The Benefit of Information Reduction for Trading Strategies

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The Benefit of Information Reduction for Trading Strategies

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Abstract
Motivated by previous findings that discretization of financial time series can effectively filter the data and reduce the noise, this experimental study compares the trading performance of predictive models based on different modelling paradigms in a realistic setting. Different methods ranging from real-valued time series models to predictive models on a symbolic level are applied to predict the daily change in volatility of two major stock indices. The predicted volatility changes are interpreted as trading signals for buying or selling a straddle portfolio on the underlying stock index. Profits realized by this trading strategy are tested for statistical significance taking into account transactions costs. The results indicate that symbolic information processing is a promising approach to financial prediction tasks undermining the hypothesis of efficient capital markets.

1 Introduction

In informationally efficient capital markets, market prices are supposed to reflect all available information relevant for valuing securities. For derivative markets, this means that all relevant information should be concentrated in the volatility of the underlying asset which is the only unobservable parameter for standard contracts such as European call options (Figlewski, 1997). This fundamental importance of volatility has been documented in an enormous body of literature over the past decades. The main question addressed is the following one: Since volatility is not directly observable, what is a good measure of volatility that can be extracted, modelled and predicted?

There are two fundamental approaches: The first approach is based on historical information on the underlying asset which is usually the series of daily asset returns. Prominent and widely used historical volatility measures are the standard deviation of returns over some time window, an exponentially weighted moving average of squared returns and the conditional standard deviation of a return series model. The applied return series model can be parametric such as autoregressive conditional heteroskedastic (ARCH) models (Engle, 1982) and generalized ARCH (GARCH) models (Bollerslev, 1986), semiparametric (Engle and González-Rivera, 1991), seminonparametric (Schittenkopf et al., 2000) or nonparametric (Pagan and Schwert, 1990).

The second approach extracts information from derivatives traded at financial markets. The standard procedure is to extract volatilities implied by observed option prices assuming a Black-Scholes world (Black and Scholes, 1973) and to average these volatilities
by some weighting scheme. The resulting volatility measure is called an \textit{implied volatility measure}.

The discussion whether historical or implied volatility measures are better measures of future volatility, which is needed for derivative pricing, has been stimulated by quite controversial empirical results. Schmalensee and Trippi (1978), for instance, find that implied volatilities yield better forecasts of future return variability whereas, e.g., Canina and Figlewski (1993) report that implied volatilities have virtually no correlation with future volatilities. The basic problem revealed by these studies is that the true volatility is unobservable. Moreover, the best volatility estimation method may differ for different users. As emphasized by Figlewski (1997), option traders and market makers, for instance, usually have a quite different understanding of volatility.

Whatever volatility measure one is willing to apply, a central question is that of predictability of volatility. In particular, predictability in a statistical sense and economically meaningful predictability must be distinguished. The former notion of predictability is based on the exploitation of statistical dependencies in a time series to predict future values by means of a time series model. The notion of economically meaningful predictability refers to the fact that if volatility is predictable to some extent in a statistical sense, this knowledge need not necessarily be turned into abnormal profits by implementing some trading strategy at the market. Transactions costs may be so large and/or the extracted dependencies may be so weak that significantly positive profits cannot be obtained. Harvey and Whaley (1992), for instance, report that implied volatility changes of options on the S&P 100 are predictable to some extent, but that abnormal profits are not raised in a trading strategy which takes transactions costs into account. This supports the notion that the market for options on the S&P 100 is informationally efficient. Noh et al. (1994) report that abnormal profits can be achieved by trading straddles on the S&P 500 if volatilities are predicted by a GARCH model. For a regression model of implied volatility, the trading strategy loses money (after considering transactions costs). Dockner and Strobl (1999) take up this approach and apply it to Bund future options. They report that the trading strategy can be optimized to obtain abnormal profits after transactions costs. Schmitt and Kaehler (1996) implement a trading strategy based on straddles on the DAX. They find that profits are significant for market makers using historical volatility models but not for those using implied volatility models.

These results indicate that the issue of efficiency of capital markets is closely related to the issue of information processing by market participants. Let $\Phi_t$ denote the information set available to some market participants at time $t$. Let $Z_t \in \Phi_t$ be a quantity of interest to these market participants which contains important information with respect to their trading strategy. For instance, $Z_t$ could be a measure of volatility at time $t$. From the market participants’ point of view, the information processing task contains the prediction of $Z_{t+1}$ conditional on $\Phi_t$. However, prior to any prediction, the market participants should be absolutely sure about what they really must predict. For instance, it might happen that they just need to know the \textit{direction} of change in volatility, i.e., whether volatility will increase or decrease, and not the precise value of volatility on the next trading day. If this is the case, they are rather confronted with a classification task than with a prediction task. Anyway, the market participants must decide on their information set $\Phi_t$. In a univariate framework, the information set contains only the previous

\footnote{See, however, Andersen et al. (1999) for a model-free volatility measure which is also free of measurement errors under general conditions. This so-called realized volatility is supposed to be a good approximation of the true volatility.}
realizations $Z_n, Z_{n-1}, \ldots$ of the quantity or the corresponding approximations if the quantity itself, such as volatility, is unobservable. In the multivariate case, other sources of information can be included by the market participants to improve their predictions.

All of these issues heavily depend on the representation of information by the market participant. In practice, this is also important due to the fact that information is usually contaminated by noise. One approach to achieve an efficient representation and processing of market information is to switch to a symbolic level. Thereby the complexity of the data is reduced while most of the relevant information can be retained. We apply a discretization strategy transforming real-valued time series into symbolic streams as an effective complexity reduction tool.

The idea of discretizing economic and financial time series has already appeared in some studies (Apte and Hong, 1994, Bühlmann, 1998, Giles et al., 2000, Papageorgiou, 1997). Apte and Hong (1994) apply a minimal rule generation system to monthly S&P 500 data discretized by a special feature discretization subsystem. Bühlmann (1998) models the extreme events of returns of the Dow Jones Industrial Average and of volume of the New York Stock Exchange given their previous histories. The original return and volume series are discretized into streams over three ordinal categories (lower extreme, usual, upper extreme). Giles et al. (2000) consider a set of five time series of major exchange rates and predict directional changes in the exchange rates by applying recurrent neural networks. The historical real-valued directional changes are discretized into series of up to seven symbols using self-organizing maps (Kohonen, 1990). Papageorgiou (1997) builds predictive models to determine the direction of change in high-frequency exchange rate tick data. The real-valued exchange rate returns are discretized into nine symbols.

