Optimal Contracts for Vertically Connected, Unionized Duopolies

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Abstract: In this paper a vertically structured duopolistic market with unionized price setting firms is analyzed. The form of the contract of the transactions between upstream and downstream firms can be linear pricing, franchising or vertical integration. It is known from literature (Irmen (1997)) that the price elasticity of the industry demand and the degree of product differentiation are the decisive factors in the determination of the profit maximizing form of the contract. In this paper it is shown that the bargaining power of the union is an additional factor. With a higher bargaining power linear pricing becomes less preferable.

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1 Introduction

What does the optimal contract form between vertically connected firms look like? For the monopoly case, it is well known, that industry profit is not as high as possible if two vertically connected monopolies choose a simple linear pricing contract, because the problem of double marginalization occurs. Integration, franchising or retail price maintenance are preferable to a simple linear pricing in this case. (See for example Tirole (1988).) The situation differs if more than one upstream or downstream firms are involved. The strategic interactions quickly become very complicated and general conclusion can be stated rarely. Lin (1988), Bonanno and Vickers (1988), Gal-Or (1991) or Irmen (1997) are examples for papers dealing with this problem. Lin (1988) shows that in a linear city framework vertically connected duopolies have no incentive to integrate vertically. In Gal-Or (1991) and Irmen (1997) a similar problem with a more general linear demand function is formulated. Irmen (1997) uses a slightly different formulation of a linear demand function. With this formulation it is possible to analyze the price elasticity of the industry demand and the degree of product differentiation separately. The result is that both, the degree of product differentiation and the price elasticity of the industry demand, are decisive for the equilibrium industry structure. The result in Lin (1988) for example is based on the assumption of an inelastic industry demand.

As is typical for partial equilibrium analysis, in all these models cost functions are given exogenously. But taking into account the existence of trade unions these results may change, because the wage setting behavior and therefore the cost function may depend on the specific contractual arrangement. There exists a long list of papers analyzing the impact of different wage bargaining arrangements on oligopolistic markets, for example Dowrick (1989), De Fraja (1993), Dobson (1994), Corneo (1995), Santoni (1996), Vannetelbosch (1997) and Grandner (2000a). Grandner (2000c) introduces a vertically structured production process and analyzes the effects of unions in vertically connected Cournot oligopolies. In all these papers it is shown that the equilibrium allocation depends on the specific institutional arrangement of the wage setting process.

An additional argument why trade unions may play a role in determining the industry equilibrium is derived in Grandner (2000b). The argument there is that the bargaining power that can be realized by the trade union may differ with the contractual arrangements. Grandner (2000b) derives the argument for vertically connected monopolies, whereas in the paper presented here, unionized price setting duopolies are analyzed.

2 Model

In the described duopolistic industry the production process is separated vertically in two parts, an upstream and a downstream sector.

In a first step the possible contracts between upstream and downstream firms are
restricted. First of all the two upstream firms choose simultaneously a downstream firm and one of the following three forms of pricing.

1. linear pricing (LP),
2. franchising (FF), or
3. integration (IN).

Downstream firms do not have the possibility to choose. These contracts are exclusive, the upstream firms cannot deal with the second downstream firm and the downstream firms cannot have any deal with the second upstream firm. The exclusivity of the dealing contracts is crucial for the model, otherwise a franchise fee contract may not be an equilibrium structure at all. (See Hart and Tirole (1990) for further discussion).

Succeeding these decisions firms and unions bargain over wages at the firm level. Finally the wholesale price (except in the IN scenario) and consumer prices are set and the goods are produced.

To make the analysis manageable the production technology is assumed to be linear in both production stages. An upstream firm produces one unit of output with the help of one unit of labor and no other input. A downstream firm produces one unit of the final output with one unit of the upstream product and \(1/c\) units of labor. \((q_{d,i} = q_{u,i} = q_i)\). Upstream and downstream firms are price setters.

The downstream firms have to choose linear consumer prices. The consumer demand for the final product of downstream firm \(i\) is given by the following inverse demand function:

\[
q_{d,i} = A - e p_{d,i} + b(p_{d,j} - p_{d,i}) \quad A > 0, \quad b, e \geq 0, b + e > 0, \quad i, j = 1, 2, \quad i \neq j \quad (1)
\]

With \(e\) the price elasticity of the industry demand changes. \(b\) is the parameter of product differentiation, \(b = 0\) describes maximal differentiation. With \(b \to \infty\) the products of the two firms become homogeneous.

