frac{1}{v} \left( 1 - \frac{1}{\Lambda} \right) < \frac{1}{v} \frac{P_w}{W}, \quad \frac{\partial P_w^N}{\partial a} = \frac{P_w}{\Lambda} \left( \frac{P_w - \mu g''}{v^2} \frac{e_{v,j}}{h''} \right) > 0
\]

\[
\frac{\partial P_w^N}{\partial b} = -\frac{P_w}{\Lambda} \frac{e_{v,j}}{h''} > 0, \quad \frac{\partial P_w^N}{\partial M} = \frac{P_w}{\Lambda} > 0, \quad \frac{\partial P_w^N}{\partial a} - \frac{\partial P_w^N}{\partial b} = \frac{P_w}{\Lambda} \left( \frac{P_w - \mu g''}{v^2} \right) > 0.
\]

By successive substitution:

\[ v = v^M \left[ W, P_w^N (W, a, b, M) \right] = v^MN (W, a, b, M); \]

(MN2) \[ \frac{\partial v^MN}{\partial W} = \frac{1}{P_w} \left( 1 - v \frac{\partial v^MN}{\partial W} \right) = \frac{v}{W \Lambda} > 0, \quad \frac{\partial v^MN}{\partial a} = -\frac{v}{P_w} \frac{\partial v^MN}{\partial a}, \quad j = a, b, M. \]

(MN3) \[ x = x^M \left[ v^MN (W, a, b, M) \right] = x^MN (W, a, b, M), \quad \frac{\partial x^MN}{\partial W} = x^R \frac{\partial v^MN}{\partial W}, \quad j = W, a, b, M. \]
y = y^R \left[ v^{MN}(W,a,b,M),a \right] = y^{MN}(W,a,b,M);

\frac{\partial y^{MN}}{\partial j} = y^R \frac{\partial v^{MN}}{\partial j}, \quad j = W,b,M,

\frac{\partial y^{MN}}{\partial a} = y^R \frac{\partial v^{MN}}{\partial a} + y^R = -\frac{1}{\nu \Lambda} \left( 1 - \mu - \frac{\nu W}{f''Y} \right) = ? (0, \text{if } A3).

r = r^R \left[ v^{MN}(W,a,b,M),a \right] = r^{MN}(W,a,b,M); \quad \frac{\partial r^{MN}}{\partial j} = r^R \frac{\partial r^{MN}}{\partial j}, \quad j = W,b,M,

\frac{\partial r^{MN}}{\partial a} = r^R \frac{\partial v^{MN}}{\partial a} + r^R = \frac{1}{\Lambda} \left[ \frac{P_w}{f''} \left( g'' + 1 \right) - \frac{e_{x,y} x}{h' i} - \frac{g''}{v^2} \right] > 0.

\frac{\partial r^{MN}}{\partial W} = z^R \frac{v}{W} < 0,

\frac{\partial r^{MN}}{\partial a} = z^R \frac{\partial v^{MN}}{\partial a} + y^R = -\frac{1}{\nu \Lambda} \left( 1 - \mu - \mu \frac{e_{x,y} x}{e_{x,y} y} - \frac{\nu W}{f''Y} \right) = ?,

\frac{\partial r^{MN}}{\partial b} = z^R \frac{\partial v^{MN}}{\partial b} > 0, \quad \frac{\partial r^{MN}}{\partial M} = z^R \frac{\partial r^{MN}}{\partial M} > 0.

i = i^R \left[ v^{MN}(W,a,b,M),a,b \right] = i^{MN}(W,a,b,M);

\frac{\partial i^{MN}}{\partial W} = i^R \frac{\partial v^{MN}}{\partial W} = ? (0), \quad \left( i^R > \right) \frac{\partial i^{MN}}{\partial a} = i^R \frac{\partial v^{MN}}{\partial a} + i^R > 0,

\left( i^R > \right) \frac{\partial i^{MN}}{\partial b} = i^R \frac{\partial v^{MN}}{\partial b} + i^R > 0, \quad \frac{\partial i^{MN}}{\partial M} = i^R \frac{\partial i^{MN}}{\partial M} = ? (0).

For the monetary variables:

\frac{\partial P}{\partial W} = P^M (P_w,r) = P^M (P_w ^{MN}(W,a,b,M),r^{MN}(W,a,b,M)) = P^{MN}(W,a,b,M);

\frac{\partial P}{\partial M} = \frac{\partial P^M}{\partial M} = \frac{\Delta P}{\Delta W} = \frac{\lambda - \Delta}{\nu \Lambda} > 0 \text{ (if A1)};

\frac{\partial P}{\partial a} = \frac{\partial P^M}{\partial a} = \frac{\lambda P_w}{\partial v^{MN}} = ?; \quad \frac{\partial P}{\partial b} = \frac{\partial P^M}{\partial b} = \frac{\partial v^{MN}}{\partial b} = \Delta > 0;

\frac{\partial P}{\partial M} = \frac{\partial P^M}{\partial M} = \frac{\partial v^{MN}}{\partial M} = \Delta > 0.

Y = Y^\prime \left[ M,i(W,a,b,M) \right] = Y^{MN}(W,a,b,M);

\frac{\partial i^{MN}}{\partial W} = M^\prime \frac{\partial Y}{\partial \bar{\sigma}}, \quad j = W,a,b; \quad \frac{\partial r^{MN}}{\partial M} = V + M^\prime \frac{\partial \bar{\sigma}}{\partial \bar{\sigma}} > 0 (?).
And for the interest rate divergences:

\[
\begin{align*}
\frac{\partial MS}{\partial W} - \frac{\partial MN}{\partial W} &= i_v \frac{1}{P_v} \left( \frac{1}{\Delta} - \frac{1}{\Lambda} \right) = ? (> 0), \\
\frac{\partial MS}{\partial a} - \frac{\partial MN}{\partial a} &= i_v \left( \frac{\partial v_{MS}}{\partial a} - \frac{\partial v_{MN}}{\partial a} \right) = i_v \frac{(1 - \lambda)\nu}{\Lambda \Delta} \left( P_w - \frac{e_{y,j}}{h' i} - \frac{e_{y,j} \delta''}{\nu^2} \right) = ? (> 0), \\
\frac{\partial MS}{\partial b} - \frac{\partial MN}{\partial b} &= -i_v \frac{\nu e_{y,j}}{\Lambda h' i} = ? (> 0).
\end{align*}
\]
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