Generally, it is found that discretization of financial time series can effectively filter the data and reduce the noise. Furthermore, the symbolic nature of the preprocessed data often enables one to interpret the predictive models as lists of clear and intuitively appealing rules (Giles et al., 2000). However, these experimental studies, apart from Apte and Hong (1994), are not performed in a realistic trading setting. Moreover, it is sometimes not clear why a particular discretization is chosen or whether the chosen partition is optimal (with respect to the performance measure), neither in terms of the number of symbols nor in terms of the corresponding discretization intervals.

The aim of our paper is to compare the trading performance of predictive models based on different modelling paradigms in a realistic setting. In particular, several different methods ranging from real-valued time series models to predictive models on a symbolic level are applied to predict the change in volatility of two major stock indices. Volatility is measured by one historical and one implied measure based on daily and intraday data. The predicted volatility changes are interpreted as trading signals for buying or selling a straddle portfolio on the underlying index. The profitability of this volatility trading strategy is analyzed in a realistic setting with special emphasis on stationarity issues, the inclusion of transactions costs, and the statistical significance of the obtained results. Additionally, we analyze possibilities of combining different models to new predictors which, in most cases, leads to higher profits.

As a result, discretization of real-valued time series and modeling of the resulting symbolic sequences is a promising approach for prediction tasks in finance. For one measure of volatility of one of the stock indices, profits after substantial transactions costs are still so high that one might conclude that the corresponding option market is not fully efficient.

The paper is organized as follows. In Section 2 the data sets and the extracted
measures of volatility are described. Section 3 explains the trading strategy and the sliding window technique employed. Some properties of the volatility series on the real-valued and on the symbolic level are studied in Section 4. Section 5 gives a detailed description of the predictive models applied to the volatility series. In Section 6 the empirical results are presented and the significance of profits is discussed. Some concluding remarks are offered in Section 7.

2 Options Data and Volatility Measures

Two large data sets are analyzed in this study. First, we describe the set of daily options data on the DAX. Then we give some details on the set of intraday option contracts on the FTSE 100. Finally, the two measures of volatility are defined which are applied in the trading strategy described in Section 3.

2.1 DAX data set

The data set is a series of daily closing values of the German stock index DAX together with a series of daily closing prices of call and put options on the DAX with different maturities and exercise prices. In particular, the first in-the-money and the first out-of-the-money call and put option maturing next month are available. The at-the-money point is assumed to be the value of the DAX at that time\(^2\). Straddle prices are obtained by adding call and put prices. The series start on 22 August 1991 and end on 8 June 1998 which corresponds to a period of 1700 trading days. The time series of continuously compounded daily returns is depicted in the upper graph in Fig. 1.

2.2 FTSE 100 option contracts

This sample comprises transactions data of FTSE 100 option contracts traded at the London International Financial Futures Exchange (LIFFE). Intraday bid-ask prices of American options on the FTSE 100 between 29 May 1991 and 29 December 1995 are available. This time period corresponds to 1161 trading days. The option prices are recorded synchronously with the FTSE 100 and time-stamped to the nearest second. Since our trading strategy is set up on a daily basis, we must fix a reference point in time on each trading day. This reference point is 3 pm on normal trading days and 12 pm on days where the stock exchange closes earlier. The first quote of the FTSE 100 after the reference point is extracted for each day and used to calculate the historical volatility measure described in Section 2.3. In addition, the first quotes of call and put options maturing the next month with the same strike price as close as possible to the value of the FTSE 100 at that time are extracted for the actual trading day (and also for the next trading day). For these options, which are roughly at-the-money, the average of bid-ask quotes is calculated as an approximation of a reasonable option price. Then the prices of call and put options are added to obtain straddle prices. The time series of daily returns (between the reference points) is depicted in the lower graph in Fig. 1.

\(^2\) It would be probably more appropriate to take the value of the futures contract with the same maturity as the at-the-money point. This is left for future studies.
2.3 Volatility measures

As mentioned in the introduction, there are basically two notions of volatility in literature: historical volatility and implied volatility. We use one volatility measure of each notion in the trading strategy.

The historical measure is the popular approach where exponentially declining weights are given to past volatilities, approximated by squared returns. That is, the historical volatility is an exponentially weighted moving average of squared returns (EWMA-measure). For the DAX, the returns are calculated from the closing values of the index whereas the returns of the FTSE 100 are calculated from the value of the index at the reference point in time. Denoting the index values $s_t$ and the continuously compounded returns $r_t = \log(s_t/s_{t-1})$ the historical measure $v_t$ is defined as

$$v_t = (1 - \alpha) \sum_{\tau \leq t} \alpha^{t-\tau} r_\tau^2.$$  \hfill (1)

The weighting factor $\alpha \in (0,1)$ determines the impact of past returns on the actual volatility: The larger $\alpha$, the larger the impact and the longer the “memory”. Our choice was $\alpha = 0.9$. The measure defined in Eq. (1) can also be specified in a recursive fashion:

$$v_t = \alpha v_{t-1} + (1 - \alpha) r_t^2$$  \hfill (2)

with $v_0 = 0$. The historical measure $v_t$ is similar to the basic volatility measure applied in RiskMetrics™ and it is well-suited for the purpose of trading options on financial indices (Figlewski, 1997).
The implied volatility measure (IV-measure) is estimated from the extracted options’ volatilities implied by the Black-Scholes model (Black and Scholes, 1973). For the DAX, the average value of the implied volatilities of the first in-the-money and the first out-of-the-money call and put option is calculated. For the FTSE 100, the average value of the implied volatilities of the call and put option closest to the at-the-money point is used. In other words, the average value of four (two) implied volatilities is calculated as the IV-measure \( v_t \) for the DAX (FTSE 100) every day.