To simplify expressions in the analysis the following abbreviations will be used:

\[
\begin{align*}
k_1 &= b^2 + 4be + 2e^2 \\
k_2 &= k_1 + (2b + e)(b + 2e) \\
k_3 &= k_1 + e(3b + 2e) \\
k_4 &= k_1^2 + e(3b + 2e)(b + 2e)(2b + e) \\
k_5 &= k_1 + 2e(b + e) \\
k_6 &= k_1 + 2(b + e)(2b + e)
\end{align*}
\]

2.1 Linear Pricing (LP)

With no vertical integration the profit functions of the downstream and the upstream firm \(i\) are given by:

---

1Hart and Tirole (1990) have presented a paper with less restrictions of possible contracts.
\[
\pi_{d,i} = (p_{d,i} - p_{u,i} - \frac{w_{d,i}}{c}) q_i - f_i
\]
\[
\pi_{u,i} = (p_{u,i} - w_{u,i}) q_i + f_i
\]

(2)

where \( f_i = 0 \) in the linear pricing scenario.

The reaction function of the downstream firm \( i \) in the price game, given a wholesale price chosen by its upstream partner, is given by:

\[
p_{d,i} = \frac{A + bp_{d,j} + (b + e) \left( p_{u,i} + \frac{w_{d,i}}{c} \right)}{2(b + e)}
\]

(3)

The equilibrium price is therefore:

\[
p_{d,i} = \frac{(3b + 2e)A + 2(b + e)^2 \left( p_{u,i} + \frac{w_{d,i}}{c} \right) + b(b + e) \left( p_{u,j} + \frac{w_{d,j}}{c} \right)}{(3b + 2e)(b + 2e)}
\]

(4)

The optimal price demanded by downstream firm \( i \) depends positively on the own marginal costs and on the marginal costs of the competing firm.

The quantity that is sold by firm \( i \) is given by:

\[
q_i = \frac{(b + e) \left[ (3b + 2e)A + b(b + e) \left( p_{u,i} + \frac{w_{d,i}}{c} \right) - k_1 \left( p_{u,i} + \frac{w_{d,i}}{c} \right) \right]}{(3b + 2e)(b + 2e)}
\]

(5)

Equation (5) also describes the demand function for the price setting upstream firm \( i \). If the contract between the upstream firm \( i \) and the downstream firm \( i \) allows only linear pricing (\( f_i = 0 \) in equation (2)) the optimal wholesale price set by upstream firm \( i \) is:

\[
p_{u,i} = \frac{(3b + 2e)A}{2k_1} + \frac{b(b + e) \left( p_{u,j} + \frac{w_{d,j}}{c} \right)}{2k_1} + \frac{w_{u,i} - \frac{w_{d,i}}{c}}{2}
\]

(6)

That means the optimal wholesale price of firm \( i \) depends on the wholesale price chosen by the competing upstream firm.\(^3\)

The equilibrium of the price game of the upstream firms is described by:

\[
p_{u,i} = \frac{(3b + 2e)A}{k_3} - \frac{k_4 \frac{w_{d,i}}{c}}{k_3k_2} + \frac{k_1 b(b + e) \left( \frac{w_{d,i}}{c} + w_{u,j} \right) + 2k_1^2 w_{u,i}}{k_3k_2}
\]

(7)

\[
q_i = \frac{k_1(b + e)A}{k_3(b + 2e)} - \frac{k_1 k_4(b + e) \left( \frac{w_{d,i}}{c} + w_{u,i} \right)}{k_3k_2(3b + 2e)(b + 2e)} + \frac{k_1^2 b(b + e)^2 \left( \frac{w_{d,i}}{c} + w_{u,j} \right)}{k_3k_2(3b + 2e)(b + 2e)}
\]

(8)

\(^3\)Here I assume that both pairs of firms choose the same form of contract. There would be the possibility of asymmetric equilibrium industry structure where one pair of firms integrates vertically. But an asymmetric equilibrium will not exist, see Irmen (1997)
With linear pricing the equilibrium profit of the downstream and the upstream firm is positively correlated with the firm’s output.

