Thomas Grandner
Product differentiation in a linear city and wage bargaining
Paper

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Abstract — Economides (1986) has shown that within a linear city an equilibrium exists in a two-stage location-price game when the curvature of the transportation cost function is sufficiently high. One important point is that not all of these equilibria are at maximal differentiation. In this paper we include an additional stage with decentralized wage bargaining. This intensifies price competition resulting in locations that are nearer to the extremes of the city. The magnitude of this effect depends on the bargaining power of the unions.

Keywords: linear city; product differentiation; wage bargaining

JEL-Classification: L13; J51
Hotelling’s (1929) article was the starting point of an intense discussion on optimal location in a heterogeneous product space. Hotelling claimed that firms will locate in the middle of a linear city to cover a hinterland as large as possible. Many authors criticised the principle of minimal differentiation (for early contributions see Lerner and Singer 1937 or Smithies 1941), others supported Hotelling’s view (for example Chamberlin 1948). Only in 1979, d’Aspremont, Gabszewicz and Thisse have shown that Hotelling’s argument is invalid, because a pure strategy price equilibrium does not exist in his case. They slightly transformed the model by introducing quadratic instead of linear transportation costs and got a reversed result in a two stage game: maximum differentiation. In an article published in Economics Letters, Economides (1986) analyzed a family of transportation cost functions where the linear and the quadratic function are special cases. He was able to derive regions in the parameter space where pure strategy price equilibria exist and some location equilibria do not exhibit maximum differentiation (minimum differentiation is never an equilibrium).

In this short note the Economides paper is extended to a three stage game, including a wage bargaining stage. In stage 1 two firms simultaneously choose their location in a linear city. In stage 2, given the location of the firms, both firms bargain with their firm unions on the wage rate (decentralized wage bargaining) and in the last stage, given location and wages, firms set prices simultaneously.

Assume firm 1 is located on the left and firm 2 on the right side of a linear city with the length of 1. $0 \leq a, b \leq \frac{1}{2}$. $a$ describes the distance between firm 1 and the left corner, and $b$ the distance between firm 2 and the right corner. Consumer $x$ is indifferent to buy from firm 1 or 2, if

$$p_1 + |x - a|^\gamma = p_2 + |1 - b - x|^\gamma \tag{1}$$

with $1 \leq \gamma \leq 2$. We get Hotelling’s case with $\gamma = 1$ where a pure strategy price equilibrium does not exist.\(^1\) The quadratic transportation costs case is included with $\gamma = 2$.

Analyzing only symmetric solutions, Economides has shown that in equilibrium the indifferent consumer $x$ will be between $a$ and $1 - b$. Using this information we are able to isolate $\frac{\partial x}{\partial p_1}$ by differentiating (1) with respect to $p_1$, respectively to $p_2$. 

$$\frac{\partial x}{\partial p_1} = -\frac{1}{\gamma [(x-a)^{\gamma - 1} + (1-b-x)^{\gamma - 1}]} = -\frac{\partial x}{\partial p_2} \tag{2}$$

All consumers have unit demands and the gross surplus is high enough for them to buy. Firms’ price decisions only determine market shares, not the overall sale. Demand for firm 1 is given by $D_1 = x(p_1, p_2)$ and for firm 2 by $D_2 = 1 - x(p_1, p_2)$. Assuming a simple production technology with one unit of labor (the only input) producing one unit of output, profit can be written as $\pi_i = (p_i - w_i)D_i$. Substituting (2) into the first order conditions of the profit maximizing problem results in

$$p_i - w_i = \gamma D_i [(x-a)^{\gamma - 1} + (1-b-x)^{\gamma - 1}] \tag{3}$$

$$\pi_i^* = \gamma D_i^2 [(x-a)^{\gamma - 1} + (1-b-x)^{\gamma - 1}] \tag{4}$$

Substituting (3) into (1) and using abbreviations $A = (x-a)^{\gamma - 1} + (1-b-x)^{\gamma - 1}$ and $B = (x-a)^{\gamma - 2} - (1-b-x)^{\gamma - 2}$ we can implicitly characterize the price equilibrium (stage

\(^1\) Osborne and Pitchik (1987) have calculated the equilibrium of Hotelling’s location game numerically allowing for mixed strategy price equilibrium in the second stage. In the preceding location game firms choose $a$, respectively $b$ around 0.27.
3 of our model) by
\[
G(x, a, b, w_1, w_2) \equiv \gamma(2x - 1)A + (x - a)^\gamma - (1 - b - x)^\gamma + w_1 - w_2 = 0
\quad (5)
\]

Wage Bargaining (Stage 2)

We present the wage bargaining process as a right to manage model with decentralized wage setting.\(^2\) The objective function of the union\(^3\) is \((w_i - \bar{w})L_i(w_i)\), the objective function of the firm is the profit.

