Incentives to Cooperate in New Product Development

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Abstract

The knowledge required for decision making in a firm is distributed across various departments. In practice cross functional teams are used to integrate this distributed knowledge. Incentive schemes are of crucial importance to encourage departments to share knowledge. In this paper, we study different incentive schemes by means of a two stage model. In the first step departments have to choose between learning and sharing knowledge, in the second stage, they bargain about a new product feature. The outcome of the bargaining process in the second stage depends on the capabilities of the agents and their uncertainty about the opponent. The result of the second stage determines the agents' payoffs which in turn influence the time allocation. In a simulation study, we investigate different incentive systems and show to which extent a firm has to reward the sharing of knowledge in order to reach its overall objectives. Furthermore, we are able to derive an analytical solution for the bargaining process under uncertainty and compute Nash equilibria for a discrete set of possible actions.

1 Introduction

The incentive system is an important parameter of an organizational structure [cf. Plambeck and Zenios 1999]. Traditionally, this aspect has been dealt with via principal-agent models. This approach frames the problem as one of incomplete information: the principal (e.g., firm owner) cannot perfectly monitor the actions of a possibly shirking agent (management, employee), both directly and indirectly via observing the results. The problem is solved through the determination of a profit-sharing scheme that rewards the agent also via the principals utility, the fixed income part being dependent on the different attitudes towards risk.

In this setting, the first best solution is that the agent adopts the principals utility function. This implies, that the principal knows about the subject matter at least as much as the agent. Total Quality Management challenges this "scientific management" view of knowledge distribution within the firm. It is a management method that tries to mobilize knowledge residing on all levels of the organization to improve products and processes. The premise of TQM is that the most valuable knowledge about the idiosyncrasies of a production process or customer preferences is held by the employees actually dealing with them daily, i.e., shop floor workers and salesmen.

Thus, the function of an incentive system here is to foster the generation of knowledge through experimentation and reflection, the explication of knowledge as a prerequisite for its integration for building common models for groups searching processes. This is reflected in a number of TQM-methods: the seven tools of Kaizen are simple data gathering and model building methods used in quality circles in operations, the House of Quality is a graphical model of the interactions between customer preferences, product and process specifications used as a road-map by multi-function product development teams.

Thus, when dealing with incentives in the TQM philosophy, one has to give up the notion that the principal knows at least as much as the agent. For a one-to-one setting, models of this type are studied in [Minkler 1993]. He shows that a profit-sharing scheme can also be explained in a setting, where the principal can perfectly observe the agent’s actions but cannot judge their effect on his utility due to limited knowledge. In this paper,

\textsuperscript{1}cf. [Vetschera 1999] who assumes incomplete information on the agents’ preferences models a situation where the principal can obtain additional preference information. In contrast, we assume that the principal cannot observe the actions of the agents.
we will extend this view to a one-to-many setting: the principal has to provide incentives for a team of agents, each one having different knowledge the principal does not have. Following a stylized model of the House of Quality, the agents have to bargain over an interface specification between their systems (e.g., a technical characteristic of a product) that determines the quality of the result and the principals profit. The only other information the principal has besides the outcome is the knowledge explicated, in the form of e.g., a House of Quality model. Clearly, one could think about rewarding the whole team based on the team effort. However, as shown e.g. in [Huberman and Glance 1996], this often leads to a prisoner’s dilemma type of situation, where shirking is rational, especially if the team size increases so that social dilemmas arise. Furthermore, the burden on the principals information system increases heavily if the same agent is part time member of different teams and also has to be rewarded for the job done regularly. We thus focus on incentive systems involving rewards based on the knowledge explicated. This issue is also important when designing a computerized organizational memory organized e.g., as an Intranet-based document management system where the employees should enter the knowledge obtained for further usage. Empirical evidence shows, that such IT investments fail, when there is no incentive to do this or worse, if knowledge sharing is punished in the sense that the knowledge is used by other team members to limit the bargaining opportunity of the employee publishing his knowledge.

We apply our model for studying incentives for marketing and production agents involved in the product development process. However, the model applies to a much wider range of settings where conflicts between departments influence a firm’s profitability.

In the following section we describe our two step model. In section (3), we apply our model to the product development process and study the impact of different incentive systems on firm’s performance. Section (4) summarizes our main results.