\[ \pi_{d,i} = \frac{q_i^2}{b + e} \]  
\[ \pi_{u,i} = \frac{(b + 2e)(3b + 2e)q_i^2}{k_1(b + e)} \]  

**Wage Bargaining with Linear Pricing**

In the next step the wage bargaining process is described by the generalized Nash bargaining. The solution of the bargaining can be found by analyzing the following maximization problem:

\[ \max_w \left\{ L(w - \bar{w})^\alpha \pi^{1-\alpha} \right\} \]  

with \( 0 \leq \alpha \leq 1 \) describing the exogenous bargaining power of the union. \( \alpha = 0 \) is equivalent with a competitive labor market and \( \alpha = 1 \) gives the monopoly union model.\(^4\) In the presented model all four unions have the same bargaining power.\(^5\) \( \bar{w} \) is the reservation wage and is also given exogenously in the model.

Firms and Unions bargain over wages only. This is the so called right to manage model. Alternatively, with efficient bargaining instead (see McDonald and Solow (1981)), the employment decision would be in conflict with a simultaneous quantity determination in oligopolistic markets. (See also Petrakis and Vlassis (2000), where efficient bargaining changes the form product market competition from Cournot duopoly to Stackelberg.)

The following reaction function describes the optimal wage in the downstream firm \( i \) given the wages in the competing downstream firm and in both upstream firms.

\[ w_{d,i} = \frac{ck_2\alpha(3b + 2e)A}{2k_4} - \alpha w_{n,i} - \frac{2 - \alpha}{2} \bar{w} + \frac{ck_1\alpha ab(b + e)}{2k_4} \left( \frac{w_{d,j}}{c} + w_{u,j} \right) \]  

In the upstream firms the optimal wages depend also on all other wages.

\[ w_{u,i} = \frac{k_2\alpha(3b + 2e)A}{2k_4} - \alpha \frac{w_{d,i}}{2} - \frac{2 - \alpha}{2} \bar{w} + \frac{k_1\alpha ab(b + e)}{2k_4} \left( \frac{w_{d,j}}{c} + w_{u,j} \right) \]  

All wage bargainings take place simultaneously, therefore the equilibrium wages in the LP scenario are given by the intersection of these four reaction functions.

\(^4\)In the non-cooperative interpretation of the Nash bargaining solution of Rubinstein (1982) \( \alpha \) is determined by the exogenously given time preferences of the bargaining partners.

\(^5\)In Grandner (2000b) the bargaining power can differ between firms.
\[ w_{d,i} = \overline{w} + \frac{k_2 \alpha (3b + 2e)c \left( A - e \left( 1 + \frac{1}{c} \right) \overline{w} \right)}{(2 - \alpha)k_4 + 2\alpha e(3b + 2e)k_2} \]  
(14)

\[ w_{u,i} = \overline{w} + \frac{k_2 \alpha (3b + 2e) \left( A - e \left( 1 + \frac{1}{c} \right) \overline{w} \right)}{(2 - \alpha)k_4 + 2\alpha e(3b + 2e)k_2} \]  
(15)

\[ q_i = \frac{k_1 k_4 (2 - \alpha)(b + e) \left( A - e \left( 1 + \frac{1}{c} \right) \overline{w} \right)}{k_3 (b + 2e)((2 - \alpha)k_4 + 2\alpha e(3b + 2e)k_2)} \]  
(16)

All wages and quantities must be positive, therefore I assume \( A > e \left( 1 + \frac{1}{c} \right) \overline{w} \).

With linear pricing wages are equal in the downstream and the upstream firm \( i \). Wages and consumer prices increase with the bargaining power of the unions, quantities decrease.

### 2.2 Franchise Fee (FF)

A franchise fee contract consists of a fix amount of money which the downstream firm has to pay to the upstream firm and a variable linear wholesale price \((f_i > 0 \text{ in equation (2)})\). With the fix part of the two-part tariff the upstream firm can absorb all the potential rent from the downstream firm. Because the franchise fee is part of the fix cost of the downstream firm, the pricing behavior of the downstream firm will be the same than within the LP scenario described in equation (4), where \( p_{u,i} \) is the variable wholesale price demanded by the upstream firm \( i \).