3 Trading Strategy and Stationarity Issues

We start with a description of the trading strategy. The basic strategy is very simple: *Every trading day, predict the change in volatility to the next trading day. If volatility is predicted to increase, buy near-the-money straddles, otherwise sell them.* On the next trading day, close the position and restart by predicting the next volatility change. In particular, the two volatility measures described in the previous section will be used to predict the “true” volatility changes, which are unobservable. This means that we will compare the performance of the trading strategy based on each measure and analyze which measure is more suitable for making profitable predictions.

The straddles bought or sold every day are near-the-money, i.e., the strike price closest to the at-the-money point is selected. Therefore, the straddle portfolio is approximately delta-neutral which means that there is no need to delta-hedge the position\(^3\). Every day, after predicting the direction of change in volatility, straddles worth a fixed amount of money are bought or sold. The choice of a fixed but otherwise arbitrary investment is intended to facilitate the interpretation of results with respect to transactions costs. Finally, only straddles maturing the following month are bought or sold, which avoids the influence of strong price movements towards the end of the contracts.

As mentioned in the introduction, an important issue in financial time series analysis is stationarity of the analyzed series. This issue is of particular relevance for trading strategies where the aim is to detect dependencies in the data which can be exploited to generate abnormal profits and which certainly change through time. A useful method to deal with non-stationarity of data is the sliding window technique. First of all, the length of the time window for which the data may be assumed to be stationary must be defined. After selecting the time window length, the following iterative procedure is applied: The models are estimated and evaluated on the first time window. Then the sliding window is shifted by one or several steps. The models are estimated and evaluated on the second time window and so on. The choice of the time window length is naturally a trade-off: On one hand, the time window should be large so that a reliable estimation of the model parameters is possible. On the other hand, the time window should be small to lower the probability of structural breaks in the time series within a time window.

In our study each time window consists of three parts: a training set, a validation set, and a test set. The training set is used to estimate the model parameters by minimization of an error function. The size of the training set is 500 which means that the volatility measure is collected over a period of roughly two years. In general, several representatives from a class of models are estimated which differ in the number of inputs used to predict the next volatility change. The performance of the estimated models with respect to

\(^3\) The sensitivities with respect to the interest rate and the time to maturity are negligible and inevitable, respectively.
profit on the validation set is the criterion for selecting the best model within its model class. The size of the validation set is 125 trading days (roughly half a year). Finally, the out-of-sample profit of the best model is determined on the test set which covers 5 trading days (one week). Then the sliding window with a total size of 630 days is shifted by 5 days, the models are reestimated etc. In particular, the test sets are non-overlapping and the obtained profits can be concatenated to form a large series of out-of-sample profits. Several model classes can be compared by means of their out-of-sample performance represented by the corresponding series of profits. The sliding window technique is illustrated in Fig. 2. More details on performance measures and on the evaluation of profits will be given in Section 6.

![Diagram](attachment:image.png)

Figure 2: The sliding window technique applied in the experimental studies: The training set, the validation set, and the test set of each time window cover 500, 125, and 5 trading days, respectively. The sliding window is shifted in steps of 5 days which yields non-overlapping test sets. The test sets are concatenated to form a large series of out-of-sample profits which is analyzed in blocks of 40 days.

4 Properties of the Volatility Series

In the previous section the issue of stationarity of financial time series was stressed and a sliding window technique was proposed to overcome this problem. For each time window, the task is to model and to predict the daily change in volatility. In principle, there are two possibilities: One can either model the volatility series and take differences to obtain
the volatility changes or one can model the volatility changes directly. The latter approach is preferable at least for the symbolic models since one can work with a binary alphabet. More precisely, the discretization of the series of volatility changes $\Delta v_t = v_t - v_{t-1}$ into sequences of symbols $s_t$ is performed by

$$ s_t = \begin{cases} 1 & : \Delta v_t \geq 0 \\ 0 & : \Delta v_t < 0 \end{cases} \quad (3) $$

An increase in volatility is thus represented by the symbol $s_t = 1$ whereas a decrease is represented by the symbol $s_t = 0$. For the real-valued models, working with the differenced volatility series can be motivated by stationarity issues: A Dickey-Fuller test applied to the volatility series indicates that the null hypothesis of a unit root\(^4\) is not rejected at the 1\% significance level for the series of IV-measures. For the series of EWMA-measures, the null hypothesis is rejected. However, it is not rejected if the test is applied to single time windows in the second half of the time series. Since stationarity of the series is crucial for model estimation, we decided to work with the differenced series of the volatility measures.

Taking the differences also changes the crosscorrelation between the series of volatility measures. For the DAX (FTSE 100) data set, the crosscorrelation between the EWMA-measure and the IV-measure is 0.792 (0.526) which is significantly larger than 0. This result is plausible since both measures quantify the volatility of the underlying index and they are therefore expected to be related in some way. For the differenced series, the crosscorrelations decrease to 0.231 and 0.029, respectively, where the latter value is not significantly positive. Looking at the information available on the symbolic level, the difference between the EWMA-measure and the IV-measure is also substantial. Only on 53\% (51\%) of all trading days, the realized direction of change in volatility (increase or decrease) of the DAX (FTSE 100) is the same for both measures. Therefore, it may be expected that profits realized by the trading strategy vary substantially with the applied measure of volatility changes.

It is also interesting to look at the time series properties of the differenced series as a function of the time window position. As an example, the statistical dependencies in the changes of the IV-measure for the FTSE 100 are depicted in Fig. 3. In the upper graph, only the significant autocorrelations of the real-valued series (up to lag 5) are depicted for each time window. The autocorrelation at lag 1 is negative for all time windows. For higher lags, the significance of the autocorrelations changes as the time window is shifted. For instance, at the beginning of the sample period there is a negative autocorrelation at lag 3, in the middle the autocorrelation is negative at lag 2, and at the end the autocorrelation becomes positive at lag 4. Harvey and Whaley (1992) also report negative autocorrelations at lag 1 and 2 for a series of implied volatility changes of the S&P 100.