2 The Model

In most organizational models, agents are viewed as essentially honest and as not knowingly communicating incorrect information or decisions although the task itself may be so ambiguous that the agent may not be able to detect whether or not other agents are lying [Carley and Gasser 1999]. Furthermore, Carley and Gasser point out that to date the concept of lying has not been incorporated into Computational Organization Theory. In the following, we describe our model which introduces the concept of lying and information hiding in an organizational learning framework with a focus on the new product development process.

In product development time to market is a key factor for product success. Therefore time is a limited resource and has to be managed efficiently. The knowledge required for product development is distributed across various departments in the organization. Sharing knowledge between departments facilitates the search for feasible products, reduces development time and increases a departments’ local knowledge.

However, since departments have distinct incentives, decisions are governed by own interests and may conflict with both, other departments and the firm’s objectives. Due to this conflict the product features have to be determined in a bargaining situation between the departments.

Typically the bargaining process is based on a win-win setting, i.e., departments have to make proposals that make both parties better off. In order to generate feasible proposals, knowledge from the other department is required. Making knowledge public decreases the other departments’ uncertainty and reduces the possibility of cheating. Therefore departments tend to hide information, negatively affecting firm’s objectives.
We model the interaction between local incentives, information hiding and the firm’s payoff by means of a competitive game with two agents. The agents can be viewed as a marketing department (agent 1), that has the incentive to maximize sales and an engineering department (agent 2) that wants to minimize costs. Figure 1 displays the structure of the model consisting of two stages: In the first stage, the agents decide how to allocate their working time (indicated by $a_i(1 - 3)$) based on the incentive scheme (Time allocation). The outcome of the first stage influences the local performance of the agents and the uncertainty about the other department. In a second stage the agents bargain about the features ($x$) of a new product (Bargaining game). The result of the bargaining process depends on the agents’ payoff functions.

![Figure 1: Model structure](image)

### 2.1 Incentive

For the purpose of simplicity, a product is assumed to be characterized by a single feature $x$. The value of $x$ determines the costs and the market share of the product. The payoff function $\Pi_1(x)$ for the marketing department is proportional to the market share of the product. It is assumed to increase linearly with $x$. For the engineering department the payoff function $\Pi_2(x)$ is proportional to negative costs per unit. The production costs are assumed to increase with $x$. The payoff functions are given by:

$$\Pi_1(x) = \delta_1 + x \quad \text{(1)}$$
$$\Pi_2(x) = \delta_2 - x \quad \text{(2)}$$

The parameters $\delta_i$ are a function of the agent’s time allocation, as time allocation influences the departments’ efficiency.

The management of the firm is interested in maximizing the global profit of the firm (eq. 3). The payoff $\Pi(x)$ for the firm is proportional to the market share $\delta_1 + x$ times contribution of margin $P + \delta_2 - x$:

$$\Pi(x) = (\delta_1 + x)(P + \delta_2 - x) \quad \text{(3)}$$

The parameter $P$ is the price of the product. It is assumed to be a constant and is set to $P = 1$. If the agents had no conflicting incentives and were interested in maximizing the
firm’s profit, a global optimum \( x^* \) could be found as:

\[
x^* = \frac{P + \delta_2 - \delta_1}{2}
\]

\[
\Pi(x^*) = \frac{1}{2}(P + \delta_2 + \delta_1)^2
\]

The optimal product attribute \( x \) increases in the difference of local knowledge between agent 2 and agent 1. If agent 2 has more local knowledge \( \delta_2 \) than agent 1, the optimal product attribute is greater than 0.5.

### 2.2 Time Allocation

In the first stage agent \( i \) allocates shares of its time \( a_i(j) \) to different types of activities \( j \):

- \( a_i(1) \): Improvement of the performance of the own department (local learning)
- \( a_i(2) \): Learning from the other agent by reading from the database
- \( a_i(3) \): Publish the own knowledge in the database

Improving the own performance means that the marketing agent is able to increase sales for the whole range of possible products. For the engineering agent improving performance results in lower costs for all possible products. Hence, local learning shifts the payoff function additively by \( a_i(1) \).