The profit maximizing franchise fee can be derived from the profit maximizing behavior of the upstream firm. A restriction for the upstream firm is, that the downstream firm’s profit has to be non-negative. In optimum it is zero.

\[ f_i = \left( p_{d,i} - p_{u,i} - \frac{w_{d,i}}{c} \right) q_i \quad = \frac{q_i^2}{b + e} \]  
(17)

Given this franchise fee the profit maximization gives:

\[ \max_{p_{u,i}} \left\{ (p_{u,i} - w_{u,i})q_i + f_i \right\} \]  
(18)

\[ \Rightarrow \quad q_i + \left( p_{u,i} - w_{u,i} \right) \frac{\partial q_i}{\partial p_{u,i}} + \frac{2q_i}{b + e} \frac{\partial q_i}{\partial p_{u,i}} = 0 \]  
(19)

\[ \frac{\partial q_i}{\partial p_{u,i}} = -\frac{(b + e)(b^2 + 4be + 2e^2)}{(b + 2e)(3b + 2e)} \]  
(20)

\[ \Rightarrow \quad p_{u,i} > w_{u,i} \quad \text{iff} \quad b > 0 \]  
(21)
Upstream duopolistic firms will not set the wholesale price equal to marginal costs, except for complete product differentiation. In this case \( b = 0 \) and therefore \( \frac{\partial q}{\partial p_{u,i}} = -\frac{e}{2} \) and firms act like monopolies. But with incomplete differentiation firms try to weaken the price competition in the consumer market by setting the wholesale price above the marginal costs. (See Bonanno and Vickers (1988).)

The reaction function of upstream firm \( i \) in the price game is:

\[
p_{u,i} = \frac{b^2(3b + 2e)A}{4k_1(b + e)^2} - \frac{b^2w_{d,i}}{4(b + e)^2} + \frac{b^3w_{d,j} + p_{u,j}}{4k_1(b + e)} + \frac{(3b + 2e)(b + 2e)w_{u,i}}{4(b + e)^2}
\]

and depends on the wholesale price of the competing firm, as in the linear pricing scenario.

Equilibrium is now described by the following equations:

\[
p_{u,i} = \frac{k_1 A}{k_5(b + e)} - \frac{k_1(k_1 + 2e(2b + e))(b + e)\left(\frac{w_{d,i}}{c} + w_{u,i}\right)}{k_5k_6} + \frac{k_1^2 b\left(\frac{w_{d,j}}{c} + w_{u,j}\right)}{k_5k_6}
\]

\[
q_i = \frac{k_1 A}{k_5} - \frac{k_1(k_1 + 2e(2b + e))(b + e)\left(\frac{w_{d,i}}{c} + w_{u,i}\right)}{k_5k_6} + \frac{k_1^2 b\left(\frac{w_{d,j}}{c} + w_{u,j}\right)}{k_5k_6}
\]

\[
\pi_{d,i} = 0
\]

As before, in equilibrium upstream profits are positively correlated with the firms’ output.

\[
\pi_{u,i} = \frac{2(b + e)q_i^2}{k_1}
\]

Wage bargaining with Franchise Fee

To analyze the wage bargaining in this scenario, one has to describe the timing structure carefully. The situation of the union changes dramatically with the specific timing of the game. Especially the time sequence of the wage setting and the fixing of the franchise fee is crucial. (See Grandner (2000b)).

Wage Bargaining Before Franchise Fee Setting (W-FF)

In this scenario the wage bargaining takes place before the upstream firm sets the fix part of the franchise fee. The profit of the downstream firm is zero, because all possible rent will be absorbed by the upstream firm. \( \pi_{d,i} = 0 \), so the downstream firm has no interest to resist union’s wage demand. But the upstream firm has to
keep in mind the restriction of the non-negative downstream firm profit. The unique subgame perfect equilibrium is given by the highest possible utility of the union. This situation is described by the monopoly union, although the union would not be so strong in a different setting (remember that bargaining power is given exogenously, for example by time preferences).