In the lower graph, the statistical dependencies of the symbolic series are depicted. The dependencies of the symbolic series are measured for each time window by means of mutual information (Cover and Thomas, 1991). The mutual information between symbol sequences is a concept from information theory and it measures how much information about one symbol sequence is contained in the other one. In our case, the mutual information between the last (the last two, the last three) symbols and the next symbol is

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\(^4\)The existence of a unit root implies that the time series is not stationary. In this case one usually calculates the differences of the time series, possibly several times, to obtain a stationary series. In our case all time series had to be differenced once.
measured. Since we have a binary sequence, the mutual information (in bits) is between 0 and 1. The former case corresponds to complete randomness\textsuperscript{5} and the latter to perfect predictability\textsuperscript{6}. Fig. 3 shows that the mutual information increases with the number of past symbols in accordance with information theory: The more is known about the past symbol sequence, the easier it is to predict the next symbol. Like the autocorrelation, the mutual information changes with the position of the time window.

![Figure 3: Statistical dependencies in the changes of the IV-measure for the FTSE 100: In the upper graph, a significant autocorrelation of the real-valued series is indicated by '×' for lag 1, '○' for lag 2, '◇' for lag 3, '△' for lag 4, and '+' for lag 5 for each time window. In the lower graph, the mutual information between the next symbol and the last symbol (dotted), the last two symbols (dashed), and the last three symbols (solid) is plotted for each time window for the symbolic series.](image)

Similar results are obtained for the EWMA-measure for the FTSE 100 and for both volatility measures for the DAX. This indicates that both the real-valued and the symbolic series of volatility changes contain dependencies that can be modeled to predict future volatility changes. Since the dependencies may vary considerably over time, methods like the sliding window technique should be employed. Finally, we emphasize that this section is intended to illustrate some of the properties of the time series of volatility changes. The insights gained did not influence our choice of structure, size or other features of the predictive models described in the next section.

\textsuperscript{5} One can interpret the symbol sequence as the outcome of repeatedly and independently flipping a coin.

\textsuperscript{6} The next symbol can be perfectly predicted knowing the past symbols.
5 Predictive Models

In this section the basic time series models used to predict future volatility changes are
described. These models are called basic models since we will also study the performance of
compound models which are introduced later. Repeating the notation of the previous
sections, returns, volatilities and volatility changes (on a daily basis) are denoted \( r_t, v_t \)
and \( \Delta v_t \), respectively.

The predictive models analyzed can be divided into four classes: real-valued models,
symbolic models, mixed models, and GARCH models. The real-valued models are
estimated on the series of volatility changes \( \Delta v_t \). In particular, linear autoregressive models
(AR models) and non-linear neural networks (NN models) are studied. The AR model of
order \( p \) is given by

\[
\Delta \hat{v}_t = a_0 + \sum_{j=1}^{p} a_j \Delta v_{t-j} \tag{4}
\]

where \( \Delta \hat{v}_t \) is the predicted change in volatility. The order of the models fit to the time
series is \( p = 1, 3, 6, 10 \). The neural networks are multi-layer perceptrons, or more precisely,
two-layered feedforward networks with a non-linear hidden layer and a linear output layer.
If the number of inputs, i.e., the order of the model in a time series context is \( p \), the
mapping of the NN models is given by

\[
\Delta \hat{v}_t = b_0 + \sum_{i=1}^{n_h} b_i \tanh \left( a_{x0} + \sum_{j=1}^{p} a_{ij} \Delta v_{t-j} \right) \tag{5}
\]

where \( n_h \) denotes the number of hidden units. It is a well-known theoretical result that
NN models of this type can approximate any smooth function with arbitrary accuracy, as
\( n_h \) goes to infinity (see, e.g., Hornik et al. (1989)). The order of the fitted models is again
\( p = 1, 3, 6, 10 \). Both, AR and NN models are estimated by minimizing the mean squared
error (MSE) of the predicted volatility changes \( \Delta \hat{v}_t \). Denoting the number of predictions
\( N \), the MSE is given by

\[
\text{MSE} = \frac{1}{N} \sum_{i=1}^{N} (\Delta v_i - \Delta \hat{v}_i)^2. \tag{6}
\]

The symbolic models operate on discretized sequences over the binary alphabet \( \mathcal{A} = \{0, 1\} \) derived
from the real-valued series of volatility changes \( \Delta v_t \) as described in the previous section. The simplest symbolic models which can be fitted to symbolic
sequences are Markov models of fixed order. Let \( s_t \) be the next symbol to be predicted and let \( w_{t-1,p} = s_{t-p} \ldots s_{t-1} \) be the block of the last \( p \) observed symbols \( (w_{t-1,p} \in \mathcal{A}^p) \). If \( \mathcal{P}_p \) denotes the true probability distribution of the blocks of \( p \) symbols, the best prediction
of the next symbol (within the class of Markov models of order \( p \)) is given by

\[
\hat{s}_t = \arg \max_{s_t} \frac{\mathcal{P}_{p+1}(w_{t-1,p} s_t)}{\mathcal{P}_p(w_{t-1,p})}. \tag{7}
\]

This means that the best prediction of the next symbol \( s_t \) is the symbol \( \hat{s}_t \) which maximizes
the conditional probability of the next symbol given the last \( p \) symbols. In other words,
for Markov models of order \( p \) only the last \( p \) symbols are relevant for making predictions
about the next symbol. Theoretically, an increase in order increases the probability of
predicting the next symbol correctly (or leaves the probability unchanged). In practice
however, this is true only up to some maximum order: Since the length of the observed
symbol sequence is always finite and since the number of possible binary symbol sequences of length $p$ grows at a rate of $2^p$, the probabilities $P_p$ cannot be estimated reliably for large $p$. In our experiments we study the performance of a class of low-order Markov models (MM-low) with order $p$ between 0 and 5 and a class of high-order Markov models (MM-high) where the order is between 0 and $10^7$.

As mentioned in the introduction, it is also possible to discretize the series of real-valued volatility changes into sequences over larger alphabets $\mathcal{A}$. However, this is more involved since additional discretization levels (besides 0) must be determined. We will comment on this issue in Section 6.