An agent \( i \) can also spent a percentage, \( a_i(3) \), of its time publishing information in a database. The amount of information published is limited to \( a_i(1) \) (an agent cannot publish more information than locally acquired). The information published, can be retrieved by the other agent from the database. This has two effects: A reduction in the uncertainty about the other agent and an increase in the own department’s efficiency. If an agent reads the amount of \( a_i(2) \leq a_j(3) \) from the database, it reduces the initial uncertainty \( \sigma_j^0 \) about the opponent’s payoff function by a fraction of \( a_i(2) \):

\[
\sigma_j = (1 - a_i(2))\sigma_j^0
\]

Learning from others increases the own knowledge and is by a factor of \( \alpha \) more efficient than learning locally. This assumption implies that learning from others increases the local performance, equally for all \( x \). If, for instance, the marketing agent learns about technical features from the production agent, it can benefit from this knowledge in the sales process. Production agents, on the other hand, can avoid over-engineering when they learn from the marketing agent about the irrelevance of some features.

The effects of time allocation on the payoff function are given by:

\[
\delta_1 = a_1(1) + \alpha a_1(2)
\]

\[
\delta_2 = a_2(1) + \alpha a_2(2)
\]

subject to the constraints:

\[
\sum_n a_i(n) = 1
\]

\[
a_i(3) \leq a_i(1)
\]

\[
a_i(2) \leq a_j(3)
\]
The firm's optimal time allocation is given by inserting (8) and (11) into (5):

$$a_1 = a_2 = \begin{cases} 
(1, 0, 0) : & \text{if } \alpha \leq 2 \\
(1/3, 1/3, 1/3) : & \text{else}
\end{cases}$$

(12)

If $\alpha \leq 2$ the net benefit from learning from others is negative, since it requires both, writing to and reading from the database. In this case the optimum is to learn locally only $(1, 0, 0)$. For $\alpha > 2$ it is optimal to allocate the minimum amount of time to local learning $(1/3, 1/3, 1/3)$. Due to the constraints reading from the database requires writing to the database (from the other agent) which in turn requires local learning. Due to the conflicting incentives the global optimum will not be reached and the question arises which incentives should be given to the agents. While learning locally and reading from the knowledge base directly increase the agents’ payoffs, writing to the knowledge base is not rewarded but causes disadvantages in the bargaining process and is time consuming. Therefore the agents must be given an incentive to write to the database. The following equations consider a reward $\beta * a_i(3)$:

$$\Pi_1(x) = a_1(1) + \alpha * a_1(2) + \beta * a_i(3) + x$$

(13)

$$\Pi_2(x) = a_2(1) + \alpha * a_2(2) + \beta * a_i(3) - x$$

(14)

2.3 Bargaining Game

In first stage of the game the agents have decided upon time allocation. This determines the payoff functions (14) and the uncertainty about the other agent (6). In the second stage the agents meet to bargain about the feature $x$ of the new product. Thus we consider a dynamic bargaining framework under uncertainty.

To introduce the basic concept of the game, we assume for both agents complete knowledge about the utility functions.

The game starts with a random selection of one agent. If agent 1 is selected it places a bid at $x_0^1 = 1$ and communicates it together with the value of its utility function $\Pi_1(x_0^1)$ to agent 2. Agent 2 responds with a value of its utility function $\Pi_2(x_0^1)$.

Agent 2 can either accept the bid, or make an alternative bid. For agent 2 it would be rational to start at $x = 0$. Agent 1 can either accept or reject. If it chooses to reject, the game enters the second round (r) and the agents have to follow the rules:

- $\Pi_i(x_r^i) > \Pi_i(x_r^j)$, $\forall r' < r$ Rationale

- $\Pi_j(x_r^i) > \Pi_j(x_r^j)$, $\forall r' < r$ Win-Win Situation

The variable $x_r^i$ is agent $i$’s bid at round $r$. The first condition states that the payoff from the own bid exceeds the payoff of all past opponent's proposals. Otherwise a rational agent must have accepted the opponent's offer previously.

The second condition enforces a win-win situation in that any bid of agent $i$ must make agent $j$ better off than all past bids of agent $i$. The end of game is reached if no more proposal is possible. In the case of no uncertainty, the final outcome is always in the middle between zero and one.

In the case of uncertainty we assume that agent $i$ only knows upper and lower bounds for agent $j$’s payoff function $\Pi_j(x)$. The true payoff function, known to agent $j$, is in the middle of the bounds, which are common knowledge.

$$\Pi_j(x)^{min} = \Pi_j(x) - \sigma_j \leq \Pi_j(x) \leq \Pi_j(x) + \sigma_j = \Pi_j(x)^{max}$$

(15)
Between the bounds agent $i$ has no knowledge about the shape of the competitor’s utility function. This enables the agents to communicate any value of the payoff function within the boundaries and therefore introduces the concept of lying as a rational strategy.