The Nash products to be maximized are given by
\[
N_i = [(w_i - \bar{w})D_i(w_i)]^\beta [(p_i - w_i)D_i(w_i)]^{1-\beta},
\quad (6)
\]
with \(\beta\) being the bargaining power of the unions. We describe a symmetric situation, with a common bargaining power for both unions. Note that with \(\beta = 0\) we have a competitive labor market, equivalent to the model of Economides (1986).

In a right to manage model the union and the firm negotiate over wages only and the firm owner can unilaterally set the employment level in the next stage. To represent this structure we have to take into account that the price will depend on the negotiated wage rate. The outcome of the bargaining can be described by the solution of the following equations.

\[
\frac{dN_i}{dw_i} = 0
\quad (7)
\]

For firm 1 we get
\[
\beta \left[ x + (w_1 - \bar{w}) \frac{dx}{dw_1} \right] (p_1 - w_1) x + (1 - \beta)(w_1 - \bar{w}) x \left[ \left( \frac{dp_1}{dw_1} - 1 \right) x + (p_1 - w_1) \frac{dx}{dw_1} \right] = 0
\quad (8)
\]

From (3) we know that \(p_1 = \gamma x A + w_1\).

\[
\frac{dp_1}{dw_1} = \gamma \left[ \frac{dx}{dw_1} A + x \frac{dA}{dw_1} \right] + 1 = \gamma (A + (\gamma - 1)xB) \frac{dx}{dw_1} + 1
\quad (9)
\]

Substituting (3) and (9) into (8) we can derive
\[
(w_1 - \bar{w}) = -\frac{\beta x A}{[(2 - \beta)A + (1 - \beta)(\gamma - 1)xB] \frac{dx}{dw_1}}
\quad (10)
\]

Equivalently, we can derive from the first order condition of the wage bargaining in firm 2
\[
(w_2 - \bar{w}) = \frac{\beta (1 - x) A}{[(2 - \beta)A - (1 - \beta)(\gamma - 1)(1 - x)B] \frac{dx}{dw_2}}
\quad (11)
\]

\(^2\) For a discussion on the right to manage model versus an efficient bargaining see Booth (1995). The cooperative bargaining solution was introduced by Nash (1950).

\(^3\) With this presentation the union tries to maximize the difference between the expected utility of the representative member and his/her disagreement utility, given the labor force is normalized to one and disagreement utility of the representative union member is the reservation wage (\(\bar{w}\)).
The next step is to isolate the effect of a marginal change in a firm’s location on the indifferent consumer. As an intermediate step we have to calculate the marginal effect of a wage change on this consumer. Using the characterization of the price equilibrium (5) and the implicit function rule, we can derive $dx/dw_i$.

$$\frac{dx}{dw_1} = -\frac{1}{\gamma [3A + (2x - 1)(\gamma - 1)B]}$$

(12)

and the equivalent manipulation for $w_2$ results in

$$\frac{dx}{dw_2} = -\frac{dx}{dw_1}$$

Substituting these results into (10) and (11), we can derive

$$w_1 - \bar{w} = \frac{\beta \gamma x A [3A + (2x - 1)(\gamma - 1)B]}{(2 - \beta)A + x(1 - \beta)(\gamma - 1)B}$$

(13)

$$w_2 - \bar{w} = \frac{\beta \gamma (1 - x) A [3A + (2x - 1)(\gamma - 1)B]}{(2 - \beta)A - (1 - x)(1 - \beta)(\gamma - 1)B}$$

(14)

Finally, the derivative of $x$ with respect to $a$ is given by

$$\frac{dx}{da} = -\frac{\partial G/\partial a}{\partial G/\partial x}$$

(15)

where

$$\frac{\partial G}{\partial x} = \gamma \left( 3A + (2x - 1) \frac{\partial A}{\partial x} \right) + \frac{\partial w_1}{\partial x} - \frac{\partial w_2}{\partial x}$$

(16)

and

$$\frac{\partial G}{\partial a} = \gamma (2x - 1) \frac{\partial A}{\partial a} - \gamma (x - a)^{\gamma - 1} + \frac{\partial w_1}{\partial a} - \frac{\partial w_2}{\partial a}.$$  

(17)

Terms $\frac{\partial w_i}{\partial x}$ and $\frac{\partial w_i}{\partial a}$ can be calculated from (13) and (14).