The results of the wage finding process in the downstream firms can therefore be described by the solution of the following problem:

$$\max_{w_{d,i}} \left\{ \frac{q_i}{c} (w_{d,i} - \bar{w}) \right\}$$

(27)

The wage bargaining in the upstream firms again can be described by the Nash bargaining solution. The wage reaction functions that can be derived from these problems are:

$$w_{d,i} = \frac{ck_6 A}{2(k_1 + 2e(2b + e))(b + e)} + \frac{ck_1 b \left( \frac{w_{d,i}}{c} + w_{u,j} \right)}{2(k_1 + 2e(2b + e))(b + e)} - \frac{cw_{u,i}}{2} + \frac{\bar{w}}{2}$$

(28)

$$w_{u,i} = \frac{\alpha k_6 A}{2(k_1 + 2e(2b + e))(b + e)} + \frac{\alpha k_1 b \left( \frac{w_{d,i}}{c} + w_{u,j} \right)}{2(k_1 + 2e(2b + e))(b + e)} - \frac{\alpha w_{d,i}}{2} + \frac{(2 - \alpha)\bar{w}}{2}$$

(29)

Finally, if again all bargainings take place simultaneously, the equilibrium is described by:

$$w_{d,i} = \bar{w} + \frac{ck_6(2 - \alpha) \left( A - e \left( 1 + \frac{1}{c} \right) \bar{w} \right)}{(1 - \alpha)(k_1 + 2e(2b + e))(b + e) + k_1 b + 3k_6e}$$

(30)

$$w_{u,i} = \bar{w} + \frac{k_6 \alpha \left( A - e \left( 1 + \frac{1}{c} \right) \bar{w} \right)}{(1 - \alpha)(k_1 + 2e(2b + e))(b + e) + k_1 b + 3k_6e}$$

(31)

$$q_i = \frac{k_1(1 + 2e(2b + e))(2 - \alpha)(b + e)}{k_5[(1 - \alpha)(k_1 + 2e(2b + e))(b + e) + k_1 b + 3k_6e]}$$

(32)

Wages are higher in the downstream firms than in the upstream firms with franchising when wage bargainings take place before the fees are set as long as $\alpha < 1$. If all union have a bargaining power of $\alpha = 1$, the described advantage of the downstream union vanishes. The upstream firm wage increases with bargaining power of the union, while the downstream firm wage decreases. Consumer price increases with $\alpha$, because the output of the firms decreases.
Franchise Fee Setting Before Wage Bargaining (FF-W)

If the upstream firm can set the franchise fee before bargaining takes place, the franchise fee can be calculated on the basis that the wage in the downstream firm is as low as possible, that is \( w \). The optimality of this action can be seen by the negative sign of the first derivative of the upstream firm’s profit function with respect to \( w_{d,i} \).

\[
\frac{\partial \pi_{u,i}}{\partial w_{d,i}} = \frac{4(b + e)q_i}{k1} \frac{\partial q_i}{\partial w_{d,i}} < 0 \tag{33}
\]

(See equation (24)). Profit will be highest if the wage rate in the downstream firm will be lowest.

So the franchise fee will be set to:

\[
f_i = \left( p_{d,i} - p_{u,i} - \frac{w}{c} \right) q_i \tag{34}\]

The reaction functions are the same as in the W-FF scenario.

Equilibrium is described by:

\[
w_{d,i} = \bar{w} \tag{35}\]

\[
w_{u,i} = \bar{w} + \frac{k_6 \alpha \left( A - e \left( 1 + \frac{1}{e} \right) \bar{w} \right)}{(2 - \alpha)k_1b + 2k_6e} \tag{36}\]

\[
q_i = \frac{k_1(k_1 + 2e(2b + e))(2 - \alpha)(b + e) \left( A - e \left( 1 + \frac{1}{e} \right) \bar{w} \right)}{k_5[(2 - \alpha)k_1b + 2k_6e]} \tag{37}\]

The wages in the downstream firms do not depend on \( \alpha \), wages in the upstream firms increase with the bargaining power of the unions, consumer prices increase and quantities fall.

The W-FF Equilibrium Relative to the FF-W Equilibrium

- The wage in the downstream firm is higher in the W-FF than in FF-W scenario, because in the W-FF scenario the downstream firms do not resist to the wage demands of their unions. See equations (30) and (35).