Again, the agents flip a coin to decide about the starter (assume agent 1 won). Agent 1 places a bid, but instead of communicating the true value of its utility function it can state every value between the corresponding bounds. Agent 2 also responds a value of its utility function, taking advantage of the other agent’s uncertainty.

A good strategy is to communicate the lower bound if one places a bid and to respond with the upper bound if one receives a bid. If agent $i$ places a bid $x^r_i$ at round $r$ then:

- $\Pi_i(x^r_i) > \Pi_i(x^r_{i'})$  $\forall r' < r$
- $\Pi_{j}^{\text{min}}(x^r_{j'}) > \Pi_{j}(x^r_{j'})$  $\forall r' < r$.

The first rule assures rationality as in the case of no uncertainty. But instead of the true values of the payoff function, $\Pi_i(.)$ denotes communicated values. The second rule is a generalization of the win-win situation. Agent $j$ can respond with any value within the bounds. Agent $i$ has to make an offer such that agent $j$’s payoff is slightly larger than all agent $j$’s previous offers, even if agent $j$ communicates the lowest possible value.

The outcome of the game can be computed analytically for the more general case of different slopes of the payoff functions $k_1$ ($k_2$) for agent 1 (2) (see Appendix).

### 3 Simulation and Results

#### 3.1 Bargaining and Information Hiding

Figure (2) illustrates the game for parameters of $\delta_1 = \delta_2 = 0$, $\sigma_1 = 0.1$ and $\sigma_2 = 0.05$. Placing a bid is indicated by a square, while responding to a bid is displayed as dots. The dotted lines indicate the boundaries of uncertainty.

Agent 1 starts, places a bid $x^0_1 = 1$ and responds the lower bound of its utility function to agent 2. Agent 2 responds with its upper bound and places a bid at $x^0_2 = 0$. Agent 1 communicates the upper bound at $x^1_2 = 0$. In the second round agent 1 must assure that its bid makes agent 2 better off than with its first bid. Therefore it places a bid such that the lower bound of agent 2’s utility function will be higher than the value responded by agent 2 in the first round. The highest possible value is $x^1_2 = 0.9$. Then it is again agent 2’s turn. The final outcome is at $x = 0.7$. As agent 1 has less uncertainty about opponent’s utility function compared to agent 2, agent 1 is better off than in the case of certainty.

Figure (3) shows the resulting product feature value $x$ for the bargaining game as a function of agent 2’s uncertainty about agent 1’s utility function. Agent 1’s uncertainty about agent 2 is fixed at $\sigma_2 = 0.1$. The outcome is nonlinearly increasing in $\sigma_1$. At a value of uncertainty of $\sigma_1 = 0.1$ the uncertainties are equal, resulting in the optimal outcome of $x = 0.5$. This suggests that the agents should have the same level of knowledge in order to avoid exploitation due to information asymmetry. This stands in contrast to previous studies on the optimal formation of teams (for an overview cf. Argote 1999) where the aggregate learning curve of the firm is maximized, if a team consists of an experienced and an unexperienced member. While this seems plausible for common incentive schemes or global optimization, our model indicates that in the case of conflicting incentives the team members should have similar experience. Otherwise the more experienced member could influence the decision to its advantage.
Figure 2: The result of the dynamic bargaining process
Figure 3: The outcome ($x$) of the bargaining process as a function of the uncertainty $\sigma_1$. 
3.2 Game vs. global optimum

At the beginning of the product development period, the agents start with a local knowledge $\delta_{1,2}$ of zero and an initial uncertainty $\sigma_{1,2} = 0.1$. The efficiency factor $\alpha$ is assumed to be 3. In the first step, the agents choose their shares of learning locally ($a_{1,2}(1)$), reading from the database ($a_{1,2}(2)$), and writing to the database ($a_{1,2}(3)$) independently. After applying the constraints the bargaining game is conducted and the agents receive a payoff. In the search for an optimal time allocation according to their local incentive (payoff function), the agents apply an iterative best response approach: alternatingly they generate time allocations and accept new solutions on a greedy basis.

The simulation yields an equilibrium for both agents. This equilibrium is characterized by a pure local learning strategy without sharing any information. This is an undesirable situation for the firm as both agents would be better off by transferring knowledge if the parameter $\alpha$ is larger than 2. However, this is avoided due to exploitation opportunities for the other agent (i.e., a prisoner’s dilemma).