The third class of models are mixed models which combine aspects of both model classes presented above. The mixed model of order $p$ is a two-layered feedforward network as above but now there are two non-linear output units. The inputs to the network are the last $p$ real-valued volatility changes (as for the NN model). The parameters of the network, however, are optimized to produce the network output $(1,0)$ whenever volatility increases and $(0,1)$ whenever volatility decreases. With a slight abuse of notation the target can be written as $(s_t, 1 - s_t)$. The network is trained to fit the “symbolic” target which corresponds to the actual volatility change $\Delta v_t$. The non-linearity in the output layer is chosen as $\sigma(x) = 1/(1 + e^{-x})$ which guarantees outputs in the unit interval $(0, 1)$. The mapping for each network output $y_{k,t}, k = 1, 2$, is given by

$$y_{k,t} = \sigma \left( b_{k0} + \sum_{i=1}^{m_k} b_{ki} \tanh \left( a_{i0} + \sum_{j=1}^{p} a_{ij} \Delta v_{t-j} \right) \right). \tag{8}$$

The order of the fitted models is again $p = 1, 3, 6, 10$. Neural networks of this type are classifiers and they are usually optimized by minimizing the cross-entropy error (CEE) (Bishop, 1995)

$$\text{CEE} = -\frac{1}{N} \sum_{t=1}^{N} \left[ \hat{s}_t \log (y_{1,t}(1 - y_{2,t})) + (1 - \hat{s}_t) \log ((1 - y_{1,t})y_{2,t}) \right] \tag{9}$$

which is also used in our numerical studies. If $s_t = 0$, the first term in the sum vanishes and the network output which minimizes the contribution to the CEE is $(0, 1)$. If $s_t = 1$, the second term vanishes and the desired network output is given by $(1, 0)$. After the training of the network the direction of the volatility changes is predicted by comparison of the network outputs $y_{1,t}$ and $y_{2,t}$: The predicted symbol $\hat{s}_t$ is 1, if $y_{1,t} > y_{2,t}$, and 0 otherwise.

Finally, GARCH models (Bollerslev, 1986) are included as a benchmark model since they have been used in related studies (Noh et al., 1994, Schmitt and Kaeckler, 1996). In contrast to the predictive models discussed so far, the GARCH models are estimated from the series of daily returns $r_t$. The conditional variance of the return series model is then interpreted as the volatility of the underlying stock index. In the numerical studies GARCH models with a conditional normal distribution (Bollerslev, 1986) and GARCH models with a conditional $t$-distribution (Bollerslev, 1987) are estimated. The conditional mean $\mu_t$ is modeled as a linear function of the previous return, i.e., $\mu_t = a_0 + a_1 r_{t-1}$.

\footnote{A Markov model of order 0 counts the number of symbols (0 and 1) in the training set and constantly predicts the more frequent symbol.}
The conditional variance is taken from the standard GARCH(1,1) specification:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$  \hspace{1cm} (10)

with $\epsilon_{t-1} = r_{t-1} - \mu_{t-1}$. For the models with a $t$-distribution, there is an additional parameter $\nu$ which determines the shape of the distribution. The parameters of both models are estimated within the maximum likelihood framework: Let $\rho(r_t; \mu_t, \sigma_t^2)$ be the conditional density of $r_t$ (with mean $\mu_t$ and variance $\sigma_t^2$) specified by the models. Then the parameters are estimated by minimizing the average negative log-likelihood function

$$LH = -\frac{1}{N} \sum_{i=1}^{N} \log \rho(r_i; \mu_t, \sigma_t^2).$$  \hspace{1cm} (11)

For the GARCH models, volatility changes are predicted as $\Delta \hat{\nu}_t = \sigma_t^2 - \sigma_{t-1}^2$.

As mentioned in Section 3, the performance (profit) of the models on the validation set is the criterion for selecting the best model within its model class (AR models, NN models, MM-low, MM-high, mixed models, GARCH models). The best model of each class is then used to predict volatility changes out-of-sample (on the test set). For the NN models and the mixed models, the potential problem of getting stuck in local minima of the error surface is reduced by repeating the model training ten times with different random initializations of the model parameters. The ten resulting models are combined to a committee of models where the individual predictions are combined to a single prediction by majority voting. This means that if at least six models predict an increase (a decrease) of volatility, the committee also predicts an increase (a decrease). If there is no majority for any direction, trading is suspended for that particular day.

6 Experimental Studies

First, we describe two additional models which are not estimated from training data in Section 6.1. The procedure for testing the statistical significance of profits is outlined in Section 6.2. Section 6.3 contains the empirical results for the different model classes for the DAX and the FTSE 100 data sets. Finally, the performance of compound models is studied in Section 6.4.

6.1 Additional models

In the experimental studies the performance of two additional models is evaluated. The reason why these models were not introduced in Section 5 is that they are not estimated from training data. The first one is called the perfect model since it provides the theoretical profit which could be achieved if all predictions were perfect. In other words, it is assumed that the change in volatility (for each volatility measure) is known a priori for all trading days. The performance of the perfect model thus yields an upper bound on the profit which can be obtained theoretically by predicting the volatility change according to some volatility measure.

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8 Due to the recursive specification, the GARCH(1,1) model takes into account the variances of all previous time steps. It is usually sufficient to fit a GARCH(1,1) model since the additional parameters of higher-order GARCH models are usually not significant.

9 It is also possible to apply other voting or combination schemes. For instance, the predictions of the networks can be averaged with weighting factors according to the model likelihood calculated on the validation set. The analysis of this more Bayesian approach is left for future research.
The second model, which is called simple, is a collection of four fixed predictors: \textit{Always predict an increase, Always predict a decrease, Copy the last change, Revert the last change} (of volatility). The four predictors operating on the symbolic sequences are Markov models: The former two models are of order 0 and the latter two are of order 1. The models are evaluated with respect to profit on the validation set. The best model is then applied to the test set. The reason for including the simple model into this study is the observation that in financial prediction tasks, simple models often outperform more sophisticated approaches.

6.2 Significance of profits

The most natural performance measure for any model with respect to the trading strategy is the average profit per day calculated from the sequence of out-of-sample profits. As an example, the realized profits of the simple model on the IV-measure of the FTSE 100 are depicted in Fig. 4. The accumulated out-of-sample profit of this model over the 525 trading days (125 test sets of 5 trading days) is 851\% (upper curve). The average profit per day is thus 1.62\% which is indicated by the upper straight line. Looking at these graphs one is tempted to claim that these profits are substantial. After subtraction of transactions costs of, say, 1\% per day, the obtained profits (depicted in the lower curves) still look promising. However, an investor observing profits only over the first 60 days (depicted in the figure inserted on the upper left-hand side) would probably be quite unsure about the profitability of the trading strategy: After 60 days the accumulated profit is about 90\% but after 40 days it is still -3\%!