- Upstream wage is lower in the W-FF than in the FF-W scenario as long as \( e > 0 \). See equations (31) and (36).

- Comparing equations (32) and (37) it can be seen that equilibrium output and therefore equilibrium profit is lower in the W-FF than in the F-WW scenario as long as \( e > 0 \).
2.3 Vertical Integration (IN)

The last scenario describes the situation, where firms integrate vertically. With integration only two firms are active and therefore only two wages are bargained. The profit function of firm $i$ is given by:

$$\pi_i = \left( p_{d,i} - \left( 1 + \frac{1}{c} \right) w_i \right) q_i$$

(38)

The price game results in the following equilibrium:

$$p_{d,i} = \frac{A}{b+2e} + \frac{\left( 1 + \frac{1}{c} \right) (b + e) (2(b + e)w_i + bw_j)}{(b+2e)(3b+2e)}$$

(39)

$$q_i = \frac{(b + e)A}{b + 2e} + \frac{\left( 1 + \frac{1}{c} \right) (b + e)(b(b + e)w_j - k_1 w_i)}{(b+2e)(3b+2e)}$$

(40)

As before the equilibrium profit is positively correlated with the firm’s output.

$$\pi_i = \frac{q_i^2}{b + e}$$

(41)

Wage Bargaining with Vertical Integration

The Nash function that must be maximized is:

$$\max_{w_i} \left( \left( 1 + \frac{1}{c} \right) q_i (w_i - \bar{w}) \right)^{\alpha} \pi_i^{1-\alpha}$$

(42)

From this the following reaction function can be derived:

$$w_i = \bar{w} + \frac{\alpha(3b + 2e)A}{2k_1 \left( 1 + \frac{1}{c} \right)} + \frac{\alpha b(b + e)w_j}{2k_1} - \frac{\alpha \bar{w}}{2}$$

(43)

Simultaneous wage bargainings in both firms result in the following equilibrium:

$$w_i = \bar{w} + \frac{\alpha(3b + 2e) \left( A - e \left( 1 + \frac{1}{c} \right) \bar{w} \right)}{\left( 1 + \frac{1}{c} \right) [(2 - \alpha)b(b + e) + 2e(3b + 2e)]}$$

(44)

$$q_i = \frac{k_1(2 - \alpha)(b + e) \left( A - e \left( 1 + \frac{1}{c} \right) \bar{w} \right)}{(b + 2e) [(2 - \alpha)b(b + e) + 2e(3b + 2e)]}$$

(45)

Again, wages and consumer prices increase with the bargaining power of the unions, and firms’ output fall.
3 Comparison

Now I can compare equilibrium profits of all four scenarios to find out which scenario gives the highest profit. In figure 1 profits for all four scenarios are displayed for varying product differentiation parameter $b$.

![Graph showing profit with varying $b$](image)

Figure 1: Profit with varying $b$

The dashed line in figure 1 shows the sum of the profits of the downstream and the upstream firm in the linear pricing scenario (LP), the crosses show profit of the franchise fee scenario where wage bargaining takes place before the fee is fixed (W-FF), and the circles show the profit of the franchise fee scenario, where the fee is fixed before wages are bargained (FF-W), and the solid line is the profit line for the integrated scenario (IN).

In figure 1 the parameters of the model are set to: $A = 10$, $c = 1$, $\bar{w} = 1$, $\alpha = 1/2$, and $e = 1$.

- As can be seen in figure 1 profit in scenario FF-W is higher than profit in the vertical integration scenario. Only when $b = 0$ profit is equal in both scenarios. This is the vertical separation result of Bonanno and Vickers (1988). As is well known, industry profit is lower in duopolistic markets than with a cartel, because consumer prices are lower in the more competitive case. Both firms could win with higher prices. With franchising the upstream duopolists will set...
their wholesale prices above marginal costs and therefore will push consumer prices up compared to integrated firms.

- Given the parameter $e$, linear pricing becomes preferable for profit owners with less product differentiation.

![Figure 2: Profit with varying $e$](image)

In figure 2 profit lines are shown for the four scenarios for varying parameter $e$. The parameters of the model are given by $A = 10$, $c = 1$, $\bar{w} = 1$, $\alpha = 1/2$, and $b = 4$.