Table 1 compares the payoffs and time allocations for this game to the hypothetical global optimum. Both, the agents’ profits and the firm’s profit are higher for the global optimum. Due to the constraints of time allocation the optimal solution is to spend the same amount of time in learning locally, learning from the other agent and transferring knowledge. However, this optimum cannot be enforced by the management since it can only observe the amount of information put into the knowledge base.

<table>
<thead>
<tr>
<th></th>
<th>$\Pi_1$</th>
<th>$\Pi_2$</th>
<th>$\Pi$</th>
<th>$a_{1,2}(1)$</th>
<th>$a_{1,2}(2)$</th>
<th>$a_{1,2}(3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>game</td>
<td>1.50</td>
<td>0.50</td>
<td>2.25</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>global optimum</td>
<td>1.83</td>
<td>0.83</td>
<td>3.36</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Table 1: The payoffs in the case of the game and the global optimum

3.3 Incentives to share knowledge

In practice, firms have reacted to this problem by rewarding the members for writing to the knowledge base. However, it is unclear under which circumstances and to what extent such rewards should be used. For this purpose, we study the impact of different levels of rewards ($\beta$) for sharing knowledge on firms’ profits. The mean results over 100 replications for different levels of $\beta$, symmetric initial knowledge about the other agent and a fixed level of $\alpha = 3$ are shown in figure 4\(^2\). In can be seen that profits rise with higher levels of $\beta$ reaching a plateau for values of $\beta$ between 2.1 and 4.2. The profits in this region are close to the optimal solution of $\Pi = 3.36$. With higher levels of $\beta$ ($\beta > 4.2$) profits start to decrease and fall sharply when $\beta$ exceeds a value of 5.4.

This analysis shows that for this setting ($\alpha = 3$), the reward for sharing knowledge must be higher than twice the expected return from local learning. This relatively high incentive for sharing knowledge has two reasons: (1) it has to compensate for less efficient bargaining power (exploitation through cheating) and (2) for the opportunity cost of local learning.

For a game theoretical analysis of our model, we limit the number of time allocation possibilities to three strategies:

- strategy 1 is to learn locally only (1, 0, 0); i.e., information hiding

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\(^2\)For the purpose of simplicity, the costs for paying the incentives are omitted in our analysis.
Figure 4: The mean profit over 100 replications as a function of the incentive to transfer knowledge.
• strategy 2 is to share the time between learning locally and writing to the knowledge base (1/2, 0, 1/2)
• strategy 3 is to spend the same amount of time to all activities (1/3, 1/3, 1/3)

This simplification leads to a normal form game with the payoff matrix displayed in table 2. Agent 1 is the row player and agent 2 the column player. For values of $\beta$ between 0 and 1 there exist two Nash equilibria for the strategies (1, 1) and (3, 3), respectively. The equilibrium at (3, 3) can not be reached due to the hiding of information. Between values of $\beta = 1$ and $\beta = 5.4$ the strategies (3, 3) are the only Nash equilibrium. For values of $\beta > 1.133$ strategy 1 is dominated by strategy 2 for both agents and by applying successive dominance, the final outcome can be found at (3, 3). For this discrete setting, when the incentive for writing into the knowledge base lies between 1.133 and 5.4, the agents end up at the firm’s optimum. This result is consistent with the simulation based on the best response approach described in the previous section. Strategy 2 dominates the other strategies for $\beta > 5.4$. In this case it is optimal for the agents to write as much as possible to the knowledge-base without reading from it. This results in a large drop of firm profit, as we observed in the simulation (see Fig. 4).

4 Conclusion

This paper offers a model which relates learning processes of a firm’s agents to both, the firms and the agents’ payoffs, respectively. We study the interrelationship between learning and incentive systems for a new product development problem where agents from different departments with conflicting objectives have to decide about a new product. We propose a two stage model. In the first stage, departments have to choose their time allocation for learning and sharing knowledge. In firms, such knowledge can, e.g., be published in an intranet based document management system or stored in form of a House of Quality. In the second stage the agents bargain about a new product feature. The outcomes of both stages are interdependent: The first stage determines the capabilities of the agents and their uncertainty about the opponent. In our model, a higher degree of uncertainty about the opponent negatively influences the agents’ payoffs. The bargaining process yields the new product and the agents’ payoffs which affects their time allocation (first step).