![Figure 4: Accumulated out-of-sample profits of the simple model on the IV-measure of the FTSE 100. The upper curve shows the profit without transactions costs. The average profit per day is indicated by the upper straight line. The two lower curves represent the profit and the average profit after subtraction of transactions costs of 1\% per day. The figure inserted on the upper left-hand side is an enlargement of these curves over the first 60 trading days.](image-url)
The standard approach to deal with this uncertainty about profits is to apply statistical tests. In related works (Harvey and Whaley, 1992, Noh et al., 1994, Schmitt and Kaeehler, 1996) a $t$-test is used to test whether profits are significantly positive or not. However, the $t$-test relies on two crucial assumptions: First, profits are assumed to be independent. Although the test sets are non-overlapping, some dependencies might be introduced because the training sets and the validation sets are overlapping. Since we did not find significant correlations in the series of daily profits, the assumption of independent profits is likely to be correct. The second assumption underlying the $t$-test is that the profits are normally distributed. Fig. 5 shows a histogram of the profits of the simple model for the IV-measure of the FTSE 100 together with the (appropriately scaled) normal density with the same mean and the same standard deviation as the sample of profits. Obviously, the distribution of profits is far from a normal distribution\footnote{A Jarque-Bera test rejects the null hypothesis of normally distributed profits at any reasonable significance level.}. In particular, the distribution has fat tails which means that large positive and negative profits are observed much more frequently than predicted by the normal distribution: For instance, the largest profit is about 34\%. The probability to observe a profit of this size (or a larger one) over the out-of-sample period of 525 trading days is about 0.01\% under the assumption of normally distributed profits. This assumption is thus hardly valid and a $t$-test should not be applied. Similar histograms are also obtained for the other models.

![Histogram of the profits of the simple model (in percent) for the IV-measure of the FTSE 100 together with the (appropriately scaled) normal density with the same mean and the same standard deviation.](image)

**Figure 5:** Histogram of the profits of the simple model (in percent) for the IV-measure of the FTSE 100 together with the (appropriately scaled) normal density with the same mean and the same standard deviation.

Since $t$-tests are not suited due to their specific assumption about the underlying distribution, one could think of using a nonparametric test such as the Wilcoxon test (Press et al., 1992). However, the Wilcoxon test calls for a transformation of the series of profits into a series of ranks meaning that the information about the absolute size of the profits is lost: For instance, it makes no difference if the largest profit is 20\% or 200\%. This property is rather undesirable for the analysis of profits of a trading strategy and
therefore we decided not to use the Wilcoxon test either.

We tested for significance of the profits obtained from the trading strategy by applying the following procedure: As described in Section 3, the profits on the individual test sets are concatenated to form a large series of out-of-sample profits. This series is divided into distinct blocks of length 40 which corresponds to a period of roughly 2 months (see Fig. 2). Due to the central limit theorem which holds under rather general conditions, the sum of profits and therefore also the average block profit, i.e., the average profit over a block of 40 trading days can be assumed to be normally distributed\(^{11}\). Hence we can subject the series of average block profits to \(t\)-tests\(^{12}\). The number of out-of-sample profits for the DAX and the FTSE 100 data set is 1060 and 525, respectively. Consequently, the number of average block profits for the DAX and the FTSE 100 data set is 26 and 13, respectively\(^{13}\).

6.3 Empirical results

Tables 1 and 2 contain the results obtained for the basic models described so far for the DAX and the FTSE 100 data sets, respectively. In particular, the following statistics of the EWMA-measure and the IV-measure are reported: The first and the second column give the mean and the standard deviation of daily profits\(^{14}\). The third column gives the highest transactions costs (HTC) in percent that may be subtracted from the profits on a daily basis so that the average block profit is still significantly positive (under a \(t\)-test with a 5\% significance level). This means that the reported transactions costs are assumed to be total daily transactions costs for buying and selling a straddle portfolio. For transactions costs below 0.01\% the table entry is '-', in the fourth column, the percentage of blocks (PB) with a positive average profit is reported.

This approach to the inclusion of transactions costs is intended to take into account the fact that market participants are subject to different levels of transactions costs. For instance, according to information provided by the German Futures and Options Exchange (Deutsche Terminbörse), normal transactions costs (for traders) are about 0.5\% per straddle. However, market makers only pay about 0.1\% and investors who are not members of the exchange face transactions costs of roughly 1\% (Schmitt and Kaehler, 1996). In addition, investors who are trading on a daily basis (as in our setup) and who are willing to invest potentially large amounts of money usually negotiate special discount rates for transactions costs at the stock exchange.

We stress that our assessment of profits is rather pessimistic since the straddle portfolio is bought and sold every day regardless of future volatility changes. For instance, if the portfolio was purchased yesterday due to a predicted increase in volatility and the volatility is again predicted to increase today, it is more profitable to simply keep the portfolio than to sell it and buy it again afterwards. This refinement of the trading strategy would thus reduce transactions costs and thereby raise profits.

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\(^{11}\) Although the central limit theorem provides an asymptotic result, the normal distribution is usually approached very fast in practice as the number of independent random variables (profits) increases. In general, 30 to 40 random variables are assumed to be sufficient.

\(^{12}\) After calculating the average block profits for the profits depicted in Fig. 5, a Jarque-Bera test does not reject the null hypothesis of a normal distribution at any reasonable significance level.

\(^{13}\) The last block contains more than 40 profits for both data sets.