**Choice of location** (Stage 1)

In stage 1 firms can freely choose their locations. For firm 1 using (4) the first order condition of this problem is given by

$$\frac{d\pi_1}{da} = \frac{\partial \pi_1}{\partial a} + \frac{\partial \pi_1}{\partial x} \frac{dx}{da} = \gamma x^2 \frac{\partial A}{\partial a} + \gamma \left( 2xA + x^2 \frac{\partial A}{\partial x} \right) \frac{dx}{da} = 0$$

$$\Rightarrow - (\gamma - 1) x(x - a)^{\gamma - 2} + (2A + (\gamma - 1)xB) \frac{dx}{da} = 0$$

(18)

Evaluating this expression for symmetric locations $(x = \frac{1}{2}, a = b, A = 2 (\frac{1}{2} - a)^{\gamma - 1}, B = 0)$, we get

$$- (\gamma - 1) 2 \left( \frac{1}{2} - a \right)^{\gamma - 2} + 4 \left( \frac{1}{2} - a \right)^{\gamma - 1} \frac{dx}{da} = 0$$

(19)

$\frac{dx}{da}$ is an unhandy term. Evaluated for symmetric locations, we get
\[
\frac{dx}{da} = -\gamma \left(\frac{1}{2} - a\right)^{-1} + \frac{3\beta \gamma (\gamma - 1)}{2\gamma - 1} \left[ \left(\frac{1}{2} - a\right)^{-2} + \frac{(1 - \beta)(\gamma - 2)(\frac{1}{2} - a)^{-3}}{2\gamma - 2}\right]
\]

Consider the extreme cases \(\beta = 0\) (competitive labor market) and \(\beta = 1\) (monopoly union) the expression becomes easily manageable.

**Competitive labor market:** Substituting \(\beta = 0\) into \(\frac{dx}{da}\) and solving for \(a\), results in

\[
a = \frac{5 - 3\gamma}{4}
\]

with an interior solution for \(\gamma < \frac{5}{3}\). This is exactly the result of Economides (1986). With \(\beta = 0\) his and our model differ only in the inclusion of constant marginal costs. The price setting stage is exactly the same in both models, we can conclude that a pure strategy price equilibrium exists for the range \(2 \geq \gamma \geq 1.26\) (see Economides 1986, Proposition 2) and the wage setting and the location games are properly defined.

**Monopoly union:** Now let us analyze the case of an extraordinarily strong union. Substituting \(\beta = 1\) into \(\frac{dx}{da}\) and solving for \(a\), we get

\[
a = \frac{23 - 21\gamma}{4}
\]

The solution would be an interior one, if \(\gamma < \frac{23}{21}\). However in this case, as mentioned above, a pure strategy price equilibrium does not exist. Thus, for all values of \(\gamma\) where a pure strategy price equilibrium exists, product differentiation is maximal when wages are set by monopoly unions in a decentralized environment.

**Intermediate case:** With an intermediate strong union, the optimal location is the nearer to the extreme points, the stronger the union is (the higher \(\beta\)). Figure 1 shows the optimal \(a\) depending on the bargaining power for \(\gamma = 1.3\) (solid line) and \(\gamma = 1.4\) (dashed line).

![Figure 1](Image)
With $\beta > 0.4$ we get maximum differentiation for both values of $\gamma$. With $\gamma = 1.3$ we get an inner solution for $\beta$’s between around 0.23 and 0.4.

Tirole (1988, Chap. 7) argues that two opposite effects are at work. The market share effect as emphasized by Hotelling is pushing the firms to the center of the city, while a large distance to the competitor allows a firm to increase its price (the strategic effect) and gives firms an incentive to move to the borders. The significance of this strategic effect increases with the bargaining power of the unions. With decentralized wage bargaining, the marginal change of demand caused by a marginal change in wage (12) is negative. This effect is stronger in regions where price competition is very high, which is in the middle of the city (with $\gamma = 1$ this effect is zero, but then a pure price strategy does not exist). A unilaterally changing wage has a stronger negative impact on profits in the center than at the borders of the city. That is why the higher the bargaining power of the unions the more firms tend to locate at the borders of the city.

References


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$^4$ These two effects can generate the non-concavity of the profit function and cause therefore the possibility of the non-existence of the price equilibrium.

$^5$ The derivative of (12) with respect to $a$, evaluated for a symmetric location is

$$\frac{dx}{dw_1} = -\frac{\gamma - 1}{12\gamma (\frac{1}{2} - a)} \leq 0$$
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