We have derived an analytical solution for the bargaining process under uncertainty and computed Nash equilibria for a discrete set of actions. In a simulation study, we have analyzed the impact of incentives to share knowledge on the performance of the firm. The major implications can be summarized as following:

• When an incentive scheme does not consider an explicit reward for making knowledge public agents prefer to hide information.

• The outcome of the bargaining process depends on the uncertainty about the other department. This leads to information hiding. We find that the agents should have the
same level of knowledge in order to avoid exploitation due to information asymmetry. This stands in contrast to previous studies [Argote 1999] on the optimal formation of teams where the aggregate learning curve of the firm is maximized, if a team consists of an experienced and an unexperienced member.

- The overall objective of the firm depends on the interdependency between the departments. When the gains in efficiency by using the common knowledge base are at least twice the gains from local learning, local rewards for sharing knowledge help firms to reach their overall objective.

References


Appendix

The outcome of the bargaining game does not depend on the levels of the payoff functions. Therefore the parameters $\delta_{1,2}$ can be set to zero. The upper and lower bounds of the utility functions are given by:

\[
\Pi^{\min}_i (x) = -\sigma_i + k_ix \Pi^{\max}_i (x) = +\sigma_i + k_ix
\]

(16)

The parameters $k_1$ ($k_2$) are the slopes of agent 1 (2)'s utility function. The possible bids at round $r$ are:

\[
x_1^r = 1 + (r - 1)\Delta_1 \text{ with } \Delta_1 = 2\sigma_2/k_2
\]

(17)

\[
x_2^r = (r - 1)\Delta_2 \text{ with } \Delta_2 = 2\sigma_1/k_1
\]

(18)
The agents strategies are to understate the true value of the payoff, if one places a bid and overstate it, if one responds to a bid.

If agent 1 starts bidding it follows from the rationality constraint that
\[
\Pi_1^{\min}(x_1^r) \geq \Pi_1^{\min}(x_2^r - 1)
\]
(19)
\[
\Pi_2^{\min}(x_2^r) \geq \Pi_2^{\min}(x_1^r)
\]
(20)

Plugging in the expressions for \( \Pi \) and \( x_i^r \) one finds for the number of possible moves:
\[
r_1 = \text{Int} \left( \frac{-2\sigma_1 + k_1(1 + \Delta_1 - 2\Delta_2)}{k_1(\Delta_1 - \Delta_2)} \right)
\]
(21)
\[
r_2 = \text{Int} \left( \frac{2\sigma_2 + k_2(1 + \Delta_1 - \Delta_2)}{k_1(\Delta_1 - \Delta_2)} \right)
\]
(22)

The function \( \text{Int} \) computes the integer value. The final outcome \( x^{\text{Start1}} \) is given by:
\[
x^{\text{Start1}} = \begin{cases} 
1 + r_2 \Delta_1 & \text{if } r_1 > r_2 \\
(r_1 - 1) \Delta_2 & \text{else}
\end{cases}
\]
(24)

If agent 2 starts bidding it follows from the rationality constraint that
\[
\Pi_1^{\min}(x_1^r) \geq \Pi_1^{\min}(x_2^r)
\]
(25)
\[
\Pi_2^{\min}(x_2^r) \geq \Pi_2^{\min}(x_1^r - 1)
\]
(26)

After some algebra one finds:
\[
r_3 = \text{Int} \left( \frac{-2\sigma_1 + k_3(1 + \Delta_1 - \Delta_2)}{k_1(\Delta_1 - \Delta_2)} \right)
\]
(27)
\[
r_4 = \text{Int} \left( \frac{2\sigma_2 + k_4(1 + 2\Delta_1 - \Delta_2)}{k_1(\Delta_1 - \Delta_2)} \right)
\]
(28)

The final outcome \( x^{\text{Start2}} \) is given by:
\[
x^{\text{Start2}} = \begin{cases} 
1 + r_3 \Delta_1 & \text{if } r_4 > r_3 \\
(r_4 - 1) \Delta_1 & \text{else}
\end{cases}
\]
(30)

Due to the randomization at the beginning the outcome of the game is given by the mean of the outcomes, if agent 1 starts and if agent 2 starts.
\[
x = \frac{1}{2} (x^{\text{Start1}} + x^{\text{Start2}})
\]
(31)