- For a given degree of product differentiation profit is higher with linear pricing than with a franchise fee or with integration if the price elasticity of industry demand is low. With an increasing elasticity franchising (FF-W) and integration becomes preferable for the firms. If industry demand is completely inelastic the integration scenario yields the lowest profit.

In figure 3 profit is shown for the four scenarios for varying bargaining power of the union. The parameters of the model are set to: $A = 10$, $c = 1$, $\bar{w} = 1$, $b = 4$, and $e = 1$. 

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In figure 3 it can be seen, that the bargaining power of the unions is also decisive for the selection of the specific form of contract between the upstream and the downstream firm. With a higher $\alpha$ linear pricing becomes unprofitable.

**IN - LP:**

\[
\pi_{IN} = \pi_{LP} \quad \Rightarrow \quad \frac{1}{b+e} q_{IN}^2 = \left( \frac{1}{b+e} + \frac{(b+2e)(3b+2e)}{k_1(b+e)} \right) q_{LP}^2
\]

\[
\Rightarrow \quad q_{IN} = \left( 1 + \frac{(b+2e)(3b+2e)}{k_1} \right)^{\frac{1}{2}} q_{LP}
\]

When equilibrium profit is equal in the integrated and in the linear pricing scenario output is higher in the integrated scenario and consumer prices are lower.

Equation (47) depends only on the parameters $p$, $e$ and $\alpha$. Irmen (1997) shows that the scenario with the highest profit depends on the quotient $\mu = b/e$ in his model. With wage bargaining the bargaining power becomes an additional factor for this determination. The relevant solution can be found numerically for given $\alpha$. 

Figure 3: Profit with varying $\alpha$
With $b/e > \pi$ firms will prefer linear pricing to integration. This critical $\pi$ becomes larger with higher $\alpha$. For higher profit in the LP scenario than in the IN scenario products must be less differentiated for a fixed value of $e$. Competition must be higher, if unions are strong.

**LP - FF:**

\[
\pi_{LP} = \pi_{FF} \Rightarrow \left( \frac{1}{b+e} + \frac{(b+2e)(3b+2e)}{k_1(b+e)} \right) q_{LP}^2 = \frac{2(b+e)}{k_1} q_{FF}^2 \quad (48)
\]

\[
q_{LP} = \frac{b+e}{(k_1 + (b+e)^2)\frac{1}{2}} q_{FF} \quad (49)
\]

- When equilibrium profit is equal in the linear pricing and the franchise fee scenario where fee setting takes place before wage bargaining, output is higher in the franchise scenario and consumer prices are lower.

Again this equation depends only on $b$, $e$, and $\alpha$. The solutions for various values of $\alpha$ is given by:

\[
\begin{array}{|c|c|}
\hline
\alpha & \text{critical $\pi$} \\
\hline
0 & 0.761557182 \\
1/4 & 1.753037696 \\
1/3 & 2.173432665 \\
1/2 & 3.210262030 \\
2/3 & 4.613785055 \\
3/4 & 5.508620941 \\
1 & 9.401658345 \\
\hline
\end{array}
\]

As before, if $b/e > \pi$ firms will prefer linear pricing to franchise fee.

If the fee is set after wage bargaining, firms will prefer linear pricing under all conditions.
IN - FF:

Figure 2 shows that profit owners prefer franchising to integration whenever the franchise fee is fixed before wage bargaining takes place. But if wages are set before the fee, franchising will be preferred only if the products are not differentiated to much. The specific solutions are summarized in the following table.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\bar{\mu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12.55341034</td>
</tr>
<tr>
<td>1/4</td>
<td>12.68680868</td>
</tr>
<tr>
<td>1/3</td>
<td>12.74062715</td>
</tr>
<tr>
<td>1/2</td>
<td>12.86722349</td>
</tr>
<tr>
<td>2/3</td>
<td>12.86722349</td>
</tr>
<tr>
<td>3/4</td>
<td>13.12469551</td>
</tr>
<tr>
<td>1</td>
<td>13.52156852</td>
</tr>
</tbody>
</table>

The interpretation of the critical values of $\bar{\mu}$ is the same as before. Profit is lower in the IN scenario than in the W-FF scenario if $b/e > \bar{\mu}$.

References