\(^{14}\) The actually invested amount of money is not specified in the experiments since profits are calculated and reported in percentages of the investment.
Table 1: Trading performance of the basic models on the DAX

<table>
<thead>
<tr>
<th>Model</th>
<th>EWMA-Measure</th>
<th>IV-Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.</td>
</tr>
<tr>
<td>Perfect</td>
<td>1.82</td>
<td>4.25</td>
</tr>
<tr>
<td>GARCH</td>
<td>0.10</td>
<td>4.38</td>
</tr>
<tr>
<td>AR</td>
<td>-0.02</td>
<td>4.42</td>
</tr>
<tr>
<td>NN</td>
<td>0.15</td>
<td>4.31</td>
</tr>
<tr>
<td>Mixed</td>
<td>0.11</td>
<td>4.14</td>
</tr>
<tr>
<td>Simple</td>
<td>0.41</td>
<td>4.42</td>
</tr>
<tr>
<td>MM-low</td>
<td>0.25</td>
<td>4.32</td>
</tr>
<tr>
<td>MM-high</td>
<td>0.33</td>
<td>3.93</td>
</tr>
</tbody>
</table>

Table 2: Trading performance of the basic models on the FTSE 100

<table>
<thead>
<tr>
<th>Model</th>
<th>EWMA-Measure</th>
<th>IV-Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.</td>
</tr>
<tr>
<td>Perfect</td>
<td>1.79</td>
<td>6.38</td>
</tr>
<tr>
<td>GARCH</td>
<td>0.00</td>
<td>6.63</td>
</tr>
<tr>
<td>AR</td>
<td>-0.16</td>
<td>6.63</td>
</tr>
<tr>
<td>NN</td>
<td>-0.12</td>
<td>6.08</td>
</tr>
<tr>
<td>Mixed</td>
<td>-0.02</td>
<td>6.48</td>
</tr>
<tr>
<td>Simple</td>
<td>0.30</td>
<td>6.62</td>
</tr>
<tr>
<td>MM-low</td>
<td>0.34</td>
<td>6.62</td>
</tr>
<tr>
<td>MM-high</td>
<td>0.45</td>
<td>6.28</td>
</tr>
</tbody>
</table>

The following main observations can be made based on the experimental results in Tables 1 and 2. The profits gained by the perfect model show that, theoretically, predicting daily volatility changes provides a good basis for automatic trading strategies buying and selling straddles: For both volatility measures and both data sets, average daily profits are significantly positive even after substantial transactions costs of more than 1%. The percentage of blocks with positive average profit is not always 100%. For the IV-measure of the DAX, the PB is 88% pointing to potential difficulties concerning the reliable estimation of implied volatilities which is discussed below. For the FTSE 100 data set, the PB is 92% for the EWMA-measure. This might indicate that the EWMA-measure does not fully reflect the market’s notion of volatility. Nevertheless, the profits of the perfect model show that for both volatility measures of both indices in theory considerable profits can be realized by the implemented trading strategy.

The performance of the GARCH model is disappointing\(^{15}\). The average profits are close to zero which is, on average, obtained by applying the random strategy where a coin is flipped every day to decide whether straddles are bought or sold.

Surprisingly, the performance of the real-valued models (AR, NN) and of the mixed

\(^{15}\) For the GARCH model, the entries for the EWMA-measure and the IV-measure are the same since the model is trained on the return series and not on the series of volatility changes.
model is not better for the EWMA-measure. Sometimes even negative profits are obtained. The results for the simple model and for the Markov models are better in the sense that the average profits are higher. However, the standard deviation of the profits is so high that only for very low transactions costs and only on the DAX data set profits might be significantly positive. Therefore, if volatility is measured by the EWMA-measure, economically meaningful predictions are hardly possible in the presented setup. The PB of the best models is about 70-80%.

For the IV-measure, the performance of the models on the FTSE 100 data set is substantially different from the performance on the DAX data set. For the DAX data set, the results are similar to the ones for the EWMA-measure described above. This is in total contrast to the FTSE 100 data set where the trading strategy is much more profitable for all models. One possible explanation for this observation is that the DAX data set consists of closing prices and closing values. As repeatedly mentioned in literature, the closing prices of options and the corresponding closing values of the underlying index are potentially recorded asynchronously, which may blur the implied volatility estimates substantially. Therefore, the FTSE 100 data set which consists of intraday option prices recorded at the same time as the underlying index provides a more realistic setup.

The best performance is achieved by the mixed model where transactions costs of up to 1.15% yield significantly positive profits. While the average profit is somewhat higher for the simple model, the lower standard deviation of the profits of the mixed model results in a higher value for the highest transactions costs (HTC). The performance of the Markov models is slightly worse but again better than the performance of the real-valued AR and NN models. For the Markov models, deeper memory does not necessarily lead to better models and higher profits. The high-order models (MM-high) are prone to overfitting since each training set contains only 500 samples. The higher predictability of the IV-measure for the FTSE 100 is also reflected in the percentage of blocks with positive average profit. For most models, the PB is 92% meaning that for all but one block, the average profit is positive.

We also estimated Markov models up to fifth order from sequences over an alphabet with four symbols. For this purpose, the series of volatility changes were discretized into Extreme Increase, Increase, Decrease, and Extreme Decrease. Optimal cut values separating Extreme Increase and Increase as well as Decrease and Extreme Decrease were determined from profits on the validation set. However, this discretization scheme lead to worse results than the binary scheme due to the restricted length of the training sequences.

For the IV-measure and the FTSE 100 data set, all models except the GARCH model produce significant profits after substantial transactions costs. For the most profitable models, an average daily profit of about 0.5% remains even after transactions costs of 1% per day. On a yearly basis, the profit of these strategies is thus about 125%. This seems to indicate that the option market for FTSE 100 index options is not fully efficient. Economically meaningful predictions are possible if the volatility of the FTSE 100 index is measured by the IV-measure.

The main insight from the experimental results is the fact that discretization of the time series of volatility changes for modelling and prediction tasks seems to be beneficial. The corresponding profits obtained from the trading strategy are higher than those of the real-valued time series models in nearly all cases. However, as typical in financial prediction tasks, it is difficult for complicated models to beat simple-minded predictors such as the simple model. Therefore, simple-minded predictors should always be considered in studies like this one.
6.4 Compound models

A potentially more powerful prediction strategy is to combine the simple model with any "sophisticated" model (AR, NN, mixed, MM-low, MM-high)\(^{16}\). For each combination, a natural criterion for selecting between the two candidates is the profit achieved on the validation set. The main reason why this approach could be interesting is that the simple model is not fitted to actual training data in contrast to all other mentioned models: If the data are stationary over the particular time window, the "sophisticated" model can extract relevant information from the training data and produce reliable predictions. If the data are not stationary, however, the simple model determined only by the profit on the validation set might be selected and yield higher profits on the test set.

The results for the compound models on the DAX and on the FTSE 100 data set are summarized in Tables 3 and 4. For most combinations of models, the average profit is higher than for the "sophisticated" model applied alone. For the DAX data set, the combination of the high-order Markov model and the simple model generates higher profits than the simple model, which is the best single model, for both volatility measures. At the same time higher transactions costs can be subtracted. On average, the PB is higher for the combined models. For the FTSE 100 data set and the EWMA-measure, the best compound model is also obtained for the high-order Markov model yielding higher profits. However, profits are still not significantly positive even by ignoring transactions costs. For the IV-measure, the combination of models is not always beneficial. The profits and HTCs of the AR and NN model raise, they remain constant for the Markov models and they decrease for the best single model, i.e., for the mixed model. The PB is roughly the same as for the basic models. All together, the combination of the simple model with the Markov models produces the best results on average.

<table>
<thead>
<tr>
<th>Model</th>
<th>EWMA-Measure</th>
<th>IV-Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.</td>
</tr>
<tr>
<td>AR</td>
<td>0.36</td>
<td>4.41</td>
</tr>
<tr>
<td>NN</td>
<td>0.38</td>
<td>4.42</td>
</tr>
<tr>
<td>Mixed</td>
<td>0.41</td>
<td>4.41</td>
</tr>
<tr>
<td>MM-low</td>
<td>0.44</td>
<td>4.42</td>
</tr>
<tr>
<td>MM-high</td>
<td>0.49</td>
<td>4.39</td>
</tr>
</tbody>
</table>

It is also interesting to analyze the results by plotting and comparing characteristics of the selected models as the time window is shifted. In Fig. 6, for example, the evolution of the memory depths of the Markov models MM-low and MM-high fitted to the IV-measure of the DAX data set is depicted (upper graph). For each trading day, the order of the selected Markov model is plotted. The simple-minded strategies Always predict an increase (I), Always predict a decrease (D), Copy the last change (C) and Revert the last change (R), which are selected on the validation set, are shown in the lower graph. Models from the class MM-high tend to use either shallow memory (order 0–1) or rather deep memory (order 8–10). Only few intermediate memory lengths are used. There also seems to be a correlation between memory depths selected for low-order Markov models

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\(^{16}\) The GARCH model is not considered due to its poor performance.
Table 4: Trading performance of the compound models on the FTSE 100

<table>
<thead>
<tr>
<th>Model</th>
<th>EWMA-Measure</th>
<th></th>
<th>IV-Measure</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.</td>
<td>HTC</td>
<td>PB</td>
</tr>
<tr>
<td>AR</td>
<td>0.27</td>
<td>6.62</td>
<td>-</td>
<td>54</td>
</tr>
<tr>
<td>NN</td>
<td>-0.03</td>
<td>6.36</td>
<td>-</td>
<td>54</td>
</tr>
<tr>
<td>Mixed</td>
<td>0.07</td>
<td>6.58</td>
<td>-</td>
<td>38</td>
</tr>
<tr>
<td>MM-low</td>
<td>0.37</td>
<td>6.61</td>
<td>-</td>
<td>62</td>
</tr>
<tr>
<td>MM-high</td>
<td>0.55</td>
<td>6.44</td>
<td>-</td>
<td>62</td>
</tr>
</tbody>
</table>

and strategies selected in the simple model. In periods dominated by Markov models of orders 1 and higher (days 1–70 and 700–970), the strategy *Always predict an increase* is selected most often. In periods dominated by memoryless Markov models (order 0), the strategies *Always predict a decrease* and *Copy the last change* seem to be preferred. The strategy *Revert the last change* is selected very rarely.

Figure 6: Characteristics of low-order and high-order Markov models and of simple models for the IV-measure of the DAX data set for each trading day. In the upper graph, the order of the Markov models is plotted (solid line: low-order Markov models, dashed line: high-order Markov models). The lower graph is a plot of the selected simple strategy.

7 Conclusions

The central conclusion from this experimental study is that discretization of financial time series can effectively filter the data and reduce the noise. In a realistic setting, where straddles on major stock indices are bought or sold depending on whether volatility
is predicted to increase or decrease, symbolic models of volatility changes on average generate higher profits than various real-valued models trying to predict volatility changes as precisely as possible.

The symbolic models are particularly simple since they are implemented over binary sequences. The only information represented in these sequences are increases and decreases of volatility, which must be estimated from the available data. As typical in financial prediction tasks, a simple model, which in our case selects one out of four simple rules to predict an increase or decrease of volatility, produces results, i.e., profits which are not easily improved by more sophisticated approaches. The four simple rules are Markov models of order zero and one. On average, a careful combination of these simple rules with high-order Markov models produces the best results. This may indicate that two memory regimes characterized by shallow and deep memory dominate the studied series of volatility changes.

Two markets are analyzed: the German market represented by the DAX and the British market represented by the FTSE 100 index. For both volatility measures of the DAX, economically meaningful predictions resulting in abnormal profits are only obtained for very low transactions costs. The market of DAX options thus tends to be informationally efficient. One source of uncertainty about the rigidity of this statement is the fact that only closing prices were available which could blur the implied volatility estimates. This point is supported by the results for the intraday FTSE 100 options data. For the IV-measure, all models except the GARCH model generate abnormal profits after substantial transactions costs. In other words, the market of FTSE 100 index options does not seem to be fully efficient.

For the future, it is planned to extend the presented results into several directions. First, one can analyze the profitability of the trading strategy with respect to other notions of volatility (e.g., with respect to the notion of realized volatility (Andersen et al., 1999)). Secondly, despite the discouraging results with Markov models over a four-symbol alphabet, it seems worthwhile to redo the experiments with a three-symbol alphabet. In this setup, the third symbol (besides 1 for Increase and 0 for Decrease) is intended to represent small changes in the level of volatility which do not really represent buying or selling signals. The third symbol can thus be interpreted as No interesting change. Finally, it is planned to compare the presented models to other simple models which are motivated by market phenomena (for instance, volatility is likely to increase if the underlying index decreases).

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